INTERVAL YIELD CURVE ESTIMATION FOR KAZAKHSTAN BETWEEN 2019-2023

BY

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Abstract

This paper is a continuation of a thesis “Estimation of the term structure of interest rates for Kazakhstan government bonds using modified Nelson-Siegel methodology.” (Issayev&Post, 2019) This paper attempts to build forecast interval estimates of the linearized Nelson-Siegel model replacing TTM with the notion of tenure as it proved to increase the precision of the estimates in the low-liquidity environment of the Kazakhstani bonds market in the abovementioned paper. This work adds to the literature on the topic as there was no extensive research done on the yield curve estimation between 2019 and 2023.

1. Introduction

Construction and timely updating of the reliable yield curve of the economy is crucial for its efficiency. Term-structure of interest rates is essential for making a wide range of financial decisions involving pricing derivatives, valuation of business projects, portfolio management, and risk management. (Diebold & Li, 2005)

In Kazakhstan, there was no publicly available information on current yield curve for government bonds between 2015, when Khalyk Finance terminated its reports, and 2019, when Kazakhstan Stock Exchange (KASE) started to publish its estimates for spot Nelson-Siegel function of zero-coupon yields.

Kazakhstan's market for government securities, as typical for an emerging economy, is characterized by low liquidity and inefficiency. In 2019, the turnover of the government securities in the secondary market was 1-2 orders of magnitude lower compared with the countries from a similar income group. The secondary market turnover of the short-term notes by the National Bank of Kazakhstan was as low as 0.02% the same year. (Khakimzhanov et al, 2019) Khakimzhanov et al (2019) point out the prevalence of captive investors among holders of long-term government debt and the government's low borrowing needs as reasons for low levels of liquidity in the primary market. Captive demand from institutional investors, in turn, decreased liquidity in the secondary market by distorting prices.

Construction of reliable point estimates of the yield curves in such inefficient and illiquid environment of Tenge-denominated government securities is a difficult task. That is why this paper follows a methodology suggested by Issayev&Post (2019) and attempts to estimate forecast intervals for the parametric Nelson-Siegel model for Kazakhstani government bonds between 2019 and 2023.

2. Kazakhstan government bonds market

In Kazakhstan, the market of government securities is supplied by the Ministry of Finance, the National Bank, and representatives of municipal power. Ministry of Finance issues 7 types of government bonds with maturities starting from 3 months, although issuing securities maturing in less than a year is not a typical function of the Ministry of Finance. The National Bank of Kazakhstan affects the market for government securities by controlling short-term liquidity through notes of its emission. Those securities can be traded on both KASE and over-the-counter (OTC) markets, with deals on OTC markets also being registered and stored in the KASE database.

Table 1: Types of Ministry of Finance Treasury department securities (Issayev & Post, 2019)

Name	Circulation Period	Characteristics
Short-term treasury bonds (MEKKAM)	3, 6, 9, and 12 months	Nominal face value – 100tg. Discount notes.
Medium-term treasury bonds (MEOKAM)	1 to 5 years inclusive	Nominal face value – 1000tg. Coupons paid twice a year.
Long-term treasury bonds (MEУKAM)	Starting from 5 years	Nominal face value – 1000tg. Coupons paid once a year.

Medium-term indexed treasury bonds (МОИКАМ)	1 to 5 years inclusive	Nominal face value – 1000tg. Coupons paid twice a year. Floating rate indexed by CPI.
Long-term indexed treasury bonds (МУИКАМ)	Starting from 5 years	Nominal face value – 1000tg. Coupons paid once a year. Floating rate indexed by CPI.
Long-term treasury savings bonds (МЕУЖКАМ)	Starting from 5 years	Nominal face value – 1000tg. Coupons paid once a year. Floating rate indexed by CPI. Sold exclusively to pension funds.
Special medium-term treasury bonds (МАОКАМ)	1 to 5 years inclusive	Nominal face value – equivalent to 10 USD. Sold exclusively to individual investors. Fixed coupon rate.

For this research, similarly to Issayev&Post (2019), only plain "vanilla" bonds are selected, i.e. non-indexed bonds with fixed coupon rates issued and served in Kazakhstani Tenge. According to KASE, total of 21,157 deals were done involving such bonds and short-term notes by the National Bank. In total 443 instruments were traded.

Table 2: Top 10 most traded instruments

Instrument	Number of deals
MKM012_0153	446
MUM120_0016	443
MUM120_0017	342
MUM120_0019	296
MUM180_0011	292
MUM132_0005	267

MUM132_0007	263
MUM168_0003	261
MUM120_0018	256
MUM084_0018	254

Table 3: Top 10 instruments by total deal volume when recounted to USD

Instrument	Volume in USD
NTK028_2833	4,209,912,924.00
NTK028_2828	3,776,125,410.98
NTK028_2836	3,717,237,257.74
NTK028_2834	3,679,973,940.48
NTK028_2826	3,624,971,599.16
NTK026_2824	3,424,340,622.54
NTK027_2831	3,408,272,572.49
NTK028_2830	3,370,935,056.95
NTK028_2832	3,269,318,903.62
NTK028_2838	3,150,003,827.87

Table 4: Top 10 instruments by total deals on KASE volume when recounted to USD

Instrument	Volume in USD
MUM120_0017	1,041,278,319.76
MUM240_0004	958,140,141.73
MUM144_0001	938,132,010.34
MUM108_0014	936,935,748.92

MUM120_0019	894,019,418.19
MUM084_0018	863,754,946.28
MKM012_0148	838,461,003.80
MUM240_0002	813,151,886.00
MUM180_0012	803,827,457.42

Among the most traded instruments are long-term bonds by the Ministry of Finance except for one-year MEKKAM bonds. (see Table 2) However, the list of the 10 most heavily traded government securities in Kazakhstan between 2019 and 2023 is fully comprised of short-term National Bank notes traded in OTC markets in large volumes. (see Table 3)

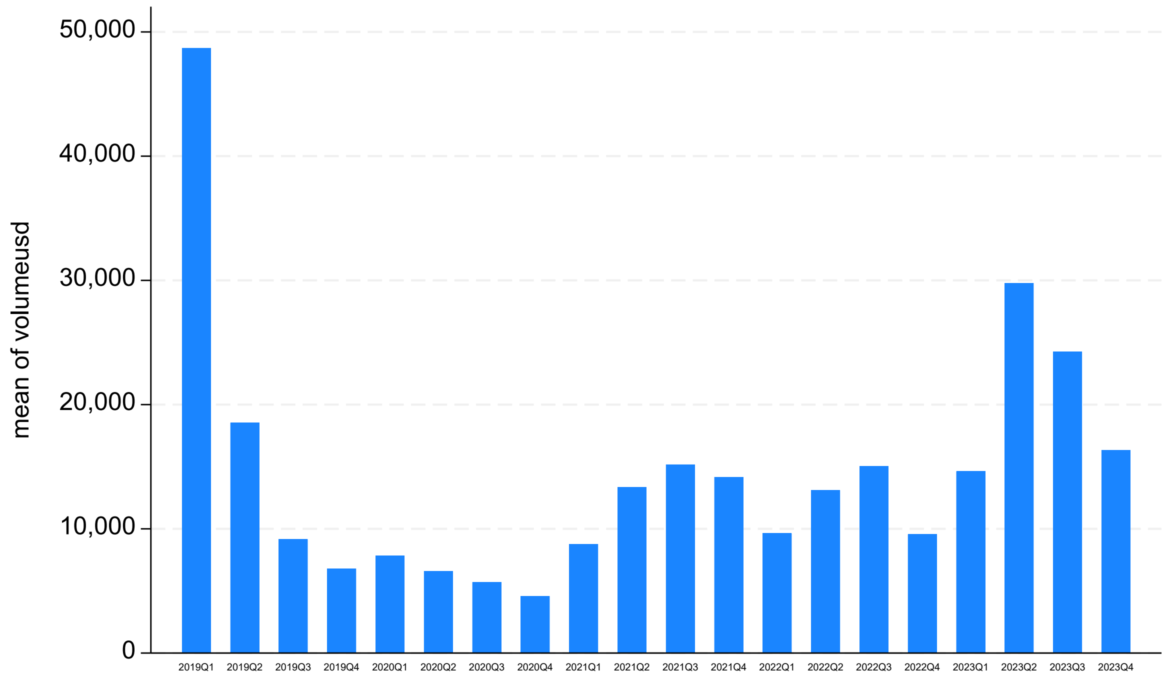

Figure 1:Total volume by quarters in millions of USD

Although total volume traded by quarter is still volatile in 2019-2024, as in 2012-2018, (Issayev&Post, 2019) there were no quarters with liquidity close to 0. Liquidity at the beginning of 2019 was extremely high which was due to an increased activity in OTC markets. (compare with Figure 2) Overall, liquidity tended to increase towards the end of considered time period, achieving its second-highest spike in second quarter of 2023.

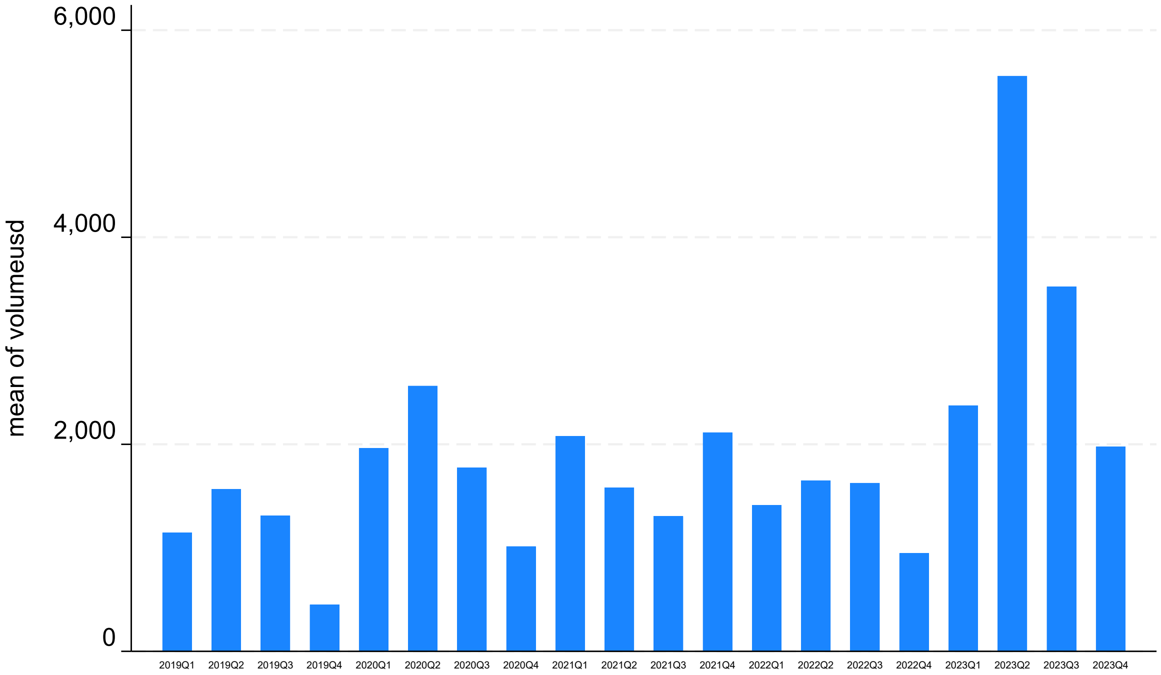

Figure 2: Total volume on KASE by quarter in millions of USD

3. Literature review

The whole body of academic literature on the topic of estimation of the zero-coupon yield curve can be divided into 2 distinct methodological groups.

The first branch of research bases the term structure modeling on either models assuming no arbitrage, such as Hull&White (1990) and Heath et al (1992), or equilibrium models, such as Vasicek (1977), Cox et al (1985), and Duffie&Kan (1996). Those models typically work by defining some state variables of random processes that would describe the general state of the economy, from which under the assumption of the absence of arbitrage opportunities or other various assumptions about the risk premium the term structure is derived.

The second body of research bases the term structure modeling on the construction of the zero-coupon yield curves from the existing coupon and zero-coupon yields. The base idea of this methodology is that each coupon payment can be considered as a distinct zero-coupon bond maturing in its payment date. This body of research can further be divided into models which are defined as piecewise polynomial functions (splines) and parsimonious models. Among the most influential spline-based models are McCuloch (1971), Schaefer (1981), Vasicek&Fong (1982), Fischer, Nychka & Zervos (1995), Waggoner (1997), and Anderson et al. (2001)

Parsimonious models work by estimating the parameters of either zero-coupon rate, discount factor function, or implied forward rate. In 1984, Chambers et al. (1984) introduced a discount factor function modeled using an exponential polynomial. But most models used nowadays by the central banks in estimating and forecasting the term structure are based on Nelson-Siegel's (1987) parsimonious model of the implied forward rate (Nymand-Andersen, 2018) from which spot rates can be derived analytically. A useful applied feature of the model is that all its four parameters possibly can be interpreted as level (long-term factor $\beta 0$) , slope (short-term factor $\beta 1$), and curvature (medium term-factor $\beta 2$) of a zero-coupon yield curve.

The model had been modified by different researchers, including Bliss (1996) who extended the model to contain 5 unknown parameters instead of 4 claiming it better performing at long-term yields estimation, Svensson (1994) who added two more parameters in the search for extra flexibility due to one more “hump.”

The model was also extended for dynamic forecasting of term structure by Diebold & Li (2006) using autoregressive models for 3 time-varying parameters of the model.

Most of the academic literature on the topic of term structure estimation considers developed markets with high liquidity, such as the US or European Union’s markets on government bonds, leaving a substantial gap in the literature around term structure estimation in environments characterized by low liquidity and inefficiency. Chou et al. (2017) compare the performance of different modifications of the Nelson-Siegel model on Taiwan bond market data, finding Svensson’s model (1994) the best fitting. Zoricic & Orsag (2013) apply the Nelson-Siegel-Svensson model to the Croatian market of government securities finding it possible to capture business cycle changes predicted macroeconomically, but still getting an overfitted model at shorter maturities.

In 2019, Khakimzhanov et al. applied Nelson-Siegel parsimonious parametrization on 10 of the most recent deals from 4 maturity ranges, getting a smoother curve than the model of KASE 2011 and being more reproducible due to fewer parameters to be estimated. In 2021, Alimbetova et al. conducted a complex overview of the Kazakhstani bond market, pointing out the difficulty of creating an adequate yield curve for Kazakhstan due to bond market illiquidity. The authors point out low demand for bonds issued by the Ministry of Finance with 3 being an average number of participants in auctions involving those assets, most of which being captive investors. The authors indicate non-competitive pricing, high fragmentation of bond emissions, and risk of Tenge depreciation as factors keeping market participants unwilling to invest in the market on government bonds.

In 2024, B. Shamar provided factor decomposition of the Kazakhstani yield curve. Following Imakubo and Nakajima (2015) he used expected nominal rates and expected inflation to estimate nominal and real rates and term premia, although unable to untangle real term premia and inflation risk premia from nominal term premia, again due to low liquidity.

4. Data description

The dataset used for the this study was obtained from KASE's informational terminal and is subject to a non-disclosure agreement. The dataset consists of all deals made on KASE between 1st January 2019 and 31st December 2023 inclusive, with security codes, exact dates of transactions, exact dates of maturity start, coupon payments and coupon frequencies (if applicable), yields to maturity, volumes, clean prices (in percentages of FV), and market of transactions as separate data entries. Additionally, coupon schedules and face values were later manually inserted into the dataset.

While data on transactions conducted on KASE was mostly internally consistent, data on deals from OTC markets was excluded from the dataset due to its poor quality. Yields to maturity were not calculated for those transactions and attempts to do so resulted in negative and implausible yields. Most of those transactions were chunk-added into the dataset in one day, usually at the end of the quarter, raising doubts regarding the exact dates of those transactions.

From the whole dataset of government bond deals, only “plain-vanilla” bonds, unprotected from inflation, issued and serviced in Tenge were selected. Thus, from 7 types of bonds issued by the Ministry of Finance, only 3 were saved in dataset (МЕККАМ, МЕОКАМ, МЕУКАМ), as well as discount notes of the National Bank. Municipal bonds, as well as bonds and notes maturing in less than a month or less than one million KZT in volume were excluded from the dataset.

5. Methodology

Estimation of yield curves using the Nelson-Siegel model is mostly done daily, however, in conditions of illiquid and inefficient emerging markets this may not be the best model of choice. In Kazakhstan such yield curve is estimated by KASE daily, however low liquidity diminishes the quality of information extracted from such a curve. (Alimbetova et al, 2021)

Because of that, following methodology by Issayev & Post (2019), monthly horizons were chosen as they allow keeping precision in a low liquidity environment.

5.1. Estimation of tenure

Classically, TTM is used in estimating yield curves using Nelson-Siegel parametrization. However, this approach does not quite capture the coupon effect on the yield to maturity. Practical literature widely uses duration instead of TTM; however, this study uses tenure as a viable alternative to both duration and TTM.

$$\text{Tenure} = \frac{\sum_{i=1}^{n} PV_i \cdot t_i^2}{\sum_{i=1}^{n} PV_i \cdot t_i}$$

Tenure therefore is a slightly modified duration, allowing to capture short and medium ends of the yield curve, slightly underestimating long-term yields. (Issayev & Post, 2019)

5.2. Weighting scheme

Following the methodology of Issayev & Post (2019) each observation is assigned two weights – a calendar weight and a volume weight. Calendar weights are as follows: days between the 11th and 20th in the month are given a weight of 1, days from the 1st to the 11th are assigned linearly increasing weight from 0.2 to 1, while weights for days from the 20th until the 31st are assigned linearly decreasing weight from 1 to 0.12.

Volume weights are calculated by dividing the logarithmic volume of each observation by the total sum of logarithmic volumes within a month. After both volume and calendar weights are calculated, their sum is used in a Nelson-Siegel regression analysis

5.3. Modified Nelson-Siegel with fixed τ

Following Issayev & Post (2019), Fabozzi et al. (2005), and Barrett et al. (1995), the conventional Nelson-Siegel model is linearized for the purpose of computational optimization by fixing variables to 3 and further modified by replacing TTM with tenure, resulting in a following equation:

$$\mathrm{YTM} = \beta_0 + \beta_1 \cdot \frac{1 - e^{-\mathrm{tenure}/3}}{\mathrm{tenure}/3} + \beta_2 \cdot \left(\frac{1 - e^{-\mathrm{tenure}/3}}{\mathrm{tenure}/3} - e^{-\mathrm{tenure}/3} \right)$$

First NS1 and NS2 components are computed for each month of data using tenure. Then a weighted linear regression is conducted with YTM as the dependent variable, from which 90% forecast intervals (FI) are computed for each month. After that following a methodology often implemented by central banks (BIS, 2005), yield curves for maturities from 1 to 10 are constructed with their respective FI.

6. Results

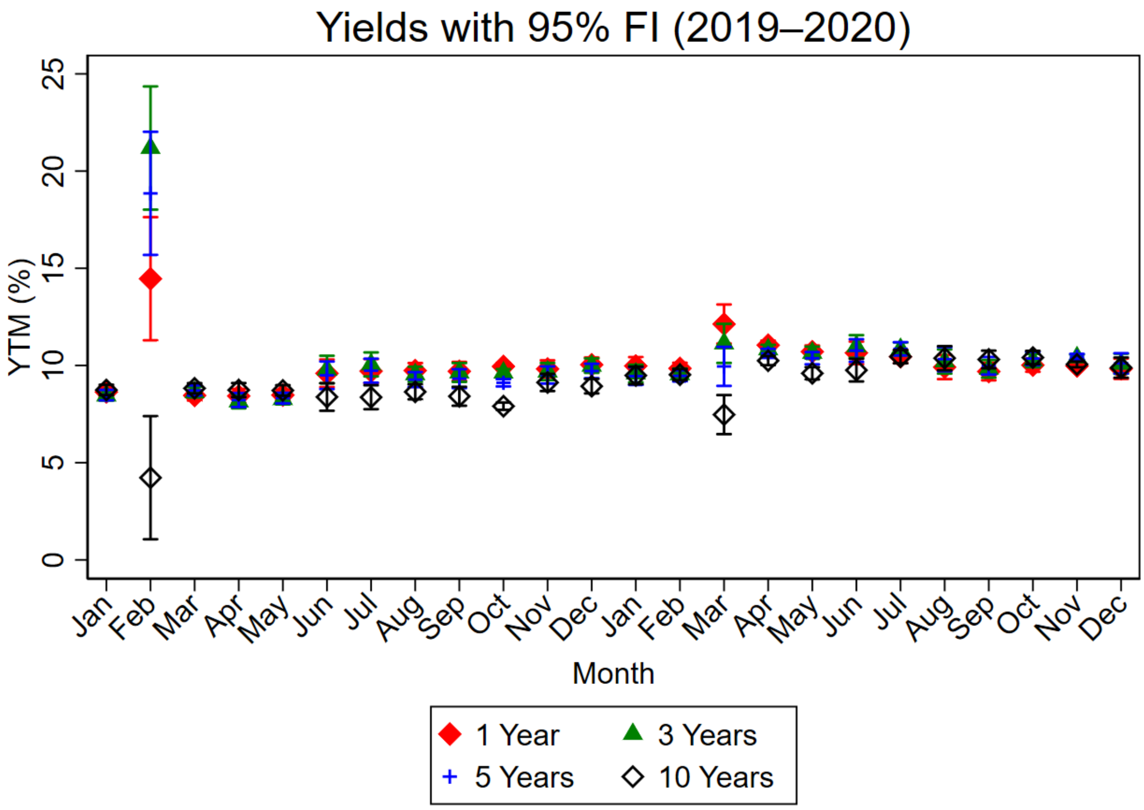

Figure 3: Yield curve 2019-2020

Similarly to the 2018 yield curve estimated by Issayev & Post (2019), the yield curve remains either flat or inverted in 2019-2020. This period is described by low liquidity in the coupon bonds market with most liquidity concentrated towards short-term notes by the National bank.

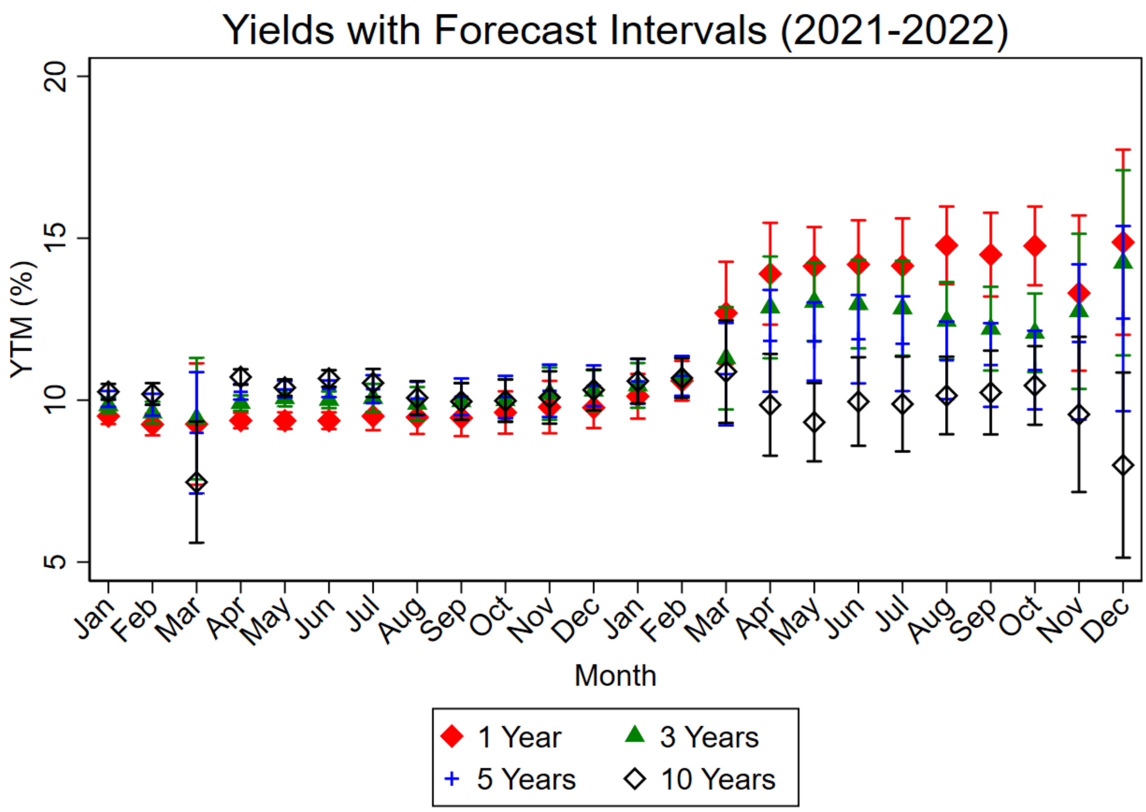

Figure 4: Yield curve 2021-2022

Except for March 2021, when the yield curve was overall flat, but with low yields on long-term bonds, the yield curve has been upward sloping until February 2022, although with a comparatively low spread between long-term and short-term rates. However, due to increased inflationary risk in the economy and increased pressure on the Tenge, yield curve became inverted and remained so until the end of 2022. Those results are consistent with the yield curve during the same period constructed by Shamar (2024) using a methodology of Khakimzhanov et al. (2019)

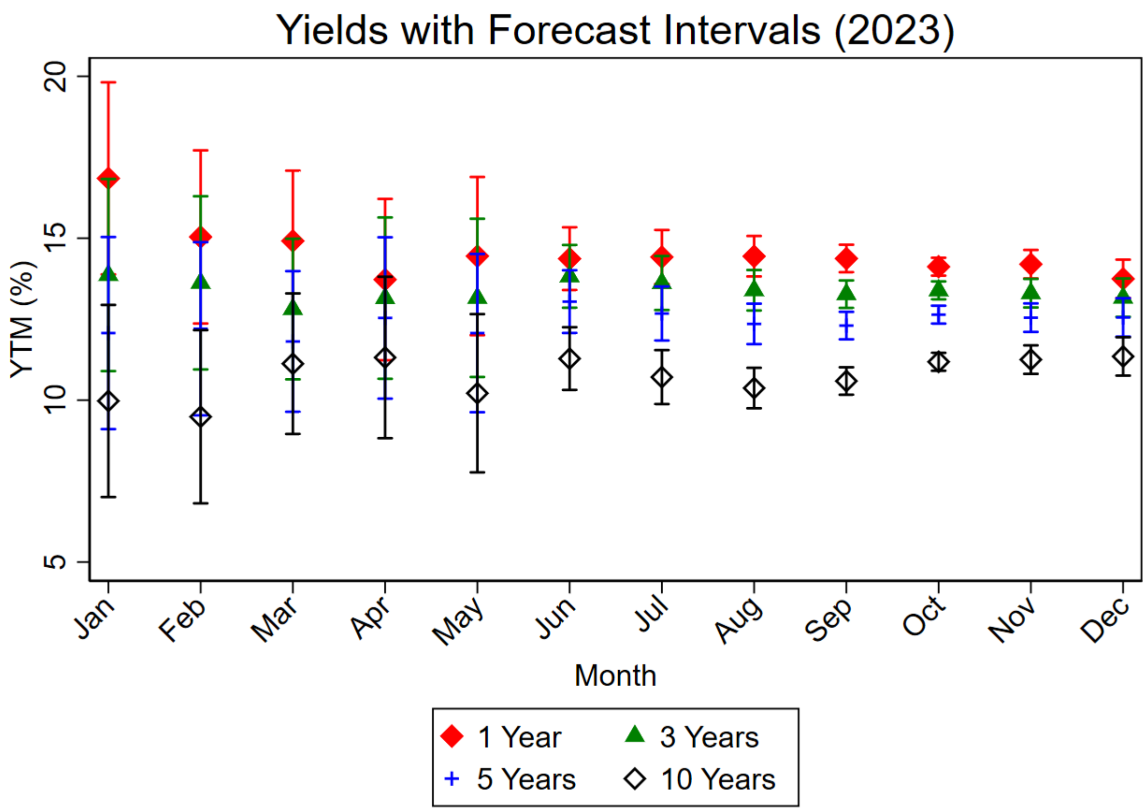

Figure 5: Yield curve in 2023

High uncertainty in the face of macroeconomic risks is reflected in large forecast intervals at the beginning of a year with clear inverted nature of a yield curve, although with overlapping intervals. Towards the end of a year, confidence intervals start to narrow, which may be a sign of lower uncertainty on the market.

One feature of forecast intervals is that their wideness reflects increased variance in yields, especially in markets with low liquidity. Figures 6-8 (Appendix 1) represent the dynamics of yield volatilities during the considered periods. Variance in yields seems to be the only parameter directly affecting forecast intervals. Although the low range of maturities traded may be interpreted by some as the reason for flat yield curves between 2019-2020, as liquidity was mostly provided by short-term notes of the National Bank.

7. Conclusion

This study continues the work of Issayev & Post (2019) and uses a modified Nelson-Siegel methodology with fixed decay parameter and tenure as a time parameter to estimate the zero-coupon yield curve for Kazakhstan between 2019-2023.

Larger volumes compared to Issayev & Post (2019) allowed us to estimate the yield curve for the whole period, which can roughly be divided into 3 subperiods. 2019-2020 can be characterized with flat yield curve, which start to look more upward sloping in 2021, but then in February 2022 under the inflationary pressure and currency risk inverts and stays as such until the end of 2023.

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Appendix 1.

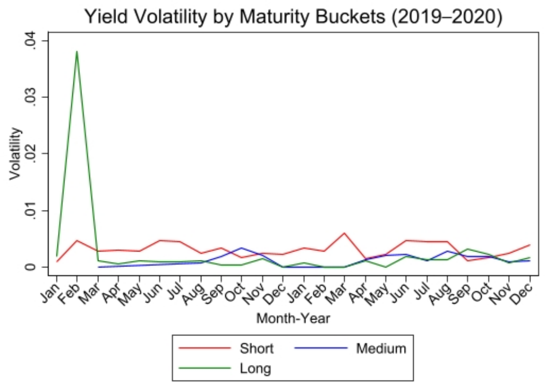

Figure 6: Yield Volatility 2019-2020

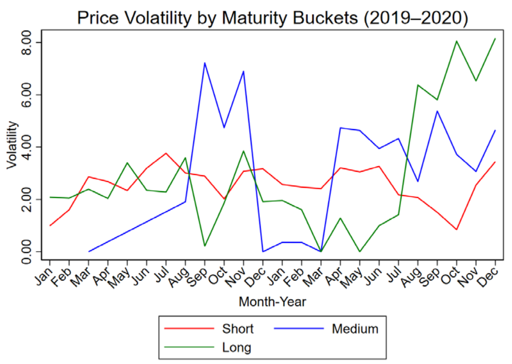

Figure 8: Price Volatility 2019-2020

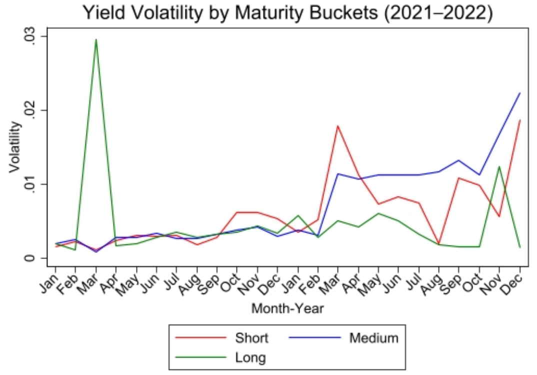

Figure 7: Yield Volatility 2021-2022

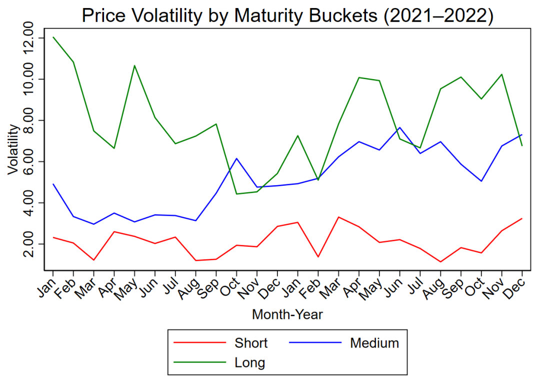

Figure 9: Price Volatility 2021-2022

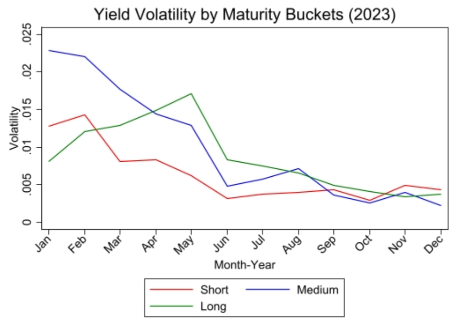

Figure 10: Yield Volatility 2023

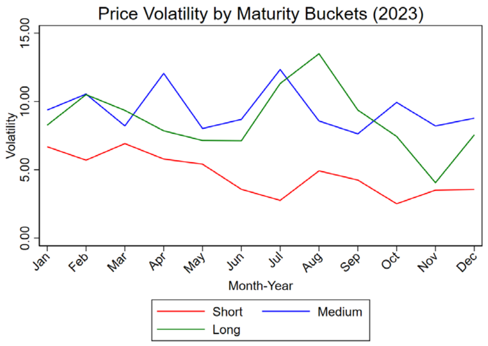

Figure 11: Price Volatility 2023