Thermodynamics and gravitational collapse

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It is known now that a typical gravitational collapse in general relativity, evolving from regular initial data and under physically reasonable conditions would end in either a black hole or a naked singularity final state. An important question that needs to be answered in this connection is, whether the analogues of the laws of thermodynamics, as formulated for relativistic horizons are respected by the dynamical spacetimes for collapse that end in the formation of a naked singularity. We investigate here the thermodynamical behaviour of the dynamical horizons that form in spherically symmetric gravitational collapse and we show that the first and second laws of black hole thermodynamics, as extended to dynamical spacetimes in a suitable manner, are not violated whether the collapse ends in a black hole or a naked singularity. We then make a distinction between the naked singularities that result from gravitational collapse, and those that exist in solutions of Einstein equations in vacuum axially symmetric and stationary spacetimes, and discuss their connection with thermodynamics in view of the cosmic censorship conjecture and the validity of the third law of black hole mechanics.

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I. INTRODUCTION

The thermodynamical behaviour of black holes in Einstein gravity was first noted by Hawking, Carter, Bardeen and Bekenstein in a series of papers, that established what are known now as the laws of black hole thermodynamics [1]. These laws link the area and surface gravity of the event horizon of a stationary black hole in classical general relativity to the entropy and temperature of a thermodynamical system at equilibrium. The connection between such diverse concepts as that of the gravity and the thermodynamical laws gained physical respectability after the discovery of Hawking radiation, which, by considering quantum corrections in a semiclassical context, showed how the analogy was deeper than that at a formal level, thus proving that black holes must indeed be considered as black bodies, radiating at the Hawking temperature [2].

Since the first formulation of the laws of black hole thermodynamics, a lot of effort has been devoted in exploring the connection between gravity and thermodynamics in the hope that the analogy can be extended to the whole theory, making it valid not only on the horizons, and therefore showing possibly how gravity could be considered as a thermodynamical theory [3].

A key issue in this respect is the role played by the gravitational collapse. The dynamical physical process that would give rise to a black hole in nature would be typically the gravitational collapse of a massive star which shrinks under the force of its own gravity at the end of its life cycle. In general, a matter of considerable interest would be the validity of thermodynamical laws during the dynamical gravitational evolutions of physical systems in the universe. As it is known now, such a collapse would terminate in either a black hole or a naked singularity, depending on the nature of the initial data from which the collapse evolves, under physically reasonable conditions. It is known that naked singularities do typically arise in solutions of Einstein field equations in a wide variety of situations and one would like to know whether these spacetimes exhibit a behaviour that is in accordance with the laws of black hole mechanics.

In stationary or static spacetimes, we already know that extremal solutions (such as in the Kerr geometry or in the Reissner-Nordstrom spacetime) can lead to the appearance of naked singularities, and a lot of investigation has been carried out in order to show whether or not these manifolds, once interpreted in the context of thermodynamical gravity, exhibit a behaviour in accordance with the laws of thermodynamics [4]. It is important, however, to notice that since a spacetime singularity, where the densities, curvature, and all physical quantities diverge, is not part of the spacetime, understanding thermodynamical properties of the manifold in the limit of approach to the singularity could imply at times some technical difficulties. The eventuality of the occurrence of naked singularities in the physical universe is a well-known theoretical open problem that is usually referred to as the cosmic censorship conjecture (CCC) [5]. It is not difficult to see how the validity of CCC and thermodynamics of naked singular spacetimes are closely related issues. Therefore the thermodynamical behaviour of spacetimes with naked singularities has been connected some times to the possibility to prove the CCC.

It is clear that in order to provide a wider perspective on the thermodynamical properties of gravity one has to include dynamical situations, thus extending the laws of black hole thermodynamics to non-stationary spacetimes. To this aim, the equivalence of Einstein equations and the first law of thermodynamics was shown by Jacobson for all local causal Rindler observers, provided that the Clausius relation holds [6]. Further, Hayward showed how the laws of black holes thermodynamics can be extended to trapping horizons in dynamical spacetimes [7]. This in turn has led to the investigation of dynamical solutions such as the Friedmann-Robertson-Walker cosmological models in order to establish the connection be-

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tween gravity and thermodynamics on a cosmological level [8].

Even then, very little is known as of today regarding the connection between the thermodynamic behavior and gravity, for general dynamical asymptotically flat spacetimes which describe isolated bodies not in equilibrium [9], such as for example, the processes that lead to the formation of a black hole as the endstate of gravitational collapse. We do know that naked singularities as well as black holes can form as the endstate of collapse in a wide variety of models. These naked singularities of collapse are entirely different from the extremal stationary ones mentioned above, and still their thermodynamical behaviour is not fully understood.

Our purpose here is to consider dynamical spacetimes involving generic spherical gravitational collapse, where the final state could be either a black hole or a naked singularity. As mentioned, this is an entirely different issue as compared to naked singularities that can be found in stationary spacetimes such as the Kerr-Newman solutions. We shall therefore investigate the thermodynamical behaviour of the apparent horizon that develops inside the collapsing cloud in the limit of approach to the singularity.

Such a study has some resemblance with the analysis of the thermodynamical properties of the cosmological horizon in the FRW models, since the homogeneous Lemaitre-Tolman-Bondi (LTB) model with a non-zero pressure is analogous to the FRW cosmological model with the time reversed. Still there are some key differences, for example in the structure of the trapped surfaces, that in collapse must consider the contribution from the outer portion of the spacetime. Further the general collapse situation allows for inhomogeneities and anisotropic pressures in the collapsing cloud to be considered. If we allow for inhomogeneities, the LTB collapse scenario exhibits very different behaviours depending on the properties of the infalling matter.

We shall show here that, in general the solutions where the spherical gravitational collapse could lead to naked singularities, do not violate the analogue of the laws of thermodynamics for dynamical spacetimes. In this light, it is important therefore to stress that such naked singularities of collapse do not exhibit the same causal structure behaviour as seen in those arising in stationary cases such as the extremal Kerr naked singularity. Furthermore, it has been noted many times how a theory of quantum gravity might smoothen the behaviour of quantities approaching the singularity, thus removing the issue of diverging matter densities. Therefore, a possible connection of thermodynamics with gravity on a completely general level would not need to imply that such singularities cannot occur, while it might rule out other kinds of naked singularities in case a relation between some formulations of the CCC and thermodynamics can be proven.

The plan for the paper is as follows. In Section II we shall briefly outline the most important features of generic spherical gravitational collapse and divide the possible final outcomes into two classes. These are black holes, in which the singularity that forms at the end of collapse is hidden within an horizon at all times, and the naked singularities, in which case the light rays can escape the singularity at the instant of its formation to reach faraway observers. In section III, we study the thermodynamical behaviour of such solutions of Einstein field equations in order to show how the analogy between gravity and the first two laws of thermodynamics holds regardless of the fact that the final outcome of collapse is a black hole or a naked singularity, while in section IV we briefly discuss the third law. Finally, in Section V, we will discuss the above results in view of a possible distinction between different types of naked singularities, mainly the ones that arise in static or stationary solutions of Einstein equations, and those that occur in dynamical collapse models.

II. GRAVITATIONAL COLLAPSE

Consider a spherically symmetric spacetime, depending on three metric functions of the comoving time t and the radius r, as given by,

$$ds^2 = h_{ab}dx^a dx^b + R(r,t)^2 d\Omega^2 \ , \tag{II.1}$$

with $h_{ab} = \text{diag}(-e^{2\nu(t,r)}, e^{2\psi(t,r)})$, $(a,b) = 0,1$ and $x_0 = t$, $x_1 = r$. The metric functions ν, ψ, and R are related to the energy momentum tensor which in general is defined by,

$$T_t^t = -\rho; \ T_r^r = p_r; \ T_\theta^\theta = T_\phi^\phi = p_\theta, \tag{II.2}$$

via the Einstein equations. It is the set of equations that, once a choice for the initial data is made, determines the final outcome of gravitational collapse in terms of either a black hole or naked singularity.

We can define the Misner-Sharp mass of the system as,

$$2E = R(1 - G + H) \ , \tag{II.3}$$

where

$$G(t,r) \ = \ e^{-2\psi} R'^2 \ , \tag{II.4}$$

$$H(t,r) \ = \ e^{-2\nu} \dot{R}^2 \ . \tag{II.5}$$

Then the Einstein equations for the $(0,0)$ component and for the $(1,1)$ component are given as,

$$\rho \ = \ 2\frac{E'}{R^2 R'} \ , \tag{II.6}$$

$$p_r \ = \ -2\frac{\dot{E}}{R^2 \dot{R}} \ , \tag{II.7}$$

and these determine the radial pressure and the energy density as functions of the physical radius R and of the Misner-Sharp mass. Together with the other two Einstein equations, namely

$$\nu' \ = \ 2\frac{p_\theta - p_r}{\rho + p_r}\frac{R'}{R} - \frac{p_r'}{\rho + p_r} \ , \tag{II.8}$$

$$2\dot{R}' \ = \ R'\frac{\dot{G}}{G} + \dot{R}\frac{H'}{H} \ , \tag{II.9}$$

they fully determine the structure of the spacetime [10].

Since the Einstein equations do not carry any information regarding the physical properties of the matter sources, in order for the solution to be physically reasonable, some energy and regularity conditions must be satisfied by the energy momentum tensor, and therefore as a consequence, by the metric functions. In particular, the weak energy condition requires positivity of ρ, $\rho + p_r$ and $\rho + p_\theta$, regularity at the center of the cloud requires that $E(r,t)$ goes like r^3 near the center, and also it is possible to prove that the pressures must behave like a perfect fluid near the center, namely we must have $p_r = p_\theta$ in the limit of r approaching zero.

A. Collapse final states

The complete gravitational collapse of such a matter distribution generally ends in the formation of a spacetime singularity, which is indicated by the divergence of the curvature and the energy density ρ. The possible outcomes of the complete collapse, in terms of either a black hole or naked singularity are then characterized by the occurrence and behaviour of the trapped surfaces developing in the spacetime as the collapse progresses. In the black hole scenario, the apparent horizon forms at an outer shell of the collapsing matter at a stage earlier than the singularity. The outside event horizon then entirely covers the final stages of collapse when the singularity forms, while the apparent horizon inside the matter evolves from the outer shell to reach the singularity at the instant of its formation (see figure 1).

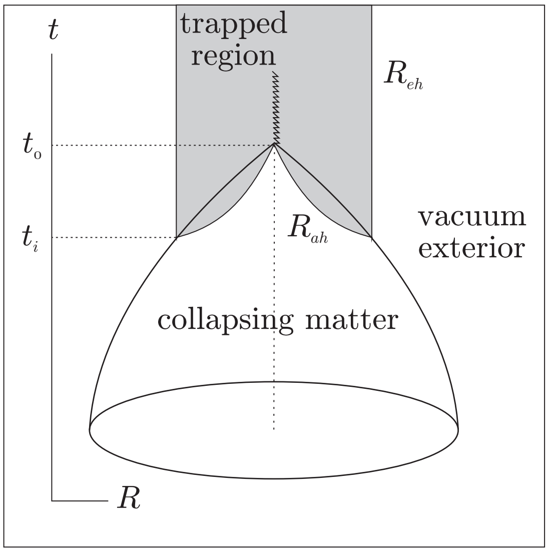

FIG. 1: Collapse leading to a black hole: The trapped surfaces form at a time t_i before the formation of the singularity. The apparent horizon moves inwards from R_{eh} at t_i to reach zero at t_0.

In the naked singularity scenario on the other hand, the trapped surfaces form at the center of the cloud at the time of formation of the singularity. The apparent horizon then moves outwards to meet the event horizon at the boundary of the cloud at a time later than the time at which the singularity formed. The instant of formation of the singularity is therefore not covered by the horizon and it can be shown that families of light rays and particles can escape the central singularity and propagate to faraway observers in the spacetime (see figure 2).

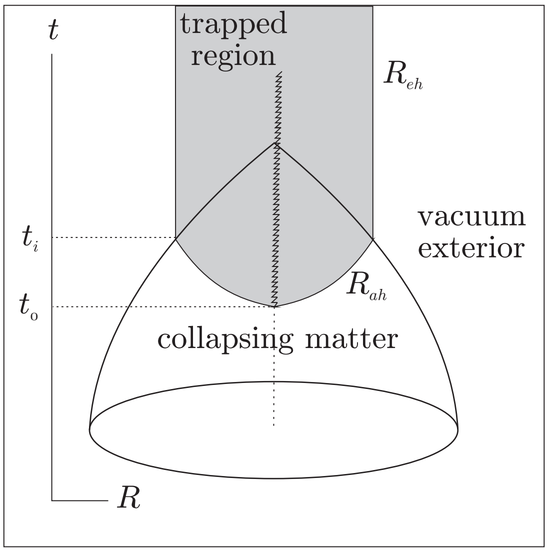

FIG. 2: Collapse leading to a naked singularity: The trapped surfaces form at the same time of formation of the singularity, t_0. The apparent horizon moves outwards from a zero physical radius at t_0 to reach a value R_{eh} at t_i.

Mathematically, it can be proven that in the latter case a set of future directed null geodesics can emanate from the singularity to reach faraway observers in the spacetime [11]. Physically this means that the ultra-dense region surrounding the singularity, where the quantum-gravitational effects would become dominant, can communicate with the outside universe. We shall note that a wider variety of scenarios is possible even when non-zero pressures are included in the collapse analysis. These include situations where the collapsing matter is radiated away during the collapse process, and the singularity curve remains uncovered for a longer time as the trapped surfaces form later than the singularity [12]. Nevertheless for our purposes here, the schematic view outlined above is enough to distinguish the black hole case from the naked singularity in gravitational collapse.

Whether naked singularities can actually arise from physically viable processes has been a matter of much debate currently and for past many years. The CCC, as formulated by Penrose, claims that all such occurrences must be covered at all times by an horizon, ruling out the possibility for singularities to be visible to faraway observers. Nevertheless, many counterexamples in models of collapse exhibit the behaviour described above, thus suggesting that naked singularities can indeed form as endstates of complete gravitational collapse of massive bodies [13]. Supporters of the CCC have therefore tried to invoke the laws of thermodynamics in order to construct a mechanism by which naked singularities cannot form. While such claims can in principle be legitimate in some cases, we shall see that the behaviour of horizons in gravitational collapse in general does not violate the laws of thermodynamics, regardless of the fact that the final outcome is a black hole or a naked singularity.

III. THERMODYNAMICS

The laws of black hole thermodynamics as stated by Hawking, Carter and Bardeen establish the connection between gravity and thermodynamics on the event horizon of a stationary black hole. In the case of vanishing charge and angular momentum (namely for the Schwarzschild black hole) these can be written as:

- 0th law: $\kappa = const.$ on the horizon.
- 1st law: $dM = \frac{\kappa}{8\pi} dA$ on the horizon.
- 2nd law: $\frac{dA}{dt} \geq 0$ on the horizon.

In the above, κ is the surface gravity on the event horizon, M is the total mass of the system (corresponding to the Schwarzschild parameter) and A is the area of the event horizon. The full analogy with equilibrium thermodynamics is then established once we relate the area to the entropy S (in geometrized units) via

$$S = \frac{A}{4} \; , \tag{III.1}$$

and the surface gravity to the temperature T via

$$T = \frac{\kappa}{2\pi} \; . \tag{III.2}$$

The generalization to rotating and charged black holes is possible, but not necessary in view of the present analysis of collapse, which does not consider the charge and angular momentum parameters.

A. Trapping horizons

Typically, the laws of black hole thermodynamics are defined on the event horizons in vacuum stationary space-times, like the Schwarzschild or Kerr-Newman solutions, since, in general, if the metric depends on t we cannot then find a preferred time direction in the form of a Killing vector. Therefore we cannot define a temperature in the sense of Hawking and Bekenstein, since it involves evaluating the surface gravity on a Killing horizon.

It is nevertheless possible to generalize the results of black hole thermodynamics to dynamical spacetimes where the metric components depend on t, if one makes use of the Kodama vector, instead of the Killing vector, to identify a preferred time direction. Then the thermodynamical quantities are defined on trapping horizons instead of the Killing horizons [14].

In this context, we are not anymore considering the laws of black hole thermodynamics at equilibrium. We are instead considering a dynamical system approaching equilibrium. It is important therefore to specify clearly what constitutes the system under consideration. In the case of gravitational collapse, the natural choice to make is that of the trapped region that develops as the collapse evolves, namely the volume of spacetime region causally disconnected from the outside universe, the boundary of this region being given by the trapping horizon. Of course, in the limit of approach to equilibrium the boundary of the trapped region approaches the usual event horizon.

At first, as matter starts collapsing under the effect of gravity, no portions of the spacetime are trapped. As certain high densities are reached, the trapped surfaces form and a trapped region develops in the spacetime. It is this part of the spacetime that evolves eventually forming the final black hole, possibly in a static or stationary configuration. Before it settles to its final state, the boundary of the trapped region is marked by the presence of an apparent horizon. As we have seen in the previous section, the apparent horizon typically develops between the time of formation of the singularity and the time at which it meets the outer Schwarzschild event horizon, and the singularity can be either causally connected or disconnected from the outside universe, which is decided by the pattern of trapped surfaces formation as the collapse evolves.

A sphere of physical radius R is said to be trapped, marginally trapped or untrapped if $h^{ab}\partial_a R \partial_b R$ is smaller than zero, equal to zero, or greater than zero, respectively. Defining the double null coordinates $\{\xi^+, \xi^-\}$ in such a way that

$$ds^2 = 2d\xi^+ d\xi^- + R^2 d\Omega^2 \; , \tag{III.3}$$

where the double null vectors are given by

$$\partial_\pm = \frac{\partial}{\partial \xi^\pm} = -\sqrt{2}\left(\frac{\sqrt{G}}{R'}\partial_r \mp e^{-\nu}\partial_t\right) \; , \tag{III.4}$$

we can write the expansion of the null geodesic congruences as,

$$\theta_\pm = 2\frac{\partial_\pm R}{R} \; , \tag{III.5}$$

from which we can see that the condition $\theta_\pm > 0$ (< 0) on a sphere of radius R is enough to ensure that the light rays on the sphere diverge (converge). It can be shown that if $\theta_+\theta_-$ has non-vanishing derivatives then the spacetime can be divided into a trapped region and an untrapped region, separated by a marginally trapped surface.

Defining the trapping horizon as the closure of the hypersurface obtained through the foliation of marginal spheres in the whole spacetime, we get the condition that R must satisfy at all times in order to describe a trapping horizon, which is given by,

$$h^{ab}\partial_a R \partial_b R = 0 \; . \tag{III.6}$$

In the above double null foliation, supposing the horizon is given by $\partial_+ R = 0$, then it is said to be future if $\partial_- R < 0$ (past, if positive), and outer if $\partial_-\partial_+ R < 0$ (inner, if positive). A black hole is defined as a future, outer trapping horizon. In the case of the apparent horizon, the requirement that the horizon be 'outer' can be dropped. In fact in the FRW models the horizon is considered to be future but negativity of the surface gravity indicates that it is an inner horizon, nevertheless, it is possible to prove the thermodynamical behaviour of such an horizon. The same reasoning holds for gravitational collapse.

We see immediately that in the general situation described by collapse, the trapped horizon in the collapsing matter is given by the apparent horizon R_{ah} and it is possible to construct the foliation such that the apparent horizon is a future horizon.

Nevertheless, the apparent horizons in the situations for black hole and naked singularity formation are different (as well as they differ from the FRW cosmological scenario). In the black hole case, we have $\dot{R}_{ah} < 0$ and the horizon equation is given by $\partial_- R = 0$, while in the naked singularity case we have $\dot{R}_{ah} > 0$, and the horizon equation is given by $\partial_+ R = 0$. By using the Misner-Sharp mass it is easy to check that in both cases we are dealing with a future horizon which is implicitly defined by $2E = R$.

B. Zeroth law

In the case of dynamical spacetimes the zeroth law can be formulated by the statement that the total trapping gravity of a future outer marginal sphere has an upper bound [7]. From this definition it is possible to retrieve the case of event horizons in stationary spacetimes considered by Hawking. It has also been noted that the surface gravity in dynamical spacetimes can be defined in many different ways [15]. Following Heyward, we can evaluate the surface gravity on the trapping horizons as,

$$\kappa = \frac{1}{2\sqrt{-h}}\partial_a(\sqrt{-h}h^{ab}\partial_b R) \ , \tag{III.7}$$

which in our case becomes

$$\kappa = \frac{E}{R^2} + \frac{1}{4}(p_r - \rho)R \ , \tag{III.8}$$

and we can relate the surface gravity of the apparent horizon with the temperature through equation (III.2). It is easy to check that for the event horizon, the equation (III.8) reduces to the well-known formula for Schwarzschild.

C. Unified first law

The Einstein equations (II.6) and (II.7) imply that we can write the variation of the Misner-Sharp mass as

$$dE = A\Psi + WdV \ , \tag{III.9}$$

where the area and volume of the sphere are the geometrical invariants used to define R from

$$A = 4\pi R^2, \ V = \frac{4}{3}\pi R^3 \ , \tag{III.10}$$

W is the work defined by

$$W = -\frac{1}{2}h_{ab}T^{ab} \ , \tag{III.11}$$

and Ψ is the energy-supply defined by

$$\Psi_a = T^b_a\partial_b R + W\partial_a R \ . \tag{III.12}$$

Equation (III.9) is the Unified First Law, and it naturally descends from Einstein equations in spherical symmetry. In fact by differentiating equation (II.3) it is easy to show that equation (III.9) is equivalent to equations (II.7) and (II.6). Evaluating equation (III.9) on a trapping horizon gives the first law of black hole thermodynamics which states,

$$\langle dE, z\rangle = \frac{\kappa}{8\pi}\langle dA, z\rangle + W\langle dV, z\rangle \ , \tag{III.13}$$

for some vector z tangent to the trapping horizon. As it has been noted, $z = z^+\partial_+ + z^-\partial_-$ is not an arbitrary vector but it must satisfy a certain condition on the horizon. Namely, if the horizon is given by $\partial_- R = 0$ then,

$$\frac{z^-}{z^+} = -\frac{\partial_+\partial_+ R}{\partial_-\partial_+ R} \ . \tag{III.14}$$

D. The Clausius relation

In order to prove that the Unified First Law is equivalent to the first law of black hole thermodynamics on the apparent horizon, one has to prove the Clausius relation,

$$\langle A\Psi, z\rangle = \frac{\kappa}{8\pi}\langle dA, z\rangle \ , \tag{III.15}$$

where the left hand side is just the heat flow δQ as defined by the energy momentum tensor, while the right hand side has the form TdS once the above mentioned identifications between T and κ and S and A are made. We can see then that if the first law holds on the horizon, the Unified First Law implies the Clausius relation $\delta Q = TdS$.

E. The Second law

We shall consider here the second law of black hole mechanics for dynamical horizons in gravitational collapse, which states that under the satisfaction of energy conditions the area of a future (outer) horizon is non-decreasing.

Nevertheless, there are examples in cosmological models, violating energy conditions but still used to describe the dark energy in the universe, where the second law at the horizon is violated. Generally one resorts to the formulation of a Generalized Second Law of black hole thermodynamics which states that the sum of the entropy of the horizon and the entropy of the matter bounded by the horizon (in the case of a cosmological horizon) does not decrease in time. These considerations suggest that one must be careful when talking about entropy in dynamical spacetimes, since a proper thermodynamical definition of the same, based on the microscopic behaviour of the spacetime, is still missing.

In the case of gravitational collapse, we would not consider the entropy of the infalling matter and therefore we will restrict ourselves to a genuine second law evaluated on horizons. For this reason we will neglect considerations regarding the microstates, as it will be sufficient for us to extend the usual definition of entropy as the area of the horizon, from the static case of the event horizon to the dynamical case of the apparent horizon. As we have seen, the apparent horizon in the black hole formation scenario in collapse is not an outer horizon, and therefore the area theorem does not hold in this case. Nevertheless we can formulate the second law of black hole thermodynamics once we consider the trapping horizon of the spacetime as the union of the inner apparent horizon and the outer event horizon, thus considering the horizons as the actual boundary of the trapped region in the spacetime. It is straightforward then to see that the volume of the trapped region is increasing in time, both in the case of the black hole and naked singularity. As said, in analogy with the static case, we shall define the entropy of the trapping horizon at any given time as the derivative with respect to R of the volume of the trapped region. Therefore, in the black hole case we obtain,

$$S_h = \pi(R_{eh}^2 - R_{ah}^2) \ , \tag{III.16}$$

where R_{eh} is the Schwarzschild radius in the exterior spacetime and R_{ah} goes to zero as t goes to t_0. Note that with this definition the apparent horizon has a negative entropy, in accordance with the entropy of the horizon in FRW models, but the entropy of the whole system is positive. In this case, the horizon forms at $t_i < t_0$ and becomes $R = 2M$ for $t \geq t_0$, with $M = E(r_b, t)$ being the total mass of the system.

In the naked singularity case we obtain,

$$S_h = \begin{cases} \pi R_{ah}^2 & \text{for } t \in [t_0, t_i) \\ \pi R_{eh}^2 & \text{for } t \in [t_i, +\infty) \ , \end{cases} \tag{III.17}$$

where again R_{eh} is the Schwarzschild radius in the exterior spacetime and R_{ah} goes from zero at the time of the formation of the horizon t_0 to R_{eh} at the time t_i, when all the matter is bounded by the horizon. In this case, the horizon forms at $t_0 < t_i$ and reaches the value $R = 2M$ for $t \geq t_i$.

In both these cases, the apparent horizon curve $r_{ah}(t)$ is determined implicitly by,

$$2E(r_{ah}(t), t) = R(r_{ah}(t), t) \ , \tag{III.18}$$

and in both cases of either the black hole or naked singularity formation, the horizon settles to the Schwarzschild event horizon once the collapse ends, at which point we regain the usual static definition of entropy. It is straightforward to check that

$$\frac{dS_h}{dt} \geq 0 \tag{III.19}$$

in both cases, and thus, if the above definition of the entropy is valid from a microscopical point of view, the second law of thermodynamics holds for the trapping horizons in the gravitational collapse scenario.

IV. THE THIRD LAW AND COSMIC CENSORSHIP

The analogy between black hole mechanics and thermodynamics would be complete if a satisfactory formulation of the third law could be given. Unfortunately the status of the third law regarding black hole mechanics remains unclear at the present juncture.

There exist two alternative formulations of the third law of thermodynamics. The 'strong' form says that the entropy S of a system goes to a constant value, which can be taken to be zero, as the temperature of the system goes to zero. A weaker statement says that it is impossible to reach T equal zero through any finite series of physical processes [16].

When applied to black hole mechanics, it is easy to find counterexamples to the analogue of the third law in its strong form. The usual model provided to illustrate how black hole mechanics does not satisfy the third law is given by the Kerr-Newman black hole. It is in fact straightforward to check that the parameters in the Kerr-Newman geometry can be varied in a way such that T goes to zero as S remains a finite function of the charge and the angular momentum, thus violating the 'strong' form of the third law [17]. This behaviour has been in turn interpreted as an indication that the extremal solutions cannot be attained by any finite physical process [18], or that a phase transition occurs when the extremal values are reached and that under these regimes matter behaves in an anti-thermodynamical way [19], or that a revision of the laws of black hole mechanics from microscopic considerations in order to reproduce the laws of thermodynamics is needed.

Given the fact that the extremal and super-extremal Kerr-Newman spacetimes present a naked singularity, as the horizon vanishes as the parameters reach the extremal value, the analysis of the third law in this context has been often associated to that of the CCC, and the validity of some form of the third law has been suggested at times as an argument in favor of the Cosmic Censorship. We had like to point out here how the discussion regarding the third law in extremal Kerr-Newman spacetimes and the validity of CCC cannot be associated to the discussion on the validity of CCC in collapse models.

In fact, the third law of black hole mechanics stands on a different footing when applied to dynamical spacetimes describing collapse or to extremal Kerr-Newman solutions, just as much as naked singularities arising in collapse models stand on a different ground from those appearing in stationary and axially symmetric spacetimes. Generally in the static, non charged case, T goes as the inverse of M and so, even for the Schwarzschild black hole, it is impossible to reach $T = 0$ with a finite physical process, a statement that is in agreement with the weak form of the third law.

In our case, neglecting rotation and charge, the third law, in its 'strong' form, still presents some issues in the sense that the identification of a temperature for a dynamical spacetime is a non-trivial task that can lead to different conclusions [15].

Approaching the singularity along the apparent horizon, the surface gravity diverges as the area goes to a constant value. This is not surprising, given the fact that at the singularity the energy density is diverging and since we have to remember that the singularity is not part of the spacetime. Therefore, the values of S and T at the singularity must be taken only in a limiting sense, or it would be correct to say that these are actually not defined there. Nevertheless this behaviour might indicate the necessity to find a more suitable definition for S or T on the apparent horizon as it approaches the singularity. For example, in the black hole case we could set the temperature as the temperature of the event horizon, as seen at spatial infinity, thus neglecting the contribution to the entropy given by the apparent horizon, that diverges as the singularity is approached but cannot be seen by any outside observer.

On the other hand, in the collapse models described above, the 'weak' formulation of the third law remains valid in a similar way as it is valid for the Schwarzschild case, since for these models the surface gravity goes to zero as R goes to infinity, which corresponds to the statement of physical unattainability.

V. CONCLUDING REMARKS

We discussed here a generic gravitational collapse going either to a black hole or to a naked singularity final state, and we examined the validity of the laws of black hole thermodynamics in such a dynamical scenario.

We note that in the Einstein theory, naked singularities arise in a variety of ways:

1. In extremal vacuum spacetimes such as Kerr-Newman or Reissner-Nordstrom.
2. In static and stationary axially symmetric vacuum spacetimes such as the Zipoy-Voorhees and the Tomimatsu-Sato metrics.
3. In dynamically evolving spacetimes such as the Lemaitre-Tolman-Bondi models.

The first category above is that of the naked singularities that appear in the extremal Kerr-Newman and Reissner-Nordstrom spacetimes. These are not dynamical spacetimes, and the naked singularity can be obtained by setting 'ad-hoc' certain values of the key parameters describing the solution. Therefore the procedure by which such naked singularities are achieved, starting from a non-extremal black hole, is not necessarily a physical dynamical process. It is essentially the variation of the basic parameters of the model (namely, the charge and the angular momentum) in a stationary spacetime. Hence, although it can be argued that the occurrence of such singularities could be the endstate of some physical processes (by which the above mentioned parameters can be varied), the procedure by which they are obtained is not in general the result of the dynamical evolution of the system as per the Einstein equations.

Therefore, the attainability of such naked singularities starting from a covered non-extremal black hole is a matter of much discussion and debate at present [20]. There are some claims that suggest that these naked singularities would violate the laws of black hole thermodynamics, and therefore, accepting the analogy between gravity and thermodynamics as a real feature of physics would imply that these singularities cannot occur in the physical universe. In this sense, this point of view links the validity of the Cosmic Censorship Conjecture to the laws of black hole thermodynamics.

The second category includes naked singularities that appear in static or stationary axially symmetric spacetimes. Again these are not dynamical spacetimes and the occurrence of singularities is a direct consequence of the chosen family of vacuum solutions. Departing from spherical symmetry immediately destroys the horizon structure, giving rise to a naked singularity. In some cases it is possible to link such spacetimes to spherical ones, for example, as in the Zipoy-Voorhees solution, also known as the gamma metric, or in the Tomimatsu-Sato metric, therefore obtaining a procedure to move from spherical symmetry to axial symmetry by the variation of one deformation parameter. Again, it has been suggested that once the parameter is changed as to depart from spherical symmetry, the thermodynamical properties of the horizon are lost. This is not surprising since the coordinates at which the horizon was located become singular and do not any longer describe a portion of the manifold. Nevertheless, once again, the variation of the deformation parameter is not a dynamical process as it can occur in the real universe. A similar situation in reality would require a dynamically evolving spacetime and the solution of Einstein equations for that case, a scenario that is still faraway from our understanding today.

The third category, which is the one we have investigated here, describes singularities arising in dynamical spacetimes evolving from some regular set of initial data. This is the case of the well-known naked singularities in gravitational collapse. For example, in the Lemaitre-Tolman-Bondi models the presence of inhomogeneities in the collapsing dust cloud can be enough to cause the collapse to end in a naked singularity. It must be stated though that naked singularities arising in many of the collapse models are naked only for a 'brief' period of time. The presence of pressures can alter the usual black hole formation picture, thus uncovering a portion of the singularity curve, but it cannot undress the singularity at all times [12].

We have shown here that the thermodynamical analysis of the apparent horizon in such cases as the third category above, is entirely possible and that it does not violate the laws of black hole thermodynamics as formulated for dynamical horizons. Therefore, a possible proof of the validity of the connection between thermodynamics and gravity on a general level would not necessarily rule out the occurrence of such singularities, whether hidden within a black hole or visible to external observers in the universe.

We emphasize again that singularities cannot be treated as part of the spacetime. Therefore one should study the thermodynamical behaviour and laws only on appropriate surfaces such as the apparent horizon and not on the singularity itself, though one can approach the singularity as close as one wishes. We also note that, considering that naked singularities do occur quite generally in Einstein's gravity, both in static or stationary situations as in the dynamical cases, it may also be the case that if they are not consistent in some situations with the thermodynamical behaviour, then gravity may not be equivalent to thermodynamics in a global and fully general sense.

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