EXCHANGE RATE DYNAMICS AND MONETARY POLICY IN A SMALL OPEN ECONOMY: A DSGE MODEL FOR KAZAKHSTAN

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Abstract

Shift to floating exchange rate regime is controversial decision to commodity oriented economy of Kazakhstan. Increase in volatility of nominal exchange rate is believed to bring deterioration of main macroeconomic variables and reduction of welfare. This paper thus aims to define optimal monetary policy in estimated small open economy of Kazakhstan. On a foundation of Taylor rule and under two specifications of the model it is found that currently National Bank of Kazakhstan reacts to movements in exchange rate on a limited extent. In addition, optimal policy simulation based on posterior distribution of model parameters and welfare loss minimization function is performed. This analysis shows that exchange rate stabilization within 5% band delivers lowest welfare loss.

Contents

1 Introduction

For the last 30 years exchange rate policy in Kazakhstan has changed dramatically from relatively floating regime in the 1990s to extremely fixed in 2014. However, in 2015, after substantial devaluation of the national currency Tenge (hereinafter, the "Tenge") National Bank (hereinafter, the Central Bank) abandoned pegging and attached to completely opposite regime - free floating. Central Bank claimed that new currency regime will be performed without further interventions in currency market by authorities.

Central Bank explained such decision by implementation of a new monetary policy inflation targeting and intention to preserve country's foreign reserves. Inflation targeting strategy has gained popularity recently and has been implemented in many countries such as New Zealand, Canada, UK, Australia, Brazil, which are considerably different in their economic development.

In monetary policy a central role is played by short-term interest rate as a comprehensive function of information and judgements. Briefly, monetary policy determines optimal value of short-term interest rate that minimizes the deviation of inflation and output from their optimal projections. Nominal exchange rate is excluded from the policy rule as long as free floating is believed to be the optimal currency regime. Such assumption is based on the model of Mundell-Fleming where floating facilitates the economy with short-term nominal rigidity to adjust to exogenous shocks much faster and with less cost than in case of pegging.

Nevertheless, such approach raises issues concerning validity of the monetary policy in developing countries. Weak development of financial markets and previously stable exchange rate have contributed to huge borrowings in foreign currency. Developing and emerging countries have managed to collect a high level of liabilities, thus unexpected and significant devaluation of national currency could threaten their financial stability. In addition, switching to a new equilibrium can demand too large devaluation and increase inflation substantially.

Thus, flexible exchange rate can work as automatic stabilizer and absorber of external shocks. For its appropriate implementation economy should satisfy such conditions as mature financial market and institutes, independence of Central Bank and its non-intervention in currency operations. It is in question whether exchange rate should be excluded from the optimal rule, particularly in these countries where the economy and financial markets are not well developed.

Local economists support that view and criticize neglecting of nominal exchange rate fluctuations for high volatility and uncertainty. Opponents of current macroeconomic policy in Kazakhstan argue that such high level of uncertainty is 'absolute evil' and provide main reasons of its inadequacy [1]:

1. Very high devaluation expectations among population and investors.

2. Very low level of trust to monetary authorities.

3. Export and export revenue are not diversified. Kazakhstan's economy mainly depends on oil volumes and prices are vulnerable to fluctuations. In the long term this volatility translates into economy.

4. Central Bank initially assured non-intervention in currency market operations, however for the last 5 years it has been involved in significant interventions in currency market according to their publications. It seems that Central Bank is still involved in control of exchange rate but the agents are absolutely uncertain about the future currency changes since Central Bank provides ex post publications. It has never warned the agents

[1] From the statement of former Vice minister of investment and development and Director of Center for Applied Research "Talap" Rahim Oshakbayev

of future possible actions in currency market.

5. Flexible exchange rate initiates additional emission of money by Central Bank to maintain stability and thus high inflation.

6. Import goods constitute the largest share of domestic consumption. Given the high exchange rate pass-through in Kazakhstan this results in welfare reduction.

The volatility of exchange rate has increased dramatically since 2015. During the last five years it fluctuated from 1% to 19% each quarter. Uncertainty in exchange rates sends worrying signals to people which eventually could impede economics and bring welfare loss that overlap benefits from free market framework.

This study aims to address the following issues:

1. Is there evidence that Central Bank intervenes in exchange rate policy under the announced inflation targeting regime, except for its publications in media?

2. Which policy is optimal for country's economy?

I develop DSGE model to answer these questions in spirit of Justiniano and Preston (2008) and Gali and Monacelli (2005). Kazakhstan is a small open economy which is dependent on the prices of commodities, especially, crude oil. DSGE models may predict the reaction of different macroeconomic variables to external shocks and contains theoretical micro-foundation as well as empirical method. As opposed to purely empirical models as VAR and its varieties DSGE initially solves general equilibrium problem and finds relations between different variables. Hence estimation of the parameters is performed based on solution of optimization problem instead of relying on historical data. This advantage of the model made DSGE one of the most spread method in describing the behavior of the economy. The number of specifications of the model can be arbitrarily large. It is imperative to select specification which is best reflects the economy of the country. By now there are not many studies with DSGE models in Kazakhstan.

One of the notable works for Kazakhstan in relevant field is the study of Algozhina (2016). She develops a complicated oil economy with two production sectors, Sovereign Fund, two types of households, monetary and fiscal instruments. The main finding of this study is that CPI targeting with flexible exchange rate is more preferable allowing exchange rate to adjust to external shocks rapidly. The intervention of Central Bank in currency market is not advisable. This work includes time series prior to implementation of inflation targeting policy in Kazakhstan and thus neglects this period. The author calibrates parameters for simulation experiments instead of using estimation which is assessed to be more advanced and reliable method. Shults and Kyssykov (2019) obtained the opposite results using a model based on the Blanchard- Kiyotaki specification and estimated it via Bayesian method. The work covers time period up to 2018 and thus inflation targeting policy is implemented in model. They conclude that integration of exchange rate in Taylor equation increases the welfare of society by 3.2%. The authors propose to use producer price index(PPI) instead of CPI (consumer price index) as a targeted inflation level of the policy. This work still does not consider alternative monetary policy other than inflation targeting and does not provide clear recommendations of the optimal regime to follow.

In this paper I try to develop DSGE model to be clear and consistent. I initially determine main business cycles patterns for Kazakhstan in section 3. Based on these findings I describe a model for small open economy for Kazakhstan with current monetary policy and sticky prices. Commodity sector is introduced through the risk premium shock. I interpret positive shock to interest rate spread as increase in commodity price which initiates the growth of domestic price level. While on the one hand, this deteriorates terms of trade making domestic goods more expensive than their foreign counterparts, on the other hand, this leads to improving of commodity revenue and therefore increase social welfare. According to study of Fernandez et al. (2015) fall in foreign demand for home goods is not sufficient to overweigh the effect of raised revenue. Foreign block is represented by the economy of Russia - the biggest and the most influential economy in the CIS region. Section 6 extends the basic specification of the model by explicit introduction of separate commodity sector. Export revenue accrues as endowment to households and accounts in overall country's GDP. I then log-linearize and solve it by using Schur decomposition in a Dynare software. I estimate parameters based on Bayesian method described in the section 7. This estimation provides answer on the first research question whether Central Bank intervenes in currency market and control fluctuations of nominal exchange rate. If the coefficient on changes in exchange rate in Taylor rule is zero, then Central Bank pursues inflation targeting policy without involvement in foreign currency market. The result clearly indicates non-zero parameter on nominal exchange rate change for both model specifications. Finally, I undertake simulations with different exchange rate regimes with the introduction of structural shocks to the business cycle fluctuations. Welfare loss is estimated under alternative policy rules. Such analysis provides evidence that optimal policy includes response to nominal exchange rate to keep the volatility of the latter within 5-6% band. Furthermore, this result is robust over both specifications. While more active stabilization of the nominal exchange rate boosts inflation level to overwhelming values, failure to response to its movements makes exchange rate more volatile.

Nominal rigidities for domestic and foreign goods prices, retailers sector importing goods and selling them in home market, implicit introduction of commodity good market, using of Taylor rule in modeling inflation targeting I believe make my model realistic in describing Kazakhstan's economy. Following Bayesian estimation which emerges prior knowledge about the parameters and the actual data should provide the most reliable estimates that may improve the output of the simulation analysis.

2 Literature review

The evolution and the history of exchange rate regimes include number of land mark events that had a great impact on the economies of many countries. Failure of gold standard and Bretton Woods agreement, the establishment of European Monetary Union and deep currency crisis in Southeast Asia are the most significant shocks that initiated a series of discussions and studies regarding the optimal monetary policy.

Further introduction and spreading of inflation targeting is assumed to be explained by explicit Taylor-type rule in relevant studies. Optimal policy coefficients are determined in order to minimize quadratic loss function including variation in inflation, output and interest rate.

According to Svensson (2010) inflation targeting is first characterized by the presence of announced inflation target value. Next, inflation forecast takes a significant part in monetary policy. Finally, high level of accountability and transparency of monetary authorities is essential framework to implement inflation targeting.

Last years have been marked by a plenty of studies on this question with the use of the general equilibrium model framework. Algorithm, first, includes a construction of comprehensive DSGE model for a particular economy. Next, optimal policy rule is included in the model. Then the model is solved and estimated by differenst methods. Finally, conclusion on optimal monetary rule is provided based on the results of simulations.

The studies for industrialized countries mainly demonstrate that coefficient on the exchange rate is almost zero. Taylor (1999) aimed to draw implications for interest rate determination for the European Central Bank. He simulated different interest rate rules for France, Italy and Germany as part of European Monetary Union (EMU) and United Kingdom, Japan, USA and Canada as outside the EMU. The author concludes that simple rule without inclusion of exchange rate provides the most robust results. Despite the fact that alternative monetary policy with exchange rate benefited in some countries, those results are not robust to the number of models proposed in the paper.

Froyen and Guender (2016) re-examined the inclusion of nominal exchange rate in Taylor rule for small open economy. By contrast, they conclude that a small weight of exchange rate response in loss function can improve the policy implications. Central Bank that follows Taylor rule and somehow wants to preserve currency market stability needs to react to the real exchange rate fluctuations.

Justiniano and Preston (2009) estimated DSGE model for Australia, Canada and New Zealand. Posterior distribution of the parameters was used for exchange rate policy simulations. They found that it was not optimal for monetary policy to respond for exchange rate fluctuations. Nevertheless, controversial result was obtained for Switzerland by using the similar DSGE model where monetary policy framework is believed to be mature and highly trusted. Baurle and Menz (2008) assessed the optimal monetary policy by using the DSGE-VAR model and came to conclusion that Swiss Central Bank sets a non-zero coefficient on the exchange rate response in the rule. Small positive coefficient is optimal for the monetary policy conducted in Switzerland.

Gali and Monacelli (2005) model exchange rate explicitly, where pegging is described by zero change in nominal exchange rate from period to period. The results demonstrate that welfare losses are higher in case of pegging and lower in case of domestic inflation targeting policy. However, the later brings substantial instability in nominal exchange rate and terms of trade.

A specific DSGE model for Russia was built by Malakhovskaya (2015). She performs the simulation of main macroeconomic variables' behavior based on negative exogenous export shock and risk premium shock. The analysis shows that the economy is mostly affected in case of pegging the currency - consumption and output have fallen more deeply than in case of floating or managed floating exchange rate. Malakhovskaya stated flatly that following inflation targeting and avoiding the support of national currency is the best possible reaction to exogenous shocks relevant for the Russian economy.

Bayesian analysis for Singapore by Chow and McNelis (2010) suggests that nominal exchange rate stability is more preferable in case of Singapore. Higher volatility of the nominal exchange rate will lead to a greater volatility in consumption, despite the trade-off small reduction in variability of inflation and investment. Taking into account high dependence on import and overall openness of the economy they conclude that fear of floating is reasonable in choosing optimal monetary policy.

There is, therefore, no clear conclusion regarding what is better - to free exchange rate and trust market forces or to intervene and control fluctuations in currency market. It seems reasonable to examine each case separately based on particular data sets.

3 Business cycles in Kazakhstan

3.1 Correlation of main commodity prices with business cycles in Kazakhstan

This section provides empirical evidence on business cycles pattern due to the commodity price fluctuations in emerging economy of Kazakhstan. Kazakhstan is characterized by substantial share of commodity export in its GDP and overall reliance on mineral resources. According to the Bureau of National statistics extracting industries comprise about 30% of GDP, 70% of export and half of government income. The statistics support poor economy diversification statement. Commodity industry patterns make Kazakhstan vulnerable to commodity price fluctuations. What are the consequences for macroeconomic variables of Kazakhstan from such instability? I study the fluctuations in consumption, output, real exchange rate and risk premium and their relations with the price for exported commodities. This analysis is able to demonstrate the dependence of Kazakhstan's economy on the prices of commodities and overall importance of the sector. It is reasonable to include the analysis of the period consistent with the one obtained for DSGE model.

It is clear that commodity prices are exogenous to country's economy since the supply of natural resources is relatively small in comparison to the rest of the world and cannot influence the prices. The idea is to assess serial correlation between main macroeconomic variables and commodity price index. Commodity price index in quarter t is commodity export price index weighted by GDP. The index is constructed by International Monetary

Fund (IMF) which represents the following:

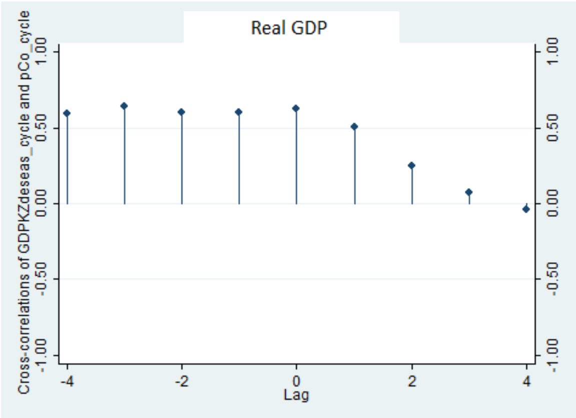
$$P_t^{Co} = \frac{\sum_{i=1}^{k} ex_{i,t}}{GDP_t}$$

where $ex_{i,t}$ is export value of commodity i in year t expressed in US dollars; GDP_t is country's nominal gross domestic product in year t expressed in US dollars.

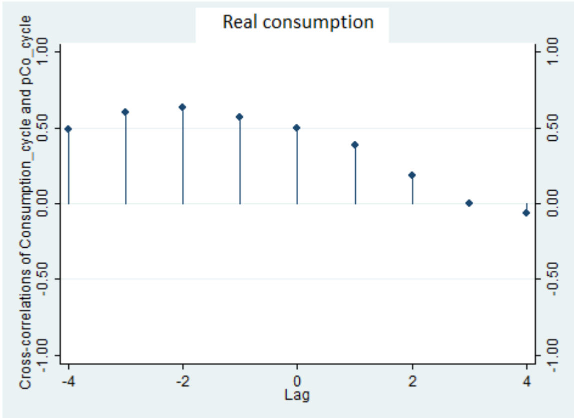
Macroeconomic variables of Kazakhstan are deseasonalised and filtered using one-sided Hodrick-Prescott filter. I extract the cyclical component of all variables and use them in primary analysis. Since change in price for commodity may have impact in the coming quarters without affecting current period, serial correlation of commodity price index and variables $\rho(X_{n,t}, P_{t+j}^{Co})$ is performed with time lags $j = -4, ..., 4$. Serial correlation plots can be examined on the Figure 1:

Figure 1: Cyclicality of commodity price index

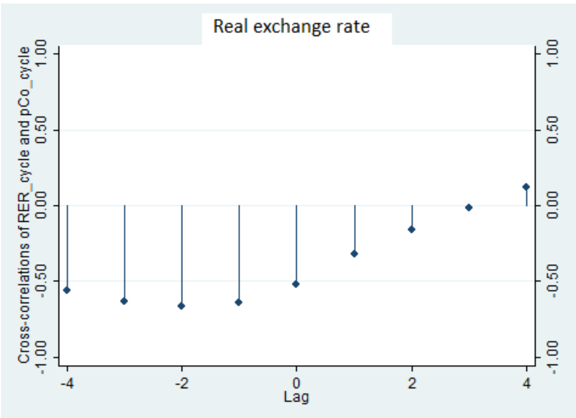
(a) GDP



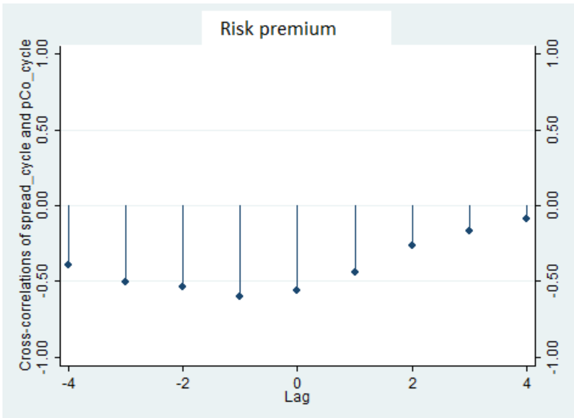
(b) Consumption



(c) Real exchange rate



(d) Risk premium



Analyzing the above panels I get the following stylized facts:

1. The country commodity price index is procyclical to real GDP and real consumption. Predictably, for consumption correlation is higher when the index is lagged two periods (quarters).

2. Real exchange rate is negatively correlated with commodity price index. This empirically proves that increase in price for commodity leads to appreciation of the Tenge. The highest coefficient is attained with one and two quarters lags.

3. One of the main subjects of interest is the link between interest rates spread and commodity price. A series of studies on commodity exporting

countries has indicated the strong negative comovement of interest rate spread and price for commodity. Shousha (2016) claims that commodity price shocks primary difference between advanced and emerging economies concerns the reaction of interest rate to the shock. The explanation for such a behavior is that country risk falls with the increase in price for exporting commodity. This outcome is justified in case of Kazakhstan - real interest rates spread moves oppositely to commodity price. Average of yields on deposit placement or KIBOR rate as one of the most reliable and open rates was chosen as local interest rate. This rate reflects the preferences of second-tier banks in placing deposits in other banks and the yields were recorded accurately each month. For the foreign interest rate 3 months US treasury bills rate is used which can be assumed as one of the most influential and driving interest rates.

To summarize, export of raw materials accounts for the bulk of Kazakhstan's export, being at around 30% of GDP for the last 20 years. Commodity price index is given exogenously and demonstrates strong relationships with the main macroeconomic variables of Kazakhstan - output, consumption, exchange rate and interest rate premium.

3.2 Synchronization of business cycles in Russian Federation and Kazakhstan

Close proximity to the most influential and largest country of the region - Russia - require to be responsive to negative shocks of Russian economy. Hence, another source of business cycles fluctuations in Kazakhstan is assumed to be Russian Federation's economy. For many years both countries shared common history and culture. Additionally, starting from 2010 Kazakhstan, Russian Federation and Belarus have formed Custom

Union with uniform external custom duties. This large integration also has been facilitated by unrestricted flows of labor and capital between two countries for the last ten years. Undoubtedly, all these facts made both economies interdependent. It might be more correct to consider the strict dependence of Kazakhstan's economy due to the size of Russian economy.

Jenish N.(2013) examines business cycles in Russia and Central Asian countries. He came to conclusion that business cycles of Kazakhstan and Russia are synchronized and demonstrate strong economic alliance between two countries. If we apply for trade statistics, Russia has been the main partner for import for the last five years (Table 1).

Table 1: Main import partners of Kazakhstan, share in %

Country	2015	2016	2017	2018	2019
Russia	34.4	36.6	39.6	39.3	36
China	16.6	14.5	15.9	16	17.1
Germany	6.5	5.7	5	4.9	3.8
USA	4.8	5	4.2	3.8	3.4

Source: stat.gov.kz

Despite the fact that Italy is the major exporting partner of Kazakhstan, almost 99% of export consist of crude oil and mineral fuel. In China the share of crude oil export for the last 5 years was about 30% and in Russia this rate was lower. In 2019 the share of crude oil export stood at 12.1%, making Russia the leading non-oil exporter.

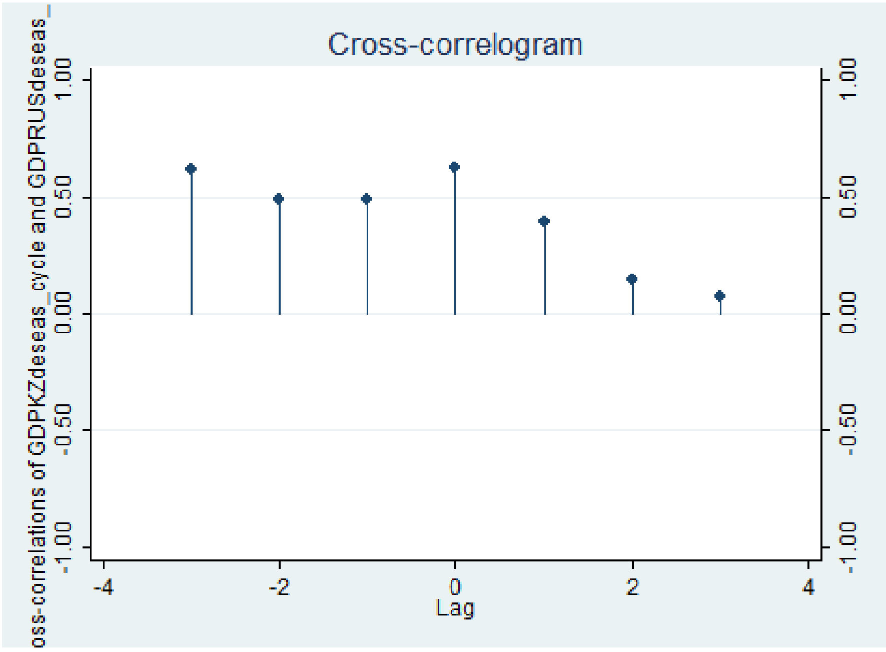
Table 2: Main export partners of Kazakhstan, share in %

Country	2015	2016	2017	2018	2019
Italy	17.7	20.4	17.9	19.2	14.4
China	11.9	11.5	12	10.3	13.8
Russia	9.9	9.4	9.6	8.6	9.8
Netherlands	10.8	8.9	9.8	10.1	7.6

Source: stat.gov.kz

External trade behavior and relationship with Russia confirm the link between two economies. Further evidence is obtained through cross-correlation between real GDP per capita in Russia and Kazakhstan as depicted in Figure 2:

Figure 2: Cross-correlation of cycles in Kazakhstan and Russia



It is clear that Kazakhstan's output is highly correlated with Russian's one. Figure 3 demonstrates almost absolute synchronization resulting from the comparison of deseasonalised GDPs:

Figure 3: Synchronization pattern of GDP in Kazakhstan and Russia

Therefore, from inspecting the above figures the stylized fact can be claimed: **analysis of cross-correlation between GDP of Russia and Kazakhstan provides ultimate evidence of strong connection and dependence. Further examination of annual output per capita confirms integration by almost absolute comovement for the last 15 years.**

Overall, the facts illustrate strong dependence of Kazakhstan's economy on commodity sector and neighbouring Russian economy as well as common behavior of internal business cycles. I incorporate these drivers in small open economy model of Kazakhstan which I describe in the next section.

4 A small open economy model

I describe the model as a small new-keynesian open economy with sticky prices. I impose the following frictions to the model that I assume to be suitable for Kazakhstan's economy:

1. Introduction of the retailers sector as in the model of Justiniano and Preston (2009). Retailers import foreign goods and resell them in domestic market. Such implication can be explained by the the high volume of import in Kazakhstan. In 2019 the import of goods from the countries of Customs Union exceeded export to those countries by 2.3 times.

2. The law of one price for imported goods does not hold. As long as retailer's sector is monopolistically competitive, each retailer has some degree of power in setting local prices in short-term.

3. Almost 70% of country's export consist of raw materials selling abroad. Therefore, export of commodity goods should be introduced in the model. In this research I provide two specifications of the model. First is represented by implicit commodity sector. Commodity price volatility is expressed through the risk premium exogenous shock process. Second specification is characterized by explicit commodity sector with its revenue accruing to households. I, therefore, assume commodity price shock to be exogenous household's income fluctuations.

4. Other sources of shocks in this economy are technology innovation, preference, import cost-push, monetary, risk premium, foreign interest rate, foreign output and foreign

inflation shocks.

Households, domestic producers, retail firms imported foreign goods, monetary authority (Central Bank) and the rest of the world act as agents. Households consume foreign and domestic goods receiving utility from it and aiming to maximize this utility. They supply paid labor to domestic firms and receive profits both from firms producing home goods and firms selling imported goods. Households hold foreign and domestic bonds which bring them income. Home producers use Cobb-Douglas production function to manufacture goods and sell them either for domestic or foreign households. Both home producers and foreign goods importing firms aim to maximize profits. Central Bank follows Taylor rule in determining monetary policy by regulating interest rate. The foreign block is exogenous to domestic economy and follows AR(1) process for inflation, output and interest rate.

4.1 Households

The economy is populated by a continuum of representative households that maximize their lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] \tag{1}$$

where β is the discount factor, σ is constant relative risk aversion, ϕ is Frisch elasticity of labor supply, N_t is labor supply and C_t is a complex consumption consists of domestic and foreign bundles of goods.

Aggregate consumption of representative household is:

$$C_t = [(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ is the bundle of domestically produced goods; $C_{F,t}$ is the bundle of imported goods; $\eta > 0$ is the elasticity of substitution between domestic and imported goods; $\alpha \in (0,1)$ is the share of foreign commodities in total consumption or a measure of trade openness of the economy.

Household maximizes lifetime utility subject to the following budget constraint:

$$P_t C_t + D_t + \tilde{e}_t B_t = D_{t-1}\tilde{i}_{t-1} + \tilde{e}_t B_{t-1}\tilde{i}^*_{t-1}\gamma_t + \Pi_{H,t} + \Pi_{F,t} + W_t N_t + T_t \tag{3}$$

where B_t is the net foreign debt in period t; D_t is the bonds denominated in domestic currency; γ_t is the function managing gross premium; W_t is the nominal wage rate; $\Pi_{H,t}$ and $\Pi_{F,t}$ - profits from domestic firms and firms that import goods (retailers); T_t is the transfers (taxes or subsidies) which is distributed/collected from households; $\tilde{e}_t$ is the nominal exchange rate defined as domestic currency for unit of foreign currency; $\tilde{i}_t$ and $\tilde{i}^*_t$ is the local and foreign nominal interest rates respectively.

Household minimizes costs for consumption for any given level of consumption:

$$\min_{C_{H,t}, C_{F,t}} P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{4}$$

where $P_{H,t}$ and $P_{F,t}$ are prices for local and foreign goods respectively

(4) is subject to a Dixit-Stiglitz composite of local and foreign goods (2).

First order conditions of the household's expenditures minimization problem for a given level of consumption provides the following demand function for domestic and foreign goods:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{5}$$

$$C_{F,t} = \alpha(\frac{P_{F,t}}{P_t})^{-\eta} C_t \tag{6}$$

Given the allocation of household's costs for home and foreign goods aggregate Consumer price index is derived:

$$P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{7}$$

The main problem of representative household is to maximize its lifetime utility (1) subject to flow budget constraint(3) and demand functions (5) and (6). The solution of this problem yields next optimality conditions:

$$C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t} \tag{8}$$

$$\tilde{i}_t = \tilde{i}_t^* \frac{\tilde{e}_{t+1}}{\tilde{e}_t} \gamma_{t+1} \tag{9}$$

$$\lambda_t = \beta E_t \left(\lambda_{t+1}(1+\tilde{i}_t) \frac{P_t}{P_{t+1}} \right) \tag{10}$$

where (8) is optimal labor supply scheme, (9) is Uncovered Interest Parity(UIP) condition and (10) is conventional Euler equation, λ_t is a Lagrange multiplier related to budget constraint.

4.2 Definitions of inflation, terms of trade, real exchange rate and deviations from the law of one price

This section summarizes relationships between domestic and foreign prices, exchange rate and the terms of trade. First, I introduce the definition of terms of trade describing price of foreign goods in terms of domestic goods:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{11}$$

Real exchange rate is defined as the relationship between nominal exchange rate, foreign and local price levels:

$$\tilde{q}_t = \frac{\tilde{e}_t}{P_t^*} P_t \tag{12}$$

The law of one price gap is given by:

$$\Psi_{F,t} = \frac{\tilde{e}_t P_t^*}{P_{F,t}} \tag{13}$$

Here I suggest that the law of one price does not hold. This means that price set by retailers for foreign foods is deviated from its price set abroad. The assumption looks reasonable since importing firms sector is monopolistically competitive and thus has a market power to influence the price for goods sold.

CPI, domestic and foreign prices inflation are simply the ratio of current period corresponding price level to the previous period price level.

4.3 International risk sharing and uncovered interest parity condition

Uncovered interest parity condition regulates the dynamics of domestic and foreign interest rates as well as nominal exchange rate. Derivatives of unconstrained household's optimization problem with respect to home and foreign bonds yield UIP condition (9). I define risk premium as the spread between mentioned interest rates. I assume that function governing risk premium is given by the functional form provided in equation (14):

$$\gamma_{t+1} = exp(-\chi A_t - \varepsilon_{st}) \tag{14}$$

where parameter χ reflects the elasticity of the risk premium with respect to foreign debt and ε_{st} is the risk premium shock.

Basic specification does not include separate commodity sector which is thus assumed to be the part of domestic firms overall output. However, since the significant parts of GDP and export comprise of mineral resources exporting revenue, it would be very rough assumption to neglect commodity part of the economy. I introduce shock to commodity price through the risk premium volatility. If commodity price rises then interest rate spread should decline due to decreasing country risk. Additionally, it is pushing down the exchange rate and Tenge is appreciated. Therefore positive commodity price shock is expected to be negative risk premium shock which leads to reduction is risk premium and appreciation of local currency.

4.4 Domestic Firms

All domestic firms are monopolistically competitive. They produce differentiated goods using a labor as input. Individual production function for the firm i is:

$$Y_t(i) = \tilde{\varepsilon}_{at} N_t(i) \tag{15}$$

where $N_t(i)$ is the labor supply of households to the firm i, $\tilde{\varepsilon}_{at}$ is technological innovation or productivity which follows exogenous AR(1) process.

Aggregate production function can be written as:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{16}$$

where ε is the elasticity of substitution between the different varieties.

As long as firms produce differentiated goods, each of them has some market power in setting price. Demand of the households for variety i changes inversely with its price:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \tag{17}$$

Prices are assumed to set in a staggered style, a la Calvo. θ_H is probability that the firm cannot change its price. Let us assume that a firm which can re-charge its price in period t will choose optimal price $\tilde{P}_{H,t}$. Under Calvo-pricing this firm will hold this price in period $t+\tau$ with probability θ_H^τ. Domestic price index under Calvo pricing scheme is:

$$P_{H,t} = [\theta_H P_{H,t-1}^{1-\varepsilon} + (1-\theta_H)\tilde{P}_{H,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{18}$$

So, such firm having an opportunity to change its price in period t will maximize its profit stream, conditional on that price:

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \theta_H^\tau E_t(Q_{t,t+\tau}[Y_{t+\tau}(\tilde{P}_{H,t} - MC_{t+\tau}^n)]) \tag{19}$$

where $MC_t^n = \frac{W_t}{\tilde{\varepsilon}_{at}}$ - nominal marginal costs; $MC_t^r = \frac{W_t}{\tilde{\varepsilon}_{at} P_{H,t}}$ - real marginal costs; $Q_{t,t+\tau}$ - stochastic discount factor for nominal pay-offs for households which is dependent on time. Complete international markets and free capital mobility imply that a full set of international contingent obligations are available for households. Therefore, stochastic discount factor can be expressed as $Q_{t+\tau} = \frac{\lambda_{t+\tau}}{\lambda_t} = \beta^{\tau}\left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+\tau}}$. Since households own and govern the firms, it forces firms to take into account this discount factor in making their decisions. Objective function (19) must satisfy the following demand curve:

$$Y_{t+\tau} \leq \left(\frac{\tilde{P}_{H,t}}{P_{H,t+\tau}}\right)^{-\varepsilon}(C_{H,t+\tau} + C^{*}_{H,t+\tau}) \tag{20}$$

Unconstrained maximization problem of the domestic firms takes the form:

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \theta_H^{\tau} E_t(Q_{t,t+\tau}[\left(\frac{\tilde{P}_{H,t}}{P_{H,t+\tau}}\right)^{-\varepsilon}(C_{H,t+\tau} + C^{*}_{H,t+\tau})(\tilde{P}_{H,t} - MC^{n}_{t+\tau})])$$

By taking FOC of the above equation with respect to $\tilde{P}_{H,t}$ one will obtain:

$$\tilde{P}_{H,t}\sum_{\tau=0}^{\infty} \theta_H^{\tau} E_t(Q_{t,t+\tau}Y_{t+\tau}[(\tilde{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1}MC^{n}_{t+\tau})]) = 0 \tag{21}$$

where $\frac{\varepsilon}{\varepsilon-1}$ is equal to real marginal costs in case of fully flexible prices.

Replacing stochastic discount factor for its consumption-price counterpart one can get:

$$\tilde{P}_{H,t}\sum_{\tau=0}^{\infty} (\beta\theta_H)^{\tau} E_t(P^{-1}_{t+\tau}C^{-\sigma}_{t+\tau}Y_{t+\tau}[(\tilde{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1}MC^{n}_{t+\tau})] = 0 \tag{22}$$

Adding $P_{H,t-1}$ in the above equation gives:

$$\tilde{P}_{H,t}\sum_{\tau=0}^{\infty} (\beta\theta_H)^{\tau} E_t(C^{-\sigma}_{t+\tau}Y_{t+\tau}\frac{P_{H,t-1}}{P_{t+\tau}}\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon-1}\Pi^{H}_{t-1,t+\tau}MC_{t+\tau}\right)) = 0 \tag{23}$$

where $\Pi^{H}_{t-1,t+\tau} = \frac{P_{H,t+\tau}}{P_{t+\tau}}$; $MC_{t+\tau} = \frac{MC^{n}_{t+\tau}}{P_{H,t+\tau}}$

4.5 Domestic retailers

Domestic retailers import foreign goods and sell them in home markets. Retail sector is characterized by monopolistic competition with some degree of market power for each retailer. There are deviations from the law of one price in the short term as mentioned earlier. Domestic retailers similarly follow staggered price setting. Price stickiness parameter for them is θ_F.

Import price index follows the same rule as in case of domestic prices:

$$P_{F,t} = [\theta_F P_{F,t-1}^{1-\varepsilon} + (1-\theta_F)\tilde{P}_{F,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{24}$$

The expected discounted profit stream for a firm that can re-set its price in period t:

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \theta_F^{\tau} E_t(Q_{t,t+\tau}[C_{F,t+\tau|t}(\tilde{P}_{F,t} - \tilde{e}_{t+\tau} P_{F,t+\tau}^{*})]) \tag{25}$$

which is subject to constraint:

$$C_{F,t+\tau|t} = \left(\frac{\tilde{P}_{F,t}}{P_{F,t+\tau}}\right)^{-\varepsilon} (C_{F,t+\tau}) \tag{26}$$

By plugging the above constraint into (25) one will obtain:

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \theta_F^{\tau} E_t(Q_{t,t+\tau} \left(\frac{\tilde{P}_{F,t}}{P_{F,t+\tau}}\right)^{-\varepsilon} [C_{F,t+\tau}(\tilde{P}_{F,t} - \tilde{e}_{t+\tau} P_{F,t+\tau}^{*})]) \tag{27}$$

Taking FOC of the above equation with respect to $\tilde{P}_{F,t}$ yields optimality condition for the local retailers:

$$\sum_{\tau=0}^{\infty}\theta_F^{\tau}E_t(Q_{t,t+\tau}C_{F,t+\tau}(\tilde{P}_{F,t}-\frac{\varepsilon}{\varepsilon-1}\tilde{e}_{t+\tau}P^*_{F,t+\tau}))=0 \tag{28}$$

5 Equilibrium

5.1 Market clearing

Market clearing condition requires equivalence of domestic output and consumption of this output either by local or foreign households:

$$Y_{H,t}=C_{H,t}+C^*_{H,t} \tag{29}$$

where $C^*_{H,t}$ - export demand from the rest of the world.

Left hand side of the equation (29) represents supply of domestic goods while right hand side is demand for domestic goods. Local and foreign consumption is represented by the following functional forms:

$$C_{H,t}=(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t \tag{30}$$

$$C^*_{H,t}=\zeta\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta^*}Y^*_t \tag{31}$$

where ζ - steady-state ratio of exports to GDP in the foreign economy which is introduced in order to obtain sensible log-linearized form of market clearing condition.

I assume that the law of one price holds for import of domestic goods to the foreign economy:

$$P_{H,t} = \tilde{e}_t P_t^* \tag{32}$$

The resource constraint (29) can be rewritten as:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \zeta(\Psi_{F,t} S_t)^{\eta^*} Y_t^* \tag{33}$$

5.2 Monetary policy

Central Bank of Kazakhstan declared that inflation targeting is based on CPI index. Also some interventions to currency market in order to control large fluctuations are possible. The current rule of Central Bank in log-linearized form can be described as:

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \phi_e \triangle e_t + \varepsilon_{\mu,t} \tag{34}$$

where $\phi_i, \phi_\pi, \phi_y, \phi_e$ are parameters defining the response of Central Bank to interest rate, CPI inflation, domestic output and change in nominal exchange rate; $\varepsilon_{\mu,t}$ is exogenous idiosyncratic monetary shock.

Estimated parameter is the coefficient on exchange rate in Taylor rule. If this is not zero, currently Central Bank somehow targets exchange rate. If it is zero, Central Bank does not intervene in foreign currency market.

5.3 Foreign economy block

For the sake of simplicity it is proposed that the output, inflation and interest rate in the rest of the world follows AR(1) process:

1. World output:

$$y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*} \epsilon_{y^*} \tag{35}$$

2. World interest rate:

$$i_t^* = \rho_{i^*} i_{t-1}^* + \sigma_{i^*} \epsilon_{i^*} \tag{36}$$

3. World inflation:

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_{\pi^*} \epsilon_{\pi^*} \tag{37}$$

5.4 Log-Linearized model

Let $x_t = \frac{X_t}{X_{ss}}$, where X_t is the variable of the model and X_{ss} is the steady-state level of that variable. The model can be described through relationship of variables percentage deviations from their steady-state. The summary of log-linearized equations of the model is provided below:[2]

1) Euler equation:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\}) + \varepsilon_{g,t} \tag{38}$$

2) Terms of trade (difference equation):

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} \tag{39}$$

3) Law of one price gap:

$$\psi_{F,t} = q_t - (1 - \alpha)s_t \tag{40}$$

[2]The derivations for log-linear equations are provided in Appendix A

4) Domestic firms optimal prices:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}[\sigma c_t + \phi y_t - \varepsilon_{a,t}(1+\phi) + \alpha s_t] \qquad (41)$$

5) Retailers optimal prices:

$$\pi_{F,t} = \beta E_t\{\pi_{F,t+1}\} + \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}\psi_{F,t} + \varepsilon_{cp,t} \qquad (42)$$

6) CPI inflation and domestic inflation relation:

$$\pi_t = \pi_{H,t} + \alpha(s_t - s_{t-1}) \qquad (43)$$

7) UIP condition:

$$i_t - i_t^* = E_t\{\pi_{t+1}\} - E_t\{\pi_{t+1}^*\} + E_t\{q_{t+1} - q_t\} - \chi a_t - \varepsilon_{rp,t} \qquad (44)$$

8) Domestic goods market clearing condition:

$$y_t = (1-\alpha)c_t + \alpha\eta s_t(2-\alpha) + \alpha\eta\psi_{F,t} + \alpha y_t^* \qquad (45)$$

9) Flow budget constraint:

$$c_t + a_t = \frac{1}{\beta}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t \qquad (46)$$

10) Taylor rule:

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \phi_e(e_t - e_{t-1}) + \epsilon_{m,t} \tag{47}$$

11) Nominal exchange rate:

$$e_t = e_{t-1} + (q_t - q_{t-1}) + \pi_t - \pi_t^* \tag{48}$$

12) Foreign economy block - world output:

$$y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*} \epsilon_{y^*} \tag{49}$$

13) Foreign economy block -world interest rate:

$$i_t^* = \rho_{i^*} i_{t-1}^* + \sigma_{i^*} \epsilon_{i^*} \tag{50}$$

14) Foreign economy block - world inflation:

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_{\pi^*} \epsilon_{\pi^*} \tag{51}$$

The log-linearized model can be expressed as linear rational expectations system and solved using Uhlig's toolkit, Blanchard and Kahn or Sims algorithms [3]:

$$x_t = A x_{t-1} + B \varepsilon_t \tag{52}$$

[3] I utilize generalized Schur decomposition which is a variation of the methods proposed by Sims, Uhlig and Klein. Schur decomposition or QZ decomposition is by default solver of rational expectation models in Dynare software.

$$y_t = Cy_{t-1} + D\varepsilon_t \tag{53}$$

$\varepsilon_t \sim N(0, \Sigma)$

where:

1) x_t is the vector of endogenous state variables $\{y \ i \ s \ \psi_{F,t} \ a \ y^* \ i^* \ e\}$;

2) y_t is the vector of endogenous jumpy variables $\{c \ \pi \ \pi_H \ \pi_F \ \pi^* \ q\}$;

3) ε_t is vector of fundamental shocks $\{\epsilon_{a,t} \ \epsilon_{m,t} \ \epsilon_{g,t} \ \epsilon_{rp,t} \ \epsilon_{cp,t} \ \epsilon_{r^*} \ \epsilon_{a^*} \ \epsilon_{\pi^*}\}$ describing domestic technology shock, monetary shock, preference shock, risk premium shock, cost-push shock, foreign interest rate shock, foreign output shock, foreign inflation shock respectively.

6 Extension to the Basic Model. Inclusion of commodity sector

Kazakhstan can be considered as a commodity exporting country which is vulnerable to this commodity's price fluctuations. Basic model assumes that commodity sector is incorporated in domestic firms production which is a rough simplification. Therefore in this section commodity sector is introduced as endowment flow to consumers and appears in household's budget constraint. Such assumption seems to be realistic as long as Kazakhstan mainly exports raw materials extracted from the earth without any value added processing.

Other things being equal, new model deviates from the basic one in the following points:

1. There is no separate production function. Commodity export is given as a long-

term fraction of GDP which is subject to global demand shocks that simultaneously affect the volume of export. A corona-virus can be thought as a kind of such shock that reduced demand for crude oil worldwide.

2. Aggregated output then consists of goods produced by domestic firms together with export of commodity:

$$GDP_t = Y_t + Com_t$$

where Com_t represents the volume of commodity export. For the sake of simplicity this variable is defined as long-run share of GDP level δGDP_{ss} which is subject to global commodity demand shock $\varepsilon_{co,t}$. Parameter δ is calibrated to fit long-run share of commodity export in GDP. Log-linearization of the above condition yields in adjusted good market clearing condition (45):

$$gdp_t = (1-\delta)y_t + \delta\varepsilon_{co,t} \tag{54}$$

where y_t is production of non-commodity goods by domestic firms; $\varepsilon_{co,t}$ is AR(1) process for exogenous global demand shock;

3. Flow budget constraint (3) now includes revenue stream obtained from the export of raw materials. Such revenue is subject to exogenous shock to oil price. The household resources fluctuate depending on the this shock and global commodity demand shock:

$$P_tC_t + D_t + \tilde{e}_tB_t = D_{t-1}\tilde{i}_{t-1} + \tilde{e}_tB_{t-1}\tilde{i}^*_{t-1}\gamma_t + \Pi_{H,t} + \Pi_{F,t} + W_tN_t + T_t + P_{co,t}Com_t \tag{55}$$

where $P_{co,t}$ is commodity price in period t which is given exogenously and follows AR(1) process.

Log-linear approximation of the above constraint states that net foreign asset obtained as a fraction of steady-state output aggregates foreign assets interest income from the previous period, domestic firms output, commodity exporting income net of domestic households' consumption which is varied with deviation from the law of one price, terms of trade and economy's openness parameter:

$$a_t = \frac{1}{\beta}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t - c_t + \delta(p_{co,t} + \varepsilon_{co,t}) \tag{56}$$

$$\varepsilon_{co,t} = \rho_{\varepsilon,co}\varepsilon_{co,t-1} + \sigma_{\varepsilon,co}\epsilon_{co,t} \tag{57}$$

$$p_{co,t} = \rho_{p,co}p_{co,t-1} + \sigma_{p,co}\epsilon_{p_{co},t} \tag{58}$$

7 Estimation

7.1 The Bayesian approach

The Bayesian method is generally accepted as the most spread way to estimate DSGE models. Schorfheide and An (2007) argue that Bayesian estimation is favorable in comparison with other methods due to the following reasons: first, solved DSGE model is matched to a vector of variables from data time series; second, DSGE generates likelihood function which is then is incorporated in the estimation; third, prior distribution of parameters is added as a source of additional information in estimation process.

Prior distribution for a vector of parameters $p(\mu)$ is set by the researcher. Then given data, likelihood function is determined. Posterior distribution which is defined as interaction between likelihood function and prior density contains the final distribution of the parameters vector:

$$p_1(\mu|X_{obs}) = \frac{p_0(\mu)p(X_{obs}|\mu)}{p(X_{obs})}$$

where $p_0(\mu)$ is prior distribution of a vector of parameters; $p(X_{obs}|\mu)$ is likelihood function or density of the observed sample conditional on parameter value; $p(X_{obs})$ is the marginal density of the sample. The posterior distribution is proportional to the product of prior distribution and likelihood function:

$$p_1(\mu|X_{obs}) \propto p_0(\mu)p(X_{obs}|\mu)$$

The estimation of the model is performed in Dynare add-in of the Matlab. The estimation steps can be generalized as:

1. Log-linearized model from the previous section is solved in the form of state-space representation (52) and (53).

2. The likelihood for a given observable variables are computed using Kalman filter and Gaussian shocks assumption.

3. Posterior kernel density is obtained as the product of likelihood function and prior distribution of parameters.

4. Steps 2 and 3 are iterated up to maximization of posterior kernel density.

5. Metropolis Hastings algorithm is used where Markov chain for μ is generated such that distribution of parameters converges to posterior distribution.

The main idea of Metropolis Hastings method lies in comparison of target or posterior parameters distribution and log likelihood of the model associated with the state-space representation of the model. Initial value for the vectors of parameters μ is chosen through the maximization of posterior kernel density. Then a new draw of parameters is obtained through the random walk:

$$\mu^{new} = \mu^{old} + e$$

where μ^{old} equals the initial value for the first draw and $e \sim N(0, \Sigma)$.

Target or posterior distribution is calculated as the sum of log likelihood and log priors. Acceptance probability is derived through the minimizing the ratio of new and old densities and is compared to the target distribution. If acceptance probability is lower than a number u drawn from a standard uniform distribution, we accept μ^{new}. Otherwise μ^{old} is kept and proceed iterations M times which is set by researcher.

7.2 Prior distribution of the parameters

Similar to Schorfheide and Del Negro (2008) I identify prior distributions under subjective assumptions. Priors can alternatively be determined from data sets excluded from the estimation sample.

Some parameters that can be derived from steady state interactions among observables are calibrated. Following Kydland and Prescott (1982), long-run averages of macroeconomic variables are commonly used to get steady-state parameters. So I calibrate discount rate β, the degree of economy's openness α and elasticity of risk premium with respect to foreign net asset χ along the lines of Justiniano and Preston(2009). Discount rate β is calibrated based on annualized real interest rate:

$$\beta = \frac{1}{r^*/4}$$

Collecting and further processing the data for inflation and Central Bank interest rate from National Statistics Bureau of Kazakhstan have enabled to calibrate the value for β on the level of 0.99.

I next calibrate α or the economy openness parameter as the average of export and import shares (sum of exports and imports is divided by double output). I obtain the data for import, export and GDP for the period from 2000 to 2019 and calibrate the value of α on the level of 22%.

I identify other parameters for endogenous variables and structural shocks based on the proposition that researcher may use different information sources to assist in building priors for these parameters, suggested by Schorfheide and Del Negro (2008). The degree of price rigidity or labor supply elasticity can occur from micro-level studies on price behavior or labor market. However in choosing priors I follow some recommendations from the previous studies. First, according to Blake A. and Mumtaz H. (2017) if a researcher has high degree of certainty in defining the prior for parameter, she is able to choose tight prior variance. Otherwise, more loose prior should be selected. Since my model specification was not previously solved and estimated for Kazakhstan, I prefer to approach higher levels of prior variances than in DSGE models studies of country's economy by other researchers. Second, distributions for parameters are determined in line with Justiniano and Preston (2009) given that my model is mainly based on their specification. I use priors for constant relative risk aversion and Frisch elasticity of labor supply parameters from the work of Adilkhanova (2020) where she calibrates parameters based on micro-data. Tables 3 and 4 summarizes prior distributions, means and standard deviations of the model's parameters:

Table 3: Prior distribution, means and standard deviations of parameters for basic model

Parameter	Notation	Distribution	Mean	Std
Constant relative risk aversion	σ	Gamma	2.71	0.50
Frisch elasticity of labor supply	ϕ	Normal	1.98	0.50
Elasticity domestic/import goods	η	Gamma	1	0.50
Calvo parameter for domestic firms	θ_H	Beta	0.5	0.10
Calvo parameter for retailers	θ_F	Beta	0.5	0.10
Smoothing parameter in Taylor rule	ϕ_i	Beta	0.50	0.25
Inflation parameter in Taylor rule	ϕ_π	Gamma	1.5	0.30
Output parameter in Taylor rule	ϕ_y	Gamma	0.5	0.25
Exchange rate parameter in Taylor rule	$\phi_{\triangle e}$	Gamma	0.42	0.10
Inertia of the technology shock	ρ_a	Beta	0.8	0.10
Inertia of the risk premium shock	ρ_{rp}	Beta	0.8	0.10
Inertia of the cost push shock	ρ_{cp}	Beta	0.5	0.25
Inertia of the preference shock	ρ_g	Beta	0.8	0.10
Foreign output inertia	ρ_{y^*}	Beta	0.8	0.10
Foreign inflation inertia	ρ_{π^*}	Beta	0.5	0.10
Foreign interest rate inertia	ρ_{i^*}	Beta	0.8	0.10
Foreign output sd	σ_{y^*}	Invg	0.8	0.20
Foreign inflation sd	σ_{π^*}	Invg	0.8	0.20
Foreign interest rate sd	σ_{i^*}	Invg	0.8	0.20
Technology shock sd	σ_a	Invg	0.8	0.20
Monetary shock sd	σ_m	Invg	0.8	0.20
Risk premium shock sd	σ_{rp}	Invg	0.8	0.20
Cost-push shock sd	σ_{cp}	Invg	0.8	0.20
Preference shock sd	σ_g	Invg	0.8	0.20

Table 4: Prior distribution, means and standard deviations of parameters for model with commodity sector

Parameter	Notation	Distribution	Mean	Std
Inertia of commodity price shock	ρ_{co}	Beta	0 8	0 10
Inertia of global demand for commodity shock	ρ_{com}	Beta	0 8	0 10
Commodity price shock sd	σ_{co}	invg	0 8	0 20
Global demand for commodity shock sd	σ_{com}	invg	0 8	0 20

7.3 Measurement equations and data processing

The second part of posterior distribution estimation is defining the likelihood function based on the density of the observed variables. The mapping between data and DSGE model is implemented through the following measurement equations:

$$x_t = g(x_{t-1}, \varepsilon_t^{str})$$

$$X_t^{obs} = h(x_t, \varepsilon_t^{obs})$$

The former equation is called state-transition equation which describes the process for state variables x_t based on past state of the system and realization of structural shocks ε_t^{str} through the policy function g. I have already obtained this set of equations by log-linearizing the equilibrium equations of the model. The latter is called observation equation and represents how observed from the data variables map into the state variables taking into account possible measurement error ε_t^{obs}. As long as the model is log-linearized, functions g and h are linear. One of the main issues in building measurement equations is that model variables like output is stationary while observed GDP is non-stationary due to growth trend. In order to avoid stochastic singularity we should apply as many observables as shocks in the model. Since the data can be poorly measured we should add measurement error term in each equation.

I use quarterly series for the period from 2007 to the third quarter of 2019. I choose the data on GDP per capita, CPI inflation, KIBOR interest rate, nominal exchange rate and terms of trade as my observables by pursuing Justiniano and Preston (2009). Data on output, inflation and terms of trade are obtained from Bureau of National Statistics of Kazakhstan[4] while interest and exchange rates come from the official website of Central Bank of Kazakhstan[5].

[4]Quarterly time series come from the official website of Bureau of National Statistics of Kazakhstan https://stat.gov.kz/

[5]Monthly time series for US Dollar to Kazakhstani Tenge exchange rate and KIBOR interest rate are provided by the official website of Central Bank https://nationalbank.kz/en

The output and inflation of foreign block is represented by the GDP per capita and CPI inflation of Russia. The statistics are produced from Federal State Statistic Service of Russia[6]. I assume interest rate to be proxied by short term US interest rate.

To construct the observation equations I follow instructions proposed by Pfeifer J. (2020) and Justiniano and Preston (2009):

1. Output in my model is variable y which is a percentage deviation from the steady state and has mean zero. Output from the data is a growing GDP variable. Since the model does not provide the trend explicitly I use output growth. To match these 2 variables I need to take the logarithm of the deseasonalised GDP per capita, detrend it using one-sided Hodrick-Prescott filter and find the first difference. The overall process is described in the following equation:

$$y_t^{obs} = log(y_t^{data}) - log(y_{t-1}^{data}) - \frac{\sum_{t=1}^{k}[log(y_t^{data}) - log(y_{t-1}^{data})]}{k} = y_t - y_{t-1} + \epsilon_{y,me}$$

where k is the last period of the time series.

2. Observed inflation is a first difference of logarithm of Consumer Price Index which has been annualized, π_t is the percentage deviation of the CPI inflation from its steady state. Measurement equation for inflation is:

$$\pi_t^{obs} = log(\pi_t^{data}) - log(\pi_{t-1}^{data}) - \frac{\sum_{t=1}^{k}[log(\pi_t^{data}) - log(\pi_{t-1}^{data})]}{k} = (\pi_t - \pi_{t-1}) * 4 + \epsilon_{\pi,me}$$

3. In the model i_t is the nominal quarterly percentage deviation of the interest rate

[6]Quarterly data are obtained from the ofiicial website of Federal State Statistic Service of Russia https://eng.gks.ru/

from its steady state. To map observed interest rate we should adjust for the time period:

$$i_t^{obs} = log(i_t^{data} + 1) - \frac{\sum_{t=1}^{k} log(i_t^{data} + 1)}{k} = 4 * i_t + \epsilon_{i,me}$$

4. My measurement equation for the real exchange rate is:

$$q_t^{obs} = log(q_t^{data}) - log(q_{t-1}^{data}) - \frac{\sum_{t=1}^{k}[log(q_t^{data}) - log(q_{t-1}^{data})]}{k} = q_t - q_{t-1} + \epsilon_{q,me}$$

5. Similarly to Tolepbergen A. (2020) I consider that terms of trade is an inverse IMF commodity terms of trade index. Each commodity is weighted as a share of net export in annual output:

$$\omega_{i,j,t} = \frac{x_{i,j,t} - m_{i,j,t}}{GDP_{i,t}}$$

where $x_{i,j,t}$ is export value of commodity j of country i in year t denoted in US dollars; $m_{i,j,t}$ is import value of commodity j of country i in year t denoted in US dollars; $GDP_{i,t}$ is gross domestic product of country i in year t denoted in US dollars.

Then all weights are aggregated in one commodity terms of trade index s_t^{Co}. In my model $s_t = p_{F,t} - p_{H,t}$. So I directly match these variables and find first difference in data and model:

$$s_t^{obs} = log(s_t^{Co}) - log(s_{t-1}^{Co}) - \frac{\sum_{t=1}^{k}[log(s_t^{Co}) - log(s_{t-1}^{Co})]}{k} = s_t - s_{t-1} + \epsilon_{s,me}$$

where s_t^{Co} is observed commodity terms of trade index.

The foreign block nominal interest rate, output and inflation are treated in the same way as domestic nominal interest rate, output and inflation respectively.

6. Specification of the model with commodity sector also includes time series for

commodity price index which is proxied by IMF constructed commodity export price index weighted by GDP:

$$p_{t,co}^{obs} = log(p_{t,co}^{data}) - log(p_{t-1,co}^{data}) - \frac{\sum_{t=1}^{k}[log(p_{t,co}^{data}) - log(p_{t-1,co}^{data})]}{k} = p_{t,co} - p_{t-1,co} + \epsilon_{p_{co},me}$$

7.4 The results of Estimation

Utilizing quarter series of data and prior distributions of the model's parameters log posterior is maximised and modes are generated. Then Metropolis Hastings Algorithm is applied to create 5 parallel chains with 500,000 draws within each. Scaling factor is adjusted to obtain current acceptance ratio of each chain within 20% - 40% range which is assumed to be reasonable in Bayesian estimation literature. For all chains convergence is detected both for univariate and multivariate cases.

First, let us examine the values for basic specification of the model. For point estimates of parameters posterior mode is used as central tendency of distribution. The estimated parameters are presented in Table 5. Intertemporal elasticity of substitution turns out to be considerably smaller than its prior taking value of 1. This coefficient tends to be at the same level as in developed economies indicating that households in Kazakhstan are moderately risk averse. The inverse elasticity of labor supply takes value of 2.69 identifying non-elasticity of labor supply which is confirmed by microeconomic studies. The posterior mode for elasticity of substitution between export and import good is less than its prior. This presumably happen due to the commodity exporting nature of Kazakhstan's economy and high dependence on imported goods. Prices for imported goods demonstrate re-optimizing every 3 quarters while prices for domestic goods are adjusted more frequently being changed every 1.5 quarters on average. Policy parameters distribution from Taylor rule demonstrate the following behavior. It seems that interest rate smoothing is a significant policy purpose in Kazakhstan's monetary policy. Posterior mode takes value of 0.84 for lagged interest rate. The estimated response to output is relatively modest which is consistent with previous findings. The parameter on nominal exchange rate movements achieves level of 0.15 which clearly supports the idea of Central Bank's response to exchange rate.

Parameters posterior modes obtained from estimation of model specification with commodity sector are consistent with ones from basic model estimation. Most of the indicators are the same for both specifications or have a slight difference reflecting the robustness of the outcome. However, estimation results indicate that the value of Taylor-rule output reaction parameter is considerably higher in case of commodity enriched model.

Table 5: Estimation summary

Parameter	Distribution	Prior Mean	Prior Std	Basic		Commodity	
				Posterior Mode	Posterior Std	Posterior Mode	Posterior Std
σ	Sigma	2.71	0.50	1.01	0.22	0.87	0.26
ϕ	Normal	1.98	0.50	2.69	0.63	2.48	0.52
η	Gamma	1	0.50	0.56	0.07	0.60	0.05
θ_H	Beta	0.5	0.10	0.35	0.06	0.41	0.06
θ_F	Beta	0.5	0.10	0.67	0.06	0.52	0.09
ϕ_i	Beta	0.50	0.25	0.84	0.12	0.89	0.13
ϕ_π	Gamma	1.5	0.30	0.95	0.24	0.89	0.19
ϕ_y	Gamma	0.5	0.25	0.31	0.19	0.63	0.26
$\phi_{\triangle e}$	Gamma	0.42	0.10	0.15	0.04	0.14	0.04
ρ_a	Beta	0.8	0.10	0.83	0.09	0.87	0.07
ρ_{rp}	Beta	0.8	0.10	0.89	0.03	0.81	0.05
ρ_{cp}	Beta	0.5	0.25	0.63	0.11	0.92	0.09
ρ_g	Beta	0.8	0.10	0.68	0.10	0.64	0.11
ρ_{y^*}	Beta	0.8	0.10	0.91	0.06	0.86	0.08
ρ_{π^*}	Beta	0.5	0.10	0.47	0.09	0.46	0.08
ρ_{i^*}	Beta	0.8	0.10	0.76	0.10	0.75	0.11
σ_{y^*}	Invg	0.8	0.20	0.70	0.13	0.64	0.12
σ_{π^*}	Invg	0.8	0.20	1.13	0.12	1.13	0.13
σ_{i^*}	Invg	0.8	0.20	0.38	0.04	0.38	0.04
σ_a	Invg	0.8	0.20	0.63	0.12	0.68	0.14
σ_m	Invg	0.8	0.20	1.61	0.43	1.51	0.31
σ_{rp}	Invg	0.8	0.20	0.92	0.18	0.75	0.19
σ_{cp}	Invg	0.8	0.20	3.25	0.64	4.27	1.16
σ_g	Invg	0.8	0.20	0.52	0.08	0.54	0.08
ρ_{co}	Invg	0.8	0.10	-	-	0.91	0.04
ρ_{com}	Invg	0.8	0.10	-	-	0.90	0.10
σ_{co}	Invg	0.8	0.20	-	-	4.85	0.52
σ_{com}	Invg	0.8	0.20	-	-	0.80	0.05

To assess whether the generated model fit the real data I compare the moments from time series used for estimation and time series simulated from the model. The results are presented in Table 6:

Table 6: Moments Summary

Variable	St.dev. model (basic)	St.dev. model (commodity)	St.dev. data
Output growth	1.25	1.19	0.98
Real exchange rate change	6.90	6.63	6.30
CPI inflation	8.33	8.11	6.92
Terms of trade change	5.42	6.12	5.19
Nominal interest rate	3.92	3.98	3.99
Foreign output growth	1.25	1.23	1.24
Foreign CPI inflation	5.51	5.51	6.09
Foreign nominal interest rate	2.53	2.50	1.10

It can be seen that both specifications provide almost the same volatility as in real data. I, therefore, conclude that DSGE model slightly overestimates home CPI inflation fluctuations.

7.5 Variance decomposition and impulse response analysis. Business cycles and shocks

This section provides variance decomposition for both specifications and identifies which shocks drive business cycle in small open economy model of Kazakhstan. Tables 7 and 8 summarize the variance decomposition for basic and commodity enriched models, respectively:

Table 7: Variance decomposition of selected variables under basic specification in %

Variable	Innovation	Monetary	Risk Premium	Cost-Push	Foreign Output
Output	70.73	6.40	6.78	14.17	0.94
Consumption	3.72	1.26	51.07	30.56	8.07
Interest rate	14.32	3.90	37.14	13.13	1.31
s_t	2.34	0.82	38.10	46.01	10.25
$\triangle q_t$	1.26	1.05	70.55	13.24	1.61
$\triangle e_t$	0.79	7.75	65.56	10.46	1.42
CPI inflation	6.99	60.07	16.40	3.11	1.74

Table 8: Variance decomposition of selected variables under commodity enriched specification

Variable	Innovation	Monetary	Commodity Price	Risk Premium	Cost-Push	Global Demand
Output	42.01	4.90	13.44	1.05	21.95	15.41
Consumption	1.20	0.42	64.44	4.44	24.74	2.84
Interest rate	23.22	5.67	2.91	22.75	20.55	5.37
s_t	0.87	0.13	65.13	2.99	26.34	2.82
$\triangle q_t$	1.34	1.27	27.90	24.42	33.58	1.17
$\triangle e_t$	0.94	7.52	24.62	24.25	28.42	1.73
CPI inflation	13.31	42.29	11.15	3.41	20.41	2.63

What is easily observable is the risk premium shock's dominant role in basic model and its considerable limitation in case of commodity economy specification. Such change in response of main macroeconomic variables is explained by the introduction of commodity global demand and price shocks which absorb part of fluctuations. The volatility in difference between domestic and foreign interest rates is likely to be main driving force for consumption, interest rate, change in real and nominal exchange rates for basic model. Risk premium shock acts as a proxy for volatility in commodity market. I compare the results of impulse response analysis on the Figure 4 to business cycles patterns of Kazakhstan given in section 3 in order to assess whether the model provides similar behavior. Positive risk premium shock is assumed to be either increase in commodity price or demand resulting in reduction of interest rates spread. Increase in local interest rate substantially reduces nominal and real exchange rate which is equivalent to appreciation of the home currency. Nominal exchange rate falls sharply in response to risk premium shock. Appreciation of currency leads to reduction of CPI inflation and rise in consumption. Therefore, consumption, exchange rate and interest rate follows common for Kazakhstan business cycle paths. Output seems to be countercyclical to risk premium shock which contradicts to empirical evidence.

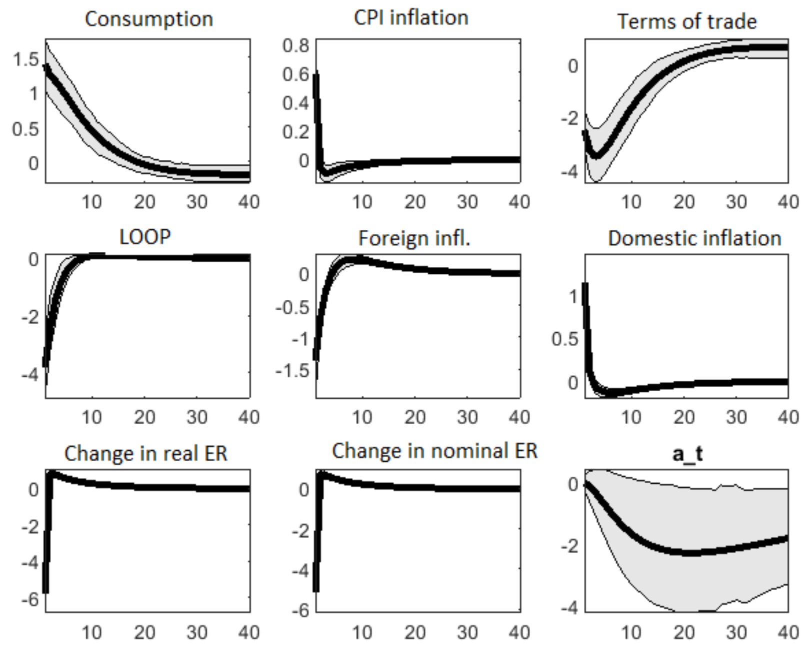
For the model with endowment flow to households from commodity export, commodity price shock reasonably becomes primary driver for business cycles. Price shock accounts for 13% variability in output, almost 65% in consumption and about 25% for both nominal and exchange rates. Inspection of IRFs yields a result consistent with variance decomposition analysis. Gross domestic product according to log-linearized equation (54) is subject to two counterforces:

$$gdp_t = (1-\delta)((1-\alpha)c_t + \alpha\eta s_t(2-\alpha) + \alpha\eta\psi_{F,t} + \alpha y_t^*) + \delta\varepsilon_{co,t} \tag{59}$$

First, increase in home consumption is a response to income shock since commodity export revenue comes as endowment. Such positive demand shock boosts prices for home goods. Therefore, domestic firms react by rising production and consequently GDP. Second, appreciation of domestic goods subsequently deteriorates terms of trade and reduces foreign demand for home goods. The magnitude of these counterforces depends on elasticity of substitution between home and domestic goods, deviation of law of one price and terms of trade. IRF shows that the former effect dominates and positive commodity shock eventually will push overall output of the economy (Figure 5). The effect on interest rate is ambiguous. While on the one hand, positive commodity price shock stimulates foreign net asset to grow pressing down interest rate, domestic and CPI inflation, on the other hand, increase significantly and Central Bank imposes restraining policy to raise the interest rate. Overall effect of commodity price shock on interest rate thus is netted off by opposing effects.

Figure 4: Impulse response of main macroeconomic variables to risk premium shock in basic model

(a)



(b)

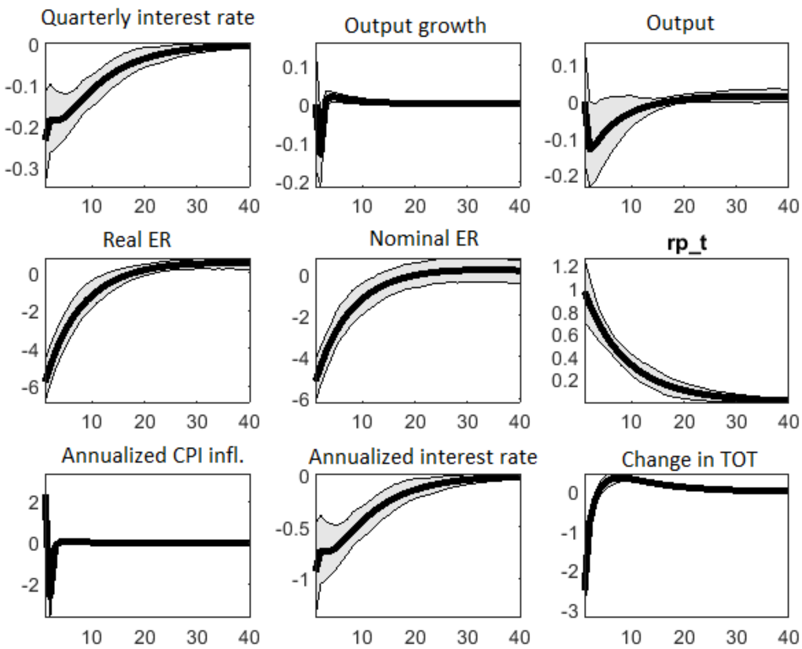
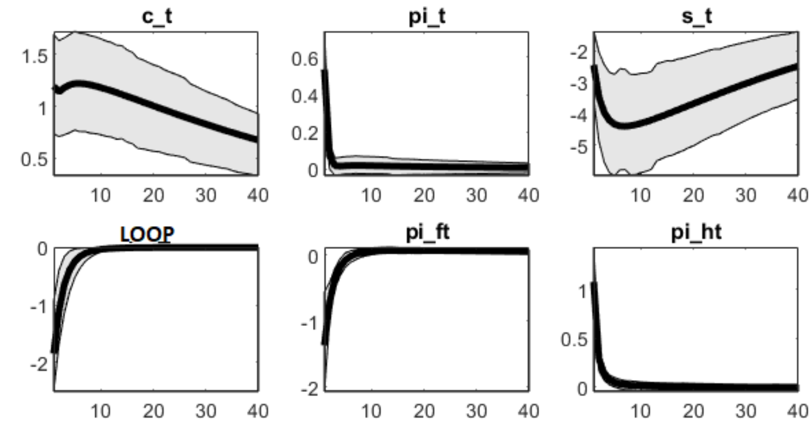
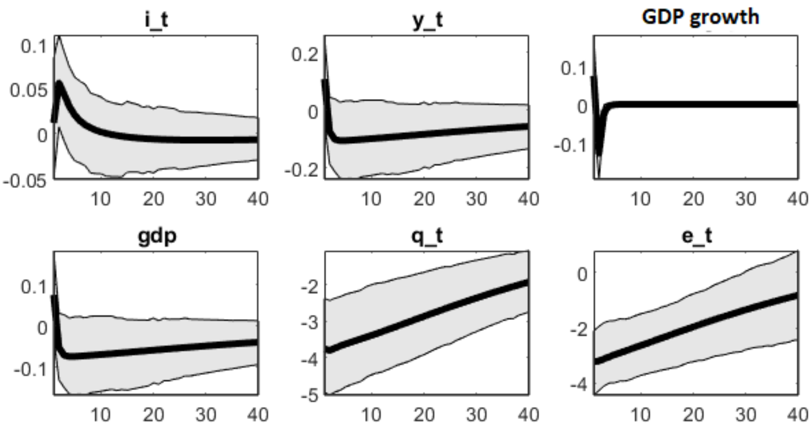


Figure 5: Impulse response of main macroeconomic variables to commodity price shock

(a)



(b)



Impulse response analysis therefore confirms the findings of section 3 that commodity price is procyclical to GDP. All other variables move in accordance with patterns obtained in section 3, except for interest rate which is almost not sensitive to movements in commodity price.

8 Policy implications of alternative exchange rate regimes

The purpose of this section is to evaluate the performance of the economy under alternative monetary policy regimes. I assess the performance based on the volatility of main macroeconomic variables - output, inflation, interest and exchange rates. Simulation of alternative monetary regimes is implemented through the parametrization of the monetary rules leaving the remaining coefficients as in estimated model (41)-(54). Estimation of the model using Bayesian approach provides a full posterior distribution $p(\mu|X_{obs})$. I divide the set of parameters into two groups - policy and non-policy parameters. Policy parameters μ_p are these from the Taylor rule which are assumed to being affected by the authority, while non-policy ones μ_{np} are the rest estimated coefficients which are assumed not to be influenced by the authority.

I evaluate the following policy regimes: pure CPI inflation targeting (CIT), policy following Taylor rule accounting for change in nominal exchange rate (CITER) and strict nominal exchange rate targeting which is also known as pegging. For CIT Taylor rule (10) is simplified by attaching a zero value to parameter on exchange rate changes ϕ_e. Policy parameters then are ϕ_y, ϕ_i and ϕ_π. CITER is described by my model's Taylor rule (10) without any modicfications. In case when monetary policy objective is to target nominal exchange rate monetary policy rule follows:

$$i_t = \phi_i i_t + \phi_e \triangle e_t + \epsilon_{m,t}$$

This rule considers only change in nominal exchange rate by fixing the remaining parameters ϕ_π, ϕ_y as zero.

Table 9 summarizes the definition of monetary policy regimes with policy parameters being calibrated for pegging and pure CPI inflation targeting and estimated for managed float:

Table 9: Regimes description

Regime	ϕ_i	ϕ_π	ϕ_y	$\phi_{\triangle e}$
CIT	0.84	1.5	0.31	0
CITER	0.84	0.95	0.31	0.15
PEG	0.84	0	0	1.5

I calibrate inflation targeting policy parameters in line with the announcement of the Central Bank of Kazakhstan which declared annual inflation band at 4-6%. To keep the volatility of exchange rate within 2%, the policy parameter ϕ_e for pegging is set at the level of 1.5. For each of the monetary rule I simulate the model and assess the macroeconomic performance based on the variability of inflation, output, domestic interest rate and nominal exchange rate. In order to assess the implementation of different policies standard deviations of variables of interest are computed and reflected in Table 10.

Table 10: Standard Deviation of Model Variables

Variable	Basic specification			commodity specification		
	CIT	CITER	PEG	CIT	CITER	PEG
Output	1.24	1.19	1.43	2.21	1.32	2.25
Nominal interest rate	3.57	3.92	8.47	3.58	3.98	8.44
Change in nominal exchange rate	7.65	6.58	1.27	6.50	6.63	1.03
CPI inflation	5.35	8.19	24.55	4.11	8.11	21.52

Overall, the presence of commodity sector provides higher volatility for output. Pegging delivers the most stable nominal exchange rate however at expense of extremely high CPI inflation and interest rate reflecting the trade-off between these volatilities. CPI strict targeting provides stable inflation and output and is followed by some moderate fluctuations in exchange rate. As a result, it is not clear which regime reaches the smallest welfare loss due to main variables instability trade-offs. To define best-performing monetary policy, loss function aggregating weighted sum of variances is imposed. Policy parameters are chosen to minimize the quadratic loss function given all other coefficients take their posterior mode values:

$$\text{min}W_0(\mu_p) = \lambda_\pi var(\pi_t) + (1-\lambda_\pi)var(y_t) + \frac{\lambda_\pi}{4}var(\triangle e_t) + \frac{\lambda_\pi}{4}var(i_t) \qquad (60)$$

The functional form of the loss function is a common and accounts for fluctuations in output, interest rate, inflation and nominal exchange rates. I assume that weight for λ_π is identified as ratio of inflation in output and inflation stabilization of the estimated Taylor rule and equals to 0.75 for basic and 0.59 for commodity models. Therefore, this exercise allows to determine optimal policy as:

$$\mu_p^* = \text{argmin}W_0(\mu_p|\overline{\mu}_s)$$

where $\overline{\mu}_s$ is non-policy parameters from Bayesian estimation. Also restrictions are placed to policy parameters: $0 < \phi_\pi < 1.5$, $0 < \phi_e < 1.5$, $0 < \phi_i < 1.5$ and $0 < \phi_y < 1$. Table 11 provides the results for optimal policy parameters obtained from minimization problem given in equation (60). The outcome shows that accounting for exchange rate fluctuations predictably excludes neglecting currency rate changes in Taylor rule. Coefficients for exchange rate changes take value of approximately 0.45 and 0.66 letting nominal exchange rate to fluctuate within 6% and 5% bands respectively. In addition, policy assumes superinertial interest rate and high coefficient on CPI inflation.

Table 11: Taylor rule coefficients for optimal monetary policy

Model specification	ϕ_i	ϕ_π	ϕ_y	ϕ_e
Basic	1	1.5	0.07	0.45
Commodity	1	1.5	0.11	0.66

Table 12: Simulation analysis results

Regime	Welfare loss, basic	Welfare loss, commodity
CIT	35.26	26.28
CITER	61.46	65.68
PEG	466.26	400.24
Optimal policy, basic	10.33	-
Optimal policy, com.	-	7.29

For both basic and commodity specifications welfare analysis reveals that optimal policy is described by Taylor rule which includes exchange rate growth. Fixing exchange rate delivers the highest welfare loss and thus is evaluated as the most inefficient policy among other possibilities.

It is worth to mention that the results are obtained under assumption of perfect capital mobility. The model does not include capital or investments which is the serious simplification. Economic literature commonly describes capital markets for developing and emerging economies to be immature and underdeveloped. So it might be questionable to assume perfect capital mobility in case of commodity exporting economy like Kazakhstan. Moreover, it seems that in many aspects the country has limited access to international capital markets. For my study this finding is important to the extent it impacts the results obtained. Does managing exchange rate still provide the lowest level of welfare losses? Husain el al. (2004) suggest that developing markets better adopt pegging of their currency: this results in both lower inflation and stable and long-term exchange rate regime. In addition, the richer the country the more flexible exchange rate regime is optimal. In this respect perfect capital mobility should favor the more flexible policy.

Also Aizenman et al. empirically come to the outcome that weak capital markets in commodity intensive economies imply inadequate and poor response to macroeconomic shocks, especially commodity price shock when stabilizing output. Eventually strong evidence is found that commodity exporters using inflation targeting policy also target exchange rate. Therefore, based on the above considerations it is most probably that inclusion of capital in the model should strengthen the obtained result towards targeting exchange rate. The extension of my model to the one that contains capital could be the subject for future studies.

9 Conclusion

In 2015, Central Bank of Kazakhstan abandoned pegging of nominal exchange rate and attached to new monetary policy - CPI inflation targeting regime. Rationality of this decision is questionable and has raised debates among local economists. Recent five years were marked of economic deterioration of main macroeconomic variables of Kazakhstan. Thus the motivation of this research is to analyze whether the current monetary policy regime is optimal.

For this purpose, I present DSGE model in two specifications. The basic is built on new-keynesian framework following Justiniano and Preston (2009) and Galli and Monacelli (2005). The second model represents the extension to basic specification by including commodity sector as endowment export revenue accrued to households. Using data on main macroeconomic variables of Kazakhstan, I further estimate parameters of the models by the Bayesian technique . In general, extended specification better replicates business cycles patterns of Kazakhstan and real data first moments. Based on estimated parameters the findings on optimal monetary policy are consistent in both specifications.

First, Central Bank currently responses to exchange rate movements. Despite the small value of the parameter on exchange rate reaction, it is significant and robust. Second, estimated Taylor rule coefficients are not optimal and switching to policy neglecting nominal exchange rate and assigning more control over inflation reduces welfare loss. Finally, when welfare loss function method is approached, inflation targeting regime with considerable response to exchange rate is defined as optimal. Moreover, considering a strong reaction to currency fluctuations yields in three times lower welfare loss in comparison to regime neglecting exchange rate changes.

Commodity enriched specification seems to serve well the tasks of the paper inspite of its relative simplicity. The issues arises from such simplicity might be addressed in future research as well as the absence of capital. One objective might be incorporation of the Government and Sovereign Wealth Fund as a substantial characteristics of Kazakhstan's economy. Another possible modification concerns the type of Taylor rule which in fact has a large number of specifications. Additionally, foreign block could follow alternative, or more complex structure as a domestic block does.

A Derivations of log-linearized equations

A.1 Euler equation

From household's costs minimization problem(4) I obtain:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{61}$$

I first take the derivative with respect to $C_{H,t}$ of the both sides of (61):

$$P_{H,t} = P_t[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{1}{\eta-1}} (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{-1}{\eta}}$$

Then I replace expression in square brackets with definition of C_t:

$$P_{H,t} = P_t C_t^{\frac{1}{\eta}} (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{-1}{\eta}}$$

The demand function for domestic goods is:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{62}$$

After taking the derivative with respect to $C_{F,t}$ I get:

$$C_{F,t} = \alpha(\frac{P_{F,t}}{P_t})^{-\eta} C_t \tag{63}$$

Plugging of (62) and (63) in (61) yields Consumer price index:

$$P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{64}$$

In order to derive optimality conditions I need to construct Lagrangian:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] - \lambda_t (P_t C_t + D_t + \tilde{e}_t B_t - (D_{t-1} i_{t-1} + \tilde{e}_t B_{t-1} i_{t-1}^* \gamma_t + \Pi_{H,t} + \Pi_{F,t} + W_t N_t + T_t))$$

1. Derivative with respect to C_t:

$$\beta^t {C_t}^{-\sigma} - \lambda_t P_t = 0 \tag{65}$$

2. Derivative with respect to C_{t+1}:

$$\beta^{t+1} C_{t+1}^{-\sigma} - \lambda_{t+1} P_{t+1} = 0 \tag{66}$$

3. Then a standard intratemporal optimality condition yields from the ratio of λ_{t+1} and λ_t:

$$\frac{\lambda_t}{\lambda_{t+1}} = \frac{1}{\beta} \left(\frac{C_t}{C_{t+1}} \right)^{-\sigma} \frac{P_{t+1}}{P_t} \tag{67}$$

4. Derivatives with respect to B_t and D_t yield:

$$\lambda_t \tilde{e_t} = \lambda_{t+1} \tilde{e_{t+1}} \gamma_{t+1} \tag{68}$$

$$\lambda_t = \lambda_{t+1} i_t \tag{69}$$

Following (68) and (69) uncovered interest parity condition is obtained:

$$i_t = i_t^* \frac{\tilde{e}_{t+1}}{\tilde{e}_t} \gamma_{t+1} \tag{70}$$

5. From (69) and (67) one can obtain a conventional Euler equation in the form of:

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} = 1 \tag{71}$$

which then log-linearized directly to:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\}) \tag{72}$$

where $\pi_{t+1} = p_{t+1} - p_t$ - log-deviation of CPI inflation from the steady-state.

6. Derivative with respect to N_t:

$$-\beta^t N_t^{\phi} + \lambda_t W_t = 0 \tag{73}$$

$$\lambda_t = \frac{\beta^t N_t^{\phi}}{W_t}$$

By substituting the above expression into (65) one will obtain optimal labor supply schedule:

$$C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t} \tag{74}$$

Direct log-linearization around the steady-state of the equation (74) yields:

$$\sigma c_t + \phi n_t = w_t - p_t \tag{75}$$

A.2 Real exchange rate, terms of trade, law of one price gap and inflation relations

Log-linear approximation of (11) around the steady state is:

$$s_t = p_{F,t} - p_{H,t} \tag{76}$$

Consumer price index (7) can be linearized straightforward to:

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t \tag{77}$$

CPI inflation is the first difference of CPI price index:

$$\pi_t = p_t - p_{t-1} \tag{78}$$

Domestic and foreign inflation are similar processes:

$$\pi_{H,t} = p_{H,t} - p_{H,t-1} \tag{79}$$

$$\pi_{F,t} = p_{F,t} - p_{F,t-1} \tag{80}$$

CPI inflation, domestic inflation and terms of trade are related according to the following functional form:

$$\pi_t = \pi_{H,t} + \alpha \triangle s_t \tag{81}$$

where $\triangle s_t = s_t - s_{t-1}$. CPI inflation is positively correlated with the degree of openness in the economy, domestic inflation and change in terms of trade.

Next, the short term deviations in the law of one price (13) in log-linearized form can be expressed:

$$\psi_{F,t} = e_t + p_t^* - p_{F,t} \tag{82}$$

where e_t is the log deviation of nominal exchange rate from its steady-state.

Finally, real exchange rate term(12) after log-linearization around the steady-state takes the form of:

$$q_t = e_t + p_t^* - p_t \tag{83}$$

Substituting equation (82) into the process for real exchange rate yields:

$$q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1 - \alpha)s_t \tag{84}$$

A.3 UIP condition

Let $A_t = \frac{\tilde{e}_t B_t}{Y_{ss} P_t}$ - net foreign debt denominated in domestic currency as a fraction of steady state output. Then log-linearization of (9) yields uncovered interest parity condition:

$$i_t - i_t^* = E_t\{\triangle q_{t+1} + \pi_{t+1} - \pi_{t+1}^*\} - \chi a_t - \varepsilon_{st,t} \tag{85}$$

A.4 Domestic Firms

Given that all firms are identical, aggregate production function in loglinearized form can be expressed as:

$$y_t = \varepsilon_{at} + n_t \tag{86}$$

where ε_{at} - technological innovation common to all firms that following AR(1) process:

$$\varepsilon_{at} = \rho_{at}\varepsilon_{a,t-1} + \epsilon_{at} \tag{87}$$

The log-linearization of the expression for real marginal costs(19) around the deterministic steady-state yields:

$$mc_t = w_t - p_{H,t} - \varepsilon_{at} \tag{88}$$

By using (88) and (75) one will obtain:

$$mc_t = w_t - p_{H,t} - \varepsilon_{at} = mc_t = w_t - p_t + (p_t - p_{H,t}) - \varepsilon_{at} = \sigma c_t + \phi n_t + \alpha s_t - \varepsilon_{at} \tag{89}$$

Plugging (86) in previous expression provides real marginal costs:

$$mc_t = \sigma c_t + \phi y_t - \varepsilon_{at}(1+\phi) + \alpha s_t \tag{90}$$

A.5 Inflation dynamics

A log-linear approximation of domestic firm's optimality condition (23) around the steady-state taking into account zero inflation and balanced trade provides (firms forecast future marginal costs and inflation rate in order to set optimal price):

$$\tilde{p}_{H,t} = p_{H,t-1} + \sum_{\tau=0}^{\infty} (\beta\theta_H)^{\tau} E_t\{\pi_{H,t+\tau}\} + (1-\beta\theta_H) \sum_{\tau=0}^{\infty} (\beta\theta_H)^{\tau} E_t\{mc_{t+k}\} \tag{91}$$

or

$$\tilde{p}_{H,t} - p_{H,t-1} = (\beta\theta_H) E_t\{\tilde{p}_{H,t+1} - p_{H,t}\} + \pi_{H,t} + (1-\beta\theta_H) mc_t \tag{92}$$

Domestic price index (18) can be log-linearized in the form:

$$\pi_{H,t} = (1 - \theta_H)(\tilde{p}_{H,t} - p_{H,t-1}) \tag{93}$$

After combining the above expression with (92) I get dynamics of domestic inflation in the form of Philips curve:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_H mc_t \tag{94}$$

where $\kappa_H = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}$

Similarly, taking into account zero inflation in steady state implies that $\Psi_F = \frac{\varepsilon-1}{\varepsilon}$ and log-linearization of retailer's optimality condition (28) yields:

$$p'_{F,t} - p_{F,t-1} = (1 - \theta_F\beta)\psi_{F,t} + \theta_F\beta E_t\{p'_{F,t+1} - p_{F,t}\} \tag{95}$$

From log-linear approximation of the import price index(24):

$$p'_{F,t} - p_{F,t-1} = \frac{\pi_{F,t}}{1 - \theta_F} \tag{96}$$

Combining the above two equations provide the following Phillips curve for import price inflation:

$$\pi_{F,t} = \beta E_t\{\pi_{F,t+1}\} + \kappa_F\psi_{F,t} + \varepsilon_{cp,t} \tag{97}$$

where $\kappa_F = \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}$ and $\varepsilon_{cp,t}$ - cost-push shock to capture inefficient variations in mark-ups.

A.6 Market clearing

Log-linear approximation around the steady state of (33) provides:

$$y_t = (1-\alpha)c_t + \alpha\eta s_t(2-\alpha) + \alpha\eta\psi_{F,t} + \alpha y_t^* \tag{98}$$

Similar to Justiniano and Preston(2009) I impose that Government chooses the amount of lump-sum taxes to neutralize the distortions from imperfect market conditions:

$$\Pi_{H,t} + \Pi_{F,t} + T_t = P_t C_t - W_t N_t + \tilde{e}_t P_{H,t}^* C_{H,t}^* - \tilde{e}_t P_t^* C_{F,t}^* \tag{99}$$

By plugging the above expression into household's budget constraint I get:

$$\tilde{e}_t B_t = \tilde{e}_t B_{t-1}(1+i_{t-1}^*)\gamma_t + \tilde{e}_t P_{H,t}^* C_{H,t}^* - \tilde{e}_t P_t^* C_{F,t}^* \tag{100}$$

Replacing $\frac{\tilde{e}_t B_t}{P_t Y}$ with A_t in budget constraint and $C_{H,t}^*$ gives the following:

$$A_t = A_{t-1}\frac{\tilde{q}_t}{\tilde{q}_{t-1}}(1+i_{t-1}^*)\gamma_t + \frac{1}{Y}\left(\left(\frac{P_{H,t}}{P_t}\right)\zeta(\Psi_{F,t}S_t)^{\eta^*}Y_t^* - \tilde{q}_t\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t\right) \tag{101}$$

In the steady state we have $(1-i^*) = 1/\beta$.

Log-linearization of the budget constraint(101) yields:

$$a_t = \frac{1}{\beta}a_{t-1} + \alpha(p_{H,t} - p_t + \eta(\psi_{F,t} + s_t) + y_t^* - q_t + \eta(p_{F,t} - p_t) - c_t) \tag{102}$$

Using the resource constraint (98) flow budget constraint is derived:

$$c_t + a_t = \frac{1}{\beta}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t \tag{103}$$

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