

Developing empirical formulas for Unplanned Dilution Estimation in Open Stope Mining Design: an Illustrative Case Study

Student's Name: Vladimir Balashov

Supervisor's Name: Amoussou Coffi Adoko, PhD

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Originality Statement

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Signed on 13.04.2026

Vladimir Balashov

Abstract

Unplanned dilution in open stope mining presents considerable economic risks. This case study evaluated stress-driven overbreak using empirical charts, numerical modeling, and regression analysis. Traditional stability graphs were inaccurate. Instead, the minor principal stress (σ_3) was identified as the strongest predictor of depth of failure. A spline regression model ($R^2 = 0.8207$) best captured the non-linear stress-failure relationship with separate equations for hanging wall and back. It was concluded that the methodology provides a data-sparse design framework to determine the depth of failure around open stopes through confinement-based criteria and hence could assist in minimize unplanned dilution.

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1. Introduction

1.1 Background

Open stope mining is one of the most used underground methods. Its popularity can be derived from its operational flexibility, relatively low development requirements, and ability to achieve high production rates. In Kazakhstan, many metallic ore deposits are exploited using this system. Namely, those are the Ridder mines and other polymetallic operations in the East and South Kazakhstan regions.

Although it is versatile, similarly to any other underground mining method, it faces several geotechnical challenges that directly influence rock stability and overall mining success. A core difficulty in mining is the quality and reliability of data. The rock mass is inherently discontinuous, heterogeneous and excessively variable at mining scale. Its properties cannot be fully characterized without extensive excavation of drilling, which is economically unfeasible. Laboratory tests may provide useful information on intact rock samples, however, these results rarely translate directly into an accurate understanding of rock mass behavior at scale. As a result, mining engineers design under conditions of limited geotechnical data and have to rely on empirical approaches that do not reflect the actual conditions present. This is common in open stope mining methods and this challenge is prominent in Kazakhstan, where comprehensive rock mass investigations are constrained.

Effective open stope design typically relies on an extensive set of geotechnical parameters. In Kazakhstan, where such data is frequently incomplete, engineers often depend on empirical tools, mostly variations of the Mathews Stability Graph or ELOS graph (Clark & Pakalnis, 1997; Mathews et al., 1980). These tools were originally calibrated using datasets from other regions, meaning their applicability is heavily limited due to difference in geological conditions. Suorineni (2012) proposed that inconsistencies and incompatible data from various mines across the world made predictions based on empirical data misleading. Still, those tools can be adapted to specific mines, and it is a very common practice, however, fine tuning of empirical charts/tables requires much data that can be obtained only during the excavation. As stated by Paixao et al. (2025) even those charts that were tuned to specific mine sites using machine learning and statistical modelling are affected by quality of data and subjective analysis.

Alternatively, numerical modelling could be used to model the stope behavior. It is a robust tool that gives precise analysis. Yet, this method is significantly more dependent on reliable data. It requires detailed material parameters that may not be unavailable. This creates significant uncertainty in both predictions, which increases the risk of excessive dilution, overbreak, stope

instability, and operational hazards. Even when data is available, as stated by Suzuki Morales et al. (2024) it additionally requires the selection of appropriate method of simulation that considers most important parameters to model the rock behavior. Even though this was said to be about cave propagation, the core idea remains the same. Typical mine operations are also limited in software availability for many irrelevant reasons, which, combined with a shortage of specialized personnel and time constraints, relegates advanced numerical modelling to a supplementary role, leaving empirical methods as the primary decision-making tool.

There is currently no consistent or validated methodology for designing open stopes in which unplanned dilution, the unintentional extraction of waste or low-grade rock surrounding the planned stope boundaries, leading to increased processing costs and reduced head grade, is minimized. Therefore, it is vital to investigate this problem by developing and testing a practical design method that is based on limited field data, empirical assessments, and numerical modelling techniques. If this can be accomplished, then more reliable designs will be achieved leading to minimum unplanned dilution (Alipov & Adoko, 2020). This should allow modelling a scenario that enables comparison between predicted and actual stope performance. Addressing the depth of failure estimation forms the main challenge that this research aims to resolve.

1.2 Objectives of the Thesis

1.2.1 Main Objectives

The aim of this study is to illustrate the development of an empirical dilution criterion capable of determining the depth of failure or overbreak around stopes in connection with open stope design at the Sokolny Mine, Kazakhstan.

1.2.2 Specific Objectives

1. Conduction of a literature review related to the topic.
2. Identify and summarize the available geological and geotechnical data from the Sokolny Mine, including core logs, laboratory test results, stope performance records, and any existing rock mass classifications.
3. Conduct numerical simulation in RS2 to simulate stope behavior and evaluate the extent of overbreak level of the stopes in the studied cases.
4. Quantify stress-failure relationship based on CMS data, evaluate all prominent correlations to the depth of failure.
5. Formulate a set of recommendations for Sokolny Mine based on the obtained results.

1.3 Hypotheses

This thesis hypothesizes that by calibrating deterministic numerical models against actual stope performance, and subsequently synthesizing the results through regression analysis, a new set of empirical equations can be derived. These equations could predict the depth of failure in a more convenient way than existing empirical or standalone numerical methods, particularly in site-specific context and early design stages.

1.4 R&D Justification

The discrepancy between planned stope boundaries and stope performance is the primary driver of financial volatility in underground mining. Conventional design practices fail to account for unique rock behavior of mining sites. Existing methods either lack site-specific calibration or demand high quality geotechnical data that may not be available.

This research addresses this challenge by developing a calibrated empirical criterion for estimating the depth of failure. It is critical to establish correlation between stress-failure relationships with field data to address the limitations of standalone empirical or numerical tools.

1.5 Scope of Work

This study focuses on the design of open stopes and dilution estimation, using the Sokolny Mine in Kazakhstan as the case study. The analysis relies on available rock mass parameters, production records, and CMS data. The research is limited to underground open stope mining and studies unplanned dilution only through empirical tools and numerical modelling. Detailed orebody modelling, long-term ground behavior, economic optimization, backfill performance, drilling and blasting accuracy as factors are not included.

2.0 Literature Review

2.1 Dilution

Dilution is defined as the contamination of ore by waste rock or material below the specific cutoff grade. In underground mining, dilution represents an economic risk. It increases the volume of material requiring hauling, crushing, and processing without a proportional increase in metal output. Scoble and Moss (1994) state that dilution impacts the efficiency of the mineral processing plant through the introduction of deleterious minerals and increased milling costs.

The industry categorizes dilution into two distinct types based on the source and the degree of control available during the design phase. Those are planned and unplanned dilution.

Planned dilution is the low-grade or waste material located within the designed stope boundaries. This material is intentionally extracted to maintain the mechanical integrity of the stope geometry or to follow the continuity of the orebody. Pakalnis (1986) defines this as "internal" because it is a byproduct of the chosen mining method and stope layout. Engineers can account for this dilution in the feasibility stage as part of the reserve estimation.

Unplanned dilution, or "overbreak," refers to the waste rock that falls from the stope walls outside the designed boundaries. This is the primary concern of geotechnical engineering. Sources include structural failure along joint sets, stress-induced spalling, or blast-induced damage. Clark and Pakalnis (1997) and Villaescusa (2014) note that unplanned dilution is the most influenced by rock mass quality and stope geometry.

2.2 Overview of the Stability Graph

Main empirical method of defining stability of open stopes was developed by Mathews et al. (1980). It was initially developed for open stope mining in massive orebodies (Mathews et al., 1980). Later its reliability was improved significantly by Potvin (1988), who did recalibration of stability factors (A, B, C) using 175 case histories with subsequent expansions to 483 cases (Mawdesley et al., 2001). Despite the seemingly good improvement of the dataset, this created a critical problem. Inconsistencies and incompatible data from various mines across the world made predictions based on empirical data misleading (Suorineni, 2012). Some attempts to fix this are initiated and implemented. Key advancements addressed limitations in fault proximity, time-dependent stability, and blast damage, which require a detailed understanding of site-specific geological structures and mining practices to be applied effectively.

Overall, the chart is based on relation of hydraulic radius (HR) and Stability Number (N') (Mawdesley et al., 2001). The primary equations governing these parameters are presented in Equations (1) – (3).

$$N' = Q' \times A \times B \times C \quad (1)$$

Where:

- N' – the Modified Stability Number,
- A – rock stress factor that accounts for induces stresses acting of the stope surface,
- B – joint orientation factor that represents the structural influence measured as relationship between the orientation of joints and stope surface,
- C – gravity factor based on the shape and dip of the stope surface,
- Q' – the Modified NGI Rock mass Quality Index.

Adjustment factors are shown in Figure 1.

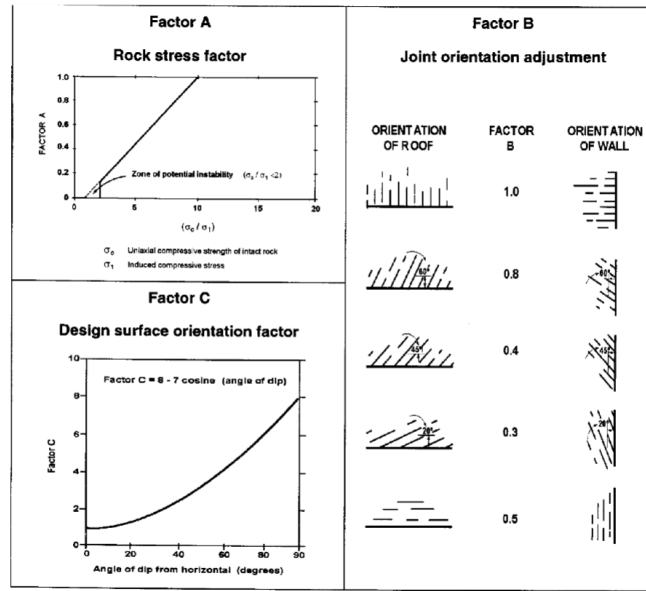


Figure 1. Determination of Adjustment Factors (Mathews et al., 1980)

Q method is a rock mass classification that is commonly used in geotechnical applications specifically for tunnel excavation. This method was introduced by Barton et al. (1974), the latest version of this method is used based on 2015 handbook by NGI. Q value could be obtained by measuring relative block size (RQD/J_n), inter-block shear strength (J_r/J_a), active stresses (J_w/SRF) (NGI, 2015). However, as Modified Stability Number already explicitly has adjustments for stress, the active stresses part of equation is dropped. This leads to an equation:

$$Q' = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \quad (2)$$

Where:

- RQD – Rock Quality Designation,

- J_n – joint set number,
- J_r – joint roughness number,
- J_a – joint alteration number.

The reliability of the Q' calculation, and the N' value, is dependent on the quality and representativeness of the input data describing jointing characteristics. This introduces epistemic uncertainty into the stability prediction from the initial characterization stage with limited data.

$$HR = \frac{\text{Surface Area}}{\text{Surface Perimeter}} \quad (3)$$

Where:

- HR – hydraulic radius,
- Surface Area – area of the slope surface,
- Surface Perimeter – perimeter of the slope surface.

The Stability Graph is divided into three zones based on stability classification: Stable Zone, Potentially Unstable Zone, Potential Caving Zone (Mawdesley et al., 2001). The boundaries between these zones are represented as transitional bands to acknowledge the uncertainty in delineating stable from unstable conditions (Fig. 2). Stable Zone is characterized as self-supporting excavation with predicted dilution being less than 10%. Potentially Unstable Zone is the condition of the slope with localized sloughage, therefore requiring some support system implementation, with dilution ranging from 10% to 30%. Potential Caving Zone is defined as slope having a high probability of failure leading to the structural collapse and high dilution and losses.

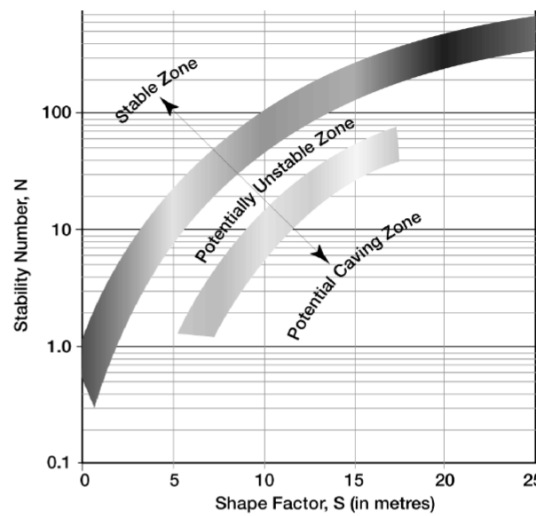


Figure 2. Original Mathew's Stability Graph (Mawdesley et al., 2001)

Later iterations of Stability Chart were based on logistic regression (Fig. 3). This graph is based on 483 back-analyzed cases. Still, the predictive power of this logistic model remains constrained by the quality and consistency of the underlying dataset. The inherent heterogeneity of the global case histories means these contours provide only a general benchmark.

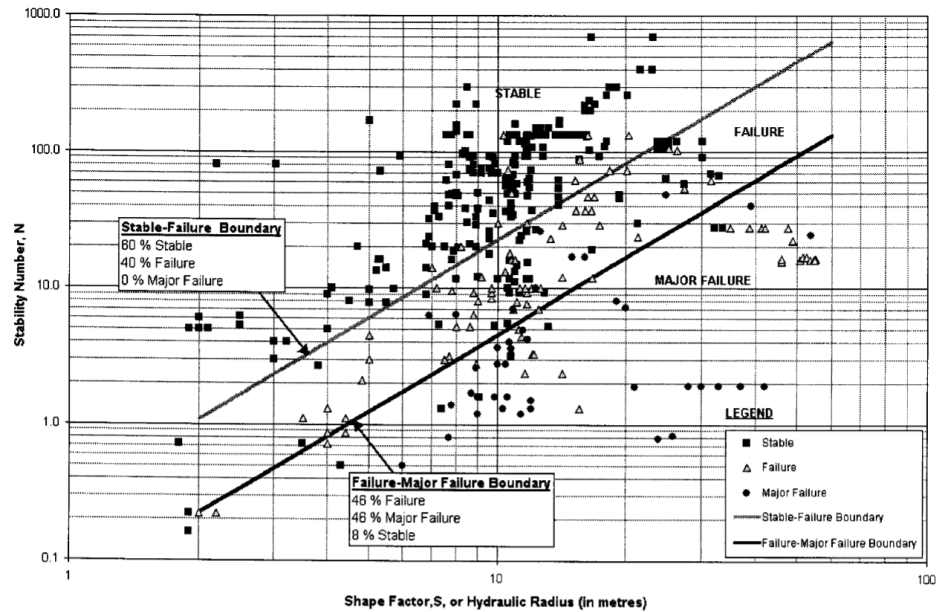


Figure 3. Log-log plot of Extended Stability Graph (Mawdesley et al., 2001)

Overall, this method is well suited for predicting slope stability, as mentioned earlier has reliability problems. Suorineni (2012) states that due to large inconsistent databases this method provides low reliability and can be used as reference point. The proliferation of different stability graph versions and calculation approaches further complicates their application and introduces additional uncertainty for practitioners, as the choice of method can itself influence the predicted outcome. On one hand, Mawdesley et al. (2001) state that statistical applications of many varieties do not change the degree of reliability and logistic regression is merely a way to quantify stability zones. On other hand, it is a common approach to fine tune the stability graph to the deposit. Such calibration allows to account for different geological conditions as it is obvious that rock mass in Canada is different to rock mass in Kazakhstan and only basic characteristics of same minerals remain similar. The site-specific stability graph can be created, and it requires more than 100 back-analyzed cases with 10-12% of them falling in the caving zone of the chart for correct calibration (Stewart & Trueman, 2001).

2.3 Overview of ELOS

A logical development of Mathews Stability Graph is a quantification of failure in the form of dilution. Clark and Pakalnis (1997) introduced the Equivalent Linear Overbreak/Slough (ELOS) parameter based on wall overbreak in terms of average failure depth. This was possible due to the development of Cavity Monitoring Systems taking place in 1990's. This empirical metric converts

the total volume of unplanned sloughage into a theoretical average depth of failure distributed uniformly across the planned stope surface. The ELOS concept enables quantitative dilution estimation for stope surfaces plotted on the stability graph.

Percentage-based dilution measurements present inherent limitations in narrow-vein or geometrically variable deposits. A fixed depth of failure is a high dilution percentage in narrow stopes and the same failure depth yields a lower percentage in wide stopes. Therefore, ELOS is a width-independent metric that enables standardized stability assessment across diverse stope geometries. Research has focused on developing quantitative dilution-based stability graphs independent of orebody width giving flexibility to design open stope sizes based on acceptable dilution amounts for specific operations (Clark & Pakalnis, 1997).

ELOS determines the average thickness of overbreak and is calculated by dividing the volume of slough by the surface area of the analyzed stope face:

$$ELOS = \frac{V_{slough}}{A_{face}} \quad (4)$$

Where:

- V_{slough} – the volume of failed rock beyond planned design boundary,
- A_{face} – surface area of stope face that is evaluated.

During the design phase, engineers predict ELOS by plotting the Modified Stability Number (N') against the Hydraulic Radius (HR). The charts use empirical isolines to estimate depths of slough (Fig. 4).

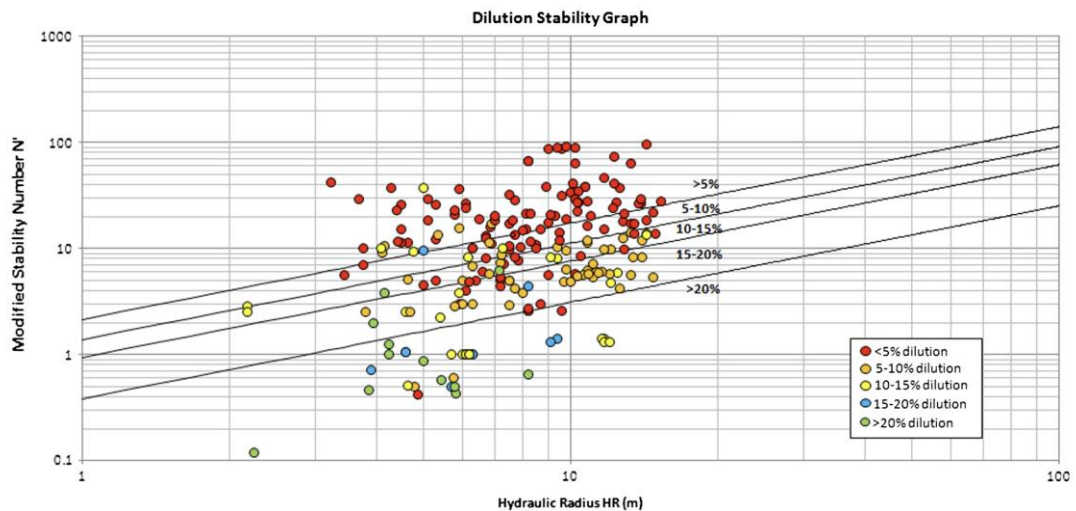


Figure 4. Dilution table (Papaioanou & Suorineni, 2016)

Originally, ELOS was based on deterministic approaches. Clark and Pakalnis (1997) established the primary empirical boundaries using Canadian underground mines. Their work

defined an ELOS of < 0.5 as stable and > 2.0 as indicative of potential caving. Later, Wang (2004) refined the Dilution Design Graph by incorporating mining-induced stress, undercutting, and blast damage into ELOS prediction. Finally, Capes (2009) introduced the Probabilistic Stability Graph. This approach recognized the scatter present in empirical data. Using a larger database, Capes assigned a probability of exceeding specific ELOS thresholds.

More recent developments include site-specific calibration of ELOS charts. Fernandes et al. (2024) demonstrated that introducing site-specific factors to account for shallow dipping orebodies and support methods returned a correlation coefficient R^2 of 0.7 between predicted and observed ELOS, compared to an R^2 of 0.1 based on published curves. This site-specific approach provided realistic and representative measures of slope behavior (Fernandes et al., 2024).

The reliability of ELOS predictions is faces the same inconsistencies identified by Suorineni (2012) for empirical databases. The precision of the depth-of-failure estimate depends entirely on the accuracy of the input parameters. Errors in calculating the joint orientation factor or induced stress at depth produce highly skewed ELOS predictions. This limitation is especially acute for this research, which addresses open slope design under limited geological data. The Sokolny Mine, like many operations in Kazakhstan, operates with constrained geotechnical investigation programs.

2.4 Existing studies on Unplanned Dilution Mitigation

The prediction and minimization of unplanned dilution in open slope remain difficult when geological data is limited. Current research has shifted toward integrating stochastic simulations with numerical modeling to bridge the gap between theoretical stability and empirical observations.

The inherent variability of rock mass properties can lead to flawed predictions in traditional deterministic designs. Shnorhokian and Mitri (2022) addressed this by using Probability Density Functions to represent the variability of rock mass properties in numerical models. Their research showed that these variations are critical for quantifying ore at risk. This suggests that stability should be viewed as a range of probabilities. It becomes possible to identify the influence of fluctuations in geomechanical parameters on slope failure. Numerical models provide a mechanical basis for understanding failure, and empirical regression analysis is the primary tool for translating these findings into metrics. Petlovanyi et al. (2020) used multiple regression analysis to show that rock hardness coefficients and the specific contact area of the hanging wall are dominant predictors of dilution. Their model accounted for over 80% of the variability in

dilution rates which emphasizes the effectiveness of regression-based approaches in identifying the most influential geological factors in a given mining environment.

2.5 Numerical Methods

Another instrument in predicting dilution is numerical modelling, which is a powerful tool for geotechnical analysis. Such software calculates in-situ parameters at any point within selected boundaries. For this research Finite Element Method (FEM) is studied and used. Finite element analysis provides a mechanistic understanding of slope stability by explicitly modeling the interaction between the excavation geometry and the surrounding rock mass.

Research by Henning and Mitri (2007) demonstrated that numerical modeling can effectively quantify the Depth of Failure (DOF) by identifying zones where induced stresses exceed the rock mass strength. This depth is often used to calculate a numerical equivalent of the ELOS parameter. The accuracy of these models is heavily dependent on the chosen failure criteria. Most studies utilize the Hoek-Brown or Mohr-Coulomb criteria to get the boundary between stable rock and the potentially failing hanging wall. Wang (2004) expanded on this by incorporating mining-induced stress and extraction sequences into the numerical framework to show how the redistribution of stress directly contributes to wall sloughage.

A key advantage of FEM is the ability to account for rock mass damage through the analysis of plastic strain. Jing (2003) noted that numerical methods are superior to empirical tools in dealing with complex geometries or heterogeneous materials. In narrow vein mining the difference in hardness between the ore and the host rock can lead to localized stress concentrations that empirical charts cannot represent. This analysis of tensile failure zones allows engineers to estimate the volume of material likely to detach from the slope faces.

Stochastic numerical modelling is the most promising research methodology. Shnorhokian and Mitri (2022) assessed the impact of varying rock mass properties on slope performance. Running hundreds of simulations with randomized geomechanical inputs generated synthetic datasets that reflected the probable range of dilution. This approach allows for the use of regression analysis to identify which specific parameters have the highest correlation with overbreak.

2.6 Stress Relaxation and Confinement

Unplanned dilution is the subject of local stress relaxation behavior near excavations. It occurs when the minor principal stress (σ_3) decreases due to the redistribution of stresses. Diederichs (2000) argued that the loss of confinement allows for the opening of pre-existing joints and fractures within the rock mass. So, if the minor principal stress reaches a state of zero or becomes

tensile, the rock mass loses its frictional resistance along structural planes and that results in gravity-driven failure.

The shear strength of rock mass is dependent on confinement provided by the surrounding stress field. Based on the Hoek-Brown failure criterion, a decrease in σ_3 results in a non-linear reduction of the rock mass strength. The precise determination of rock strength parameters is difficult and introduces uncertainty. However, the distribution of minor principal stress around an excavation is primarily controlled by the geometry of the stope and the stress ratio. This characteristic makes σ_3 a better indicator of potential instability than parameters that require extensive laboratory testing or complex site characterization. Numerical modeling tools allow for the visualization of these relaxation zones. Many researchers have observed a strong correlation between the boundary of the tensile zone and the actual depth of failure recorded by cavity monitoring systems (Cordova et al., 2022)

Wang (2004) noted that the spatial extent of the relaxation zone gives a physical basis for estimating the ELOS. The depth of failure is a direct consequence of the mechanical response of the rock to the removal of support. And for this reason, the minor principal stress accurately shows the mechanical state of the hanging wall. It gives a border of the volume of rock that is no longer held in place by compressive forces.

In a discontinuous rock mass, the blocks are held together by the frictional forces that act along the joint surfaces. These forces are proportional to the normal stress acting on those joints, which is in turn governed by the minor principal stress. When σ_3 is reduced to zero, the normal stress on many joint sets also drops. This leads to a total loss of shear strength and allows the rock blocks to fall under gravity. This is a reason why unplanned dilution often occurs even in rock masses that are classified as having high intact strength. Kaiser et al. (2000) showed that the depth of failure in brittle rock is better predicted by looking at the relaxation limit. This approach simplifies the design process. Identification of zones where minor principal stress is less than a certain threshold is the key to quantification of unplanned dilution.

The integration of σ_3 allows for the calibration of numerical models against actual stope performance. The model parameters can be refined for specific site conditions when compared the predicted relaxation zones with the actual failure boundaries. This iterative process helps to reduce the epistemic uncertainty that appears from limited data and the use of stress-based correlation reduces the complexity of the design process.

2.7 Concluding Remarks

Generalized empirical methods fail to match the specific demands of mines that operate with limited geological data. The Stability Graph and ELOS approaches offer a starting point for predicting dilution, but they depend heavily on large, high-quality historical databases. That dependence limits its effectiveness. The move from deterministic to probabilistic models has been an important step forward.

Looking at current research, the best way to compensate for limited physical data is to generate synthetic datasets using numerical models. Running simulations across a wide range of stope geometries and rock mass conditions makes it possible to build a database that captures the complexity of mining environments. This method also allows individual influencing factors to be isolated and studied separately.

It is possible to obtain a dense set of geotechnical parameters by simulating many stope cross-sections that traditional field exploration cannot provide. A regression analysis then identifies which factors drive dilution the most and gives a reliable empirical basis for design. The result is a proactive way to estimate unplanned dilution.

3.0 Methodology

3.1 Introduction

To address the main problem of estimating unplanned dilution in open stope design available geological and geotechnical data from the Sokolny Mine is compiled and characterized. Empirical design methods are then applied to establish baseline stability assessments and dilution predictions. Numerical models are developed to simulate stope behavior. The results of these simulations are calibrated against CMS data. Regression analysis is used to synthesize the modeling outputs into a site-specific empirical equation that relates key geotechnical parameters to unplanned dilution.

3.2 Ridder-Sokolny Mine Description

3.2.1 General Information

The Sokolny Mine is located within borders of Ridder, Eastern Kazakhstan and operates as an underground polymetallic mine. It is operated by Kazzinc Corporation (Kazzinc, 2026). Discovered in 1784 by Filipp Ridder this mine has been in operation since 1789. Extraction of oxidized ore continued until 1861. In 1885 the production of sulfide ore began. Now the main product is gold of 2.0 g/ton with a rate of 1.6 Mt per year (Yakubov & Adoko, 2020).

Open stope mining with delayed backfill is the primary extraction method at Sokolny Mine. The method uses automated equipment to collect ore from massive non-entry excavations. Personnel entry into open stopes is strictly prohibited due to the hazard of waste rock failure associated with stope surface instability (ZHALEL, 2019).

3.2.2 Geology of the Deposit

The lithology of Ridder-Sokolny deposit is defines as a gold-polymetallic Volcanogenic Massive Sulfide (VMS) system located within the Rudny Altai metallogenic belt of East Kazakhstan. It is situated within the Leninogorsk volcano-tectonic depression. The geological history of this region involves intensive Devonian volcanism followed by significant deformation during the Hercynian orogeny (Ganzhenko et al., 2018). All deposits are confined to the arched parts of dome structures composed mainly of hydrothermal sedimentary quartzites and layers of chlorite rocks and sericitolites.

The mine itself consists of a series of specific deposits (Fig. 5), which are the Zavodskaya, Ridderskaya, 2-nd Ridderskaya, Central, Pobeda, Perspective, Belkin, 1-st Southwest, 2-nd Southwest, 3-rd Southwest, and Bystrushinskaya areas. These orebodies exhibit complex geometric shapes that vary by depth. Mineralization in the upper sections of the deposit lies in accordance with the host rock stratigraphy. The lower sections have uneven and secant contacts where the ore cuts across the primary bedding of the host lithology (Smirnov, 1965).

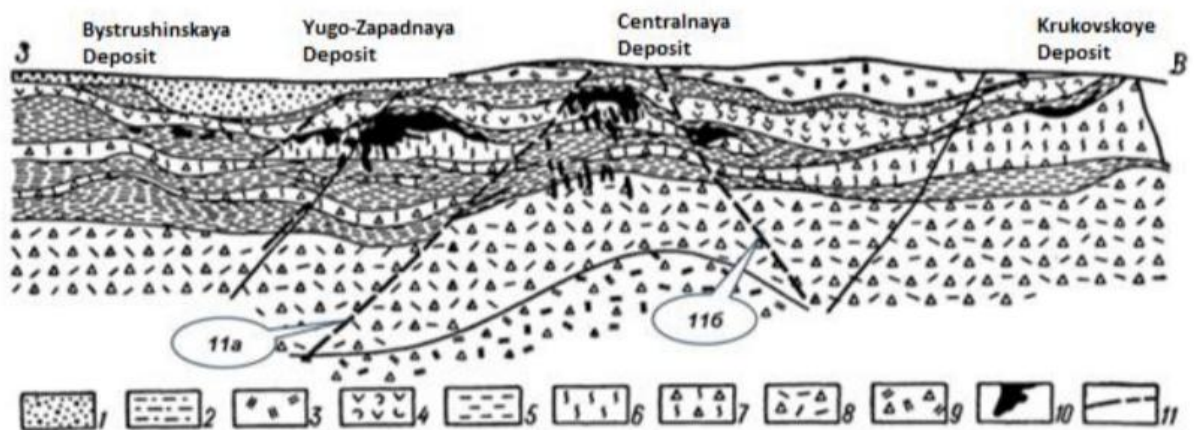


Figure 5. Geological cross-section of the deposits (Bolegenov, 2022)

Sphalerite, galena, and chalcopyrite are the main ore-forming minerals. Minor constituents include tetrahedrite, tennantite, marcasite, and gold. Rare mineral occurrences such as bornite, molybdenite, and native bismuth are also documented within the system. Nonmetallic minerals in the deposit are dominated by quartz, carbonates, sericite, barite, and chlorite. A vast majority of the remaining balance reserves are concentrated in the lower floor of mineralization. These

reserves are hosted in steeply dipping veins, lenses, and layers. The total number of identified ore bodies exceeds ten thousand. Most of these bodies are characterized by small thickness and limited length along both the strike and the dip (Spirin, 2018).

3.3.3 Geomechanical Parameters

Geomechanical properties of the rocks on the Ridder-Sokolny mine have been studied previously and were carried out in 1962-1964, 1977, 1982 and in 1992-1994 (Spirin, 2018).

Rocks with similar properties were combined into 6 domains. Stopes of interest for this study are in domains 9+10 and 13, which are microquartzite, sericitic microquartzite and sericite-chlorite-quartz rock respectfully. Their properties are shown in the following table.

Table 1. Geomechanical properties of rock mass

Rock Domain	Microquartzite, Sericitic Microquartzite	Sericite-chlorite-quartz Rock
Density, t/m³	2.72	2.68
Intact rock properties		
σ_{ci}, MPa	158 (130 reported)	82
GSI	61	57
mi	20	7
D	0	0
E_i, GPa	73	61
Poisson ratio	0.2	0.15
Hock-Brown Criterion		
mb	4.9673	1.5071
s	0.0131	0.0084
a	0.5026	0.5035
Mohr-Coulomb Fit		
c, MPa	11.3	4.2
ϕ, deg	40	30
Rock Mass Parameters		
σ_t, MPa	-0.4	-0.5
σ_{cm}, MPa	48.3	14.3
E_{rm}, GPa	39.6	27.6

3.3 Data Collection

Stope geometries were retrieved from available data provided by Kazzinc. These include Datamine files for planned geometries and CMS data files for actual stope performance. These 3D models then were cut into appropriate two-dimensional sections for the numerical analysis. The selection itself followed strict quality control. Only stopes with available both actual and planned geometries were chosen. Additionally, it is required for each stope to have a report on rock mass parameters and unplanned dilution. For these appropriate stopes the profiles with low levels of ore losses were included in dataset. High ore loss indicates that the planned ore was not fully extracted from the stope boundaries. This condition creates a physical barrier that obscures the true failure surface for the stope.

High ore losses are typically the result of operational factors, mainly poor blasting practices. These factors are unrelated to the stress-driven failure mechanisms studied in this research. The presence of a significant ore bridge also alters the stress redistribution around the excavation. This discrepancy makes the comparison between the planned numerical model and the actual cavity survey invalid. The use of stopes with high extraction ratios provides a more accurate representation of the natural stability limits. This data filtering step minimizes the noise in the regression analysis.

3.3.1 Case Study Selection

Stope 172, Stope 198 and Panel 42 are chosen. Only these cases have all the required data for further analysis. Planned geometries (orange) and CSM data (blue) are presented in respected figures below for each stope.

Panel 42 is characterized by strike length of 32 m, width of 1.2 m, and height of 10 m. The extraction unit is situated at a near-vertical dip of 89 degrees within the modeling domain. Planned geometry for this section is restricted to the upper sublevel of the panel. Observations indicate that the unit is subject to severe localized caving failure during the mining cycle.

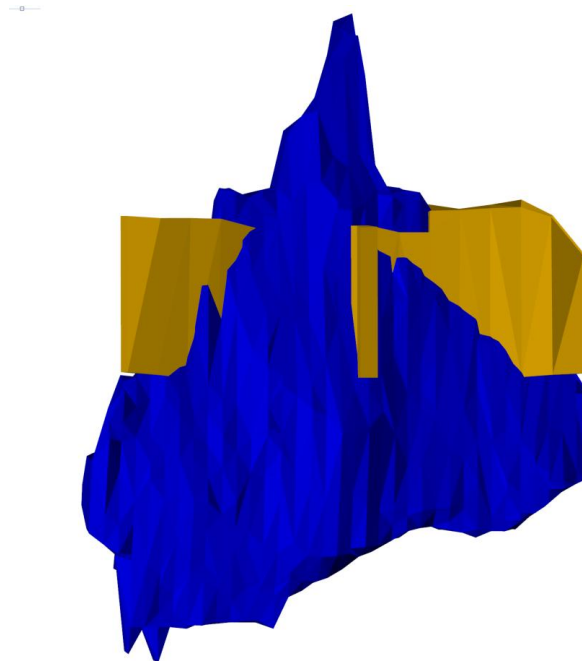


Figure 6. Panel 42 Planned and Actual Geometry

Stope 198 is characterized by strike length of 51 m, width of 2.2 m and height of 21 m. This stope is oriented at a dip of 81 degrees and has a highly irregular planned geometry. CSM data shows a wide spectrum of failure modes ranging from ore loss to massive caving.

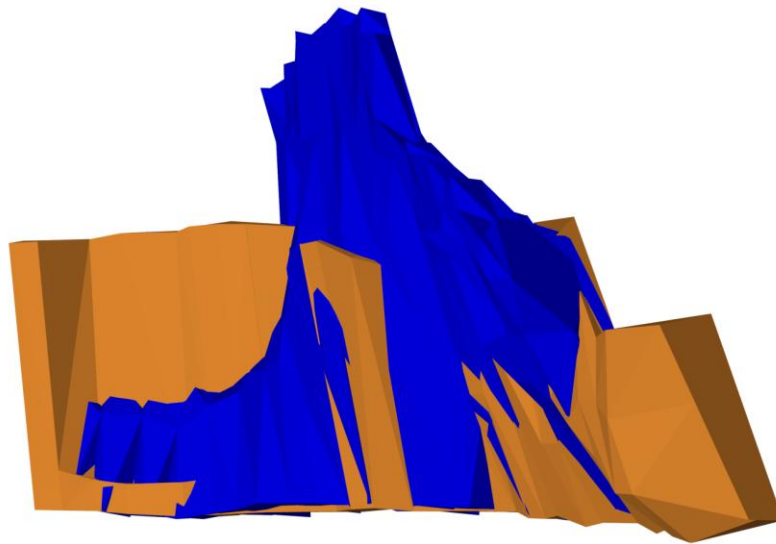


Figure 7. Stope 198 Planned and Actual Geometry

Stope 172 is defined by 28 m strike length, 2.0 m width, and maximum height of 24 m. This unit is inclined at an 85-degree dip but maintains a planned geometry that is closer to regular design standards. Comprehensive analysis is constrained by limited CSM data, which covers only an 8 m interval for unknown reasons.

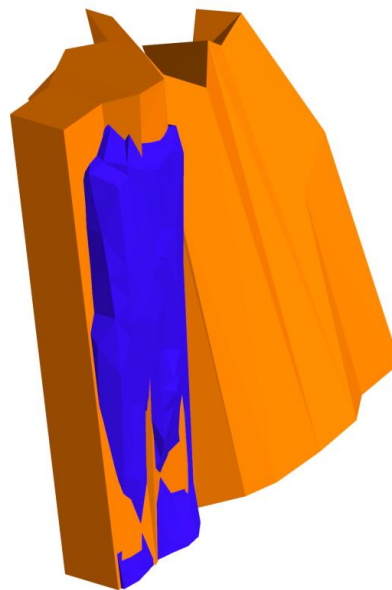


Figure 8. Stope 172 Planned and Actual Geometry

3.3.2 Model Configuration and Failure Criteria

The planned geometry is imported in RS2 software. The models calculated the stress state at the moment of stope completion. The main problem with modelling occurred due to practically no information on the rock mass: discontinuities were unknown at the location of the stopes, orebody geometry was unknown, descriptions are vague at best, the UCS was different to the reported intact rock parameters. So, it was chosen to model the planned geometry, fill most of the space with microquartzite and place sericite at the bottom of the model exactly as was written in

reports. Elastic Generalized Hoek-Brown failure criteria were chosen. All known intact rock parameters mentioned earlier were used with the exception of UCS, which was taken from the reports on slope performance. Example of a numerical model is provided below.

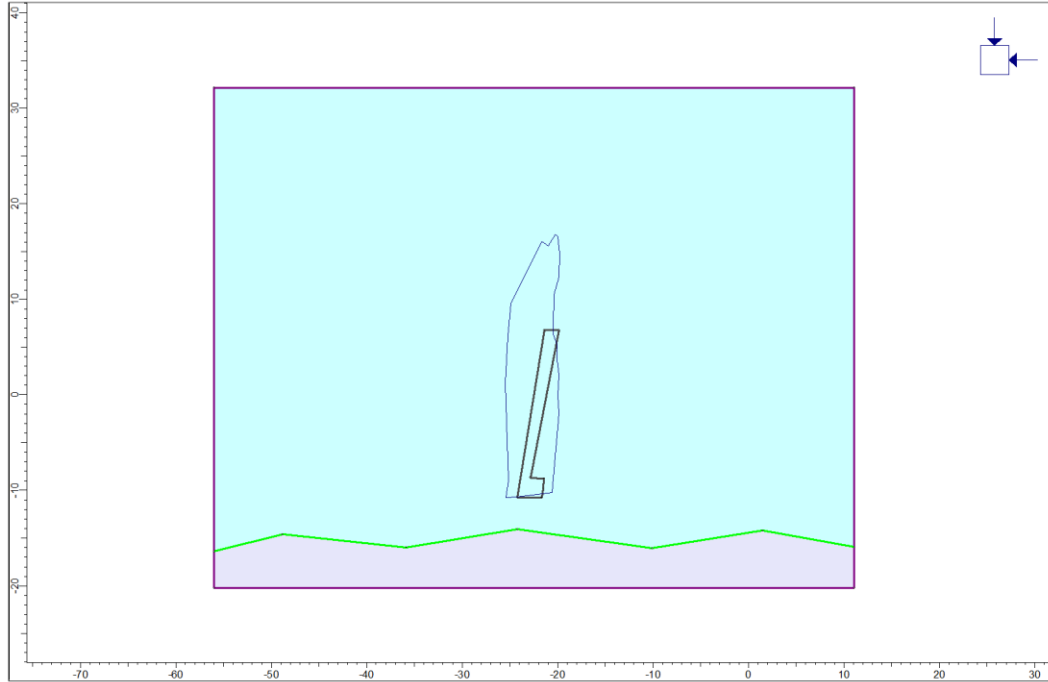


Figure 9. An example of Stope 198 Profile 1 without mesh.

The in-situ stress state for the numerical models was defined using a constant stress ratio, $k = 1.0$, to represent the regional tectonic environment of the Ridder-Sokolny deposit. The vertical stress was calculated based on the depth of the stopes and the average density of the rock domains, set to be 2.70 t/m^3 . All stopes were on the same elevation.

A graded triangular 6-noded mesh was applied to each model with a high density of elements concentrated around the stope boundaries. The external boundaries of the models were placed at a sufficient distance to prevent boundary interference with the stress redistribution.

Each profile geometry was initially assessed for ELOS based dilution prediction to compare with the actual stope performance. The classical Hoek-Brown failure criteria was utilized:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a \quad (5)$$

Then the actual performance data was then incorporated to validate these numerical predictions. The actual stope profiles were mapped directly onto the corresponding two-dimensional numerical sections. This allowed to retrieve calculated stress values at the location of the physical failure boundary. The major principal stress and minor principal stress were recorded

for every point along the failure surface. The depth of failure was calculated as the distance between the planned design and the measured overbreak.

3.3.3 Sensitivity Analysis

A sensitivity analysis was conducted to evaluate the impact of rock mass property variability on numerical modeling outcomes. For both elastic and plastic models the uniaxial compressive strength and the geological strength index values were adjusted within a range of $\pm 20\%$ to represent the inherent uncertainty of the geotechnical data. In Generalized Hoek-Brown failure criterion every constant is calculated using GSI and the failure criterion itself is using UCS of intact rock (Hoek & Brown, 1997). So, testing this range allows to assess the reliability and account for aleatory uncertainty of the rock (Behnia et al., 2018).

Total of 50 iterations were made for all possible combinations. This testing procedure aimed to determine if the predicted stress redistribution was excessively dependent on the precision of the input parameters. The results of these iterations demonstrated that the numerical models behave in a consistent manner regardless of these fluctuations. There was no significant variation in the regression analysis results between the minor principal stress and the measured depth of failure.

The spatial distribution of the relaxation zones is primarily governed by the stope geometry and the in-situ stress field and not localized strength variations. So, this successfully mitigates the epistemic uncertainty that arises from incomplete site characterization.

3.4 Statistical Analysis

3.4.1. Geomechanical Dataset Establishment

The recorded values for the major principal stress and the minor principal stress were organized into a central database for processing. The differential stress was also included as an independent variable in the analysis.

The data points located in the immediate vicinity of the stope abutments were excluded from the statistical analysis. These regions are characterized by high geometric confinement. This stress state prevents the initiation of sloughage in the corners of the stope. The structural stiffness of these zones provides stability that is distinct from the behavior of the hanging wall mid-span. Including these points in the regression would introduce a significant bias. The research targets the areas where the rock mass is most vulnerable to the loss of confinement. Focus on the primary source of unplanned dilution in the hanging wall and the back is maintained. The resulting dataset provides a representative correlation for the development of the design methodology.

3.4.2 Regression Analysis

Limited geological data requires significant simplifications for quite an obvious reason, which is epistemic uncertainty. Synthesizing data that is either currently unavailable or unknown brings too much unwanted randomness. So, it is only natural to attempt to limit the framework. This justification brings this research to the choice of simple linear regression models. Such models behave in the form of function:

$$y = \beta_0 + \beta_1 x \quad (6)$$

Where β_0 is the intercept, β_1 is the slope.

This model is easily applied as it requires only two empirical coefficients that mining engineers can correlate with physical rock mass properties instead of relying solely on complex numerical outputs.

Still, it is assumed that simple linear regression models cannot describe the relation between the depth of failure and minor principal stress. So, in attempts to fix that without over/underfitting the behavior, several methods of regression analysis are planned to be done. Firstly, it is natural to assume that this relation is non-linear and therefore, it can be captured with polynomial regression model, that behaves in the form of a function:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \dots + \beta_n x^n \quad (7)$$

Where β_0 is the intercept, β_1 is the slope, $\beta_2 - \beta_n$ are the curvature coefficients.

This regression model is a subject to overfitting as it becomes very complex when capturing the fluctuations of the dataset. If the polynomial model is overfit, then K-means clustering and regression spline models are suggested to use.

For K-means clustering the data is grouped based on minor principal stress ranges. The method is used to partition a dataset into K distinct, non-overlapping clusters. It works by minimizing the distance between data points and the center of their assigned cluster. The number of clusters is chosen, either randomly as an educated guess or based on reasoning. Each data point is assigned to the nearest centroid. This is usually calculated using the Euclidean Distance equation. After certain iterations the centroids are moved to the mean position of all data points assigned to that cluster. Clustering could possibly be realized in 3 distinct zones as relaxation zone, high stress zone in the back and transitional stress zone. Regression analysis is then performed on clusters separately.

As for regression spline model, the dataset can be divided on 2 zones for hanging wall and back. Depending on behavior of data in these zones, different models can be chosen to account for that. This leads to the highest fit with less chance of overfit.

All statistical analysis is done in Python with related libraries or Excel.

The coefficient of determination is the metric for the evaluation of the model performance for each parameter. A higher R^2 value shows that the specific variable explains a greater proportion of the variance in the unplanned dilution. The initial linear regression models identified the strongest correlation between minor principal stress and depth of failure.

3.4.4 Reliability Evaluation of Regression Models

Overfitting is a big concern in predictive modelling and for this research specifically. It occurs when a model becomes too complex and captures noise in data. It happens when high-order terms have high p-values, in other words statistically insignificant. However, it is only a marker for overfitting and is not conclusive. Therefore, it is important to verify the correct state of models. The Akaike Information Criterion (AIC) is utilized for this reason. It rewards the goodness of fit imposing a penalty for the number of estimated parameters. This prevents the selection of overly complex models that may overfit the data. A lower AIC score indicates a more desirable model. AIC is calculated using the following formula:

$$AIC = 2k - 2 \ln(L) \quad (8)$$

Where k is the number of estimated parameters in the model, L is the maximum likelihood of the model that represents the probability of the observed data given the model parameters.

That should be a solid groundwork to correctly choose the model that better represents the relation between minor principal stress and depth of failure.

4.0 Results

4.1 Empirical Charts

For stope 172, 198 and panel 42 no information about discontinuities is available. Therefore, it is assumed that the worst-case scenario is applicable for each stope profile. For that state the orientations of joints are nearly parallel to the excavation face. This leads to the fact that all stope profiles are considered stable, which is, in fact, far from true. The empirical data presented in table and graph below.

Table 2. Stability Chart Data

Stope Profile	Stope ID	HR	N	Predicted Class	Actual Class
172_1	1	1.379	22.969	Stable	Stable
172_2	2	1.817	23.619	Stable	Stable
172_3	3	1.897	24.303	Stable	Stable
198_1	4	0.616	23.977	Stable	Caved
198_2	5	0.869	22.969	Stable	Caved
198_3	6	0.895	22.969	Stable	Caved
42_1	7	0.800	24.661	Stable	Caved
42_2	8	0.782	24.303	Stable	Caved
42_3	9	0.800	24.335	Stable	Caved

Mine-specific stability chart was obtained to demonstrate the position of those stope profiles in the chart.

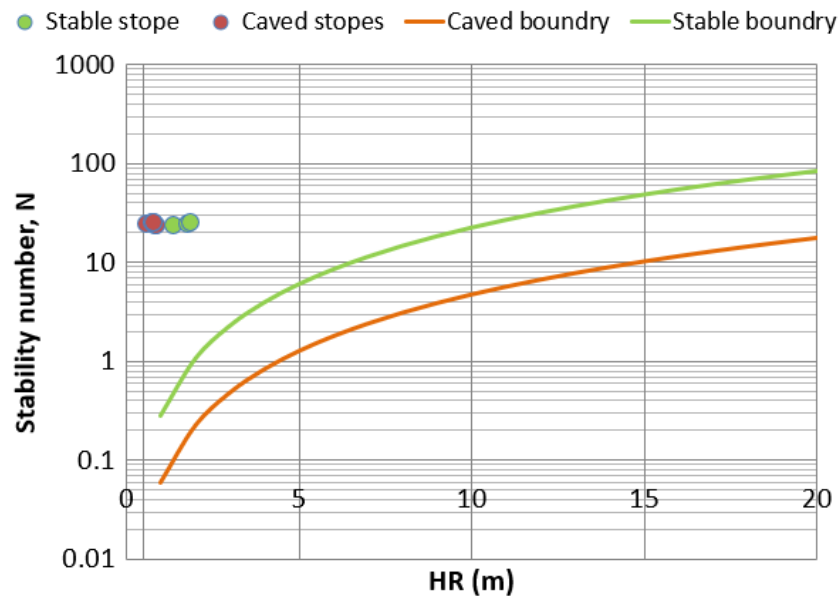


Figure 10. Stability chart with stope profiles

4.2 ELOS Approach

The planned geometries for stopes 172, 198, and panel 42 were subjected to individual numerical analysis to quantify the expected overbreak for the excavations. Geometries of planned and actual open stopes are presented in Appendix A. The boundary of the potential slough zone for each profile was traced using an isoline corresponding to the damage threshold of one-third the rock mass compressive strength. The cross-sectional area of slough was then calculated based on

the spatial propagation of this stress-driven threshold into the hanging wall relative to the design boundary.

ELOS was determined for each section according to Equation (4). The predicted dilution was subsequently derived by relating the ELOS to the planned stope width. Across all analyzed profiles, the ELOS values and the resulting predicted dilution levels were consistently lower than the actual performance figures reported. The comparative results for the 9 analyzed profiles and their respective mean predicted values against reported dilution are summarized in Table 3.

Table 3. Comparison of predicted ELOS and dilution against actual mine performance

Stope	ELOS	Predicted dilution	Mean Predicted Dilution	Reported dilution
Stope 172 P1	0.949	29.98%	20.47%	35.78%
Stope 172 P2	0.696	17.58%		
Stope 172 P3	0.606	13.86%		
Stope 198 P1	0.723	55.91%	42.34%	57.25%
Stope 198 P2	0.704	32.75%		
Stope 198 P3	0.913	38.37%		
Panel 42 P1	0.32	17.37%	17.54%	39.35%
Panel 42 P2	0.33	17.27%		
Panel 42 P3	0.349	17.98%		

4.3 Simulation Results

The simulations for all 9 stopes were performed under the conditions described in the methodology. The CSM data were then used to validate the numerical predictions. Mapped overbreak profiles were superimposed onto the corresponding numerical sections, and calculated stress values were retrieved at each point along the physical failure surface. Visual example of simulation results for Stope 172 is presented below.

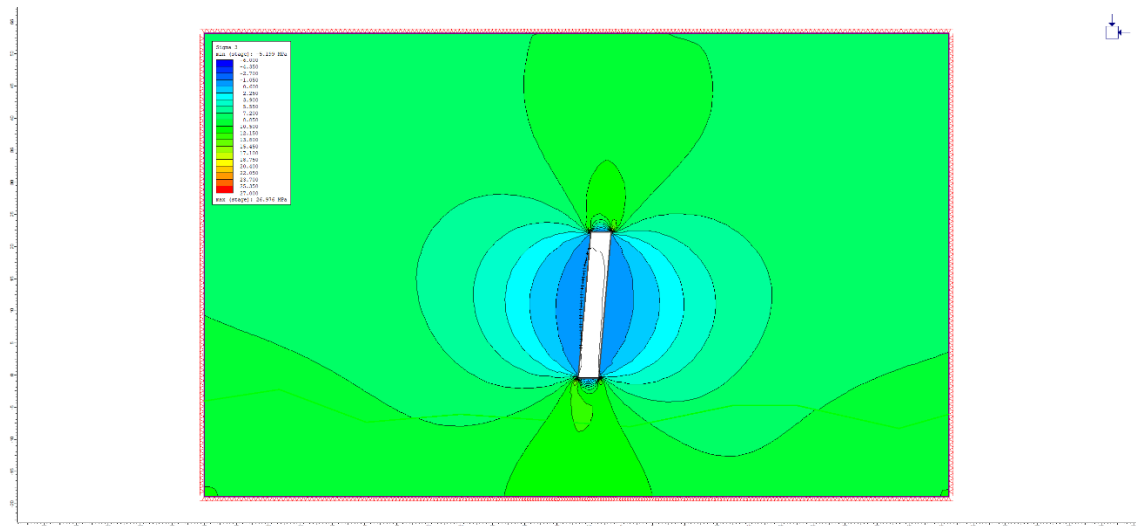


Figure 11. Minor Principal Stress Distribution for Stope 172 P1

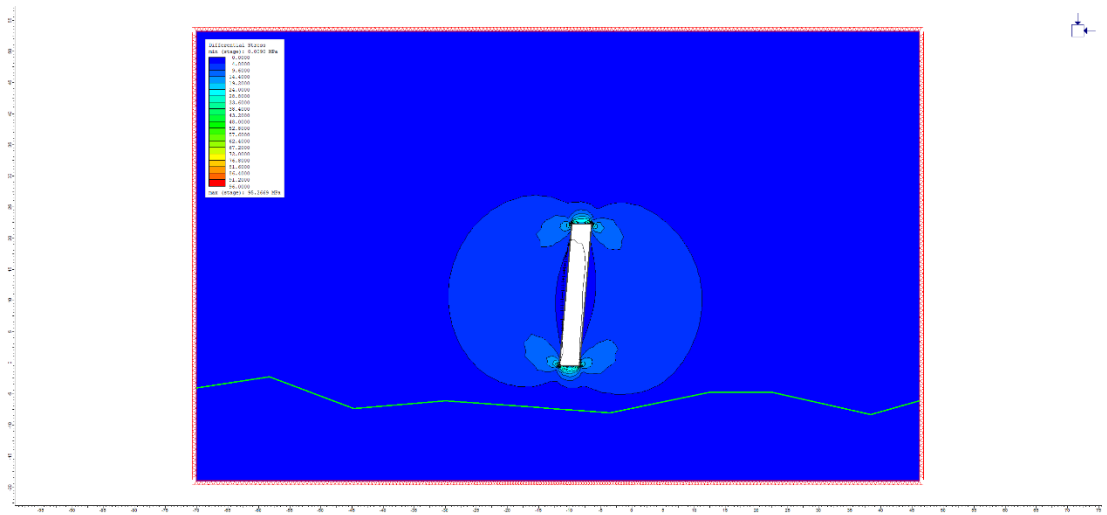


Figure 12. Differential Stress Distribution for Slope 172 P1

Additionally, the example of retrieved data is provided. It consists of data point coordinates, differential stress, major/minor principal stress and calculated depth of failure relative to the planned slope geometry. Based on preliminary regression models the minor principal stress was determined to be the best predictor of overbreak.

Table 4. Simulation Results for Slope 172 P1

Point_x	Point_y	$\sigma_1 - \sigma_3$	σ_1	σ_3	Depth of Failure
-9.875	19.803	4.827	4.822	-0.005	0.101
-10.17	19.482	5.577	5.584	0.006	0.267
-10.273	19.032	5.243	5.249	0.006	0.33
-10.313	18.571	4.804	4.805	0.000	0.329
-10.433	18.123	4.681	4.682	0.001	0.411
-10.547	17.674	4.551	4.560	0.009	0.485
-10.66	17.225	4.465	4.472	0.007	0.559
-10.709	16.764	4.250	4.260	0.010	0.567
-10.755	16.302	4.074	4.080	0.006	0.572
-10.806	15.842	3.932	3.941	0.008	0.584
-10.877	15.384	3.856	3.861	0.005	0.614
-10.94	14.925	3.784	3.789	0.005	0.637
-10.993	14.464	3.698	3.706	0.009	0.65
-11.025	14.002	3.611	3.616	0.005	0.641
-11.075	13.541	3.556	3.562	0.006	0.651
-11.134	13.082	3.532	3.538	0.006	0.67
-11.182	12.622	3.503	3.509	0.006	0.678
-11.182	12.159	3.413	3.418	0.005	0.638
-11.182	11.695	3.335	3.339	0.004	0.597
-11.224	11.234	3.326	3.331	0.005	0.599
-11.253	10.771	3.307	3.312	0.005	0.588
-11.278	10.308	3.295	3.299	0.003	0.572
-11.272	9.845	3.241	3.245	0.004	0.526
-11.262	9.382	3.190	3.193	0.003	0.476
-11.261	8.918	3.158	3.161	0.003	0.434
-11.269	8.455	3.148	3.151	0.002	0.403
-11.292	7.993	3.174	3.176	0.001	0.385

-11.284	7.53	3.154	3.155	0.001	0.337
-11.266	7.067	3.120	3.121	0.001	0.278
-11.255	6.603	3.103	3.103	0.000	0.227
-11.247	6.14	3.098	3.098	0.000	0.179
-11.235	5.677	3.088	3.089	0.000	0.126
-11.209	5.214	3.053	3.053	0.000	0.061
-11.22	4.751	3.104	3.103	-0.001	0.032
-11.246	4.288	3.212	3.211	-0.001	0.017
-11.274	3.826	3.354	3.353	-0.001	0.006
-11.698	-0.272	31.063	32.266	1.203	0.072
-8.221	-0.445	33.263	43.230	9.968	0.267
-8.278	0	19.796	20.189	0.393	0.157
-8.351	0.457	10.328	10.309	-0.020	0.048

4.4 Statistical Analysis

Minor principal stress has a bimodal tendency with 2 distinct peaks at 0 MPa and 10 MPa. There are two zones that could be described as relaxation zones near hanging wall and induced stress zones near stope back. Depth of failure has a right-skewed closely aligned with log-normal distribution. Histograms are presented below.

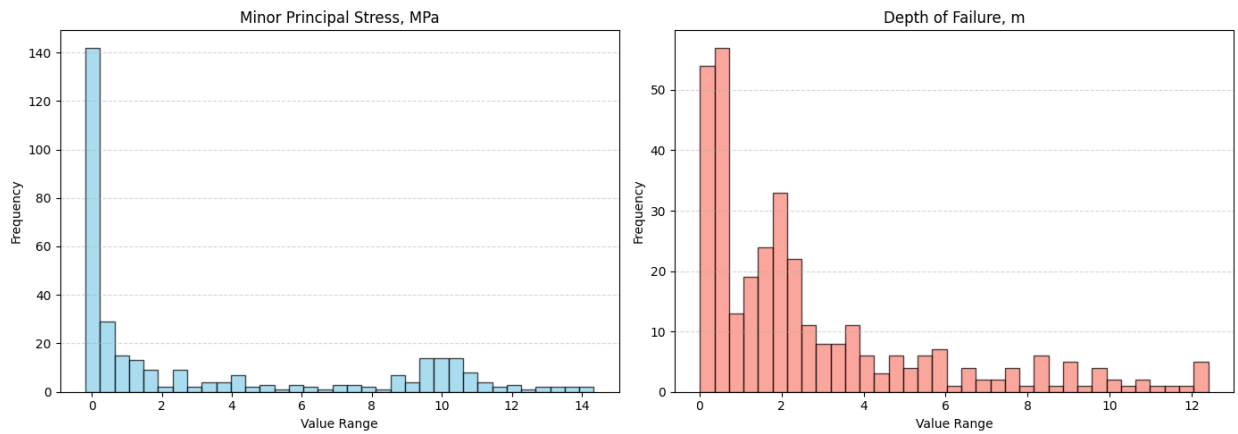


Figure 13. Data Distribution

The points are mostly localized within the hanging wall, and some are localized on the backs of the stopes. This in turn creates a high variance for minor principal stress that ranges from tensile zones with negative values to high confinement zones. The mean value of 3.30 MPa is notably higher than the median of 0.57 MPa, pointing to a positively skewed distribution. Similar situation is for the depth of failure. It exhibits a broad range from near-zero to a maximum of 12.41 meters, with a mean depth of 2.72 meters. Overall, 3 stopes have different failure reasoning and propagation with a full spectrum of stope wall behavior, from minor surface sloughing to significant structural instability. Descriptive statistics of the variables are presented in Table 3.

Table 5. Descriptive statistics of the dataset

Metric	Minor Principal Stress	Depth of failure
Mean	3.305	2.716
Standard Error	0.238	0.160
Median	0.567	1.797
Standard Deviation	4.36487	2.93599
Sample Variance	19.0521	8.620
Range	14.526	12.403
Minimum	-0.181	0.006
Maximum	14.344	12.409
Count	336	336

Additionally, a Paired Two Sample t-Test was performed for the dataset. The two-tail p-value of 0.0001896 with 95% confidence was achieved. It confirms statistical significance between correlation of the observed difference between stress level and the depth of failure. The Pearson Correlation is 0.7806, which is a strong linear relationship for such dataset. This validates the dataset for further regression analysis.

4.5 Regression Analysis

The statistical analysis identified the induced minor principal stress as the most significant predictor of wall overbreak. So, only it was chosen regression analysis. First, the linear regression is performed. The resulting equation is:

$$DOF = 0.5116\sigma_3 + 1.0253 \quad (9)$$

Furthermore, Figure 6 shows the regression results. The highest overbreak gradually increases up to 10.8 MPa and later decays with lowest recorded DOF near the values corresponding to the rock mass UCS.

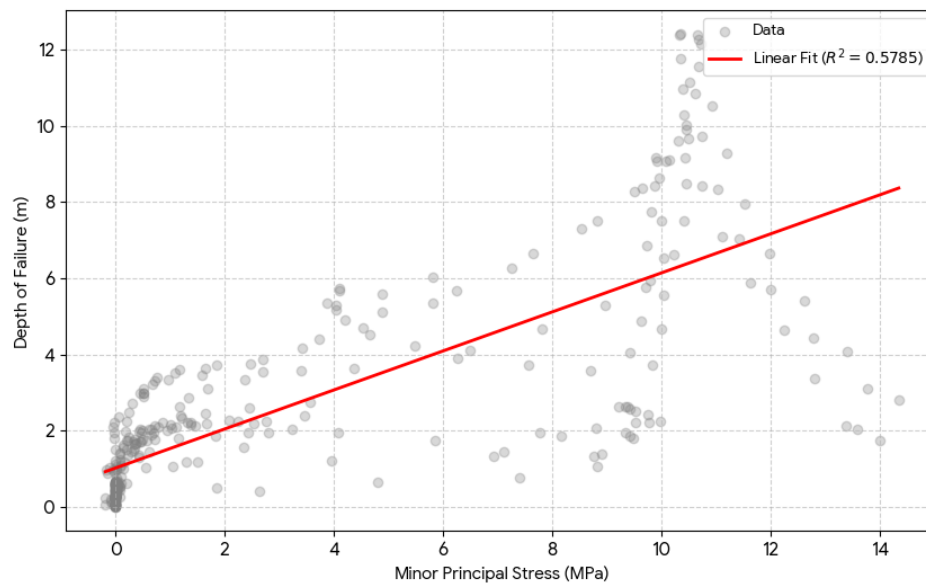


Figure 14. Global linear regression model

The model achieved a coefficient of determination (R^2) of 0.5785. The linear fit under-predicts the depth of failure in the high-stress zone and over-predicts it on tails. In a sense, it is a better version of a coinflip. Yet, the reliability of this equation is frankly significant. There is extremely low probability that the observed relationship is due to random chance. Moreover, there is a possibility to utilize the conservative design approach when using the upper 95% values to predict maximum probable depth of failure. The above-mentioned data is presented in Table 4.

Table 6. Sensitivity and Reliability of Linear Regression Analysis

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	1.02527	0.13070	7.8445	5.92E-14	0.76817	1.28236
σ_3	0.51161	0.02389	21.4107	1.25E-64	0.46460	0.55861

The next model is polynomial. The resulting equation is:

$$DOF = -0.0054\sigma_3^4 + 0.1353\sigma_3^3 - 1.0503\sigma_3^2 + 2.9701\sigma_3 + 0.559 \quad (10)$$

The polynomial model captures the dataset dips and peaks better. Within a relaxation zone it does a very good job showing a clear trend of rapid increase in depth of failure based on minor increase of principal stress. The dip between 2 peaks lacks representing the overbreak and cuts a significant portion of major confinement failures out of the fitted line. Still, it has a higher coefficient of determination (R^2) equal to 0.6995, which is significantly better. Graph is presented below.

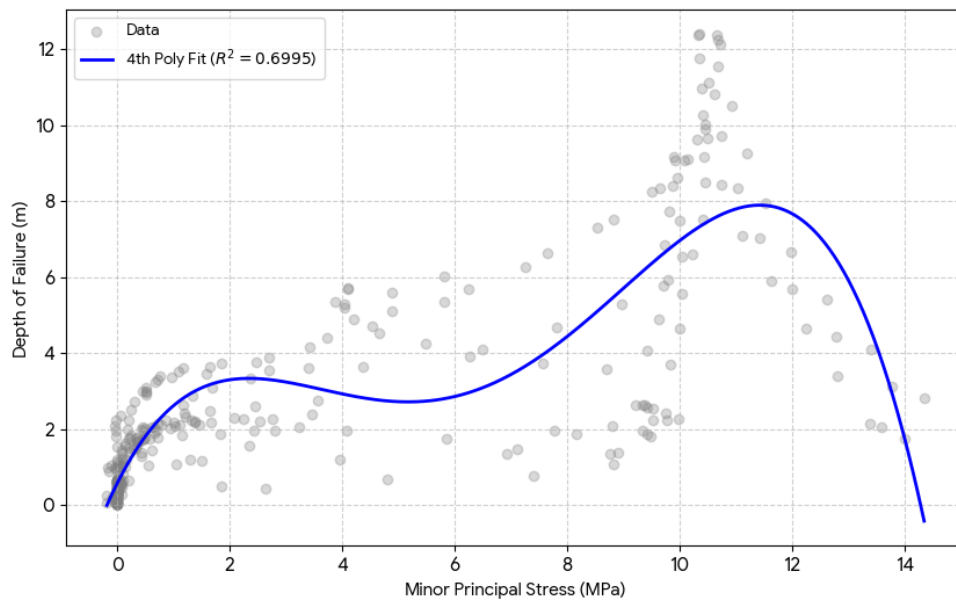


Figure 15. Global Polynomial Regression Model

As for coefficients, each of them is statistically significant. The results are reported in Table 6.

Variable	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.559	0.13137	4.2556	2.72E-5	0.30063	0.81746
σ_3	2.9701	0.30723	9.6672	1.23E-19	2.36571	3.57445
σ_3^2	-1.0503	0.11933	-8.8009	7.66E-17	-1.28500	-0.81550
σ_3^3	0.1353	0.01457	9.2846	2.20E-18	0.10664	0.16398
σ_3^4	-0.0054	0.00055	-9.7425	6.94E-20	-0.00644	-0.00428

K-means Clustering performed poorly. Centroids are 0.52 for Cluster 0 in the relaxation zone, 10.37 for Cluster 1 in high stress zone, 8.43 for Cluster 2 in transitional zone. Coefficients of determination (R^2) for clusters are as follows: 0.3466, 0.0248 and 0.0849. Equations:

$$\text{Cluster 0: } DOF = 0.6545\sigma_3 + 0.8825 \quad (11)$$

$$\text{Cluster 1: } DOF = 0.4383\sigma_3 + 4.6821 \quad (12)$$

$$\text{Cluster 2: } DOF = -0.1577\sigma_3 + 5.0642 \quad (13)$$

No further investigation of this model is required. The graph is presented below for visual review.

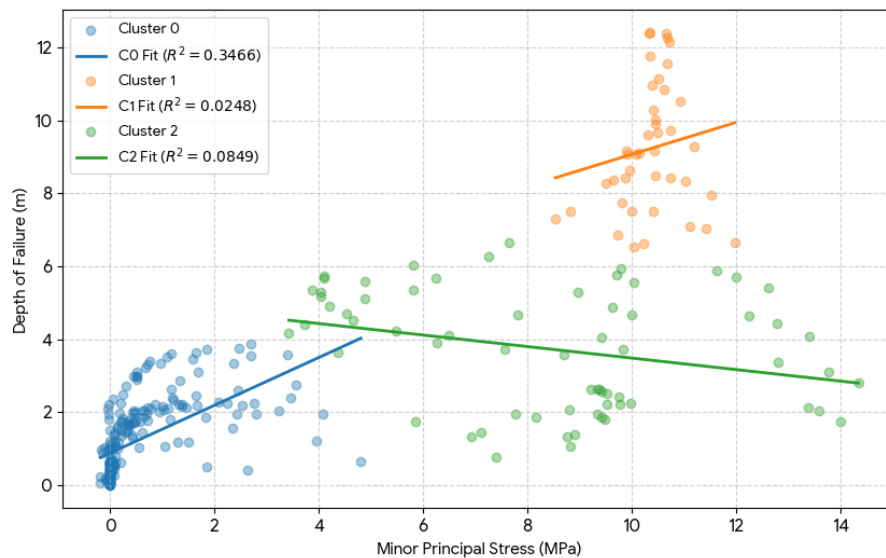


Figure 16. K-means Regression Model

The most promising model is regression spline. It was decided to use 3rd degree polynomial for hanging wall zone and linear model for back zone. The model achieved a global coefficient of determination (R^2) of 0.8207, which is the best result now. First zone R^2 is 0.609, second zone R^2 is 0.7792. Overall, it could be considered good. The resulting equations are:

$$DOF = 0.0268\sigma_3^3 - 0.4203\sigma_3^2 + 1.9930\sigma_3 + 0.6458 \quad (14)$$

$$DOF = -2.3059\sigma_3 + 34.1943 \quad (15)$$

The initial reliability check based on parameters is positive. All of them are significant for both segments. The data is presented in Table 7 and 8.

Table 8. Sensitivity and Reliability of Spline HW Zone Regression Analysis

Variable	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.6458	0.0969	6.666	1.58E-10	0.4551	0.8365
σ_3	1.9930	0.1920	10.379	5.14E-21	1.6150	2.3709
σ_3^2	-0.4203	0.0574	-7.318	1.22E-11	-0.5333	-0.3072
σ_3^3	0.0268	0.0041	6.488	1.68E-9	0.0187	0.0350

Table 9. Sensitivity and Reliability of Spline Back Zone Regression Analysis

Variable	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	34.1943	2.2143	15.442	1.89E-18	29.7190	38.6696
σ_3	-2.3059	0.1941	-11.880	1.07E-14	-2.6982	-1.9136

The graph is presented below. It can be seen that the model better captures the initial spike in depth of failure than linear fit and still underperforms within the stress transition zone.

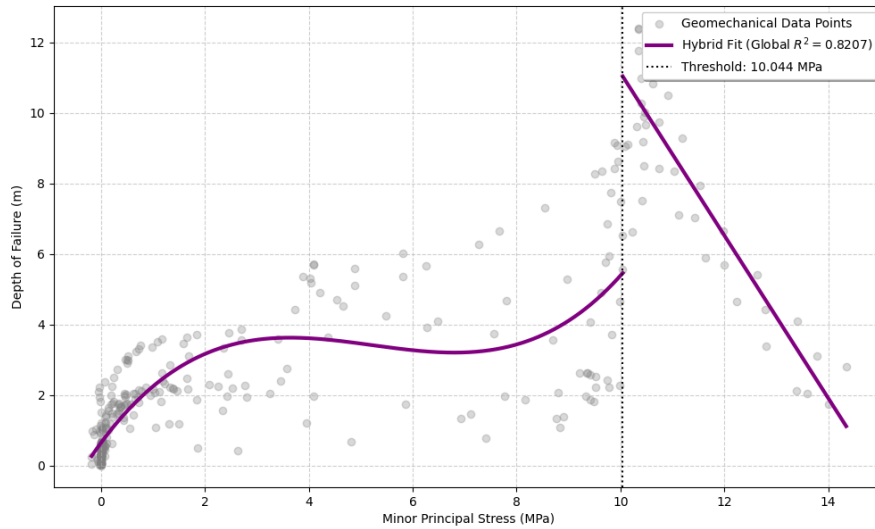


Figure 17. Global Spline Regression Model

The AIC check is done only for 3 models: linear, polynomial and spline. The K-means Cluster is excluded for poor performance. The choice based on AIC is spline regression. Results are presented in Table 9.

Table 10. AIC Reliability Results

Model type	k	RSS	R ²	AIC
Linear	2	1217.158	0.5785	438.49
Polynomial	5	867.729	0.6995	330.79
Spline	6	823.211	0.8207	313.09

5.0 Discussion

An important detail that was found during the writing of this thesis is that no failure criteria applies consistently to 2D analysis. The main problem could be framed as using intact rock UCS yields incorrect assumptions about rock mass stability. Martin's and Hoek-Brown's failure criteria use it and based on their respectable equations the it appears that the failure starts only at 52 and 45 MPa respectfully for the domain of microquartzite. Such stress values do not appear at any point of numerical models. However, the substitution of intact UCS to rock mass UCS yields results that do correlate to failure propagation found in stopes from case study.

It can be suggested that failure is likely governed by brittle spalling processes which initiate at a fraction of the laboratory UCS. Therefore, the modeling requires a calibration to account for damage initiation and the inherent reduction in strength caused by the scale effect.

5.1 Geomechanical Analysis of Stress-Failure Regression Models

The linear fit shows an R^2 of 0.5785. It is a baseline for comparison and does not serve as a definitive behavior model. It can only capture general trend of correlation between higher confining stress and higher depth of failure. The linear fit forces a straight line through the entire stress range, averaging across the non-monotonic behavior. Therefore, the central dip between stress peaks and the flattening response at high confinement are misrepresented.

The fourth-degree polynomial equation demonstrates a coefficient of determination of 0.6995. The model performs very well in the relaxation zone by showing a rapid increase in the depth of failure as confining stress increases slightly. This trend reflects the mechanical unravelling of the jointed rock mass. The polynomial curve follows the initial spike of data points near the zero-stress intercept. However, the central dip between the two major stress peaks represents a limitation for this approach. Significant portions of failure data in the high-confinement regions remain unrepresented by the fitted line.

K-means clustering yields poor results for the dataset. Centroids for the three clusters do not align with clear geomechanical boundaries. The partition of data into non-overlapping groups ignores the continuous nature of stress redistribution around open stopes. Coefficients of determination for these clusters are extremely low. This approach generally lacks predictive power.

The regression spline model offers the best representation of geomechanical data. Global R^2 reaches 0.8207 with segmented approach. The model fits the initial depth of failure spike and the decay at high confinement levels. The spline model is chosen as the main result of the research due to its best fit and confirmed reliability based on AIC approach.

This method uses a third-degree polynomial to describe the hanging wall behavior. It has been chosen based on observations of polynomial models that fit relaxation zone data better. And a linear function is chosen to represent the response of the slope back to high horizontal stresses. Overall, this model outputs the best predictive power across the model for the whole range. The threshold at 10.044 MPa for minor principal stress has been chosen based on datapoint coordinate separation relative to the slope geometry to correctly establish the zone borders.

Linear fit shows interesting correlations for its parameters. The global slope coefficient of 0.5116 is the rate of overbreak propagation relative to the reduction in induced confinement. It can be hypothesized this parameter is related to the rock mass curvature constant (a) defined within the Hoek-Brown failure criterion. Calculations for the microquartzite domain at the Sokolny Mine yield a theoretical a value of 0.5026 for a GSI of 61. The close agreement between the empirical slope and the theoretical curvature parameter indicates that the failure zone propagates through the rock mass at a rate governed by the non-linear reduction in strength within low-confinement zones (Diederichs et al., 2004). However, it is noted that it may not align for any rock type nor is it possible to define the statistical significance of this correlation at the moment. The convergence between the empirical linear slope and the Hoek-Brown curvature parameter requires further investigation beyond the scope of this case study. If this relationship proves generalizable across different rock types, it could provide a tool for rapid dilution estimation in data-sparse environments. Engineers could estimate DOF sensitivity using only GSI and intact rock properties, bypassing the need for extensive numerical modeling.

The intercept of 1.0253 meters is the minimal predicted depth of failure when the induced confining stress is zero. This value physically corresponds to the skin damage zone resulting from blast-induced fracturing and immediate stress redistribution (Kaiser et al., 2011). This parameter could be derived from ELOS, preferably site-specific one. Overall, it should account for the unavoidable unravelling of brittle jointed rock masses during the excavation cycle (Diederichs et al., 2004).

As for the chosen segmented fit any power parameter cannot be explained by such correlation. At least there is no definitive way to relate any parameter to known rock mass quality parameters. On the other hand, the free parameter for hanging wall of 0.6458 can be explained in a similar way to linear fit. It can be concluded that this is the minimal depth of failure that occurs for those slopes and can be treated as ELOS for the base case scenario.

For roof equation the same parameter is 34.1943, which is due to the nature of negative slope is the maximum depth of failure that may possibly happen within the designated geometry

of the stope. Additionally, the roof behavior is mostly linear with rapid decrease of failure when the stress approaches rock mass UCS of 14.96 MPa calculated with Hoek-Brown failure criteria assumptions. It is possible to hypothesize that confinement similar to rock mass strength is sufficient to propagate stable behavior within roof zone.

5.2 Stability Graph

The empirical analysis of stope profiles showed high level of inconstancy between predicted stability and actual performance. Due to unknown geological conditions regarding discontinuities, it was impossible to apply correctly any type of stability graph. However, assuming worst case scenarios every stope profile is predicted to fall in stable class. Only Stope 172 performed in accordance with the initial classification.

The reason why this approach appeared unreliable for this dataset is that the 2D section analysis fails to account for the out of plane span of the stope. The empirical method relies on the Hydraulic Radius (HR) to quantify excavation size, and for strike lengths exceeding 30 to 50 meters, the HR value asymptotically approaches half the stope width and does not reflect the increase in excavation scale. So, a 50 m long stope and a 100 m long stope yield nearly identical HR values and are erroneously assigned the same 'Stable' classification, despite the latter exposing a vastly larger surface area and volume of rock to gravitational unloading.

Furthermore, the 2D section analysis inherently assumes that the confinement provided by the stope end-walls extends infinitely along the strike. The stabilizing arching effect of the abutments disappears and leaves the central portion of stopes unsupported. This absence of 3D confinement enables the gravity-driven unraveling and caving observed in six of the nine cases. This failure mechanism that cannot be quantified in standard stability charts explains why profiles predicted as Stable experienced caving or excessive dilution in practice.

5.3 Overbreak Interpretations

5.3.1 Stope 172

The 35.78% dilution observed in Stope 172 is primarily driven by operational inaccuracies during the production cycle. Analysis of the CMS data shows that the actual extraction boundary deviates in the hanging wall side from the planned boundary, which indicates that improper drill hole orientation or excessive explosive energy directly contributed to the detachment of waste rock (Wang, 2004). The irregular geometry of the excavation increases this instability because narrow spans in jointed microquartzite are sensitive to the vibration profiles of heavy production blasting (Alipov & Adoko, 2020). Geomechanical stress redistribution around these slender profiles often

results in the opening of sub-vertical joints which leads to the sloughing of the hanging wall rock into the stope void (Diederichs, 2000).

5.3.2 Stope 198

The high magnitude of unplanned dilution and the instances of localized caving in Stope 198 are attributed to the propagation of structural instability under conditions of extreme stress relaxation. The presence of caving zones suggests that the induced horizontal stresses were insufficient to maintain the structural stability (Kaiser et al., 2011). These failures were likely accelerated by heavy production blasting (Wang, 2004).

5.3.3 Panel 42

Researching the upper level of Panel 42 was a necessity caused by the absence of complete geometry data for the lower mining horizons. Numerical models generated an ELOS range between 0.32 and 0.35 meters. This is possibly a stable initial condition when the stopes are evaluated as independent excavations (Clark & Pakalnis, 1997). The measured overbreak was a delayed response to sublevel excavation.

The observed instability in the upper sections of Panel 42 developed as a secondary response to the complete extraction of all three vertical levels. Successive removal of lower ore blocks increases the effective hydraulic radius of the overall hanging wall surface (Alipov & Adoko, 2020). This growth in the vertical span causes redistribution of stresses where the minor principal stress drops across the entire panel height (Wang, 2004). The upper stope walls remained stable during their individual extraction phase because the surrounding rock mass maintained sufficient geometric confinement (Kaiser et al., 2011). Progressive mining of the lower stopes triggered a large-scale relaxation zone that propagated upward and induced delayed caving of the previously stable upper horizons (Diederichs, 2000).

5.4 Limitations of the Study

This research is subject to several constraints related to the nature of the available data.

1. The use of two-dimensional sections in RS2 is a simplification of the three-dimensional stress environment found in underground mining. This approach assumes that the stress state remains constant along the strike of the stope geometry. The reliance on CMS data introduces a dependence on the precision of the underground survey equipment.
2. The study is based on a single case study from the Ridder-Sokolny Mine, which limits the generalizability of the findings to other mines. Additionally, the site-specific nature of the dataset may influence the identified relationship.

3. Statistical analysis relies on a limited dataset of open stopes. Its size allowed to establish the trend but may not fully show the complete model of stress-driven overbreak. In regression models data sparsity can and will affect its robustness.
4. The study primarily focuses on stress-driven failure mechanisms, with limited consideration of other contributing factors.
5. The proposed spline regression model remains empirical in nature and does not explicitly account for underlying failure mechanisms. The model should be interpreted as a predictive tool only.

5.5 Recommendations

Traditional strength-based factors of safety are insufficient in jointed rock masses where failure is driven by stress relaxation. The following guidelines conclude the key findings from this case study into actionable recommendations for operations facing similar data constraints. They are organized into 4 areas corresponding to the primary drivers of unplanned dilution identified in this research

1. Adopt Stress-Based Design Criteria

It is recommended to shift from traditional empirical charts to stress-driven calibrated methods of estimating dilution. It works especially well with limited geological data. Only the stress state of rock mass is required, which can be derived from depth of the location and stress ratio that can also be estimated.

2. Integrate Spline Regression Models into Design

It is recommended to apply regression analysis. Resulting equations could be used to estimate depth of failure. The minor principal stress is estimated using numerical modelling and obtained values across hanging wall and back are used as input. The mean overbreak is calculated, which can be then used as ELOS and calibration of site-specific stability chart.

3. Differentiate Design Criteria by Excavation Surface

This study illustrates the need for separate predictive equations for hanging wall and back behavior. Mine designers should explicitly account for these differences and avoid applying uniform stability criterion across all excavation surfaces.

4. Integrate Dilution Control into Mine Planning

The predicted depth of failure should be systematically incorporated into stope shape optimization and ore reserve calculations to better quantify and manage unplanned dilution.

6.0 Conclusion and Recommendations

Estimation of depth of failure as key driver of unplanned dilution has established a framework for stope design under constraints of limited data. The study confirms that, in this geological setting, traditional empirical tools underestimate the actual overbreak experienced.

Reliability in stope design can be made after moving from generalized rock mass classifications to site-specific stress propagation. In the case study the minor principal stress appears to be the predictor of wall instability and therefore dilution. Numerical modelling shows that the volume of relaxation zones is more of a function based on excavation geometry and the in-situ stress field. The regression analysis yields site-specific governing equations:

- for hanging wall: $DOF = 0.0268\sigma_3^3 - 0.4203\sigma_3^2 + 1.9930\sigma_3 + 0.6458$
- for back: $DOF = -2.3059\sigma_3 + 34.1943$

Results derived from the localized regression models indicate that different stope surfaces require distinct models: hanging wall behavior is governed by a non-linear relaxation, stope back is defined by confinement.

Mining operations should adopt the practice of generating synthetic datasets. This methodology allows for the isolation of influential factors and the derivation of site-specific empirical correlations. Regular updates to these models are much needed to maintain the accuracy of predicting dilution.

Future research should expand this framework to include the time-dependent degradation of the rock mass and the influence of backfill performance on stress redistribution. Large-scale relaxation resulting from successive extractions may induce delayed caving in previously stable stopes. This delayed caving can be prevented by monitoring the growth of the effective hydraulic radius across multiple mining levels. The integration of these variables will further refine the predictive power of the site-specific empirical equations.

Finally, uncertainty quantification was not explicitly included into the modeling framework, and the range of geomechanical properties considered is limited. Therefore, further studies may want to aim to incorporate a more comprehensive set of these parameters.

7.0 References

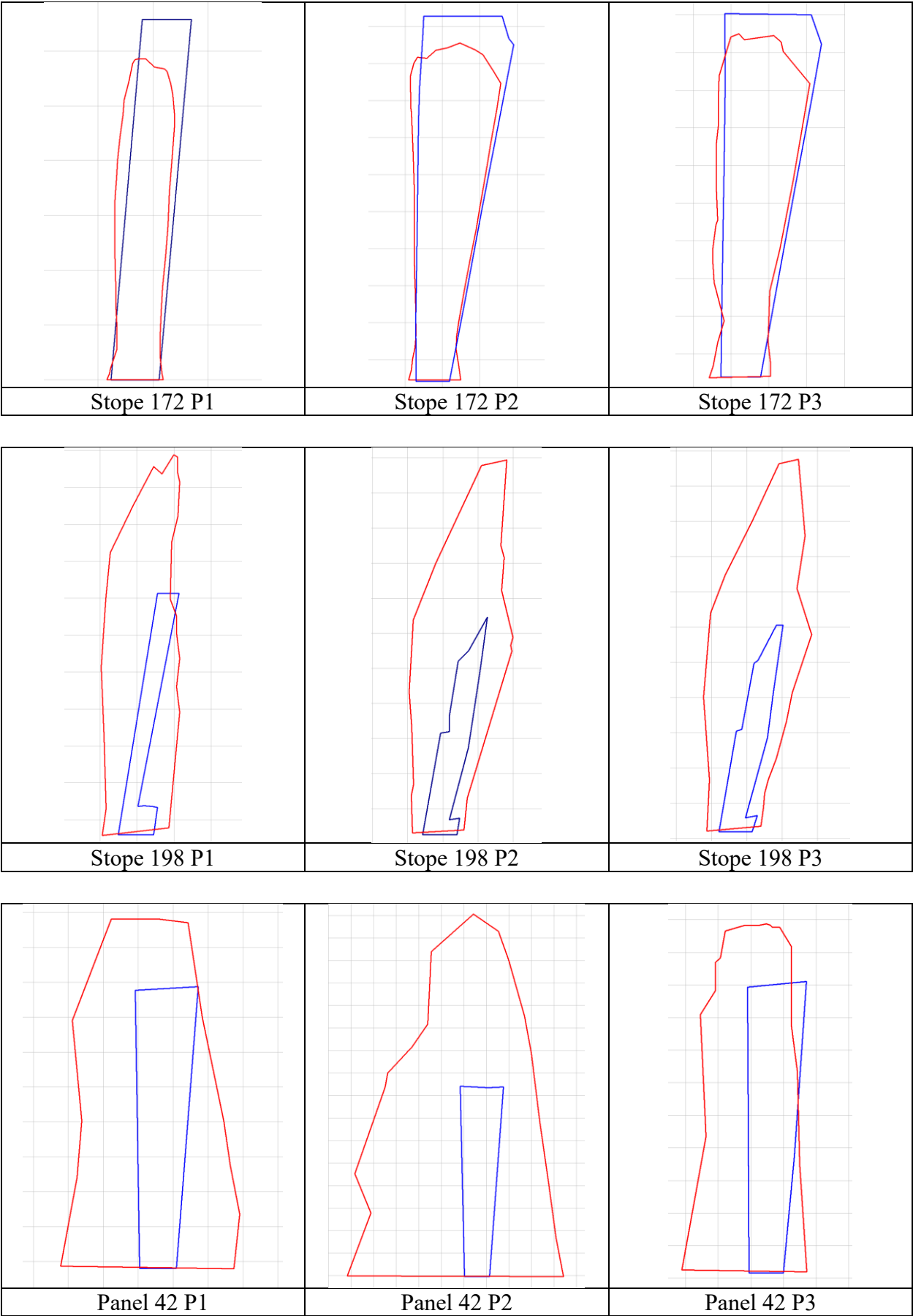
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Appendix A

Figures below present planned geometry in blue and actual geometry in red.



Appendix B

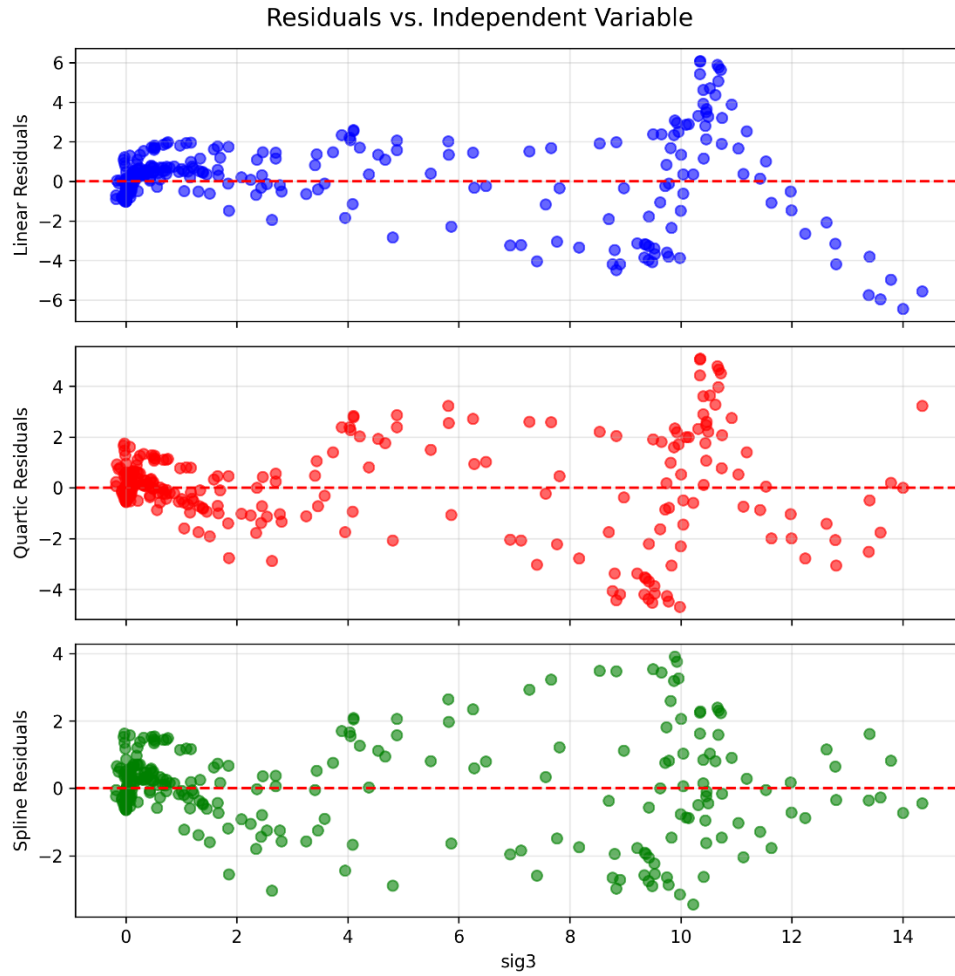


Figure 18. Residual vs. Independent variable distribution

Table 11. Regression and AIC data

Sig3	DOF	Linear RSS	Poly4 RSS	Spline RSS
-0.181	0.052	-0.88069	0.06576	-0.01057
-0.178	0.237	-0.69722	0.24068	0.16580
-0.162	0.978	0.03559	0.92825	0.86102
-0.135	0.88	-0.07622	0.74139	0.68658
-0.104	1.035	0.06292	0.79636	0.75508
-0.075	0.218	-0.76892	-0.11232	-0.14162
-0.073	0.165	-0.82294	-0.17158	-0.20008
-0.055	0.095	-0.90215	-0.29749	-0.31891
-0.047	2.09	1.08876	1.67288	1.65453
-0.039	0.992	-0.01333	0.55040	0.53505
-0.028	2.227	1.21604	1.75195	1.74067
-0.028	0.029	-0.98196	-0.44605	-0.45733
-0.027	1.949	0.93753	1.47092	1.46001
-0.02	0.936	-0.07906	0.43678	0.42841
-0.013	1.809	0.79036	1.28875	1.28288
-0.01	0.566	-0.45417	0.03676	0.03195
-0.009	0.077	-0.94368	-0.45523	-0.45968

-0.008	0.302	-0.71919	-0.23322	-0.23732
-0.007	0.009	-1.01271	-0.52920	-0.53296
-0.007	1.136	0.11429	0.59780	0.59404
-0.005	0.101	-0.92173	-0.44317	-0.44622
-0.004	0.284	-0.73924	-0.26315	-0.26585
-0.003	0.075	-0.94875	-0.47512	-0.47748
-0.002	0.342	-0.68226	-0.21110	-0.21311
-0.001	0.079	-0.94578	-0.47707	-0.47874
-0.001	0.043	-0.98178	-0.51307	-0.51474
-0.001	0.017	-1.00778	-0.53907	-0.54074
-0.001	0.043	-0.98178	-0.51307	-0.51474
-0.001	0.032	-0.99278	-0.52407	-0.52574
-0.001	0.006	-1.01878	-0.55007	-0.55174
0	0.061	-0.96429	-0.49804	-0.49937
0	0.139	-0.88629	-0.42004	-0.42137
0	0.435	-0.59029	-0.12404	-0.12537
0	0.227	-0.79829	-0.33204	-0.33337
0	0.126	-0.89929	-0.43304	-0.43437
0	0.329	-0.69629	-0.23004	-0.23137
0	0.107	-0.91829	-0.45204	-0.45337
0	0.179	-0.84629	-0.38004	-0.38137
0	0.051	-0.97429	-0.50804	-0.50937
0.001	0.2	-0.82580	-0.36201	-0.36299
0.001	0.111	-0.91480	-0.45101	-0.45199
0.001	0.259	-0.76680	-0.30301	-0.30399
0.001	0.278	-0.74780	-0.28401	-0.28499
0.001	0.17	-0.85580	-0.39201	-0.39299
0.001	0.411	-0.61480	-0.15101	-0.15199
0.001	0.337	-0.68880	-0.22501	-0.22599
0.001	0.236	-0.78980	-0.32601	-0.32699
0.001	0.265	-0.76080	-0.29701	-0.29799
0.001	0.242	-0.78380	-0.32001	-0.32099
0.001	0.385	-0.64080	-0.17701	-0.17799
0.002	0.248	-0.77831	-0.31698	-0.31761
0.002	0.543	-0.48331	-0.02198	-0.02261
0.002	0.271	-0.75531	-0.29398	-0.29461
0.002	0.23	-0.79631	-0.33498	-0.33561
0.002	0.277	-0.74931	-0.28798	-0.28861
0.002	0.253	-0.77331	-0.31198	-0.31261
0.002	0.288	-0.73831	-0.27698	-0.27761
0.002	0.283	-0.74331	-0.28198	-0.28261
0.002	0.403	-0.62331	-0.16198	-0.16261
0.003	0.18	-0.84682	-0.38794	-0.38823
0.003	0.294	-0.73282	-0.27394	-0.27423
0.003	0.476	-0.55082	-0.09194	-0.09223
0.003	0.488	-0.53882	-0.07994	-0.08023
0.003	0.42	-0.60682	-0.14794	-0.14823
0.003	0.434	-0.59282	-0.13394	-0.13423
0.003	0.313	-0.71382	-0.25494	-0.25523

0.003	0.375	-0.65182	-0.19294	-0.19323
0.003	0.572	-0.45482	0.00406	0.00377
0.004	0.436	-0.59133	-0.13491	-0.13486
0.004	0.344	-0.68333	-0.22691	-0.22686
0.004	0.597	-0.43033	0.02609	0.02614
0.004	0.526	-0.50133	-0.04491	-0.04486
0.004	1.521	0.49367	0.95009	0.95014
0.004	0.476	-0.55133	-0.09491	-0.09486
0.005	0.513	-0.51485	-0.06087	-0.06047
0.005	0.599	-0.42885	0.02513	0.02553
0.005	0.637	-0.39085	0.06313	0.06353
0.005	0.588	-0.43985	0.01413	0.01453
0.005	0.638	-0.38985	0.06413	0.06453
0.005	0.614	-0.41385	0.04013	0.04053
0.005	0.641	-0.38685	0.06713	0.06753
0.006	0.678	-0.35036	0.10117	0.10191
0.006	0.651	-0.37736	0.07417	0.07491
0.006	0.33	-0.69836	-0.24683	-0.24609
0.006	0.67	-0.35836	0.09317	0.09391
0.006	0.316	-0.71236	-0.26083	-0.26009
0.006	0.572	-0.45636	-0.00483	-0.00409
0.006	0.609	-0.41936	0.03217	0.03291
0.006	0.267	-0.76136	-0.30983	-0.30909
0.007	0.406	-0.62287	-0.17378	-0.17271
0.007	0.272	-0.75687	-0.30778	-0.30671
0.007	0.559	-0.46987	-0.02078	-0.01971
0.007	0.34	-0.68887	-0.23978	-0.23871
0.008	0.584	-0.44538	0.00126	0.00268
0.008	0.631	-0.39838	0.04826	0.04968
0.008	0.08	-0.94938	-0.50274	-0.50132
0.009	0.65	-0.37989	0.06431	0.06606
0.009	0.485	-0.54489	-0.10069	-0.09894
0.009	0.437	-0.59289	-0.14869	-0.14694
0.009	0.385	-0.64489	-0.20069	-0.19894
0.009	0.614	-0.41589	0.02831	0.03006
0.01	0.567	-0.46340	-0.02164	-0.01955
0.01	1.204	0.17360	0.61536	0.61745
0.011	0.572	-0.45891	-0.01959	-0.01716
0.012	0.568	-0.46343	-0.02653	-0.02377
0.016	0.469	-0.56447	-0.13730	-0.13319
0.02	0.778	-0.25752	0.15997	0.16540
0.02	0.363	-0.67252	-0.25503	-0.24960
0.022	0.534	-0.50254	-0.08988	-0.08379
0.024	1.081	0.04343	0.45128	0.45802
0.024	1.023	-0.01457	0.39328	0.40002
0.025	0.673	-0.36508	0.04036	0.04743
0.025	0.994	-0.04408	0.36136	0.36843
0.037	0.64	-0.40422	-0.02750	-0.01656
0.038	0.456	-0.58873	-0.21440	-0.20314

0.044	0.685	-0.36280	-0.00270	0.01045
0.049	0.502	-0.54836	-0.20007	-0.18536
0.054	0.603	-0.44991	-0.11339	-0.09714
0.068	2.365	1.30492	1.60881	1.62927
0.075	1.199	0.13534	0.42305	0.44557
0.076	0.543	-0.52117	-0.23576	-0.21295
0.078	0.546	-0.51919	-0.23838	-0.21500
0.078	0.268	-0.79719	-0.51638	-0.49300
0.08	0.448	-0.61822	-0.34200	-0.31803
0.089	1.239	0.16818	0.42384	0.45037
0.09	1.05	-0.02133	0.23206	0.25887
0.091	1.434	0.36216	0.61328	0.64036
0.092	1.385	0.31264	0.56149	0.58886
0.099	0.63	-0.44594	-0.21292	-0.18361
0.106	0.815	-0.26452	-0.04723	-0.01602
0.113	1.086	0.00290	0.20455	0.23764
0.135	1.573	0.47865	0.63181	0.67056
0.138	1.629	0.53311	0.67973	0.71923
0.139	1.007	-0.08940	0.05505	0.09479
0.169	1.191	0.07925	0.15936	0.20624
0.192	1.989	0.86548	0.89747	0.94941
0.199	1.74	0.61290	0.63044	0.68387
0.205	2.237	1.10683	1.11207	1.16673
0.209	1.499	0.36679	0.36386	0.41934
0.21	0.638	-0.49472	-0.49969	-0.44401
0.219	1.758	0.62067	0.59747	0.65494
0.229	1.374	0.23156	0.18828	0.24767
0.247	2.495	1.34335	1.26440	1.32711
0.252	1.826	0.67179	0.58305	0.64664
0.294	1.462	0.28630	0.11714	0.18757
0.317	2.733	1.54553	1.33373	1.40746
0.324	1.496	0.30495	0.08036	0.15504
0.334	1.733	0.53684	0.29414	0.37012
0.345	1.553	0.35121	0.08881	0.16615
0.353	1.769	0.56312	0.28652	0.36481
0.36	1.646	0.43653	0.14762	0.22671
0.378	1.672	0.45333	0.13313	0.21416
0.383	1.68	0.45877	0.12999	0.21152
0.429	1.296	0.05123	-0.35442	-0.26888
0.434	1.367	0.11968	-0.29411	-0.20820
0.442	1.995	0.74358	0.31689	0.40335
0.452	1.713	0.45647	0.01378	0.10091
0.455	2.053	0.79493	0.34748	0.43479
0.46	2.998	1.73737	1.28202	1.36964
0.474	1.944	0.67621	0.19897	0.28735
0.484	1.733	0.46010	-0.03258	0.05630
0.491	2.018	0.74151	0.23814	0.32733
0.501	1.777	0.49540	-0.02311	0.06648
0.507	2.993	1.70833	1.18081	1.27063

0.508	2.893	1.60782	1.07881	1.16866
0.513	3.101	1.81326	1.27681	1.36682
0.52	2.974	1.68268	1.13587	1.22610
0.561	1.048	-0.26430	-0.87008	-0.77907
0.572	1.748	0.43007	-0.19105	-0.09996
0.615	1.445	0.10508	-0.57412	-0.48327
0.615	2.043	0.70308	0.02388	0.11473
0.661	1.883	0.51954	-0.21844	-0.12876
0.661	2.053	0.68954	-0.04844	0.04124
0.67	3.227	1.85894	1.10985	1.19919
0.707	1.941	0.55401	-0.23940	-0.15178
0.721	3.312	1.91785	1.10823	1.19505
0.723	1.785	0.38982	-0.42208	-0.33539
0.73	2.133	0.73424	-0.08563	0.00063
0.758	3.402	1.98892	1.13792	1.22225
0.781	2.112	0.68715	-0.18852	-0.10600
0.867	2.222	0.75315	-0.20779	-0.13375
0.953	2.003	0.49015	-0.54537	-0.48231
0.976	3.349	1.82439	0.77067	0.83041
1.005	2.174	0.63455	-0.44107	-0.38575
1.039	2.08	0.52315	-0.57670	-0.52687
1.046	1.071	-0.48943	-1.59408	-1.54542
1.089	3.518	1.93557	0.80285	0.84406
1.116	2.094	0.49776	-0.65136	-0.61508
1.156	1.822	0.20530	-0.96642	-0.93775
1.17	2.626	1.00213	-0.17701	-0.15109
1.172	3.598	1.97311	0.79292	0.81844
1.193	2.401	0.76537	-0.42546	-0.40416
1.215	2.331	0.68411	-0.51729	-0.50051
1.306	1.191	-0.50244	-1.74143	-1.74437
1.322	2.232	0.53037	-0.71423	-0.72078
1.331	2.861	1.15477	-0.09286	-0.10147
1.38	2.208	0.47670	-0.78581	-0.80585
1.388	2.176	0.44060	-0.82408	-0.84602
1.458	2.123	0.35179	-0.92898	-0.96792
1.504	1.175	-0.61974	-1.90826	-1.95871
1.586	3.462	1.62531	0.32832	0.25682
1.648	3.635	1.76659	0.46757	0.37977
1.651	2.471	0.60105	-0.69798	-0.78657
1.666	2.178	0.30038	-0.99855	-1.09113
1.694	3.097	1.20505	-0.09314	-0.19318
1.84	1.859	-0.10764	-1.39073	-1.53020
1.846	3.719	1.74929	0.46720	0.32611
1.853	0.503	-1.47029	-2.75116	-2.89415
2.081	2.291	0.20106	-1.01949	-1.22394
2.246	2.249	0.07465	-1.07951	-1.32680
2.346	1.558	-0.66751	-1.77331	-2.04535
2.361	3.331	1.09781	-0.00024	-0.27591
2.433	1.956	-0.31402	-1.37325	-1.66589

2.449	2.605	0.32679	-0.72345	-1.01977
2.466	3.762	1.47509	0.43455	0.13436
2.537	2.189	-0.13423	-1.13280	-1.44870
2.632	0.43	-1.94183	-2.88068	-3.21636
2.694	3.868	1.46445	0.56660	0.21884
2.697	3.559	1.15391	0.25808	-0.09024
2.77	2.261	-0.18143	-1.02705	-1.38864
2.802	1.953	-0.50581	-1.32882	-1.69590
3.247	2.047	-0.63947	-1.11950	-1.53954
3.407	3.591	0.82267	0.47474	0.04759
3.432	4.16	1.37888	1.05182	0.62417
3.462	2.387	-0.40946	-0.71143	-1.13944
3.575	2.745	-0.10928	-0.31629	-0.74346
3.726	4.411	1.47947	1.39964	0.97925
3.885	5.348	2.33513	2.38843	1.98228
3.95	1.205	-1.84113	-1.73394	-2.13213
4.031	5.299	2.21143	2.38516	1.99861
4.046	5.187	2.09176	2.27772	1.89354
4.086	1.969	-1.14671	-0.92826	-1.30580
4.097	5.722	2.60067	2.82800	2.45237
4.099	5.683	2.56064	2.78959	2.41431
4.209	4.896	1.71737	2.03417	1.67997
4.374	3.631	0.36795	0.81240	0.49642
4.546	4.709	1.35796	1.92906	1.66117
4.668	4.513	1.09954	1.75588	1.52704
4.81	0.666	-2.82011	-2.07000	-2.24843
4.879	5.111	1.58959	2.38300	2.23091
4.881	5.594	2.07157	2.86620	2.71490
5.488	4.236	0.40302	1.50432	1.63028
5.809	6.016	2.01880	3.22190	3.51936
5.812	5.351	1.35226	2.55611	2.85522
5.863	1.74	-2.28483	-1.06905	-0.74169
6.252	5.672	1.44816	2.71615	3.26259
6.277	3.915	-0.32163	0.94733	1.50786
6.485	4.102	-0.24105	1.02486	1.70122
6.919	1.342	-3.22308	-2.02726	-1.12706
7.121	1.465	-3.20343	-2.06868	-1.07881
7.266	6.259	1.51639	2.59668	3.64275
7.402	0.771	-4.04119	-3.01969	-1.92836
7.566	3.728	-1.16809	-0.22695	0.90779
7.658	6.641	1.69784	2.58960	3.74266
7.764	1.956	-3.04139	-2.21014	-1.04193
7.808	4.673	-0.34690	0.45814	1.63064
8.168	1.86	-3.34408	-2.77507	-1.61771
8.536	7.309	1.91665	2.21237	3.24375
8.701	3.573	-1.90376	-1.73773	-0.80637
8.769	1.337	-4.17455	-4.06280	-3.18142
8.803	2.072	-3.45695	-3.37247	-2.51810
8.832	7.521	1.97722	2.03837	2.86863

8.834	1.073	-4.47181	-4.41227	-3.58371
8.903	1.39	-4.19011	-4.18624	-3.41934
8.969	5.284	-0.32987	-0.37939	0.32296
9.211	2.623	-3.11468	-3.35951	-2.94342
9.341	1.969	-3.83519	-4.18339	-3.95556
9.345	2.635	-3.17124	-3.52258	-3.30096
9.371	2.628	-3.19154	-3.56331	-3.38258
9.416	1.878	-3.96456	-4.37147	-4.26400
9.422	4.064	-1.78163	-2.19320	-2.09574
9.426	2.58	-3.26768	-3.68235	-3.59159
9.481	1.811	-4.06482	-4.52190	-4.52596
9.497	8.26	2.37600	1.90668	1.87411
9.52	2.525	-3.37077	-3.85759	-3.93188
9.529	2.23	-3.67037	-4.16401	-4.25487
9.629	4.893	-1.05853	-1.62679	-1.91091
9.647	8.351	2.39026	1.80882	1.48808
9.713	5.768	-0.22651	-0.85557	-1.31547
9.737	6.846	0.83921	0.19313	-0.31931
9.75	2.418	-3.59544	-4.25067	-4.79200
9.775	2.226	-3.80023	-4.47291	-5.07069
9.785	5.932	-0.09934	-0.77895	-1.39963
9.812	7.737	1.69184	0.99369	0.31025
9.831	3.713	-2.34188	-3.05294	-3.78136
9.874	8.42	2.34312	1.60329	0.77054
9.89	9.169	3.08394	2.33356	1.46109
9.926	9.07	2.96652	2.19277	1.22910
9.955	8.621	2.50268	1.71046	0.67147
9.981	2.259	-3.87262	-4.68112	-5.78905
9.993	4.659	-1.47876	-2.29469	-3.43489
10.005	7.498	1.35410	0.53081	-0.64196
10.043	6.528	0.36466	-0.48156	-1.75939
10.044	5.561	-0.60285	-1.44967	-2.73030
10.09	9.063	2.87562	2.00190	0.82595
10.138	9.113	2.90106	2.00031	0.93256
10.218	6.616	0.36313	-0.58021	-1.47010
10.311	9.618	3.31755	2.32889	1.64157
10.338	12.366	6.05174	5.05082	4.42141
10.344	11.754	5.43667	4.43308	3.81649
10.351	12.409	6.08809	5.08141	4.47974
10.399	10.971	4.62553	3.59848	3.09835
10.403	10.279	3.93148	2.90280	2.41107
10.413	7.508	1.15537	0.12264	-0.34814
10.436	9.173	2.80860	1.76680	1.34398
10.451	8.496	2.12393	1.07639	0.68467
10.456	9.888	3.51337	2.46396	2.08257
10.459	10.028	3.65183	2.60130	2.22611
10.491	9.663	3.27046	2.20835	1.89884
10.523	11.125	4.71609	3.64307	3.39858
10.623	10.832	4.37193	3.26937	3.22351

10.658	12.379	5.90102	4.78983	4.81178
10.673	11.564	5.07835	3.96373	4.01447
10.684	12.26	5.76872	4.65170	4.72344
10.72	12.147	5.63730	4.51308	4.65290
10.736	8.428	1.91012	0.78302	0.95277
10.742	9.73	3.20905	2.08093	2.26184
10.917	10.506	3.89552	2.75064	3.24422
11.032	8.341	1.67168	0.53061	1.21484
11.123	7.105	0.38913	-0.73989	0.08615
11.188	9.273	2.52387	1.40864	2.33080
11.426	7.027	0.15611	-0.86890	0.36547
11.529	7.941	1.01742	0.05234	1.40094
11.629	5.891	-1.08374	-1.97758	-0.53113
11.975	6.651	-0.50076	-1.03915	0.63690
11.991	5.698	-1.46195	-1.97950	-0.29723
12.241	4.647	-2.64085	-2.77734	-1.05341
12.623	5.404	-2.07928	-1.41237	0.15408
12.781	4.426	-3.13811	-2.05195	-0.63760
12.798	3.385	-4.18781	-3.05329	-1.65855
13.384	2.131	-5.74161	-2.51529	-2.22149
13.402	4.086	-3.79582	-0.49120	-0.24526
13.593	2.033	-5.94654	-1.75476	-2.07302
13.78	3.109	-4.96621	0.19821	-0.77649
13.999	1.741	-6.44625	-0.00345	-1.88623
14.345	2.806	-5.55826	3.23452	-0.41320