THE IMPLEMENTATION OF THE COMPUTER SYSTEM ALEKS FOR PLACEMENT IN MATHEMATICAL COURSES AT NAZARBAYEV UNIVERSITY

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Abstract. The ALEKS computer system, well-known on the American continent as an individualized teaching tool and placement test into a series of high school and college level courses, was implemented by the Mathematics Department of Nazarbayev University, most likely as the first experiment of this sort in the heart of Asia. This paper summarizes the results of the placement testing of several hundreds of Kazakhstani students passing from the preparatory study focused on biology, economics or engineering mathematics to the typical curriculum of science departments at the School of Science and Technology of Nazarbayev University.

1. Introduction

Nazarbayev University (NU) opened in the new capital of Kazakhstan, Astana, in 2010. Its first cohort of 500 undergraduate students entered the Foundation program which was designed similarly to the preparatory program of the University College London (UCL). UCL was simultaneously a partner of the Engineering School, one of three undergraduate schools in Nazarbayev University. The first cohort finished the preparatory study and entered three undergraduate schools of NU in 2011.

Only a third of students from the Foundation program were accepted to the Engineering School, and all of them were trained within the math-physics track of the Foundation program; the other half of this track as well as students from two other tracks: bio-chemistry and economics-international relations - chose to study at the School of Science and Technology (SST) and the School of Humanities and Social Sciences (SHSS). Remarkably enough, these two latter tracks did not have any mathematical training during the preparatory year. At the same time, the programs of SST and the economics program of SHSS, designed similarly to the science program of American schools, assumed Calculus I as a first required mathematical class.

It turned out that the majority of students entering SST and SHSS were either "overqualified" to take Calculus I or were not prepared for the class. The Mathematics Department of SST included the Pre-calculus course into its offerings, for the purpose of closing the gap in mathematical preparation for Calculus I, but it was in general hard to estimate who should be placed into Pre-calculus and who into Calculus I or Calculus II classes, based on the general and the mathematics scores received in preparatory study.

The solution to the placement problem was suggested through the automated assessment system ALEKS, which was known in many American Universities already in the early 2000s.

ALEKS stands for Assessment and Learning in Knowledge Spaces, which is a web-based, artificially intelligent assessment tool and learning system [**?**]. In the foundations of this tool is the Knowledge Spaces theory that was developed since its introduction in 1985 by mathematical psychologists J.-P. Doignon and J.-C.Falmagne [5].

The commercial use of ALEKS took off at the end of the 90s, and its usage in the higher education system in the US continues to grow. In 2013 it was acquired by McGraw-Hill Education. The knowledge space concept at the core of the system allows the computerized tutoring system to be used in any subject, where the system of concepts is established, together with dependencies of pre-requisite concepts.

In this paper we will present the results of implementation of ALEKS as a testing and placement tool introduced by the Mathematics Department of SST/NU in 2013-2014. We believe that the experience of Nazarbayev University may provide valuable advice to other young teaching institutions striving for the effective teaching experience of their students.

2. Description of pre-calculus course

What is customarily referred to as Pre-calculus comprises of the content of two courses offered to high-school students in the US during their studies in 9th and 10th grade. Common names of these courses are Algebra I, Algebra II, or Algebra and Trigonometry. During middle school study in the US, students are expected to cover material that is needed for these courses. We could call it Basic Algebra and Geometry and it comprises the following topics:

- sets of numbers,
- ratios and percentages,
- operations with numbers in their fractional and decimal forms,
- Descartes coordinate systems and basic graphing,
- solutions to linear equations and inequalities,
- solutions to quadratic equations,
- operations with polynomials such as factoring and expansion,
- operations with roots and powers,
- basic geometry on the plane and in three dimensions.

The High school Algebra course concentrates on the study of important functions which will be the centerpiece of further Calculus series in college. It includes:

- The general study of functions and their properties, symmetry and asymptotes,
- Basic types of functions transformations,
- Linear functions and their graphs,
- Graphical solutions to systems of linear inequalities,
- Parabolas and polynomials of higher degree,
- Rational functions (ratio of two polynomials),
- Power functions,
- Logarithmic and Exponential functions.

Students normally have some initial knowledge of most functions from middle school, but logarithmic and exponential functions are studied for the first time and constitute a considerable part of the course. The course on Trigonometry takes the Algebra course further into exploring trigonometric functions. This assumes some

initial knowledge of plane geometry. Beside the 6 to 8 major trigonometric functions, the course devotes time to the geometry of triangles, trigonometric identities and equations.

Overall, these two courses make necessary preparation for the first math college course of Calculus I, where the main goal is the introduction of the concepts of limit, continuous function, derivative and anti-derivative, and Riemanns integral. Since all these concepts are applied to polynomial, rational, power, logarithmic, exponential and trigonometric functions, students need to possess enough knowledge and skills about these functions to be successful in understanding the next level of analysis or calculation they can perform with the given functions.

Since Calculus I course is mandatory for all science, engineering and economics majors in the US, Algebra and Trigonometry courses are usually mandatory for high school students. Note that most medical training in the US occurs on the basis of a Bachelor of Science degree, thus, assumes Calculus I course at the foundation of mathematical education for a medical degree.

Students in high school may take additional mathematics courses, such as a Calculus course, which is close or equivalent to college Calculus I. In this case, students may apply for advanced placement (AP) testing for Calculus I during their junior or senior year and obtain the college credit for the course. Some high schools also offer the second Calculus course to their mathematically most advanced students. This course deals with methods of integrations and series, and it is equivalent to Calculus II offered in college. Students may obtain the AP placement for this course as well and start their college education by taking Multivariable Calculus.

Aside from the Calculus series, high schools may offer their students courses in matrix algebra, probability and statistics, geometry and logic (or mathematical reasoning), but normally none of these courses are compulsory. Currently, some states are adopting a 4-year core program math curriculum which combines the units of both Pre-calculus courses, matrix algebra, geometry, mathematical reasoning and statistics [1].

3. Placement into mathematical courses in American universities and colleges

In many US universities, SAT or ACT scores were the base for the admittance of students into Calculus course. High SAT scores typically imply high level reasoning abilities, because the problems offered there are far from standard. However, they do not require mathematical knowledge beyond middle school preparation. For example, in a SAT practice test there were no questions about logarithmic, exponential or trigonometric functions but contains questions related to plane and space geometry, statistics and logical reasoning. [1]

Moreover, SAT and ACT tests are normally taken in the Fall semester of the 11th grade, and students sometime do not enroll into any mathematics course

[1]High Schools in Kazakhstan provide similar mathematical preparation for students who plan to enter science programs in college. The study of functions is done in 9th grade, trigonometry in 10th grade, logarithmic and exponential functions in 11th grade. Moreover, a beginning analysis (differentiation and integration) is covered in 10th and 11th grades. Besides, students have a required course of geometry starting from 7th grade, which is devoted to planar and three-dimensional shapes, also analytic and vector methods.

during their last year of high school. This practice weakens their mathematical skills by the time they enter college.

For this reason, many universities in the US started administering their own placement exams when deciding about the proper placement of students into mathematics courses. Based on their scores, students are placed in one of several remedial mathematics courses. These are

- Basic Mathematics (usually the content of the 6th grade mathematics curriculum)
- Beginning Algebra
- Intermediate Algebra
- College Algebra (equivalent to Algebra I of high school)
- Trigonometry

The last two courses are customarily offered as a Pre-calculus course. Ivy League universities do not offer any remedial course below the Pre-calculus course, and many small private schools would offer mathematics courses starting from Calculus, while the low remedial math courses are delegated to community colleges and, sometimes, to large public schools.

We underline that the students need to take remedial courses, including Pre-calculus, with the future goal to take and pass the required Calculus I course. For most majors in humanities, Calculus I is not required, and students only need to take one course of college level, which is generally not devoted to the study of functions, and may include units on sets, logical reasoning, geometry and statistics.

4. ALEKS as a tool of choice for math placement

Universities in the US started using ALEKS in the late 90s. One of the pioneers of implementing the placement of ALEKS on a large scale was the University of Illinois (Urbana-Champaign), a leading public university of the State of Illinois. For decades, the university used SAT scores for the placement of about 3,500 incoming freshmen into either Calc I or Pre-calculus.

The Mathematics Department at UIUC was not satisfied with the low retention rates and high withdrawals in Calculus I, at the same time observing many students who were bored in Pre-calculus [2]. We refer to the publications on ALEKS implementation at UIUC by A. Ahlrgen and M. Harper in [3,4]. Main conclusions: ALEKS placement improves the success rates of students in Calculus and makes the overall students' experience better in both Pre-calculus and Calculus. As a result, the enrollment into other mathematics classes at UIUC increased.

In recent years, more universities made a full transition of their placement system to ALEKS. Examples of recent transitions (in 2012-2013) are: The University of Arizona, the University of Connecticut and Washington State University. [2]

For example, check the information from the University of Connecticut web-site to outline ALEKS requirements for placement.

All University of Connecticut undergraduate students who have chosen a major in the following disciplines or who intend to do so must take the Mathematics Placement Assessment (ALEKS).

[2]http://www.tricity.wsu.edu/placement/math.html
http://placement.uconn.edu/math.html
http://math.arizona.edu/academics/placement/exams

The list of 40 majors include all science (Math, Physics, Chemistry, Biology etc.), all engineer majors, all economics majors and, for example, such as Pre-teaching, Pre-pharmacy, Pre-kinesiology etc.

The information on the web further specifies:
Even students who have taken math AP courses in high school and/or have high SAT scores should take ALEKS.

Note that:
(1) There are two levels of placement in UConn;
(2) Students, who presumably took AP placement exam in Calculus I during high school and who are planning to take Calculus II in University, still need to pass the ALEKS test.

Your ALEKS score will be used for placement in required Mathematics courses. A minimum score of 76% for placement in MATH 1131 (Calculus I) and 1132(Calculus II).
A minimum score of 61% for placement in MATH 1060 (Pre-Calculus) and 1071 (Mathematics for Business and Economics).
Students who failed to achieve these scores are encouraged to spend time on the preparatory and learning modules before taking ALEKS again or register for a lower level Mathematics course.

5. ALEKS preparatory and learning module

In the foundations of ALEKS is the Knowledge Spaces theory that was developed since its introduction in 1985 by mathematical psychologists J.-P. Doignon and J.-C.Falmagne [5].

According to Wikipedia [7]:
In mathematical psychology, a knowledge space is a combinatorial structure describing the possible states of knowledge of a human learner. To form a knowledge space, one models a domain of knowledge as a set of concepts, and a feasible state of knowledge as a subset of that set containing the concepts known or knowable by some individual. Typically, not all subsets are feasible, due to prerequisite relations among the concepts. The knowledge space is the family of all the feasible subsets.

Based on this model, two computerized tutoring systems were developed: RATH and ALEKS [8].

One initial applications of Knowledge Space software in the higher education domain was developed for the system of mathematical concepts needed for the Calculus course. These are concepts studied in elementary through middle school up to Algebra I and Trigonometry in American high-school, per our description in section 2. Among the ALEKS products, a unit known as the ALEKS Preparatory and Learning Module(PLM) was the one that was implemented at the School of Science and Technology at Nazarbayev University, starting from the Fall semester of the 2013-2014 academic year. Other products of ALEKS were used later by other schools of NU, such as the Graduate School of Business.

The list of topics from Knowledge Space of ALEKS PLM currently implemented as the placement for Calculus I and pre-calculus courses at NU includes topics in arithmetic (section 1); sections 2-7 cover the needed knowledge of polynomial functions and basic geometry; sections 8-10 treat rational, power, exponential and logarithmic functions; section 11 is devoted to trigonometry.

Note that the list of topics included into the ALEKS PLM can be adjusted. For example, the topics devoted to complex numbers, which is often taught in school, may or may not be included into ALEKS PLM.

ALEKS PLM testing starts with the initial students assessment via a 30-questions test. The goal of this testing is to make an accurate placement of an individual students knowledge within the Knowledge Space associated with the pre-calculus content. It starts with very basic questions from section 1, then provided a student gives correct answers, progresses towards more advanced topics. The questions are randomly selected from the large pool of questions related to each of the topics. Students give open end answers using the palette of mathematical symbols. The system assumes that students may give an incorrect answer on each level of progression due to minor mistakes, so it will test them with another question close to their current position in the knowledge space.

Thus, the ALEKS assessment is not a pre-determined set of questions, but rather a sequence of questions which is dynamically generated during the test, based on students answers.

After the completion of 30 questions requiring an average of 35-45 minutes of a students time on a computer, the result on a 1-100 scale is immediately available and also available in the form of a pie chart, containing several sectors designated to various clusters of topics. Students may visualize their own placement results, similar to the one shown in Fig.1.

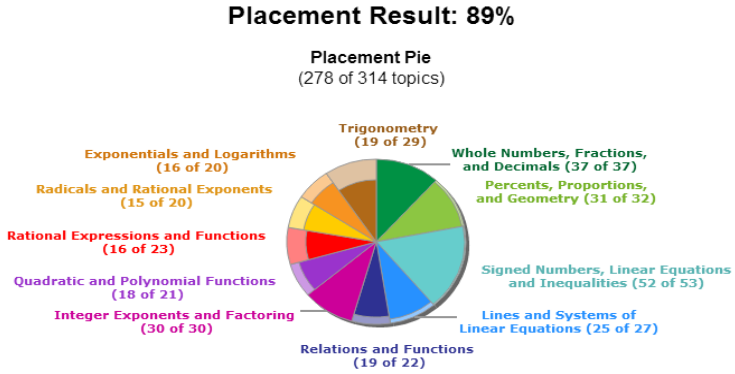

FIGURE 1. Pie chart in the ALEKS test

6. PLACEMENT OF STUDENTS OF NAZARBAYEV UNIVERSITY INTO CALCULUS I COURSE OF SST IN THE ACADEMIC YEAR OF 2012-13

In this section we will describe the placement practices of the School of Science and Technology of NU in 2012-13 academic year, i.e. during the second year of the school's operation. While the setting for the placement might not be typical for start-up schools, it may reflect the general situation when preaparatory study, or expectations at the exit of senior school, is not in accordance with the entry requirement of secondary education institutions. In such general situations the introduction of computerized practice and placement system such as ALEKS may become beneficial.

In August 2012, the placement of freshmen students at SST and SHSS into Pre-calculus and Calculus I courses was mostly based on their mathematical training during the Foundation year (College Preparatory Study, or CPS), and relied, in big part, on the scores of the CPS Exam in Core Mathematics, [10].

In addition, the Mathematics Department was offering paper tests in Pre-calculus to directly admitted students and select groups of CPS students. In rare cases, the decision about placement into Calculus was made based on SAT score.

The majority of the 500 CPS students during the 2011-2012 academic year, were enrolled in the Math/Physics track. Assumptions about the content of their mathematics training were based on their CPS Final Exam in Core Mathematics, administered on June 19, 2012 [11].

Section A of the Final Exam contains 9 mandatory problems:

- 1 problem on complex numbers
- 1 problem on logarithmic differentiation
- 1 problem on indefinite integration
- 1 problem on using integration for finding the area
- 1 problem on finding local maxima and minima of a polynomial function (the applications of differentiation)
- 2 problems on series and approximation
- 1 problem on solving a differential equation of the first order
- 1 problem on operations with vectors in three-dimensional space

In the mathematics curriculum of higher education in US the first 5 problems are studied in Calculus I (except for complex numbers, which may be covered in Pre-calculus and treated again in later Calculus courses), 2 problems belong to Calculus II, another problem is from the course of Differential equations, and the last problem is in the section on vectors from Calculus III.)

Section B contains 15 problems from 6 topics, of which students need to answer only 4. These are

- 3 problems in algebra dealing with solutions of quadratic equations and the binomial theorem
- 2 problems on differentiation in application to point or curve locations
- 2 problems on integration, for methods and the application of integration
- 3 problems on complex numbers
- 3 problems on series
- 2 on analytic geometry and vector algebra

Of these problems, only the complex numbers and algebra problems related to quadratic equations would be covered in the Pre-calculus course. Still, the complex numbers are often removed from the content of Pre-calculus and treated at the deeper level in later mathematical courses.

In 2012, CPS also administered the Final Examination in advanced mathematics. Problems in the advanced exam were on the level of solutions to differential equations of second order, improper integration, using integration for finding the length of a curve, curves in polar coordinates, double and triple integrals, modeling the elasticity by differential equations.

Thus, the mathematical content that was taught to students in Math/Phys track is normally covered in Multivariable calculus and Differential Equations courses. Therefore, the topics of both tests were in Calc I-III series and Differential equation rather than Pre-calculus topics.

For this reason, a CPS Math Core exam score was not an accurate measure for any placement decision for Pre-calculus and Calculus I courses.

While this conclusion could be made easily based on the analysis of the two CPS tests above, in August 2012 scores in Mathematics Exams provided the only mechanism for placing the graduates of a Math/Phys track from CPS into Calculus I and Calculus II.

Two other tracks at CPS are the Biology/Chemistry and Economics/IR, with a total of about 160 students either did not have or had rather limited mathematical training during Foundation year.

The Department of Mathematics offered to these incoming CPS students a voluntary Pre-calculus paper test. Three groups were tested.

- 24 Biology students were selected to take the test, 16 took it and only 8 passed;
- among 39 SHSS students taking the test only 13 passed;
- among 15 in the second group only 1 passed.

Based on previous experiences with chemistry students, and upon the recommendation of the Chemistry Department, all incoming chemistry students were enrolled into Pre-calculus without testing. For biology and SHSS incoming students, only those who passed the Pre-calculus test were placed into Calculus I, and the others took Pre-calculus.

For other majors of SST (Mathematics, Physics, Computer Science and Robotics) the cut-off in CPS Math core exam for the enrollment into Calculus I was established at 50 (out of 100). For those who got the scores of 60 (out of 100) or above in Core exam got credit for the Calculus I course, and for scores of 60 or above in both Core and Advanced exams, credits for both Calculus I and II courses were awarded.

With these cut-off scores, the Department of Mathematics had to run 5 sections of the Pre-calculus course in the Fall of 2012, with a total enrollment of 195 students, and two more sections in Spring 2013 with a total enrollment of 80 students. In Fall 2012, Pre-calculus was the full or partial load of 4 professors of the Mathematics Department; also, one of the 3 teaching assistants of the Department was fully devoted to the teaching /recitations of 195 students in this course.

It was reported in [10] that during the Fall 2012 semester there were occasions of moving students from Calc I down to Pre-calculus, since students were falling behind in learning material and could not progress in Calculus. On the other hand, there were many students in Pre-calculus course, mainly Computer Science and Robotics students, with scores from the CPS below 50, who were clearly bored in that class, because they knew the material well.

This shows the similarities with the observations described in UIUC report on placement [3], at the times when the placement into Pre-calculus was based on SAT scores.

Moreover, many students in Mathematics, Economics, Computer Science and Robotics were delayed by one semester in their progression in mathematical courses. There was an urgent need of additional offering of Calculus I and II sections in Summer 2013. For example, many Economics students took only Pre-calculus course by summer 2013, while they needed economics course with Calculus I pre-requisite in the Fall 2013. There was a high demand of places in Calculus I, and placement was handled through a waiting list.

Clearly, the main problem of transition between high school (for directly admitted students), or preparatory studies, and the first year of college education was a mismatch between the exit testing and entry requirement: while mathematical training and testing at the Foundation program of NU was well designed for the entry requirement into the School of Engineering with its ties to a program of UCL, it was not appropriate for two other undergraduate schools, whose curriculum was based on traditions of American education.

7. Pilot testing at Nazarbayev University in spring 2013

In March 2013 the Department of Mathematics of SST, in collaboration with CPS, started planning the testing of 168 CPS students of two non-mathematics tracks with the assistance of ALEKS system. By the time the Department established contact with the ALEKS Corporation, and received a 3 months trial period for Nazarbayev University to explore the system for the purpose of the placement into Calculus.

The setting of this testing was established with the following parameters:

- the cut-off for ALEKS test for placement into Calculus I was established at the score of 75
- the first attempt was not qualifying for placement
- students needed to practice at least 5 hours between the first test and the second test, which was administered a week later.

This decision was based on the following considerations:

- Certainly, all students in the two tracks of CPS would benefit from 5 hours of refresher pre-calculus training, given that they either do not have or have limited mathematical training at CPS
- the report from the earlier testing of ALEKS [12] indicated that a few students may get inflated score in the first assessment.

Indeed, the very first assessment of ALEKS in placing a knowledge set of a particular student within the knowledge space may be inaccurate in rare occasions, when the student gets lucky in replying correctly to critical questions, while not having proper knowledge. Usually, such errors are quickly corrected when the students starts practicing in PLM, and ALEKS makes the adjustment given more information from the students answers.

In the vast majority of cases, the scores of students increased between their first attempt and second attempt. We present the information on students who failed, i.e. scored < 75, on their 1st attempt separately for Bio-Chem track and Economics-IR track.

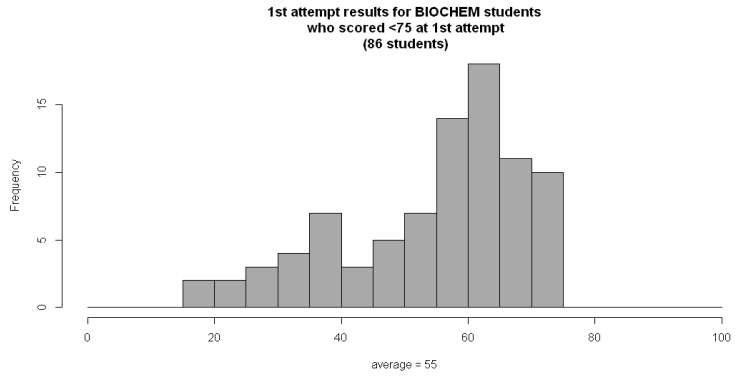

FIGURE 2. Bio-Chem test N1

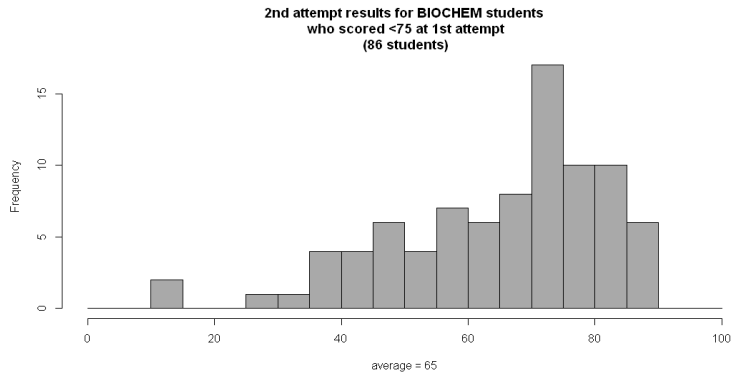

FIGURE 3. Bio-Chem test N2

Economics-IR track showed much better results on both tests, given that students in this track had to demonstrate a proficiency in mathematics upon entry to the CPS program, and some mathematics was incorporated into the course content for this track.

After running the second round of testing in the Spring of 2013, we made a decision of offering a third round of ALEKS assessment to a select group of students: it included

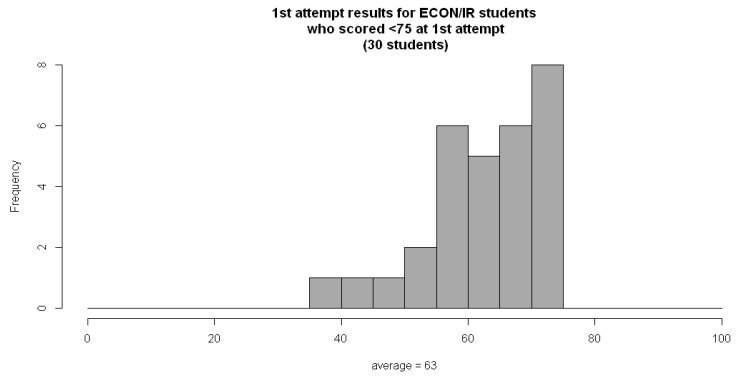

FIGURE 4. Economics-IR test N1

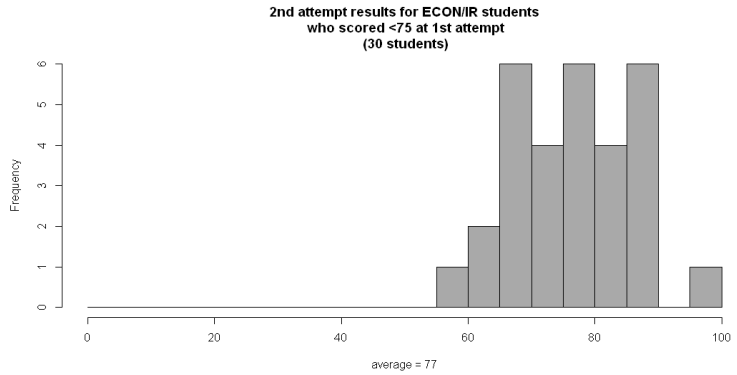

FIGURE 5. Economics-IR test N2

- students who, for any reason, could not attend either the first or the second round
- students who showed good progress between two attempts, and had their ALEKS score between 65 and 74.

The third round of tests was conducted 10 days after the second round, and later it was also offered in August, prior to the start of the Fall semester, i.e. about 3 months later than the second round. Those who would have their third ALEKS assessment had a requirement of an additional 3 hour practice in PLM, between their second and third attempt. We provide the scores for all students who took ALEKS Placement 3 times separately: for those who had their 3rd attempt in the Spring and for those who had their 3rd attempt in the Fall.

In the first picture we see that the pattern is the same for all but two students. In fact, these two students did not complete their 3rd placement - it was timed out, for various reasons. Thus, the pilot testing clearly indicated that students continue progressively to master pre-calculus skills, when they are given practice in PLM between tests, and the tests are given with about a week in between them.

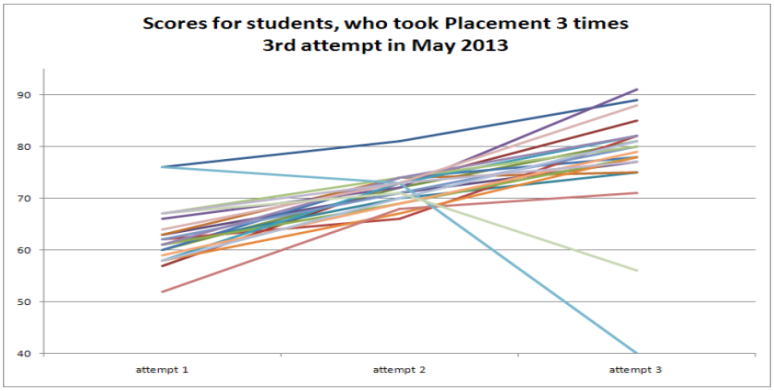

FIGURE 6. Three tests consecutive

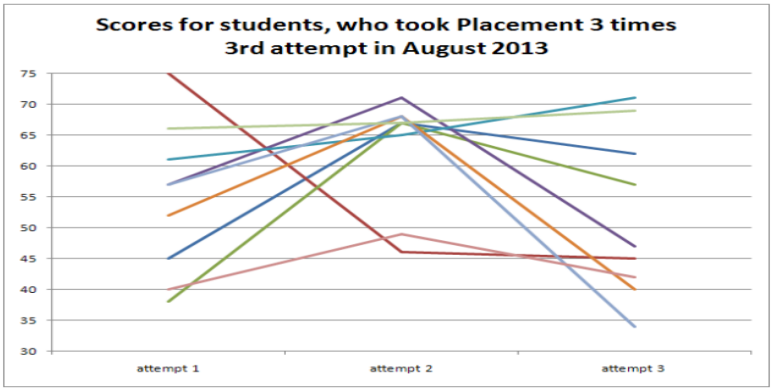

FIGURE 7. Third test after a gap

Quite to the opposite, the long gap between two tests essentially returns a student to the level she has been at the time of the first testing.

We emphasize that all the tests in these pilot experiment were *proctored*: students physically attended computer rooms on the university campus, all the internet access was blocked, except for the ALEKS site, and several assistants were checking that students perform their tests without the help of any devices or printed materials. Students were given a two-hour window, when they could come to a computer room and complete their test before the cut-off time. Any cheating on the test is practically impossible, since students are given individual problems, and they can start the test only when given a unique password. The latter is created just prior the test, when they show up in the testing room.

We note that the minimum requirement for the number of hours spent in ALEKS PLM is checked at the time of start of an assessment: if a student did not spend enough time in the PLM unit prior to an assessment, then he/she will be blocked from entering the test.

We also note that some of the students were not well motivated to pass the test successfully: at the time when the test was administered, Pre-calculus course was

offered by the Mathematics Department, and SHSS students could take that course in order to complete mathematics requirement in their programs.

8. Placement by ALEKS in the Fall of 2013

The Fall ALEKS testing was planned with the Registrar office since spring 2013, and it was set for the second week of August, 2013, during the freshmen orientation week.

Students from CPS, in two non-mathematics tracks, who already had 1 or 2 assessments with ALEKS and did not pass the 75 cut-off, were allowed to take the assessment again. Beside this group, all SST students coming from the Math/Phys track and whose Math Exam score was below 50 were designated for ALEKS testing.

The policy on placing Math/Phys students from CPS did not change much from the academic year 2012-2013:

- students scoring above 60 on the CPS Math Core exam would be given credit in Calc I and placement in Calc II
- students, who scored 50-59 on the CPS Math Core exam, would be placed in Calc I.

Unlike the pilot testing with students from non-mathematics tracks, *the first attempt was a qualifying test for placement* when students were coming from the math-physics track. The reason for that was considerable mathematics training that these students took during their Foundation year. In the past all of these students would be placed into Pre-calculus course, while the majority of them would enter SST as majors in math, physics, computer science or robotics. These are the programs, where the delay in taking Calculus I would wreak havoc in scheduling consecutive required courses, for which Calc I is a pre-requisite.

Out of 28 students in this cohort, 20 were able to pass ALEKS with the score of ≥ 75, thus, avoiding taking the Pre-calcuclus course, which would be inevitable one year earlier.

We will finish this section with the summary of all groups that were tested and the number of students in each group who passed the test on each of the given opportunities. The group of SHSS students in the middle of the table was tested prior to the pilot practice and their passing score was established at 70 rather than 75 in a later large pilot testing.

Out of 234 students who went into testing 159 (or 68%) were placed into the Calculus I course. One year earlier, these students would fill 5 sections of the pre-calculus course. In 2013 they were saved from the need to take it.

Of course, the success of any placement can be evaluated only sometime later, when students who were placed in a particular course will finish the course, and the grades could be compared to the results of an earlier placement. We will address this comparison in the next section.

9. The performance of students in Calculus I course placed by ALEKS versus CPS math score

One important observation in [2] made during the study of effectiveness of the ALEKS placement was an increase in the retention rate and an improvement in Calculus I grades.

We made a comparison of two grade distributions for the population of students taking Calculus I in Fall 2013:

	total # of tested students	# who showed >=75 in first try	# who showed >=75 in second try	# who showed >=75 in third try	# who showed >=75 in fourth try
CPS Chem/Bio	109	22	51	16	0
CPS Econ/IR	58	27	44	3	1
CPS Math/Phys	28	13	7	0	0
Directly Admitted	17	10	1	0	0
	total # of tested students	# who showed >=70 in first try	# who showed >=70 in second try	# who showed >=70 in third try	# who showed >=70 in fourth try
SHSS	22	10	10	0	0
TOTAL	234	82	113	19	1

FIGURE 8. Fall 2013 placement

- The first comprises students who are placed into the class by ALEKS.
- The second is formed by 83 students with CPS scores between 50 and 59, and also contains 6 directly admitted students: 2 of them were placed based on their high SAT1 Math scores (800/800) and the other 4 were placed using a paper-based pre-calculus test.

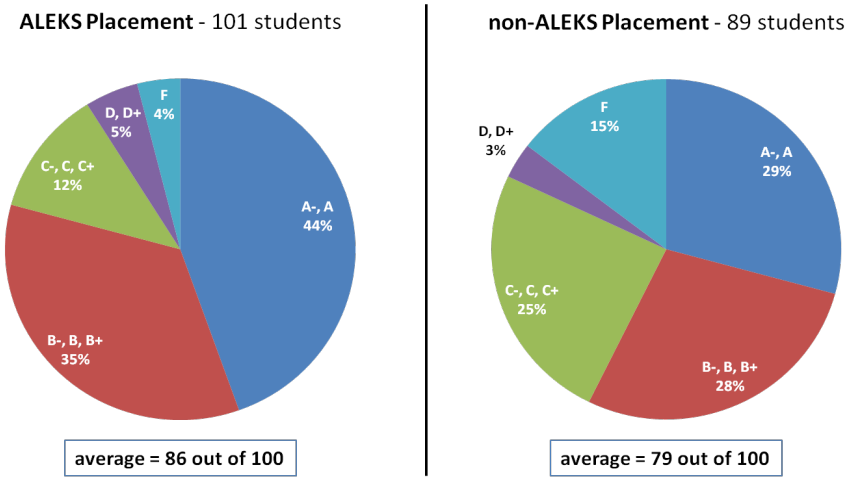

FIGURE 9. ALEKS group versus non-ALEKS placement

In the second grade distribution comparison we look into the group of students in the 2012-2013 academic year who took Pre-calculus with mathematics instructors, then enrolled and completed the Calculus I course. We make a comparison with the Summer and Fall 2013 groups in Calculus I placed by ALEKS.

In addition to higher grades for the group placed by ALEKS, the retention rate was also better in this group: only 2 students (out of 101, or 2%) withdrew from the Calculus I course in the Fall of 2013. In the previous academic year, 13 students (out of 251, or 5%) withdrew from Calc I after taking the Pre-calculus course with

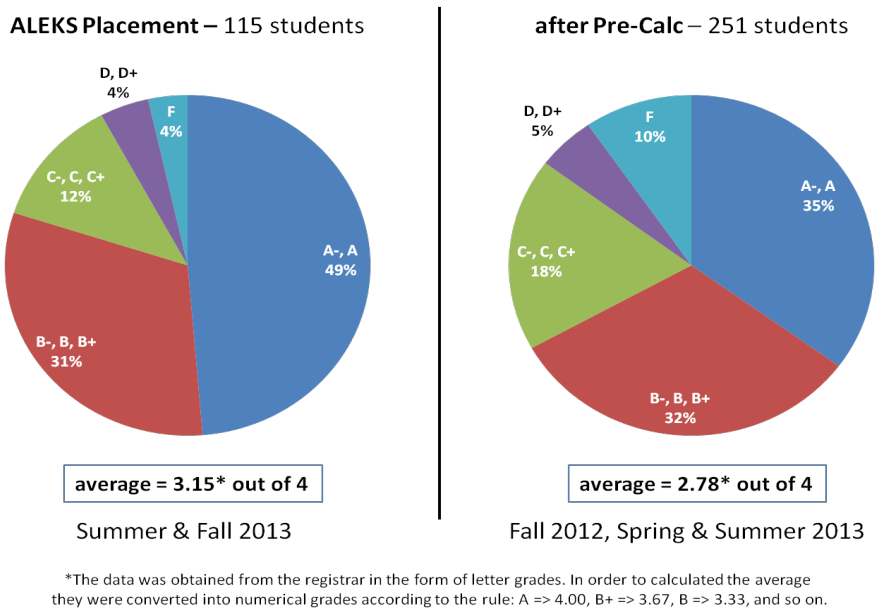

FIGURE 10. ALEKS group versus non-ALEKS placement

instructors. The data supports the conclusion similar to one obtained during the study of ALEKS placement at UIUC: *ALEKS makes more accurate placements and successful predictions of students in Calculus I than any other math testing.*

The CPS core math exam evaluates the skills of students in more advanced topics of Calculus and Differential Equations, but does not give a clear indicator whether students know all the pre-calculus topics well. This, in particular, applies to students with a low CPS score. As we mentioned in section 8, 20 students with low (< 50) CPS Math scores were able to pass ALEKS and were then placed into Calculus I.

The lower performance of students who took Calculus after the Pre-calculus course could be explained by the fact that the courses were not always taken in successive semesters, thus allowing a semester or more gap in between. Unlike this setting, the ALEKS test was mostly performed right before taking the Calculus I course.

10. PRE-CALCULUS HYBRID COURSE

In the Fall 2013 the Department of Mathematics SST offered a pilot Pre-calculus course taught by math instructors with the assistance of the ALEKS system. Unlike ALEKS PLM that is used for placement purposes, Pre-calculus with the ALEKS course is designed as an on-line tool of teaching, with or without class instructions. In case of NU, it was an 11 week course, with class instructions, that used the same text book as in the Pre-calculus courses of the previous year.

We made an initial analysis of a cohort of 88 students who were enrolled into the course.

In the first group there were 30 CPS students who took 2 or 3 assessments by ALEKS, but were not able to reach the score of 61 in their last assessment.

In the second group, there were 5 students who had only one ALEKS assessment with a score below 61.

The third group comprises students who never had thr ALEKS assessment. These were students from the Math/Phys track, directly admitted at SHSS, or students from SHSS of the second or third year.

One of the observations made during this class was that a large group of students was making very slow progression in ALEKS. Such slow progression is measured by parameters in the ALEKS system:

- time spent by student in the system
- number of topics mastered in hour of work in the system.

For students who were progressing slowly, the first parameter would be very high, and the second one very low. Note that similar observations were made in ALEKS performance for math remedial classes at Texas A&M university [12], where students were placed in classes lower than Pre-calculus.

In our case, there was a correlation of this slow progression within the first and second group. Mention that, according to the placement decision of the University of Connecticut that we referenced in section 4, the cut-off 61 in ALEKS test was considered for placing students into a remedial class *lower* than Pre-calculus.

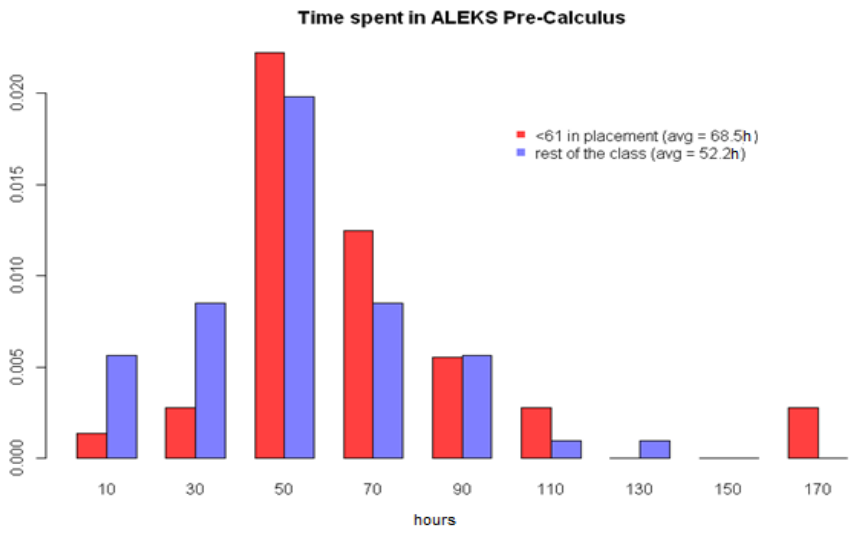

FIGURE 11. Time spent in ALEKS

From the histogram above we see that students from the first and second groups (in red) tend to spend more time in ALEKS than the rest of the class.

Below we provide the confirmation of the statistical significance of the difference between these groups.

Testing for normality According to the Kolmogorov-Smirnov Test we fail to reject normality for observations from the first and the second groups: $D = 0.1997$, p-value $= 0.09819$, and normality is not violated in the third group as well: $D = 0.0953$, p-value $= 0.6855$.

Testing for difference between sample means

Let μ_1 be the mean time spent in ALEKS Pre-Calculus by students who score < 61 in Placement, and let μ_2 be the mean time spent in ALEKS Pre-Calculus by the rest of the students. We want to test a hypothesis

$H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 > \mu_2$

Since we can assume that both samples come from normal distributions we apply the Welch t-test, which gives

$t = 2.6396$, $df = 64.855$, p-value $= 0.005192$

Sample estimates are: $\hat{\mu}_1 = 68.51481$ and $\hat{\mu}_2 = 52.26604$.

Thus we reject the null-hypothesis at 0.05 significance level.

So, it seems that students who score < 61 *in ALEKS Placement indeed spend more time in ALEKS Pre-Calculus than the rest of students.*

To complete this analysis of Pre-calculus with the ALEKS course, we provide the grade distribution. Out of 88 students who were enrolled at the start of the 11-week course, 4 withdrew, 2 failed and others completed it with a grade of at least D.

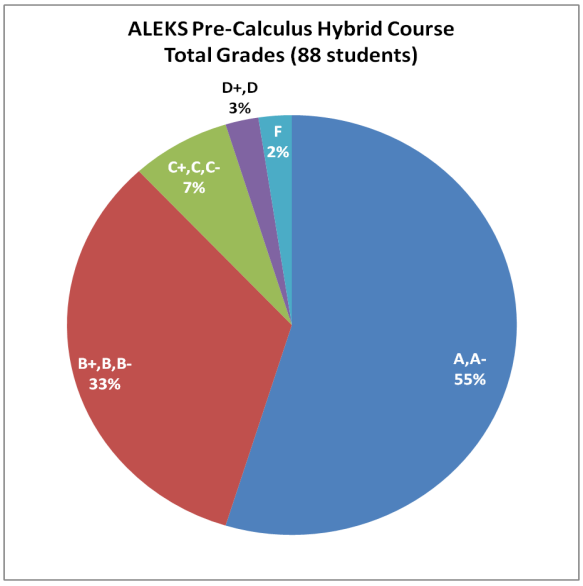

FIGURE 12. Results of hybrid ALEKS Pre-calculus class

11. Development of placement by ALEKS and modifications to Foundation program

The success of the first placement by ALEKS brought to the expansion of placement practices at SST to include all the students from CPS who scored below 60 on Math Core Exam into ALEKS testing. From 2014, the Pre-calculus course was offered only to about 80 students in each Fall semester, while no sections of the course were taught in Spring and Summer. While Calculus I was still offered in Summer 2014, there was no need for it in consecutive summer semesters, since all students who needed the course were able to complete it during regular semester in Fall and Spring.

For example, in the Fall 2014, 258 students were tested with ALEKS and 226 of them passed the test with score 75 or above, which accounted for an 88% passing rate. In the Fall of 2015 more students entering SST and the economics program of SHSS qualified for Calculus II with a CPS score of 60 or above, which resulted in less students testing with ALEKS (230).

The increasing success of students in Calculus I course brought to higher confidence and increased the enrollment in consecutive mathematics courses. Between 2013 and 2016 the enrollment in the majority of mathematics courses at NU increased on average by 50%.

Meanwhile, the new Foundation program was implemented starting from the 2015-16 academic year, where pre-calculus course was made mandatory for all students in their first semester. This brings to the end of the implementation of ALEKS as a placement tool into the SST Calculus I course. It may actually shift the placement strategies with ALEKS for one year earlier, i.e. when accepting students into courses in their Foundation program. We believe our experience trying ALEKS products for placement purposes could serve for the benefit of educators in various systems and programs, in Kazakhstan and beyond.

Acknowledgments. Many faculty and teaching assistants of the Department of Mathematics spent their time on proctoring the assessments by ALEKS for CPS and incoming freshmen at SST since 2013 year. Dr. G. Akishev was the first instructor to teach Pre-calculus with ALEKS in the Fall of 2013. We express gratitude to the staff of CPS for assistance in the first testing with ALEKS administered to 168 CPS students in April-May 2013. COO of SST Patrick Bauer has been a supportive force for the implementation and analysis of testing with ALEKS, for the future freshmen of SST and SHSS. Lynn Schweber, an Instructor of mathematics at SHSS in 2013, advised us on mathematics curriculum at her school and contributed to the effort of testing SHSS students. We thank Matthew Adarichev for the editing work on the paper.

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