

Representations of Convex Geometries

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- 1 Yin and Yang of convex geometry
- 2 Edelman-Jamison structure theorem for finite convex geometries
- 3 Representations of convex geometries
- 4 Implicational Bases for convex geometries

Alignment

Definition

Given any (finite) set X , an alignment on X is a family \mathcal{L} of subsets of X which satisfies two properties:

- (1) $X \in \mathcal{L}$;
- (2) \mathcal{L} is stable under intersection, i.e., $Y \cap Z \in \mathcal{L}$, if $Y, Z \in \mathcal{L}$.

Closure operator

Definition

Given any (finite) set X , a closure operator on X is a mapping $\mathcal{L}^* : 2^X \rightarrow 2^X$ with the following properties:

- (1) $Y \subseteq \mathcal{L}^*(Y)$, for every $Y \subseteq X$;
- (2) if $Y \subseteq Z$, then $\mathcal{L}^*(Y) \subseteq \mathcal{L}^*(Z)$, for $Y, Z \subseteq X$;
- (3) $\mathcal{L}^*(\mathcal{L}^*(Y)) = \mathcal{L}^*(Y)$, for $Y \subseteq X$.

Alignment and closure operator

Proposition

- (1) *Let \mathcal{L} be an alignment on X . For every $Y \in 2^X$, define $\mathcal{L}^*(Y) = \bigcap \{Z \in \mathcal{L} : Y \subseteq Z\}$. Then \mathcal{L}^* is a closure operator on X .*
- (2) *Let $\mathcal{L}^* : 2^X \rightarrow 2^X$ be a closure operator on X . Then $\mathcal{L} = \{Y : \mathcal{L}^*(Y) = Y\}$ is an alignment on X .*

Convex geometry as an alignment

Definition

Pair (X, \mathcal{L}) is called a convex geometry, if \mathcal{L} is an alignment on X with additional properties:

- $\emptyset \in \mathcal{L}$
- *if $Y \in \mathcal{L}$ and $Y \neq X$, then there exists $x \in X \setminus Y$ such that $Y \cup \{x\} \in \mathcal{L}$.*

Convex geometry as a closure system

Definition

Pair (X, \mathcal{L}^*) is called a convex geometry, if \mathcal{L}^* is a closure operator on X with additional properties:

- (1) $\mathcal{L}^*(\emptyset) = \emptyset$
- (2) if $Y = \mathcal{L}^*(Y)$ and $x, z \notin Y$, then $z \in \mathcal{L}^*(Y \cup \{x\})$ implies that $x \notin \mathcal{L}^*(Y \cup \{z\})$. (Anti-Exchange Property)

Convex geometry as yin and yang

Theorem (folklore)

Let X be set, \mathcal{L} be an alignment and \mathcal{L}^ be a corresponding closure operator. (X, \mathcal{L}) is a convex geometry iff (X, \mathcal{L}^*) is a convex geometry.*

Examples of convex geometries

The following set systems form convex geometries:

- convex bodies in Euclidean spaces;
- convex sets in partially ordered sets;
- down-sets in partially ordered sets;
- subsemilattices in a semilattice;
- path-closed subgraphs of a graph.

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Antimatroids

- Antimatroid is a set system $\langle A, \mathcal{F} \rangle$, where elements of \mathcal{F} are complements of members from alignment of convex geometry on set A .
- In mathematical psychology, antimatroids have been used to describe feasible states of knowledge of a human learner.
- Doignon, Jean-Paul, and Falmagne, Jean-Claude, *Knowledge Spaces*, Springer-Verlag, 1999.
- This theory is behind several automated teaching and testing tools such as ALEKS.

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New volume for the Grätzer's book on lattices this year

KA and J.B.Nation: 4 chapters including one on Convex Geometries.

Work on infinite convex geometries

- KA and J.B.Nation, *A class of infinite convex geometries*, arxiv 2015
- KA and M. Pouzet, *On scattered convex geometries*, arxiv 2015
- S. Sakaki, *Infinite closure systems and convex geometries*, thesis 2015

Join of alignments

Let \mathcal{L} and \mathcal{M} are two alignments on the same set X . Define a new family of subsets of X :

$$\mathcal{L} \vee \mathcal{M} = \{L \cap M : L \in \mathcal{L}, M \in \mathcal{M}\}$$

Proposition

If \mathcal{L} and \mathcal{M} are two alignments, then $\mathcal{L} \vee \mathcal{M}$ is the smallest alignment containing \mathcal{L} and \mathcal{M} .

Proposition (Edelman, 1980)

If \mathcal{L} and \mathcal{M} are two alignments of convex geometries on X , then $\mathcal{L} \vee \mathcal{M}$ is such as well.

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Structure via order alignments

Definition

Monotone alignment \mathcal{D}_E on (finite) set X is the order alignment corresponding to some total order E on X : if E is $x_1 < x_2 < \dots < x_n$, then $\mathcal{D}_E = \{\{x_1, \dots, x_k\} : k \leq n\}$.

Theorem (Edelman and Jamison, 1985)

Every convex geometry (X, \mathcal{L}) is the join of some monotone alignments:

$$\mathcal{L} = \bigvee_{E \in \text{CompOrd}} \mathcal{D}_E$$

where CompOrd is the set of compatible orderings of \mathcal{L} .

Minimal number of compatible orderings in such representation is called *convex dimension* of a geometry.

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Algebraic case

- N. Wahl, *Antimatroids of finite character*, J. Geom. 70 (2001), 168–175.
- KA, *Algebraic convex geometries revisited*, arxiv 2014

Theorem

Every algebraic convex geometry (X, \mathcal{L}) is the s_p -join of some monotone alignments from compatible orderings E of \mathcal{L} :

$$\mathcal{L} = \bigvee_{s_p} \mathcal{D}_E$$

Representations in Algebra

Theorem

(Cayley, 1854) Every group is a subgroup of some group of permutations.

Theorem

(Gelfand, 1969) Every commutative C^ -algebra is isomorphic to an algebra of continuous functions on its Gelfand spectrum.*

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Representation for convex geometries

- *embeddings*: finding injective maps from arbitrary *finite* convex geometry to convex geometry of special form (from special class), preserving meets and joins
- *isomorphisms*: finding exact representations via bijections preserving joins and meets

Convex geometries and join-semidistributive lattices

Embedding representation for convex geometries provides such representation for finite *join-semidistributive* lattices as well:

- every finite convex geometry is join-semidistributive;
- every finite join-semidistributive lattice can be embedded into atomistic such lattice with the same number of join-irreducibles;
- any atomistic join-semidistributive lattice is a convex geometry.

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Lattices of algebraic subsets

- KA, V.A. Gorbunov and V.I. Tumanov, *Join-semidistributive lattices and convex geometries*, Adv. Math. 2003
- embedding of every finite convex geometry into $S_p(L)$, for some doubly algebraic lattice L
- problem was raised in the paper to find representation in the class of *finite* convex geometries.

Representation for lower bounded lattices

Lower bounded lattices are embeddable into $Co(R^n, X)$, for some finite set of points X :

- F. Wehrung, M. Semenova, *Sublattices of lattices of convex subsets of vector spaces*, 2004
- KA, *Join-semidistributive lattices of relatively convex sets*, 2004
- lower bounded convex geometries are studied in KA, *Optimum basis of a convex geometry*, arxiv 2012, in relation to tractable optimum implicational basis.

Representation in relatively convex sets

K. Kashiwabara, M. Nakamura and Y. Okamoto, *The affine representation theorem for abstract convex geometries*, *Comp. Geom.* 30 (2005):

- proved that every finite convex geometry is a generalized convex shelling;
- equivalently: every finite convex geometry is an interval in $Co(R^n, X)$, for some finite X .

Generalized convex shelling: the closure system (P, ϕ_Q) , where $P = X \setminus Q$ for some $Q \in Co(R^n, X)$, and $\phi(S) = ch(S \cup Q) \cap P$, for $S \subseteq P$.

Proposition (KA and J.B.Nation, chapters for Grätzer's volume)

Assume that X is any set of points in R^n .

- ϕ_Q satisfies the anti-exchange property.
- convex sets of (P, ϕ_Q) form a complete lattice isomorphic to interval $[Q, X] \subseteq Co(R^n, X)$.

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Representation by circles on a plane

G. Czédli, *Finite convex geometries of circles*, Disc. Math. 330 (2014).

- points are replaced by circles;
- only convex geometries of convex dimension = 2;
- in fact, this can be done with intervals on the line;
- is it possible to represent the convex geometries of arbitrary convex dimension?

Representation by the polygons on the plane

M. Richter, L. Rogers, *Embedding convex geometries and a bound on convex dimension*, arxiv 2015

- used Edelman-Jamison representation via order alignment;
- polygons are not necessarily convex;
- reproved Kashiwabara et al's theorem for embedding into $Co(\mathbb{R}^k, X)$, $k = cdim$.

Motivation

Use representation results for investigation of the *optimum* implicational basis of convex geometry.

Closure systems and implications

- An implication σ on X : $Y \rightarrow Z$, for $Y, Z \subseteq X$, $Z \neq \emptyset$.
- σ -closed subset A of X : if $Y \subseteq A$, then $Z \subseteq A$.
- Closure system $\langle X, \phi_{\mathcal{S}} \rangle$ defined by set \mathcal{S} of implications on X : A is closed, if it is σ -closed, for each $\sigma \in \mathcal{S}$
- Every closure system $\langle X, \psi \rangle$ can be presented as $\langle X, \phi_{\mathcal{S}} \rangle$, for some set \mathcal{S} of implications on X .
- Example: $\mathcal{S} = \{A \rightarrow \phi(A) : A \subseteq X, A \neq \phi(A)\}$.

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Operator and sets of implications

Note:

- Every set of implications \mathcal{S} on X defines *unique* closure operator on X .
- There exist *numerous* sets of implications that define the same operator on X .

Example: Let $X = \{a, b, c\}$. Consider $\mathcal{S}_1 = \{a \rightarrow bc\}$ and $\mathcal{S}_2 = \{a \rightarrow bc, ab \rightarrow c, ac \rightarrow b, a \rightarrow b, bc \rightarrow c\}$.

The closure systems defined by \mathcal{S}_1 and \mathcal{S}_2 are the same.

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Optimum basis of a closure system

Definition

A basis $\mathcal{S} = \{X_i \rightarrow Y_i : i \leq n\}$ is called optimum, if number $s(\mathcal{S}) = |X_1| + \dots + |X_n| + |Y_1| + \dots + |Y_n|$ is smallest among all sets of implications for the same closure system.

G. Ausiello, A. D'Atri and D. Saccá, 1986:

- Every optimum basis is right and left-optimum and minimum.
- Problem of finding optimum basis from any given is NP-complete.

Optimum basis in case of convex geometry

For some classes of closure systems the optimum basis can be found efficiently:

- when the lattice of closed sets is modular (M. Wild, 2000)
- for quasi-acyclic Horn Boolean functions (P.L. Hammer and A. Kogan, 1995)
- affine convex geometries (K. Kashiwabara and M. Nakamura, 2013)
- several subclasses of convex geometries (KA, 2012 arxiv)

Hypothesis: Optimum basis can be found from any basis of a convex geometry in the time polynomial in the size of the basis.

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