EXTREME POINTS OF THE UNIT BALL OF AN OPERATOR SPACE

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ABSTRACT.

Answering Pedersen's question [1], I characterized extreme points of the unit ball of an operator space introducing the new notion "quasi-identity" [2]. An application of extreme points is also given in [3].

RESULTS.

Definition [2]: A quasi-identity of a ring RR is an element $e \in Re \in R$ such that $r = er + re - ere, \forall r \in R$ $r = er + re - ere, \forall r \in R$.

Theorem [2]: Let XX be a nonzero operator space and $z \in \mathrm{Ball}(QM^*(X)) z \in \mathrm{Ball}(QM^*(X))$, and let $(X, m_z)(X, m_z)$ be the corresponding operator algebra. Then the following assertions hold.

(1) $(X, m_z)(X, m_z)$ has a quasi-identity of norm *1* if $z^* \in \mathrm{ext}(\mathrm{Ball}(X)) z^* \in \mathrm{ext}(\mathrm{Ball}(X))$.

(2) $(X, m_z)(X, m_z)$ has a left identity of norm *1* if and only if $z^* \in \mathrm{ext}(\mathrm{Ball}(X)) z^* \in \mathrm{ext}(\mathrm{Ball}(X))$ and $z^* \cdot z = 1_{11} z^* \cdot z = 1_{11}$.

(3) $(X, m_z)(X, m_z)$ has a right identity of norm *1* if and only if $z^* \in \mathrm{ext}(\mathrm{Ball}(X)) z^* \in \mathrm{ext}(\mathrm{Ball}(X))$ and $z \cdot z^* = 1_{22} z \cdot z^* = 1_{22}$.

(4) $(X, m_z)(X, m_z)$ has a two-sided identity of norm *1* if and only if $z^* \in \mathrm{ext}(\mathrm{Ball}(X)) z^* \in \mathrm{ext}(\mathrm{Ball}(X))$ \$ and $z^* \cdot z = 1_{11} z^* \cdot z = 1_{11}$ and $z \cdot z^* = 1_{22} z \cdot z^* = 1_{22}$.

Corollary [2]: Let XX be a nonzero operator space. Then the following assertions hold.

(1) Some algebrization of XX has a quasi-identity of norm *1* iff $\mathrm{ext}(\mathrm{Ball}(X)) \cap QM^*(X)^* \neq \emptyset$ $\mathrm{ext}(\mathrm{Ball}(X)) \cap QM^*(X)^* \neq \emptyset$.

(2) Some algebrization of XX has a left identity of norm *1* if and only if $\mathrm{ext}(\mathrm{Ball}(X)) \cap \{z^t \in QM^*(X)^* \mid z^* \cdot z = 1_{11}\} \neq \emptyset \mathrm{ext}(\mathrm{Ball}(X)) \cap \{z^t \in QM^*(X)^* \mid z^* \cdot z = 1_{11}\} \neq \emptyset$.

(3) Some algebrization of XX has a right identity of norm *1* if and only if $\mathrm{ext}(\mathrm{Ball}(X)) \cap \{z^* \in QM^*(X)^* \mid z \cdot z^t = 1_{22}\} \neq \emptyset \mathrm{ext}(\mathrm{Ball}(X)) \cap \{z^* \in QM^*(X)^* \mid z \cdot z^t = 1_{22}\} \neq \emptyset$.

(4) Some algebrization of XX has a two-sided identity of norm *1* if and only if $\mathrm{ext}(\mathrm{Ball}(X)) \cap \{z^* \in QM^*(X)^* \mid z^t \cdot z = 1_{11}, z \cdot z^t = 1_{22}\} \neq \emptyset$ $\mathrm{ext}(\mathrm{Ball}(X)) \cap \{z^* \in QM^*(X)^* \mid z^t \cdot z = 1_{11}, z \cdot z^t = 1_{22}\} \neq \emptyset$.

Theorem ([3]): Let XX be a TRO which is also a dual Banach space. Then XX can be decomposed into the direct sum of TRO's $X_T X_T$, $X_L X_L$, *and* $X_R X_R$:

$$X = X_T \oplus_\infty X_L \oplus_\infty X_R$$

such that there is a complete isometry ι from XX into a von Neumann algebra in which $\iota(X_T)\iota(X_T)$, $\iota(X_L)\iota(X_L)$, and $\iota(X_R)\iota(X_R)$ are weak*-closed two-sided, left, and right ideals, respectively, and

$$\iota(X) = \iota(X_T) \oplus_\infty \iota(X_L) \oplus_\infty \iota(X_R).$$