QUASI-MULTIPLIERS AND ALGEBRIZATIONS OF AN OPERATOR SPACE

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BACKGROUND.

One of the most interesting questions in the theory of operator spaces was: What are the possible operator algebra products a given operator space can be equipped with? The author answered the question using quasi-multipliers defined in [2]. That is, the operator algebra products a given operator space can be equipped with are precisely the bilinear mappings implemented by contractive quasi-multipliers. Furthermore, the operator algebra products were characterized in terms of matrix norms with the Haagerup tensor product. This result is remarkable in the sense that an algebraic property (products) is deduced from a geometric property (norms).

RESULTS.

Theorem ([1]): Let X be a nonzero operator space with a bilinear mapping m: X×X→ X, and let $I(S_x)$ be an injective envelope C* -algebra of Paulsen's operator system S_x with identity denoted by 1.

We canonically identify X with the (1,2) -corner of S_x. Define

$$\begin{array}{cccc} & \mathbb{M}_2\big(I(S_X)\otimes_h I(S_X)\big) & & \mathbb{M}_2(X) \\ & \cup & & \cup \\ \Gamma_m\colon & \begin{bmatrix} X\otimes_h \mathbb{C}1 & X\otimes_h X \\ 0 & \mathbb{C}1\otimes_h X \end{bmatrix} & \to & \begin{bmatrix} X & X \\ 0 & X \end{bmatrix} \end{array}$$

by

$$\Gamma_m\left(\begin{bmatrix} x_1\otimes 1 & x\otimes y \\ 0 & 1\otimes x_2 \end{bmatrix}\right) := \begin{bmatrix} x_1 & m(x,y) \\ 0 & x_2 \end{bmatrix}$$

and their linear extension. Then the following are equivalent:

(i) *(X,m)* is an (abstract) operator algebra (i.e., there exists a multiplicative complete isometry from X into B(H) for some Hilbert space H, hence, in particular, *m* is associative);

(ii) there exists a contractive quasi-multiplier *z* such that , m(x,y)=x·z·y, ∀x,y ∈ X;

(iii) Γ_m is completely contractive.

Moreover, such a z is unique.

REFERENCES.

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2. M. Kaneda and V.I. Paulsen. (2004). Quasi-multipliers of operator spaces. Journal of Functional Analysis, 217 (2): 347-365.