# Direct-Torque Control System Design Using Maximum Torque Per Ampere method for Interior Permanent Magnet Synchronous Motors

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***Abstract*—The paper proposes a method for controlling interior Permanent Magnet Synchronous Motor (IPMSM) drives, that combines Direct-Torque control (DTC) and Maximum Torque Per Ampere (MTPA) in a synchronously rotating (d-q) frame. It has been proven that the increase of electromagnetic torque is proportional to the increase of angle between rotor and stator flux linkages. The use of the DTC method for controlling IPMSM shows a quick and robust response with torque and flux ripples. Combined with MPTA the efficiency of the system is increased as there is no need for prior estimation of flux. Also, speed control loop with PI controller is also introduced to the system for speed regulation. Here, we introduce a simple MTPA-based DTC scheme with speed control with satisfied performance. The feasibility of the system is confirmed by Simulink model, the results of which are presented in the paper.**

**Index terms: Direct Torque Control(DTC), hysteresis comparator, Permanent Magnet Synchronous Motor (PMSM), Proportional-Integrator regulator (PI), Maximum Torque Per Ampere (MTPA).**

## I. INTRODUCTION

Permanent Magnet Synchronous Motors (PMSMs) have become prevalent in recent years and are commonly used in industrial automation, robotics, and aerospace. This is due to their high-power capability that comes with lower mass and moment of inertia. As a permanent magnet is used to generate rotor flux, there are no losses in a rotor and this makes this type the most efficient among all AC machines [1].

Generally, there are two types of PMSM: Surface-mounted PMSM (SPMSM) and Interior PMSM (IPMSM). The key difference between them is in the location of the set of permanent magnets and this leads to a difference in performance, hence in applications. Comparisons of operations of these motors reveal that IPMSM shows slightly higher power efficiency at light load, while higher output power in high-speed regions and higher power factor of SPMSM will decrease in field weakening mode, which is not a case of IPMSM [2].

For controlling IPMSM there are two main competing strategies: Field Oriented Control (FOC) and Direct Torque Control (DTC).

FOC controls torque and flux via regulating currents (d-q-axis currents) and is known for its high torque quality and high performance at low speeds. These advantages are accompanied by drawbacks like the need for accurate position sensor feedback, complex transformation and knowledge of motor parameters. In comparison to FOC, DTC controls torque and flux directly and independently, which results in fast dynamic response in addition to high accuracy and less dependence on machine parameters [3] - [5]. The DTC method addresses drawbacks of the previously mentioned method as there is no need for current controller, which results in elimination of time delay caused by a current loop; and more importantly, knowing rotor position is not required anymore, which means no encoder and uncertainties associated with it [6]. Despite the benefits described above, it also has drawbacks in comparison with FOC, for example, high ripples. The ripples are a result of the hysteresis controllers, which are employed to select the proper voltage vector [6]. To overcome it, in other words, decrease ripples, more inverters must be introduced, which in its turn, increase the price and make implementation more complex [7]. There are many papers that compare and suggest to use either FOC or DTC, but generally, the choice depends on the application requirements. If quality of torque matters, it is better to chose FOC, while for fast response, DTC will be preferable [4]-[10].

To improve the performance of DTC, different methods were introduced [11]-[18]. The techinique in [11] used auxilary hardware to reduce the torque and flux ripples. It was proved to be effective but the method is complicated. In [12], although the method can reduce the total harmonic distortion in stator currents, a field-programmable-gate-array-based control platform is required to avoid the computation delay. New duty-cycle techniques were presented in [13], [14] to reduce torque and flux ripples, unfortunatly, the effect of parameter variations was not discussed. The predictive control algorithms are introduced in [15] to reduce ripples. But it required a lot of computation efforts. The hybrid techniques using space-vector modulation (SVM) [16]-[19] to reduce the torque ripple. However, this techique needs coordinate transformation with advanced control approaches. It made the control systems complicated and might not be feasible to implement in practice.

Conventional DTC requires reference torque and flux and by using the Maximum Torque Per Ampere (MTPA) method, we will not only eliminate the need for prior calculation but also increase the efficiency of a motor for high current applications [20]. MTPA minimizes copper losses but it only can work on low speed ranges, because of the small back electromotive force[21], [22].

There are different ways to implement MTPA based on chosen frames: stationary ($\alpha - \beta$) or synchronous: a rotor (d-q) or stator flux linkage (M-T). Each can be used efficiently with DTC depending on the purpose and among these, using a rotor synchronous frame is mostly preferred for the simplicity of its mathematical model and high efficiency for wide speed ranges [23].

In this paper, DTC control technique will be applied to IPMSM with MTPA trajectory, which will be used to increase the efficiency of the system. In the proposed DTC system, a synchronously rotating d-q frame will be used to develop the controller. First, to simplify the controller design, both torque can flux control is implement by comparator with hysteresis. Although this method is well-known, we implement it to show that with appropriate design, the torque and flux ripples still in acceptable ranges. Moreover, In order to control the speed of the IPMSM, an outer PI speed controller is added. This speed loop also help to reduce the ripple of the speed respones. It is noted that this approach is much more simple than the technique called space-vector modulation based DTC (SVM-DTC) such as in [5], [19]. The MTPA is also included to increase the efficiency of the system. This propose MTPA is dependent to q-axis inductance, d-axis inductance, and flux linkage; however, this parameters is quite insensitive to temperatures [24], so it is still effective in enhancing the efficiency. The effectiveness of the proposed control system is validated via MATLAB/Simulink simulation.

The organization of the remaining parts of the paper is the following: the mathematical model of IPMSM will be discussed in Section II, Section III contains explanations and the design of DTC and MTPA, which will be followed by Simulation in Section IV and finalized with conclusions.

## II. Mathematical model of IPMSM

Synchronous motors are capable of running at a constant speed irrespective of a load acting on them. They are machines with high efficiency and are mainly used in high-precision applications[25]. The constant speed characteristic is achieved by the interaction between a constant and rotating magnetic field. A rotor of synchronous motor produces a constant magnetic field and stator produces a revolving magnetic field. The field coil of the stator is excited by a three phase AC supply. This will produce a revolving magnetic field which rotates at synchronous speed. On the contrary, the rotor is excited by a DC power supply, so it acts like a permanent magnet[26]. Therefore, a dq-frame was adopted for convenience. The basic principle of this model is that the rotor flux is aligned with the d-axis, while the q-axis is leading by 90 electrical degrees (no rotor flux along q-axis). The transformation from three-phase frame to dq is performed by Clarke and Park Transformations.

After adjusting dq as the reference frame, the PMSM stator equations referenced to the rotor are as following:

Voltage equations are given by

$$\begin{aligned} V_q &= R_s i_q + \omega_r \lambda_d + \frac{d\lambda_q}{dt} \\ V_d &= R_s i_d - \omega_r \lambda_q + \frac{d\lambda_d}{dt} \end{aligned} \tag{1}$$

Flux Linkages are given by

$$\begin{aligned} \lambda_q &= L_q i_q \\ \lambda_d &= L_d i_d + \lambda_f \end{aligned} \tag{2}$$

Combining these two system of equations gives

$$\begin{aligned} V_q &= R_s i_q + \omega_r (L_d i_d + \lambda_f) + \frac{dL_q i_q}{dt} \\ V_d &= R_s i_d - \omega_r L_q i_q + \frac{d(L_d i_d + \lambda_f)}{dt} \end{aligned} \tag{3}$$

The developed motor torque is

$$T_e = \frac{3}{2}(\frac{P}{2})(\lambda_d i_q - \lambda_q i_d) \tag{4}$$

The mechanical torque is given by

$$T_e = T_L + B\omega_m + J\frac{d\omega_m}{dt} \tag{5}$$

Solving for the rotor mechanical speed gives

$$\begin{aligned} \omega_m &= \int \frac{T_e - T_L - B\omega_m}{J} dt \\ \omega_m &= \omega_r (\frac{2}{P}) \end{aligned} \tag{6}$$

, where $\omega_r$ is rotor electrical speed and $\omega_m$ - mechanical speed[9].

## III. The principle operation of DTC and MTPA

The most commonly used control strategies of IPMSM are MTPA and DTC control methods. DTC control strategy is based on direct control of stator flux and torque by selecting the appropriate inverter state. The MTPA approach is mainly focused on obtaining maximum possible torque using the minimum amount of current. This can be achieved by controlling stator current. These two methods can be combined for better operation of the motor.

a) Direct Torque Control

The working principle of the basic DTC is to select a voltage vector based on the errors between the

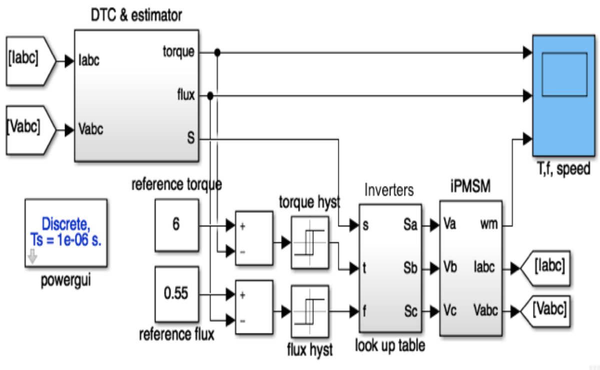


Fig. 1. DTC scheme for IPMSM

reference and actual values of torque and stator flux linkage. A basic scheme for DTC can be seen in fig.1

First, abc current measured from the motor $i_{abc}$ is transformed into its quadrature and direct axes components according to Park Transformation. The mapping is presented in the equation below:

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} * \begin{matrix} i_a \\ i_b \\ i_c \end{matrix} \tag{7}$$

By rearranging eq.(3) and setting $w_m = 0$, we can derive formula for flux,

$$\lambda_{dq} = \int (V_{dq} - R_s i_{dq}) dt \tag{8}$$

$$\lambda = \sqrt{\lambda_q^2 + \lambda_d^2} \tag{9}$$

By knowing value of flux, we can calculate the angular position of the stator flux vector. Overall, there are 6 sectors, numbered as shown in Fig.4 and each of them span 60deg.

$$\tan(\theta_\lambda) = \frac{-\lambda_d}{\lambda_q} \tag{10}$$

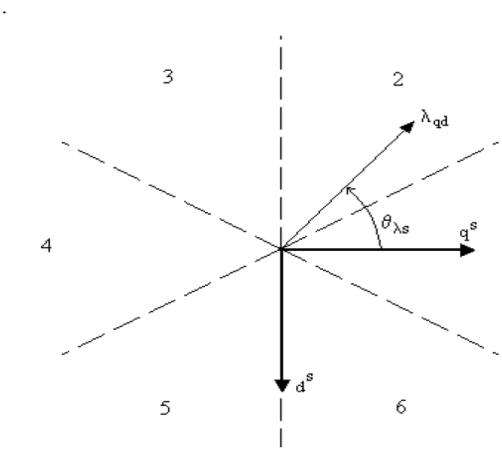


Fig. 2. DTC sectors

We define $e_T = T_e * T_e$ as the error between the reference torque$T_e*$ and actual torque $T_e$. $e_\Lambda = |\lambda_s| * |\lambda_s|$ :as the error between the magnitude of the reference stator flux $|\lambda_s|*$ and actual stator flux $|\lambda_s|$. Depending on the difference, there are two possible instantaneous values of hysteresis comparator (1 and -1 ) for both flux and torque. Further, these data will be used in the switch table (table 1) to choose the appropriate set of vectors to control the motor.

| $e_\lambda$ | $e_t$ | **Sector** $\theta(N)$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** |
| -1 | -1 | V5 | V6 | V1 | V2 | V3 | V4 |
| -1 | 1 | V3 | V4 | V5 | V6 | V1 | V2 |
| 1 | -1 | V6 | V1 | V2 | V3 | V4 | V5 |
| 1 | 1 | V2 | V3 | V4 | V5 | V6 | V1 |

TABLE I
DTC SWITCH TABLE

There are six non-zero voltage vectors:V1(100), V2(110), V3(010), V4(011), V5(001) and V6(101). The primary voltages ($V_a$, $V_b$ and $V_c$) are determined by inverter switches $S_a$, $S_b$ and $S_c$ (fig. 3),whose values are represented in Table I.

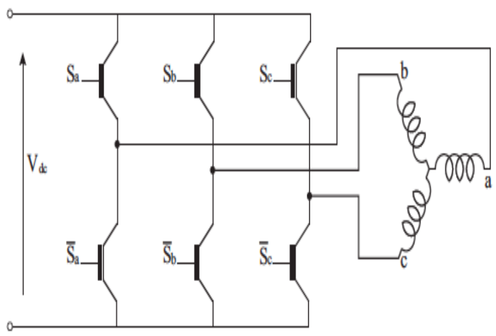


Fig. 3. Inverter scheme

The value of the torque at the motor is calculated from the three-phase voltage delivered to it. Although the DTC dynamic response is fast, the torque and flux linkage ripple needs to be reduced and for this purpose, the method can be combined with MTPA approach, which is widely used for its high efficiency[27].

b) Maximum Torque Per Ampere

The DTC method requires the values of the reference torque and flux. By adding speed loop control, the need for the explicit calculation of torque is eliminated. To maximize efficiency, the reference flux is determined by motor parameters and reference torque, which affects the controlling current. After converting the stator current into the synchronous frame $(i_d, i_q)$ and knowing parameters like the rotor magnet flux linkage constant($\Lambda_f$), the d and q axis inductances ($L_d$, $L_q$), the value of the reference flux is calculated as follows:

$$\lambda_s = \sqrt{(\lambda_f + L_d i_d)^2 + (L_q i_q)^2} \tag{11}$$

Also, expression of torque can be modified:

$$T = \frac{3}{2}P(\lambda_f + (L_d - L_q)i_d)\sqrt{i_d^2 - \frac{\lambda_f}{L_q - L_d}i_d} \qquad (12)$$

The MTPA requires introductions of some constraints on the voltage and current to enclose a specific region of stable operation. Mathematically, these conditions can be stated the followings:

$$\sqrt{v_d^2 + V_q^2} \leqslant V_{max} \qquad (13)$$

where $V_{max}$ can be calculated using rotor electrical speed ($\omega_{re}$), base speed ($\omega_b$) or crossover speed ($\omega_c$) with stator flux linkage ($\lambda_s$), rated stator flux linkage ($\lambda_{rs}$) or rotor magnet flux linkage constant ($\lambda_f$) respectively, if the stator resistance is neglected.

$$V_{max} = \omega_{re}\lambda_s = \omega_b\lambda_{sr} = \omega_c\lambda_f \qquad (14)$$

$$\sqrt{i_d^2 + i_q^2} \leqslant I_{max} \qquad (15)$$

$$i_d = I_{max}cos\theta i_q = I_{max}sin\theta \qquad (16)$$

where $\theta$ varies from 0 to 360deg

The difficulties that come with this method include sensitivity to machine parameters, which may change during the process due to temperature, rotating speed and saturation effect. Another downside of this method is the complexity of calculations due to the non-linear relationship between the electromagnetic torque and the dq currents. The solution to this issue might be the usage of lookup table: the desired electromagnetic torque with the corresponding *id* and *iq*. However, look up tables are considered as less effective and reliable, while computations are slow and heavy for the system in comparison with other methods [27]-[29].

## IV. SIMULATION OF DTC

In order to examine the performance of the DTC with MTPA, the MATLAB/SIMULINK model was created. The structure of the model is based mainly on the scheme described by Sariati Dalib [26]. However, this model took the torque and flux reference values as an input signal. In the model proposed in this paper PID controller was added next to the torque reference signal to regulate speed of the IPMSM(fig. 4). Table II shows the parameters of the IPMSM used for simulation of the DTC, which was used in the work of Foo Hock Beng [5]. Sampling time $T_s$ of the simulation was $5\mu s$.

As shown in fig. 5, the reference speed was 100 rad/s. The transient response of the DTC was approximately 20 milliseconds and the speed ripples were around ±3.14 rad/s.

For further improvement of the model, MTPA technique was applied. The flux reference was calculated via MTPA technique. In comparison with previous results, the speed ripples were reduced significantly (fig. 6), from 3.14 rad/s to around 1 rad/s.

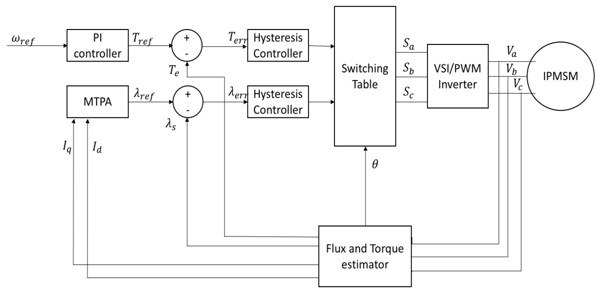


Fig. 4. DTC scheme with speed control and MTPA

TABLE II
IMPSM PARAMETERS

| | |
|---|---|
| Number of pole pairs $P$ | 2 |
| Stator resistance $R$ | 5.8 Ω |
| Magnetic flux linkage $\lambda_f$ | 0.533 $Wb$ |
| $d$-axis inductance $L_d$ | 0.0448 $H$ |
| $q$-axis inductance $L_q$ | 0.1024 $H$ |
| Phase voltage $V$ | 132 $V$ |
| Phase current $I$ | 3 $A$ |
| Base speed $\omega_b$ | 1260 $rpm$ |
| Base speed $\omega_c$ | 1460 $rpm$ |
| Rated torque $T_b$ | 6 $N.m$ |
| Rated power $P_r$ | 1 $kW$ |
| Friction coefficient $D$ | 0.0006 $Nm/rad/s$ |
| Rotor inertia $J$ | 0.000329 $kg.m^2$ |

Figures 7 and 8 show the system response to load torque alteration. The load torque changed from 1 N.m. to 2 N.m. at 0.3 seconds. The response to the change was inconspicuous. This shows that the IPMSM runs with constant speed regardless of the change in the load torque.

In both cases, the flux linkage ripples constituted ±0.1 Wb with and without applied MTPA technique in the DTC. However, the torque ripples were relatively high at steady state regions, approximately 1 N.m, although the hysteresis band was set to 0.2 N.m. On the other hand, at transient response regions (red square zones on fig. 9) torque ripples remained around the set hysteresis region. Such behavior proceeded because of the negative property of Derivative component to amplify noises in the system. It improved the transient response but increased magnitude of the ripples in the steady-state region. It also turns out that in the steady-state region the D-component was creating fast switching in torque reference signal, which resulted in multiple transient response regions, which can be seen as large spikes in torque plot (fig. 9). These large spikes signified that the controller responded very fast to the switches.

Replacement of the PID controller with the PI block halved the amplitude of the ripples, while the signal overshot in the transient region. It worth noting that the overshooting occurred in a fraction of a second, 0.2 seconds, and in the next tenth of a second signal approached its reference value. Therefore, we concluded that the overshooting is acceptable for this control method but, for the sake of convenience, the Derivative component was included in the final version of the controller.

Further reduction of the ripples can be obtained by adding filters, such as Kalman filter [30]. But as filtering adds complexity to the system. The purpose of the project was to build simple DTC for IPMSM which can give acceptable performance. For more detail of implementing Kalman filter, please refer to [30].

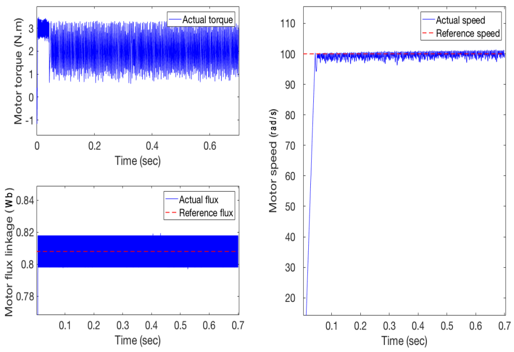


Fig. 5. Graph of the motor speed, torque and flux linkage; DTC without MTPA

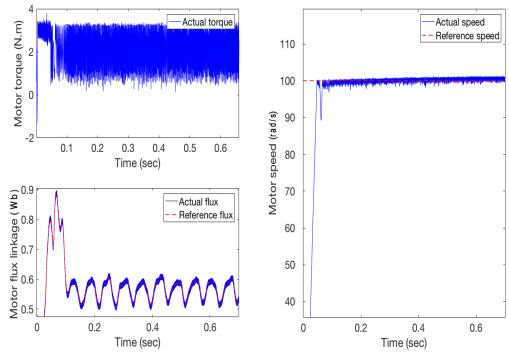


Fig. 6. Graph of the motor speed, torque and flux linkage for DTC with applied MTPA

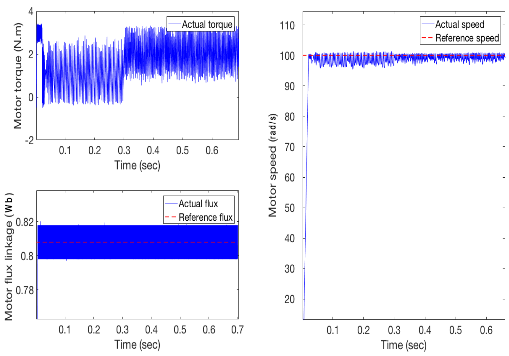


Fig. 7. Motor response to load torque change at 0.3 sec from 1 N.m. to 2 N.m; Controller without MTPA

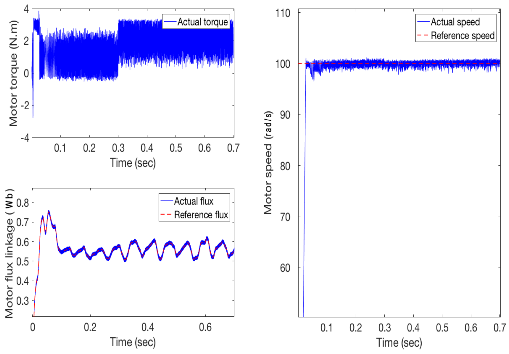


Fig. 8. Motor response to load torque change at 0.3 sec from 1 N.m. to 2 N.m; MTPA was applied in the controller

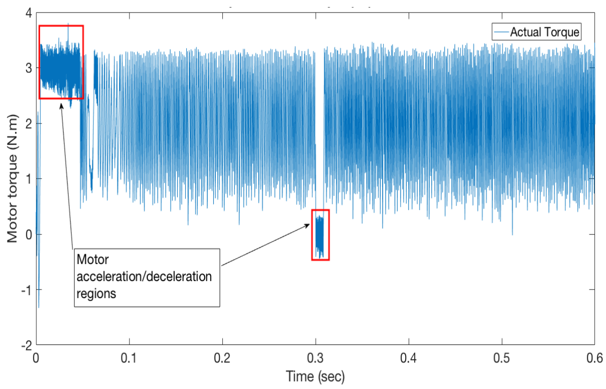


Fig. 9. Graph of the electromagnetic torque of the motor; Red squares - transient response zones

## V. Conclusion

This paper presents the DTC of IPMSM with speed control and applied MTPA technique. The mathematical model of the IPMSM and its parameters used in the simulation were described. The DTC and MTPA methods were also outlined in this paper. According to the results of the simulation, the proposed model performed better than the conventional one. The main differences from the conventional model were: ability to control directly the speed of the motor and applied MTPA method in order to increase efficiency of the system. After applying the MTPA speed ripples decreased threefold, from 3 rad/s to around 1 rad/s. Using the PI controller instead of the PID resulted in twofold reduction of the ripples amplitude, though, it caused overshoot in the response signal. However, the overshooting was acceptable in the scope of the research, due to its relatively low magnitude and small restoration time.

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