θ-D Approximation Technique for Nonlinear Optimal Speed Control Design of Surface-Mounted PMSM Drives

Ton Duc Do, *Student Member, IEEE*, Han Ho Choi, *Member, IEEE*, and Jin-Woo Jung, *Member, IEEE*

***Abstract*—This paper proposes nonlinear optimal controller and observer schemes based on a θ-D approximation approach for surface-mounted permanent magnet synchronous motors (PMSMs). By applying the θ-D method in both the controller and observer designs, the unsolvable Hamilton–Jacobi–Bellman equations are switched to an algebraic Riccati equation and state-dependent Lyapunov equations (SDLEs). Then, through selecting the suitable coefficient matrices, the SDLEs become algebraic, so the complex matrix operation technique, i.e., the Kronecker product applied in the previous papers to solve the SDLEs is eliminated. Moreover, the proposed technique not only solves the problem of controlling the large initial states, but also avoids the excessive online computations. By utilizing a more accurate approximation method, the proposed control system achieves superior control performance (e.g., faster transient response, more robustness under the parameter uncertainties and load torque variations) compared to the state-dependent Riccati equation-based control method and conventional PI control method. The proposed observer-based control methodology is tested with an experimental setup of a PMSM servo drive using a Texas Instruments TMS320F28335 DSP. Finally, the experimental results are shown for proving the effectiveness of the proposed control approach.**

***Index Terms*—Hamilton–Jacobi–Bellman equation (HJBE), nonlinear optimal control, nonlinear optimal observer, permanent magnet synchronous motor (PMSM), speed control.**

I. Introduction

LATELY, electric motors have been widely employed in diverse applications such as traction motors, machine tools, industrial fans, blowers and pumps, household appliances, disk drives, etc. [1]–[3]. As energy problems become more and more serious, the improvement in the efficiency of electric motors is the main focus. Nowadays, induction motors are one of the most popular motors in industrial applications; however, permanent magnet synchronous motors (PMSMs) have been gradually replacing induction motors due to many advantages such as higher power density, higher torque to inertia ratio, higher efficiency, and simpler control [4]–[9].

One of the most well-known control schemes for the PMSMs is the cascaded proportional-integral (PI) controller. However, the main problem of using the PI controllers is their sensitivity to inevitable uncertainties such as parametric variations, modeling errors, and external load torque disturbances [10]. To overcome these difficulties, advanced control methods are needed for precise control of the PMSMs. So far, the control performance of the PMSMs has been enhanced through various control techniques like sliding mode control (SMC), internal model control (IMC), fuzzy control, predictive control, etc. The most prominent characteristic of the SMC schemes in [11]–[13] is their robustness to the parameter variations and the external disturbances. But, in practice, it is very difficult to achieve the bounds of the uncertainties, which are essential in the designing of these SMC schemes. Next, the IMC method in [14] also possesses the disturbance rejection ability, however, the accuracy of the reference model is not guaranteed when the motor parameters vary. In [15], a discrete-time fuzzy control system is theoretically analyzed. Unfortunately, the transient speed responses under the load torque variations are not shown in either simulation or experimental studies. The predictive control methods in [16]–[18] are successfully applied to the PMSM drives. However, this approach requires a huge computation effort to solve the optimization problem at each sampling instant.

Three centuries ago, optimal control (OC) was founded by J. Bernoulli. However, the studies on the OC just started blooming in the 1960s with the invention of the computer [19]. Although the theory of the OC has been developed perfectly for the linear time-invariant systems, the implementation of the OC in the nonlinear systems is still a challenging problem. Many research works have been carried out already to find the solution of the Hamilton–Jacobi–Bellman equation (HJBE) or at least to achieve a suboptimal control law for the nonlinear dynamical models. The methods that find an approximated solution to the OC problem of the nonlinear systems can be relatively divided into four groups: power series methods, state-dependent Riccati equation (SDRE) methods, Pontryagin's maximum-principle-based methods, and successive approximation methods. In the first group, the power series methods [20], [21] are based on considering the nonlinear systems as a perturbation of the linear systems, so the controller can be obtained by extending the linear control theory to the nonlinear one. However, the method in [20] cannot assure the convergence of the series with the highly nonlinear systems, and the algorithms in [21] become very complicated when the system order is high. Next, the second group explains that the SDRE methods [22]–[24] are the extension of the algebraic Riccati equation (ARE) to the nonlinear systems. In [22], the controller leads to a large control effort when the

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The authors are with the Division of Electronics and Electrical Engineering, Dongguk University, Seoul 100-715, Korea (e-mail: jinwjung@dongguk.edu).

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values of the initial states are large, and the SDRE method in [23] requires a real-time computation. The technique in [24] can avoid the aforementioned drawbacks, but it is only applicable to a few of the nonlinear systems. In the third group, the main focus of the Pontryagin's maximum-principle-based methods [25], [26] is to find an approximated solution to a nonlinear two-point boundary value problem. However, the complex techniques are required to solve the boundary value problem. In the last group, the iterative processes are utilized by the successive approximation techniques [27]–[29] to approach the solution of the HJBE. In [27], the problem is discussed in a general context. A numerical solution to the generalized HJBE is obtained by using the Galerkin method in [28]. Meanwhile, the approximation techniques in [29] focus on finding a solution for a specified class of the nonlinear systems.

The OC has been widely utilized in numerous fields such as aircraft [21], [30], tandem hot-metal-strip rolling [23], robotics [31], induction motor [32], electricity market [33], electric vehicle [34], etc. A power series method is applied in [21], while an SDRE-based method is utilized in [23]. In [33] and [34], the Pontryagin's maximum principle is used to derive the optimal control law. A successive approximation method is employed to design the controller in [30]. In [31] and [32], just the linear optimal controller [i.e., the linear quadratic regulator (LQR)] is designed. As mentioned earlier, although the OC is a well-developed approach with huge applications, there is not much work focused on applying it to the PMSMs. In [35], an LQR is presented for the PMSM drives, but the results show that it is very sensitive to the load torque disturbance. Recently, an SDRE-based optimal control approach has been successfully applied to the speed tracking of the PMSMs [36], [37]. That is, it is shown that the SDRE-based method can improve the control performance compared to the classical methods like a PI controller and an LQR.

This paper introduces nonlinear optimal controller and observer schemes based on a θ-D approximation method for the surface-mounted permanent magnet synchronous motor (SPMSM) drives. First, the θ-D-based approximation technique is applied for transforming the HJBE to much more simpler equations, which include an ARE and state-dependent Lyapunov equations (SDLEs) [29]. In these papers, the SDLEs were solved by using a complicated technique named as the Kronecker product, which generates redundant computations. Fortunately, in this paper, by properly choosing the coefficient matrices, these SDLEs become the algebraic equations, so the Kronecker product technique is avoided. Next, the semiglobal asymptotic stability and suboptimality properties of the proposed control scheme are fully analyzed. The proposed strategy not only avoids the complex control solution to which the large-initial-states problem gives rise in some Taylor-series-expansion-based methods, but also does not require any excessive online computations like recent SDRE techniques. Furthermore, the proposed observer-based control scheme uses a more precise approximation method, therefore, it has an ability to attain faster transient response and more robustness under the parameters uncertainties and load torque variations than the SDRE-based control method and the conventional PI control method. To prove the validity of the proposed control approach, the experiment is performed on a prototype SPMSM servo drive with a Texas Instruments TMS320F28335 DSP.

II. Nonlinear Optimal Speed Controller Synthesis

A. Problem Formulation

In the field-oriented control, the dynamic equations of a SPMSM can be given by

$$\begin{cases} \dot{\omega} = k_1 i_{qs} - k_2\omega - k_3 T_L \\ \dot{i}_{qs} = -k_4 i_{qs} - k_5\omega + k_6 V_{qs} - \omega i_{ds} \\ \dot{i}_{ds} = -k_4 i_{ds} + k_6 V_{ds} + \omega i_{qs} \end{cases} \tag{1}$$

where

$$k_1 = \frac{3}{2}\frac{1}{J}\frac{p^2}{4}\lambda_m,\ \ k_2 = \frac{B}{J},\ \ k_3 = \frac{p}{2J}$$
$$k_4 = \frac{R_s}{L_s},\ \ k_5 = \frac{\lambda_m}{L_s},\ \ k_6 = \frac{1}{L_s}$$

ω is the electrical rotor speed, i_{ds} and i_{qs} are the *d*-axis and *q*-axis currents, respectively, V_{ds} and V_{qs} are the *d*-axis and *q*-axis voltages, respectively, T_L is the load torque, *p* is the number of poles, R_s is the stator resistance, L_s is the stator inductance, *J* is the rotor inertia, *B* is the viscous friction coefficient, and λ_m is the magnet flux linkage.

In the system model (1), it is noted that ω, i_{qs}, and i_{ds} are the state variables, V_{qs} and V_{ds} are the control inputs, and T_L is defined as the external disturbance. To design an observer-based nonlinear optimal control law, the following assumptions are established.

Assumption 1: 1) ω, i_{qs}, and i_{ds} are measurable and 2) T_L is unknown and varies very slowly in practice [38]–[40].

To develop the θ-D-based nonlinear optimal controller for the SPMSM, the nonlinear model of the SPMSM in (1) first needs to be transformed to the appropriate error dynamics. Let us define the speed error, *q*-axis current error, and *q*-axis current reference as

$$\tilde{\omega} = \omega - \omega_d, \tilde{i}_{qs} = i_{qs} - i_{qsd}$$
$$i_{qsd} = (k_2\omega_d + \dot{\omega}_d + k_3 T_L)/k_1 \tag{2}$$

where ω_d is the rotor speed reference, $\tilde{\omega}$ is the rotor speed error, i_{qsd} is the *q*-axis current reference, and $\tilde{i}_{qs}$is the *q*-axis current error. Note that the load torque T_L which is used to calculate i_{qsd} can be accurately estimated by a nonlinear optimal observer that will be presented in detail in the next section.

Remark 1: In the constant torque region of the SPMSM, the preferred *d*-axis current is usually considered as zero; this not only simplifies the control system but also optimizes the operation conditions. Accordingly, in this paper, the *d*-axis current reference is also chosen to be zero.

Next, the control input signals V_{qs} and V_{ds} are decomposed into the compensating and stabilizing terms as

$$V_{qs} = u_{cq} + u_{sq},\ \ V_{ds} = u_{cd} + u_{sd} \tag{3}$$

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where u_{cq} and u_{cd} are the *q*-axis and *d*-axis compensating control terms, respectively, and u_{sq} and u_{sd} are the *q*-axis and *d*-axis stabilizing control terms, respectively.

Then, the compensating control terms u_{cq}, u_{cd} are designed as

$$\begin{aligned} u_{cq} &= \frac{1}{k_6}(k_4 i_{qsd} + k_5\omega_d + i_{ds}\omega_d + \dot{i}_{qsd}) \\ u_{cd} &= -\frac{1}{k_6}(\tilde{i}_{qs}\omega_d + \omega i_{qsd}). \end{aligned} \tag{4}$$

Using (2) into (4), the model (1) could be transformed to the following error dynamics:

$$\dot{x} = f(x) + Bu \tag{5}$$

where $x = \begin{bmatrix} \tilde{\omega} \ \tilde{i}_{qs} \ i_{ds} \end{bmatrix}^T$, $u = \begin{bmatrix} u_{sq} \ u_{sd} \end{bmatrix}^T$, $f(x) = A(\tilde{\omega})x$

$$A(\tilde{\omega}) = \begin{bmatrix} -k_2 & k_1 & 0 \\ -k_5 & -k_4 & -\tilde{\omega} \\ 0 & \tilde{\omega} & -k_4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ k_6 & 0 \\ 0 & k_6 \end{bmatrix}.$$

By considering the system (5), the objective is to find the stabilizing control terms that minimize the cost function.

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \tag{6}$$

where the matrix $Q \in R^{3\times 3}$ is symmetric positive semidefinite, and the matrix $R \in R^{2\times 2}$ is symmetric constant positive definite. According to [41], the optimal solution of this infinite-horizon nonlinear control problem can be obtained by solving the following HJB partial differential equation:

$$\frac{\partial V^T}{\partial x} f(x) - \frac{1}{2}\frac{\partial V^T}{\partial x} BR^{-1}B^T \frac{\partial V}{\partial x} + \frac{1}{2}x^T Q x = 0. \tag{7}$$

Assuming that $V(x)$ is positive and continuously differentiable with $V(0) = 0$, then the control law is given by

$$u = -R^{-1}B^T \frac{\partial V}{\partial x}. \tag{8}$$

However, the HJB equation (7) is very difficult to solve in general.

B. Nonlinear Optimal Speed Controller Design Based on θ-D Technique

An approximation technique called a θ-*D* method [29] is introduced in this section to find the solution to the HJB (7).

First, the weighting matrix Q is rewritten as

$$Q = Q_0 + \sum_{i=1}^{\infty} D_i \theta^i \tag{9}$$

where $Q_0 \in R^{3\times 3}$ and $D_i \in R^{3\times 3}$ are a symmetric constant and a symmetric state-dependent matrix, respectively, θ is a scalar, and i is an integer. Note that θ and D_i are chosen such that Q is symmetric semipositive definite.

Now, $\partial V/\partial x$ is decomposed into a power series of θ as

$$\frac{\partial V}{\partial x} = \sum_{i=0}^{\infty} T_i \theta^i x \tag{10}$$

where T_i are symmetric matrices.

Substituting (9) and (10) into (7), and then, equating the coefficients of the powers of θ to zero, the following equations are achieved

$$T_0 A_0 + A_0^T T_0 - T_0 BR^{-1}B^T T_0 + Q_0 = 0 \tag{11}$$

$$T_1 A_1 + A_1^T T_1 + \frac{T_0 \Delta A}{\theta} + \frac{\Delta A^T T_0}{\theta} + D_1 = 0 \tag{12}$$

$$\vdots$$

$$T_n A_1 + A_1^T T_n + \frac{T_{n-1}\Delta A}{\theta} + \frac{\Delta A^T T_{n-1}}{\theta} - \sum_{i=1}^{n-1} T_i BR^{-1}B^T T_{n-i} + D_n = 0 \tag{13}$$

where $A_1 = A_0 - BR^{-1}B^T T_0$ and

$$A_0 = \begin{bmatrix} -k_2 & k_1 & 0 \\ -k_5 & -k_4 & 0 \\ 0 & 0 & -k_4 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\tilde{\omega} \\ 0 & \tilde{\omega} & 0 \end{bmatrix}.$$

It is clear that (11) is an ARE, while (12) and (13) are SDLEs. By setting the matrices

$$D_i = -k_i e^{-l_i t}\left(\frac{T_{i-1}\Delta A}{\theta} + \frac{\Delta A^T T_{i-1}}{\theta} - \sum_{j=1}^{i-1} T_j BR^{-1}B^T T_{i-j}\right) T_i = \frac{\bar{T}_i}{\theta^i} \tag{14}$$

where k_i and l_i are positive adjustable design parameters, and $\bar{T}_i$ are symmetric matrices. It should be noted that $\bar{T}_0 = T_0$.

Then, the SDLEs (12) and (13) are transformed to

$$\bar{T}_1 A_1 + A_1^T \bar{T}_1 + \left(T_0 \Delta A + \Delta A^T T_0\right)\varepsilon_1 = 0 \tag{15}$$

$$\vdots$$

$$\bar{T}_n A_1 + A_1^T \bar{T}_n + \left(\bar{T}_{n-1}\Delta A + \Delta A^T \bar{T}_{n-1} - \sum_{i=1}^{n-1} \bar{T}_i BR^{-1}B^T \bar{T}_{n-i}\right)\varepsilon_n = 0 \tag{16}$$

where $\varepsilon_i = 1 - k_i e^{-l_i t}$.

Finally, by establishing $\bar{T}_i = T_i^C \varepsilon_i \tilde{\omega}^i$ and $\Delta A = \tilde{\omega}\Delta A_C$, the following algebraic Lyapunov equations are obtained

$$T_1^C A_1 + A_1^T T_1^C + T_0 \Delta A_C + \Delta A_C^T T_0 = 0 \tag{17}$$

$$\vdots$$

$$T_i^C A_1 + A_1^T T_i^C + T_{i-1}^C \Delta A_C + \Delta A_C^T T_{i-1}^C - \sum_{k=1}^{i-1} T_k^C BR^{-1}B^T T_{i-k}^C = 0 \tag{18}$$

where $\Delta A_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$.

Then, the approximate control law is given by

$$u = -R^{-1}B^T \sum_{i=0}^{N} T_i^C \varepsilon_i \tilde{\omega}^i x \tag{19}$$

where $T_0^C = \bar{T}_0 = T_0$, $\varepsilon_0 = 1$, and N is the number of series calculated offline.

Remark 2: In [29], the SDLEs (12) and (13) are solved by using the Kronecker product. However, in case of the PMSMs, these equations can be further transformed to algebraic equations (17) and (18), which can be much more easily solved. This improvement does not include any drawback or limitation.

Remark 3: The proposed θ-D-based suboptimal control law (19) can more precisely approximate the solution than the SDRE-based control law in [36] with more tuning parameters (ε_i). In this context, the proposed control law is a generalized form of the SDRE-based control law. As a result, the proposed θ-D-based control scheme can achieve better transient behavior compared to the SDRE-based control method in [36].

Remark 4: First, the well-known fact should be noted that an optimal regulator minimizing the cost function *J* (6) for a linear system can usually guarantee a suitable robustness with minimum −6 dB gain margin and 60° phase margin. Also, since the proposed θ-D-based nonlinear optimal control law is more effective in dealing with nonlinearities of the system and more flexible in tuning the gains, it is more robust to the parameter uncertainties than the conventional control methods such as the PI and SDRE-based controller. For extending the proposed optimal controller to the robust optimal controller, the cost function *J* is needed to be modified. This issue is presented in detail via theorems in [30] and [31].

C. Stability Analysis

Referring to [29], both *Theorem* and *Lemma* can be given as follows:

Lemma 1: The series $\sum_{i=0}^{\infty} T_i \theta^i$ is the pointwise convergent and positive definite series.

Proof: This *Lemma* can be proved similarly to [29]. ■

Theorem 1: The closed-loop system obtained by the error dynamics (5) and the nonlinear feedback control law (19) is semiglobally asymptotically stable.

Proof: Define the following Lyapunov function

$$L(x) = \frac{1}{2} x^T \sum_{i=0}^{\infty} \bar{T}_i x. \tag{20}$$

From the *Lemma 1*, $\sum_{i=0}^{\infty} \bar{T}_i$ is positive definite, so $L(x) > 0$. Its time derivative can be obtained as

$$\begin{aligned} \frac{dL(x)}{dt} &= \left[\frac{\partial L(x)}{\partial t}\right]^T \dot{x} = \left[\frac{\partial L(x)}{\partial t}\right]^T [f(x) + Bu] \\ &= \left[x^T \sum_{i=0}^{\infty} \bar{T}_i + \frac{1}{2} x^T \sum_{i=0}^{\infty} \frac{\partial \bar{T}_i}{\partial x} x\right] [f(x) + Bu]. \end{aligned} \tag{21}$$

On the other hand, $V_x = \sum_{i=0}^{\infty} \bar{T}_i x (V_x = \partial V / \partial x)$ satisfies the following HJB equation:

$$V_x^T [f + gu] + \frac{1}{2} u^T R u + \frac{1}{2} x^T \left(Q_0 + \sum_{i=1}^{\infty} D_i \theta^i\right) x = 0. \tag{22}$$

Then, the aforementioned equation is equivalent to

$$V_x^T [f + gu] = -\frac{1}{2} u^T R u - \frac{1}{2} x^T \left(Q_0 + \sum_{i=1}^{\infty} D_i \theta^i\right) x. \tag{23}$$

Therefore

$$\begin{aligned} \frac{dL(x)}{dt} = &\ \frac{1}{2} u^T R u - \frac{1}{2} x^T \left(Q_0 + \sum_{i=1}^{\infty} D_i \theta^i\right) x + \\ &+ \frac{1}{2} x^T \sum_{i=0}^{\infty} \frac{\partial \bar{T}_i}{\partial x} x [f + Bu]. \end{aligned} \tag{24}$$

Since $u = -R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i x$, so

$$\begin{aligned} &-\frac{1}{2} u^T R u - \frac{1}{2} x^T \left(Q_0 + \sum_{i=1}^{\infty} D_i \theta^i\right) x \\ &= -\frac{1}{2} x^T \left[Q_0 + \sum_{i=0}^{\infty} \bar{T}_i B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i + \sum_{i=1}^{\infty} D_i \theta^i\right] x. \end{aligned} \tag{25}$$

Using the Courant–Fischer theorem [42], we achieve

$$\begin{aligned} &-\frac{1}{2} x^T \left[Q_0 + \sum_{i=0}^{\infty} \bar{T}_i B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i + \sum_{i=1}^{\infty} D_i \theta^i\right] x \\ &\leq -\frac{1}{2} \lambda_{\min} \left[Q_0 + \sum_{i=0}^{\infty} \bar{T}_i B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i + \sum_{i=1}^{\infty} D_i \theta^i\right] \|x\|^2 \end{aligned} \tag{26}$$

where $\lambda_{\min}$ is the minimum eigenvalue of the matrix in the square bracket in (26). Thus

$$\begin{aligned} &\frac{dL(x)}{dt} \leq +\frac{1}{2} x^T \sum_{i=0}^{\infty} \frac{\partial \bar{T}_i}{\partial x} x [f + Bu] - \\ &-\frac{1}{2} \lambda_{\min} \left[Q_0 + \sum_{i=0}^{\infty} \bar{T}_i B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i + \sum_{i=1}^{\infty} D_i \theta^i\right] \|x\|^2 \\ &\leq -\frac{1}{2} C_\lambda \|x\|^2 + \frac{1}{2} \|x\|^2 \left\|\sum_{i=0}^{\infty} \frac{\partial \bar{T}_i}{\partial x} x\right\| \left\|A - B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i\right\| \end{aligned} \tag{27}$$

where

$$C_\lambda = \lambda_{\min} \left[Q_0 + \sum_{i=0}^{\infty} \bar{T}_i B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i + \sum_{i=1}^{\infty} D_i \theta^i\right] > 0.$$

For achieving specific values of $\bar{T}_i$ to satisfy the following inequality, a small enough ε_i is chosen.

$$C_\lambda > \left\|\sum_{i=0}^{\infty} \frac{\partial \bar{T}_i}{\partial x} x\right\| \left\|A - B R^{-1} B^T \sum_{i=0}^{\infty} \bar{T}_i\right\|. \tag{28}$$

Finally, the inequality $dL(x)/dt < 0$ is also satisfied. Hence, the closed-loop system is semiglobally asymptotically stable.

III. Nonlinear Optimal Load Torque Observer Synthesis

A. Nonlinear Optimal Load Torque Observer Design Based on θ-D Technique

Based on the *Assumption 1* and the SPMSM model (1), the equations for designing a nonlinear optimal load torque observer can be achieved as follows:

$$\begin{aligned} \dot{x}_o &= f_o(x_o) + B_o u_o \\ y_o &= C_o x_o \end{aligned} \tag{29}$$

where $f_o(x_o) = A_o(\omega)x_o$, x_o is the state variables vector, u_o is the inputs vector, and y_o is the measurable outputs vector.

$$x_o = \begin{bmatrix} T_L \\ \omega \\ i_{qs} \\ i_{ds} \end{bmatrix}, \quad A_o(\omega) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -k_3 & -k_2 & k_1 & 0 \\ 0 & -k_5 & -k_4 & -\omega \\ 0 & 0 & \omega & -k_4 \end{bmatrix}$$

$$B_o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_6 & 0 \\ 0 & k_6 \end{bmatrix}, \quad u_o = \begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix}, \quad C_o = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, the load torque observer can be designed by

$$\begin{aligned} \dot{\hat{x}}_o &= f_o(\hat{x}_o) + L(\hat{\omega})\left[y_o - C_o \hat{x}_o\right] + B_o u_o \\ \hat{T}_L &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \hat{x}_o \end{aligned} \tag{30}$$

where superscript "^" denotes the estimated quantities and the matrix $L(\hat{\omega})$ is the observer gain.

In the same manner with Section II, the observer gain matrix can be constructed as

$$L(\hat{\omega}) = \left(\sum_{i=0}^{N_o} H_i^C \varepsilon_{oi} \hat{\omega}^i\right) C_o^T R_o^{-1} \tag{31}$$

where $\varepsilon_0 = 1$, $\varepsilon_{oi} = 1 - k_{oi} e^{-l_{oi} t}$ ($i \geq 1$, k_{oi}, and l_{oi} are positive adjustable design parameters), N_o is the number of series calculated offline, and the coefficient matrices H_i^C are the solution of the following algebraic equations:

$$A_{o0} H_0^C + H_0^C A_{o0}^T - H_0^C C_o^T R_o^{-1} C_o H_0^C + Q_{o0} = 0 \tag{32}$$

$$A_{o1} H_1^C + H_1^C A_{o1}^T + H_0^C \Delta A_{oC}^T + \Delta A_{oC} H_0^C = 0 \tag{33}$$

$$\vdots$$

$$A_{o1} H_i^C + H_i^C A_{o1}^T + H_{i-1}^C \Delta A_{oC}^T + \Delta A_{oC} H_{i-1}^C - \sum_{k=1}^{i-1} H_k^C C_o^T R_o^{-1} C_o H_{i-k}^C = 0 \tag{34}$$

where

$$A_{o0} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -k_3 & -k_2 & k_1 & 0 \\ 0 & -k_5 & -k_4 & 0 \\ 0 & 0 & 0 & -k_4 \end{bmatrix}$$

$$\Delta A_{oC} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_{o1} = A_{o0} - H_0^C C_o^T R_o^{-1} C_o$$

and Q_{o0} and R_o are the weighting matrices with the sufficient dimensions.

B. Stability Analysis

Referring to [43], both *Theorem* and *Lemma* can be given as follows:

Lemma 2: The series $\sum_{i=0}^{\infty} H_i^C \varepsilon_{oi} \hat{\omega}^i$ is the pointwise convergent and positive definite series.

Proof: This *Lemma* can be proved similarly to [43]. ∎

Theorem 2: The estimation error dynamics defined by $e = \hat{x}_o - x_o$ is asymptotically stable.

Proof: Based on (29) and (30), the derivative of the estimation error is given by

$$\dot{e} = \left[A_{o0} - L(\hat{x}_o)C\right] e + \left[f_o(\hat{x}_o) - f_o(x_o)\right]. \tag{35}$$

By applying (31) into (35), we have

$$\dot{e} = A_{o1} e - \sum_{i=1}^{N} F_i C^T R_o^{-1} e + \left[f_o(\hat{x}_o) - f_o(x_o)\right] \tag{36}$$

where $F_i = H_i^C \varepsilon_{oi} \hat{\omega}^i$. Since A_{o1} is a Hurwitz matrix, for any given positive definite matrix $M \in R^{4\times4}$, there exists a unique positive definite P such that

$$A_{o1} P + P A_{o1} = -2M. \tag{37}$$

Now consider the positive definite Lyapunov function $V(e) = e^T P e$, then its time derivative is obtained as

$$\dot{V} = -2e^T M e + 2e^T P \left[f_o(\hat{x}_o) - f_o(x_o) - \sum_{i=1}^{N} F_i C^T R_o^{-1} C e\right]. \tag{38}$$

With the definition of $f_o(x_o)$ in (29), the following Lipschitz condition is achieved:

$$\left\| f_o(\hat{x}_o) - f_o(x_o) \right\| \leq \|e\|. \tag{39}$$

Then

$$\begin{aligned} \dot{V} &\leq -2\lambda_{\min}(M)\|e\| + 2\|P\|\|e\|^2 + 2\left\|\sum_{i=1}^{N} F_i\right\| \left\|C^T R_o^{-1} C\right\| \|P\| \|e\|^2 \\ &= -2\left\{\lambda_{\min}(M) - \|P\| \left[1 + \left\|\sum_{i=1}^{N} F_i\right\| \left\|C^T R_o^{-1} C\right\|\right]\right\} \|e\|^2. \end{aligned} \tag{40}$$

The proper ε_{oi} and large enough $\lambda_{\min}(M)$ are chosen such that

$$\lambda_{\min}(M) - \|P\| \left[1 + \left\|\sum_{i=1}^{N} F_i\right\| \|C^T R^{-1} C\|\right] > 0. \quad (41)$$

Then, $\dot{V} < 0$. Therefore, $e = 0$ is an asymptotically stable equilibrium point. ■

C. Load Torque Observer-Based Controller and Design Procedure

With the estimated load torque information $\hat{T}_L$ from the proposed observer, the control inputs can be achieved as

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = -R^{-1} B^T \sum_{i=0}^{N} T_i^C \varepsilon_i \tilde{\omega}^i \bar{x} + \begin{bmatrix} \bar{u}_{cd} \\ \bar{u}_{cq} \end{bmatrix} \quad (42)$$

where

$$\bar{x} = \begin{bmatrix} \tilde{\omega} & \tilde{\bar{i}}_{qs} & i_{ds} \end{bmatrix}^T$$

$$\bar{u}_{cd} = \frac{1}{k_6}\left(k_4 \bar{i}_{qsd} + k_5 \omega_d + i_{ds}\omega_d\right)$$

$$\bar{u}_{cq} = -\frac{1}{k_6}\left(\tilde{\bar{i}}_{qs}\omega_d + \omega \bar{i}_{qsd}\right)$$

$$\bar{i}_{qsd} = \frac{1}{k_1}\left(k_2 \omega_d + \dot{\omega}_d + k_3 \hat{T}_L\right)$$

and

$$\tilde{\bar{i}}_{qs} = i_{qs} - \bar{i}_{qsd}.$$

The observer-based approximated control law (42) includes T_i^C and ε_i as the control gains. The matrices T_i^C can be indirectly tuned by adjusting the weighting matrices Q_0 and R. On the other hand, ε_i is tuned by choosing the design parameters k_i and l_i. Note that when $\varepsilon_i = 1$, the proposed nonlinear optimal controller becomes the SDRE-based controller [36]. The similar analysis can be applied to the nonlinear optimal load torque observer (30) and (31). Also, it is noticed that the proposed nonlinear optimal load torque also turns into the SDRE-based load torque observer [36], if $\varepsilon_{oi} = 1$. Therefore, the controller and observer gains are tuned carefully by the following procedure.

1) Set $\varepsilon_i = 1$ and $\varepsilon_{oi} = 1$. Then, tune Q_0, R by the tuning rule in [44], and Q_{o0}, R_o by the rule in [45] to achieve the satisfactory control and observation performance.
2) With the aforementioned Q_0, R, Q_{o0}, and R_o, select k_i, l_i, k_{oi}, and l_{oi} by the method in [29] to improve the control and observation performance.

Fig. 1 shows the flow chart for the controller and observer gains tuning, which describes the aforementioned procedure.

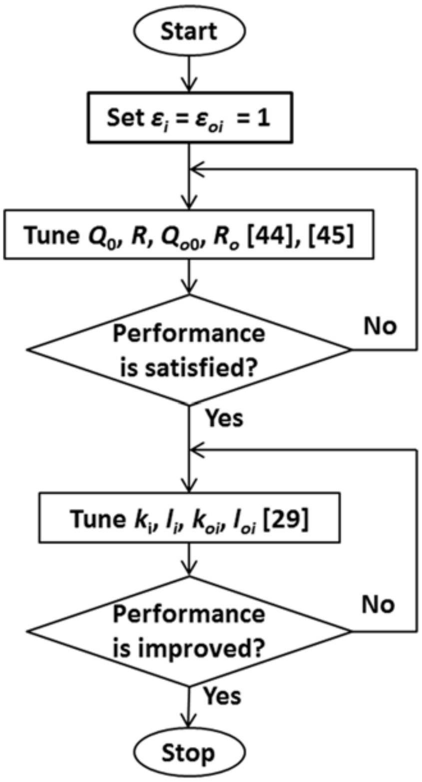

Fig. 1. Flow chart for controller and observer gains tuning.

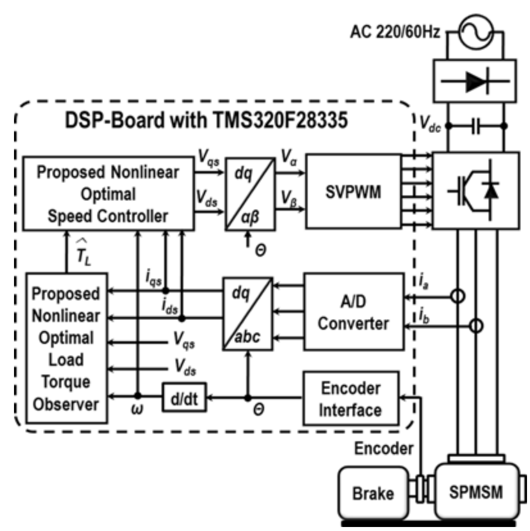

Fig. 2. The SPMSM servo drive test-bed configuration.

IV. Experimental Validation

To validate the usefulness of the proposed observer-based nonlinear optimal control strategy, the experimental investigations are done in this section, and their results are completely analyzed. Fig. 2 shows the configuration of a SPMSM servo drive system, which comprises a SPMSM, an electrical brake, an encoder, and a three-phase PWM inverter with a Texas Instruments TMS320F28335 DSP. It should be noted that the TMS320F28335 DSP has been extensively used in the area of the digital motor control applications because of its high-speed real-time signal processing and tightly integrated peripherals. Consequently, it can easily implement some complicated control algorithms such as fuzzy control, intelligent control, and sensorless control. Also, it can significantly reduce the overall

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TABLE I
PARAMETERS OF A PROTOTYPE SPMSM DRIVE

Symbol	Description	SI Value	SI Unit
$P_{\rm rated}$	Rated power	750	W
$T_{\rm rated}$	Rated torque	2.4	N·m
$I_{\rm rated}$	Rated current	4.3	A
p	Number of poles	8	-
R_s	Stator resistance	0.43	Ω
L_s	Stator inductance	3.2	mH
λ_m	Magnet flux linkage	0.085	V· s/rad
J	Equivalent rotor inertia	0.0018	kg· m^2
B	Viscous friction coefficient	0.0002	N· m·s/rad

system costs by modeling some hardware functions in software. Table I illustrates the nominal parameters of the SPMSM drive. Considering the compromise between system performance and efficiency, both the sampling and PWM frequency are chosen as 5 kHz.

The gains of the observer and controller are tuned by the procedure described in Section III-B.

1) In the first step, using the tuning rule in [44] and [45], the weighting matrices can be realized as: $Q_{o0} = \mathrm{diag}(1, 1000, 50000, 50000)$, $R_o = 10^{-5} \times \mathrm{diag}(1, 1, 1)$, $Q_0 = \mathrm{diag}(0.1, 10, 10)$, and $R = \mathrm{diag}(1, 1)$. As stated in [36], it is observed that the control and observation performance are not remarkably enhanced when N and N_o are higher than one. Therefore, N and N_o are selected to be one with considering the tradeoff between the control performance and complexity.
2) In the second step, via the method in [29], the remaining gains of the observer and controller are chosen as: $k_i = 0.3$, $l_i = 0.5$, $k_{oi} = 0.3$, and $l_{oi} = 0.5$.

To compare the superior performance of the proposed nonlinear optimal control scheme, the SDRE-based control scheme in [36] and the conventional PI control scheme are also tested. The gains of the PI controllers are tuned based on the general rule in [46]. The bandwidths of the PI speed controller and the PI current controller are selected as $\omega_s = 2\pi \cdot 2$ rad/s and $\omega_c = 2\pi \cdot 20$ rad/s, respectively.

In this paper, the parameters of the SPMSM are changed in the experiments of all control schemes to verify the robustness of the proposed control approach. The details of how to change the motor parameters are stated in [13]. According to [47] about electrical parameters variations, the stator resistance and inductance variations are chosen to be +50% and −10%, i.e., $R_s = 0.43 \times 1.5 = 0.645\ \Omega$ and $L_s = 3.2 \times 0.9 = 2.88$ mH. Meanwhile, the variations in the mechanical parameters can be higher when the motor drive is connected to an external mechanical load. Therefore, the equivalent rotor inertia and viscous friction coefficient variations are chosen to be +50% and +100%, i.e., $J = 18 \times 10^{-4} \times 1.5 = 27 \times 10^{-4}$ kg·m^2 and $B = 2 \times 10^{-4} \times 2 = 4 \times 10^{-4}$ N·m·s/rad.

The details of the conditions under the electrical and mechanical parameters variations mentioned previously are given as follows:

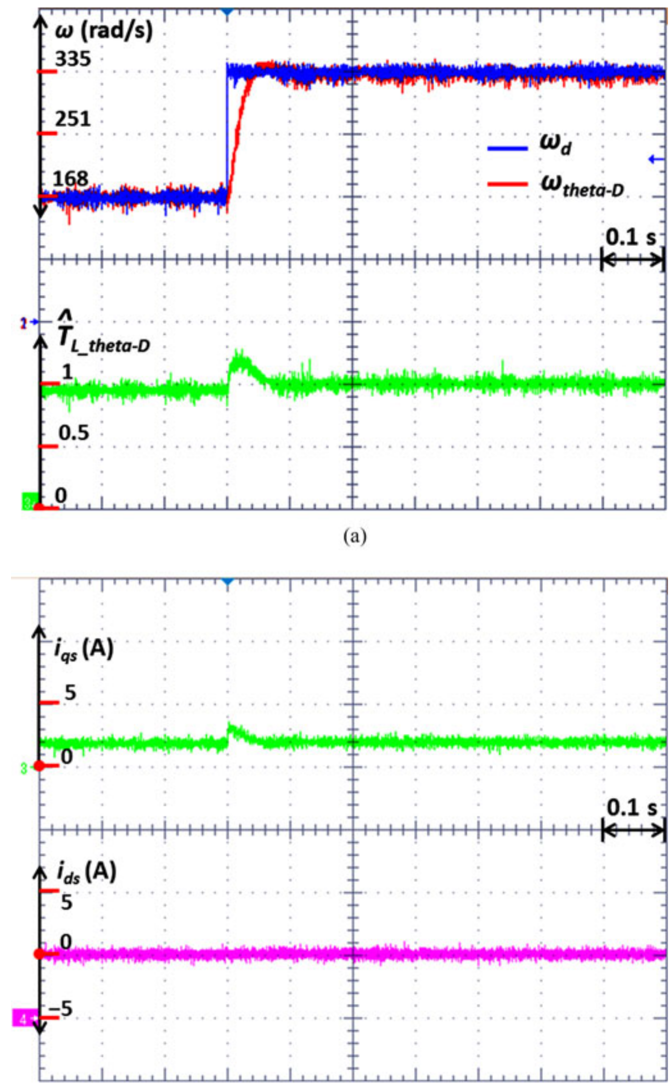

Fig. 3. Experimental results of the proposed θ-*D*-based nonlinear optimal control method for *Condition 1*. (a) Speed reference (ω_d), measured speed (ω), and estimated load torque ($\hat{T}_L$). (b) *dq*-axis currents (i_{qs} and i_{ds}).

Condition 1—Speed tracking with step-wise speed reference: The desired speed (ω_d) = 168 rad/s → 335 rad/s; Load torque $T_L = 1.0$ N·m. Although the trapezoidal speed reference is popular in industry, the step-wise speed reference is chosen to clearly testify the superior performance of the proposed control scheme over the conventional SDRE-based control method during transient time because it belongs to the worst case of the speed references.

Condition 2—Speed tracking with step-wise load torque: The desired speed ($\omega_{\rm d}$) = 209 rad/s; Load torque T_L : 1.0N·m → 0 N·m.

It should be noticed that these two conditions fully represent possible situations of the industrial motors in the constant torque region.

Figs. 3–6 demonstrate the experimental results of the both control methods under *Conditions 1 2*. That is, Figs. 3 and 4

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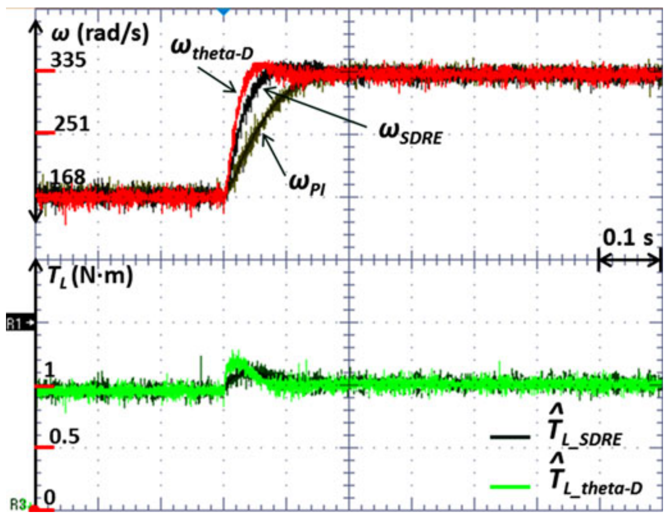

Fig. 4. Comparative experimental results among the proposed θ-D-based nonlinear optimal control scheme, SDRE-based control scheme, and PI control scheme for *Condition 1*.

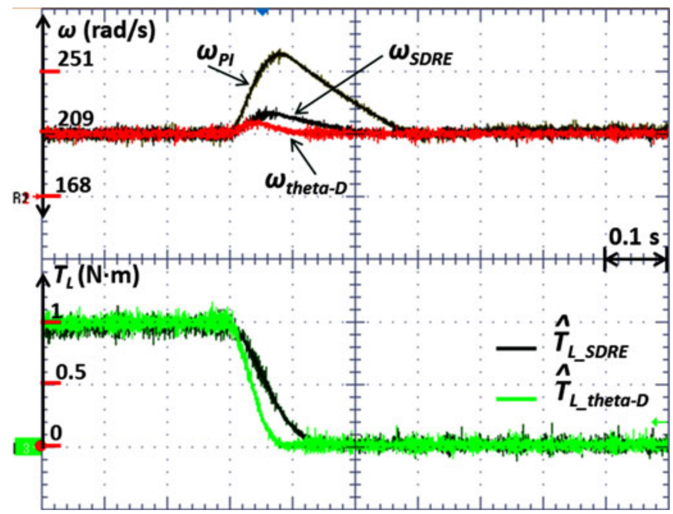

Fig. 6. Comparative experimental results among the proposed θ-D-based nonlinear optimal control scheme, SDRE-based control scheme, and PI control scheme for *Condition 2*.

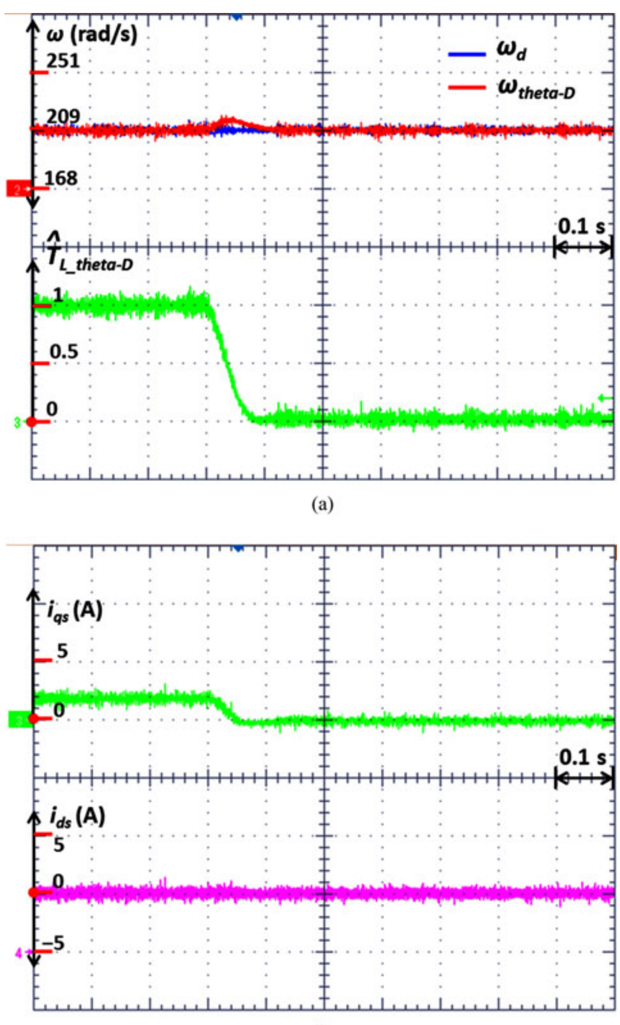

Fig. 5. Experimental results of the proposed θ-D-based nonlinear optimal control method for *Condition 2*. (a) Speed reference (ω_d), measured speed (ω), and estimated load torque ($\hat{T}_L$) (b) dq-axis currents (i_{qs} and i_{ds}).

TABLE II
PERFORMANCE COMPARISON AMONG THREE CONTROL SCHEMES BY EXPERIMENTS

Criteria		Settling time (ms)		Overshoot (%)	
Method		*Condition 1*	*Condition 2*	*Condition 1*	*Condition 2*
The proposed nonlinear control scheme	Speed	40	90	0	4
	Torque	62	75	40	0
The SDRE-based control scheme	Speed	72	190	0	8
	Torque	110	135	30	0
The PI control scheme	Speed	160	270	0	26
	Torque	-	-	-	-

are associated with the experimental results under *Condition 1*, while Figs. 5 and 6 are obtained under *Condition 2*. Also, Figs. 3 and 5 illustrate the experimental results of only the proposed control scheme, whereas Figs. 4 and 6 show the comparative experimental results among the proposed nonlinear optimal control method, the SDRE-based control method, and the PI control method. The detailed comparative performance of the three methods is summarized in Table II. Based on the experimental results of Figs. 3–6, the settling times and the overshoots of the proposed control scheme are smallest among those of three control schemes in the speed tracking (settling time of *Condition 1*: 40/72/160 ms, *Condition 2*: 90/190/270 ms; overshoot of *Condition 2*: 4%/8%/26%). Although in *Condition 1*, the estimated load torque of proposed observer are a little bigger than that of the SDRE-based control method (40%/30%), the settling times of both *Conditions 1* and *2* are much smaller than those of the SDRE-based control scheme in the torque estimation (settling time of *Condition 1*: 62/110 ms, *Condition 2*: 75/135 ms).

V. Conclusion and Future Work

In this paper, nonlinear optimal speed controller and load torque observer for the SPMSM drives were proposed. By using the two immediate variables, called θ and D, the HJBE was decomposed into simple equations. Then, by choosing the appropriate coefficient matrices, all of them became algebraic equations. Therefore, the proposed controller and observer not only avoided the complex online algorithms of the other nonlinear optimal control methods, but also eliminated the complicated Kronecker product technique in the previous θ-D-based control schemes. By utilizing a more accurate approximation technique, the proposed control scheme achieved better transient performance than the SDRE-based control scheme in [36]. The stability analysis of the controller was completely presented, and that of the observer could be similarly obtained. The experimental studies proved the feasibility of the proposed controller-observer scheme. The results also proved that the proposed control schemes could be more robust to the parameter uncertainties and the load torque disturbance as compared to the SDRE-based control method and the conventional PI control method.

Even though this paper designed a nonlinear optimal speed control system for SPMSMs in a constant torque region, the proposed method can be extended to the flux weakening region in which the effects of overmodulation harmonics are needed to be considered. Moreover, the interaction between the proposed control method and the inverter nonlinearities (e.g., dead time and switching delay) is also an opened issue. Finally, the proposed θ-D-based nonlinear optimal control method can be applied to the direct torque control of PMSM drives.

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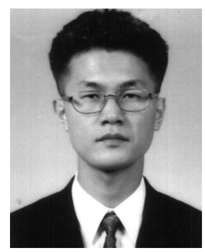
Ton Duc Do (S'12) received the B.S. and M.S. degrees in electrical engineering from the Hanoi University of Science and Technology, Hanoi, Vietnam, in 2007 and 2009, respectively, and the Ph.D. degree from the Division of Electronics and Electrical Engineering, Dongguk University, Seoul, Korea, in 2014.

From 2008 to 2009, he worked at the Division of Electrical Engineering, Thuy Loi University, Hanoi, as a Lecturer. He also worked at the Division of Electronics and Electrical Engineering, Dongguk University, as a Postdoctoral Researcher. Currently, he is a Senior Researcher at the Pioneer Research Center for Controlling Dementia by Converging Technology, Gyeongsang National University, Jinju, Korea. His research interests include the field of robotics, electric machine drives, and control of distributed generation systems using renewable energy sources.

Han Ho Choi (M'03) received the B.S. degree in control and instrumentation engineering from Seoul National University, Seoul, Korea, in 1988, and the M.S. and Ph.D. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology, Daejeon, Korea, in 1990 and 1994, respectively.

From 1994 to 1998, he was a Team Leader with the Advanced Technology Laboratory, DaeWoo Electrical Company. He is currently with the Division of Electronics and Electrical Engineering, Dongguk University, Seoul. He spent his sabbatical with the Department of Electrical and Computer Engineering, California State Polytechnic University, Pomona, CA, USA. He teaches introductory electrical engineering courses on microprocessors, robotics, sensors, and instrumentation engineering. His research interests include linear-matrix-inequality-based control system design, microprocessor-based control systems, and variable structure systems.



Jin-Woo Jung (S'97–M'06) received the B.S. and M.S. degrees in electrical engineering from Hanyang University, Seoul, Korea in 1991 and 1997, respectively, and the Ph.D. degree in electrical and computer engineering from The Ohio State University, Columbus, OH, USA, in 2005.

From 1997 to 2000, he was with the Digital Appliance Research Laboratory, LG Electronics Co., Ltd., Seoul. From 2005 to 2008, he worked as a Senior Research Engineer at the R&D Center and with the PDP Development Team, Samsung SDI Co. Ltd., Korea. Since 2008, he has been an Associate Professor with the Division of Electronics and Electrical Engineering, Dongguk University, Seoul. His current research interests include DSP-based electric machine drives, distributed generation systems using renewable energy sources, and power conversion systems and drives for electric vehicles.