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Evolutionary Optimization Using Equitable Fuzzy Sorting Genetic Algorithm (EFSGA)

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ABSTRACT This paper presents a fuzzy dominance-based analytical sorting method as an advancement to the existing multi-objective evolutionary algorithms (MOEA). Evolutionary algorithms (EAs), on account of their sorting schemes, may not establish clear discrimination amongst solutions while solving many-objective optimization problems. Moreover, these algorithms are also criticized for issues such as uncertain termination criterion and difficulty in selecting a final solution from the set of Pareto optimal solutions for practical purposes. An alternate approach, referred here as equitable fuzzy sorting genetic algorithm (EFSGA), is proposed in this paper to address these vital issues. Objective functions are defined as fuzzy objectives and competing solutions are provided an overall activation score (OAS) based on their respective fuzzy objective values. Subsequently, OAS is used to assign an explicit fuzzy dominance ranking to these solutions for improved sorting process. Benchmark optimization problems, used as case studies, are optimized using proposed algorithm with three other prevailing methods. Performance indices are obtained to evaluate various aspects of the proposed algorithm and present a comparison with existing methods. It is shown that the EFSGA exhibits strong discrimination ability and provides unambiguous termination criterion. The proposed approach can also help user in selecting final solution from the set of Pareto optimal solutions.

INDEX TERMS Multi-objective optimization, evolutionary algorithms, equitable fuzzy sorting genetic algorithm.

I. INTRODUCTION

Connotation of optimization in the realm of multi-objective problems differs from its usual context of maximization or minimization. Real world problems set forth challenges wherein a best compromised solution set is normally accepted. In the face of multiple conflicting goals which are incomparable and incommensurable, the optimal solution is normally derived in the form of a non-singular set of equitable solutions [1]. Owing to the varied perception of human end user, it is desired to obtain a wider set of equitable solutions comprising combinations of trade-offs established between objectives. The process of obtaining such set of solutions is cumbersome using classical optimization methods. Consequently, in the past two decades, evolutionary algorithms (EAs) have emerged as a plausible alternative to the classical approaches. While carrying out simultaneous optimization of multitude of objectives, EAs are able to provide equitable solutions in few simulation runs [2]. Contrary to the classical optimization approaches, EAs work with population of solutions and owing to their inherent mechanism of evolution, which is derived from natural evolution process, they facilitate exploration of improved Pareto solutions. Furthermore, EAs do not require derivatives of objective functions and has robust operators such as reproduction and regeneration to avoid convergence to local optima. Applications ranging from engineering design, groundwater monitoring, and autonomous vehicle navigation to polymer extrusion, city planning and many more have been benefited significantly by the use of EAs [3]. Apparently, the concept of Pareto dominance is conceptualized during optimization of mutually conflicting objectives [4] and therefore EAs have been preferred in most of the real world multi-objective problems (MOPs) [5], [6]. The concept of Non-dominated sorting based genetic algorithm (NSGA) [2] in EAs has been predominantly used by many researchers.

While EAs have been successfully used in last two decades to solve multi-objective optimization problems (MOP), there are certain issues which need immediate attention. As mentioned above, the concept of non-dominance is predominantly used in EAs while selecting/sorting better solutions during the optimization process. However, while dealing with many objective optimization problems, the concept of non-dominance loses its significance [2]. As a result many solutions become non-dominated and populate the Pareto front giving a pseudo Pareto Front (PF). Subsequently, the algorithm converges prematurely and optimality is never achieved. Therefore, EAs become inefficient in optimizing many objectives optimization problems [1], [7]–[9]. A number of other concerns regarding EAs have been raised in the present work and it has been stressed that in applications involving many objectives, the conventional sorting and selection approaches need to be revisited and alternatives be explored. The fuzzy dominance criterion, proposed in this paper, has been found to be a robust analytical approach which can address most of the issues concerning EAs. Various issues, identified during the present research, are discussed in the following Sections.

A. DETERIORATION OF THE SEARCH ABILITY

Evolutionary algorithms, such as nondominated sorting genetic algorithm (NSGA II) and strength pareto evolutionary algorithm (SPEA II), have been successful in two-objective optimization scenarios, however, they appear to be inefficient while dealing with three or more objectives [8], [9]. While optimizing problems involving many objectives, the popular non-domination approach may not work well owing to its weak discrimination capabilities [10]–[12]. With advancement of the algorithm many solutions become non-dominated and populate the Pareto front. This leads to a quick convergence of the algorithm to a pseudo non-dominated front.

It has been explained using an example (Table 1) here, as how, using non-dominated sorting, diverse solutions from a population become non-dominated and undesirably share the same PF. Despite the fact that these solutions give significantly different objective function values, they are all considered as Pareto optimal. Obviously, solutions 1&2 (Table 1) are far better than other solutions (for minimization goal) and therefore should hold a better rank. It is clear, therefore, that while dealing with many objectives, the concept of non-dominance loses its significance and some other approach, for better discrimination among solutions, is required in place.

TABLE 1. Weak discrimination as a result of non-dominated sorting (for minimization goal).

Solutions	Objectives				ND Front index
	f_1	f_2	f_3	f_4	
Sol. 1	3.600	3.50	3.00	3.50	1
Sol. 2	4.000	3.49	3.00	4.00	1
Sol. 3	6.000	6.00	6.00	3.49	1
Sol. 4	8.000	3.48	6.00	9.00	1
Sol. 5	3.599	9.00	9.00	9.00	1

Consequently, many approaches have been proposed in past and modification of the PF is mostly investigated. In this direction a variation operator, called segment-based search (SBS) has been proposed in order to improve the performance of EAs [13]. In another attempt [14], number of non-dominated solutions has been reduced, in order to improve the PF solution's quality. Different ranking of the non-dominated solutions has also shown improvement in the PF [15]. In order to improve PF quality, *substitute distance assignment schemes* are proposed [16], which replaces the crowding distance operator to enhance performance of EA. Koppen and Yoshida [16] have proposed several indicators, besides non-dominance ranking, to evaluate and establish the superiority of a solution over others. Further, various metrics, to ascertain and quantify quality of solutions, have been provided in [17] to compare prospective solutions and select the most competitive one. Unfortunately, approaches based on set quality measure using some indicators cannot be used during optimization owing to their extreme computational overhead.

A dynamical multi objective evolutionary algorithm (DMOEA) [9] had been proposed and compared with other extant algorithms. However, there are two predicaments while using this method; firstly, all the objectives are considered equally important. Secondly, the extreme values of all the objectives are required to be known prior to the optimization which sometimes is not possible. Differential evolution approach has been extended to be used for multi-objective optimization problems by proposing a grid-based adaptive multi-objective differential evolution algorithm [18]. Individual rank for a solution is assigned using three indices, such as, grid fitness, grid density, and grid-objective-wise standard deviation. However, the proposed algorithm is computationally intensive and does not outperform many existing approaches. An approximate non-dominated sorting algorithm has been proposed for optimization problems involving more than three objectives [19]. Here the dominance between two solutions is decided by comparing up to three objectives with respect to one of the objectives. Improved efficiency and search performance has been claimed. An archive-based steady-state micro genetic algorithm (ASMiGA) [20] has also been proposed which maintains a set of nondominated solutions in the archive to a minimum allowable size. The mating and selection schemes are also improved to enhance performance. Further, a hybrid algorithm (FP–NSGA-II) [21] is proposed combining the NSGAII with a front prediction algorithm which is claimed to provide better approximation of the PF. A hybrid multi-objective evolutionary algorithm (HMOEA) for real-valued MOPs has been proposed by [22] wherein each solution in the population maintains a non-dominated archive of personal best during evolution. The proposed method may not be the best choice, owing to its increased computational complexity. Lately, NSGA III [23], [24] has also been proposed to solve generic constrained many-objective optimization problems.

B. AMBIGUOUS TERMINATION CRITERION

Apart from improving the sorting mechanism, an additional research motivation, in the existing EAs, is the need of a clear termination criterion. An optimization algorithm should terminate when either the global optima has been reached or the values of objectives in their acceptable ranges are obtained. Most algorithms lack a mechanism to ascertain the global optimality of their final solutions and hence a termination scheme. This is evident from [25], wherein rigorous experiments are performed using NSGA-II with varied algorithm parameters. Solutions obtained from these experiments are further clustered and few wide spread trade-off solutions are again optimized using genetic local search. Finally, to validate global optimality, each of the objectives is optimized independently treating it as a single objective and compared with the NSGA-II final solutions.

One of the few research efforts in this direction [26] proposed use of an optimization convergence curve to terminate the optimization. Standard deviation of maximum crowding distance criterion and other metrics have been proposed by [26] and [27] to notice the stagnation reached in the algorithm. In another approach, rate of improvement in the solutions has been used and the optimization is stopped when this rate falls below a previously measured threshold value [32], [37]. However, this approach requires a parameter defined by the user to estimate the rate of improvement which is difficult to obtain for higher dimension optimization problems. A statistical test (Kolmogorov–Smirnov test) is proposed by Fernandez *et al.* [33], wherein the convergence can be evaluated by the use of performance metrics such as generational distance, hyper-volume and spread of the competing solutions from succeeding generations. Further research in this approach has been carried out by Liu *et al.* [34] to develop an online convergence detection criterion. According to this criterion, the optimization can be stopped when statistical tests such as *t-test* and *2-variance test* suggest the similarity in the mean and variance of competing solutions, which in turn indicates the convergence. Recently, Deb *et al.* [28] proposed KKT (Karush-Kuhn-Tucker) proximity measure as a termination criterion for an evolutionary multi-objective algorithm. In another approach [29], a hybrid framework of EA has been proposed to address uncertainty in termination. Recently, a global stopping criterion, MGBM, has also been proposed [30] which combines a mutual domination rate (MDR) indicator, with a simplified Kalman filter, for evidence-gathering purposes.

However, the underlying assumption in all the above approaches is that the EAs improve solutions in the initial stage of evolution which may not be always correct.

C. SELECTION OF FINAL SOLUTION FROM THE PF

Third and a very important issue, concerning EAs, is the selection of a final solution from the assortment of Pareto optimal solutions. Though, all the final PF solutions are non-dominated, end user or the decision maker wants a singular solution which is best among the better ones. Few research works have been done in past, wherein apart from other methods, fuzzy inference has also been used to select a final solution from the Pareto solutions [31]–[34]. Subsequent to a simultaneous optimization, reduction in the number of objectives is another way to possibly reduce the cognitive burden off the decision maker [35]. Several other visualization techniques have been mentioned in the literature [36] wherein the objectives are mapped into a low dimensional space for better visualization. Recently, a classification of various approaches to find final solution has been provided by [37].

It is quite apparent from the above discussion that while solving many objective optimization problems, existing EAs suffer from reduced discrimination between solutions, do not have a clear termination criterion and need a mechanism to select best compromised solution at the end of optimization. Until now, some of the above mentioned issues have been attempted by researchers as isolated research issues [1], [21], [24], [25], [28], [32], [33], [38]–[47]. Nevertheless, a single approach, which can effectively address all the above improvement opportunities of EA, does not exist. A multi-objective evolutionary algorithm based on decomposition (MOEA/D) had been proposed as an improvement endeavor over existing EAs [48]–[52]. This algorithm decomposes the multi-objective problem into a number of single objective optimization problems and later all these scalar optimization problems are solved concurrently to obtain a PF. However, for problems having non-convex solution space, exploration of the entire objective space using MOEA/D is not possible (explained later in Section VA) since it will require infinite aggregations of the objective functions with different weight vectors.

Therefore, the research being presented here is a major step in addressing the improvement opportunities identified in evolutionary algorithms. This research is important in the sense that all the issues with EAs (mentioned above) can be addressed by a single approach. Here, a concept of *fuzzy dominance* is introduced as a better alternative to the existing *non-dominance criterion* which is normally used in EAs. The fuzzy dominance based sorting and selection approach is referred here as the *equitable fuzzy sorting genetic algorithm* (EFSGA). In order to analyze and evaluate various aspects of the proposed approach vis-a-vis some of the existing approaches, experiments have been conducted on several benchmark optimization problems [53] including the test problems presented in CEC'09 [54]. Performance indices have also been used to evaluate and compare the quality and exactness of the PF solutions obtained from various methods. The proposed algorithm is further explained in the next Section using illustrations and examples to demonstrate its implementation.

II. FUZZY BASED SORTING GENETIC ALGORITHMS

Fuzzy set theory was given by *Lotfi Zadeh* in his seminal paper [55], whereby qualitative numbers can be treated vividly with conventional mathematical operators. Lately, fuzzy logic has become a popular heuristic approach in modelling non-linear, uncertain and ambiguous systems [56]. Evolutionary optimization methods, such as NSGA-II, have been used to optimize fuzzy systems by optimally partitioning the universe of discourse of fuzzy variables and extracting an optimal set of fuzzy rules [5], [47], [57], [58]. Similarly, fuzzy systems have also been used in the past to improve the overall performance of evolutionary optimization methods [32]–[34], [38], [59], [60]. However, the role of fuzzy logic in EAs has been limited either to the selection of a suitable solution from the set of PF solutions, or to incorporate user preference to guide the convergence of EAs. Recently, fuzzy dominance criterion has also been used in conjunction with MOEA/D and is referred as MOEA/DFD in the literature [61]. Two solutions are compared using the concept of fuzzy Pareto dominance and in case any solution fails to dominate, the scalar decomposition method is adopted. Uniformly distributed weight vectors are used to maintain diversity in the offspring. Fuzzy logic has also been used in solving many real world problems, owing to its capability of handling qualitative and quantitative data effectively [62]–[71]. The present work attempts to use fuzzy logic based approach in order to comprehensively address limitations of EAs mentioned in the previous Section.

Steps involved in the implementation of the proposed approach are explained as follows. To begin with, the objective functions are defined as fuzzy variables and this process is termed as the *fuzzification of objectives*. Other steps involved in the process are *identification of fuzzy dominant fronts*, *equitable fuzzy sorting* and *implementation of genetic operators* such as, selection, mutation, crossover etc. These steps are further discussed in detail under the following subsections.

A. FUZZY OBJECTIVES

Following the standard paradigm of EAs, an initial population of solutions is randomly initialized and their respective objective function values are computed. During fuzzification stage, these objective function values are converted into fuzzy objectives using fuzzy sets. This process has been illustrated in Figure 1, wherein two objective functions, with limiting values 0 and 10, have been defined as fuzzy objectives. Both the objectives are represented using four Gaussian activation functions (AFs). Other shapes for the activation functions, such as triangular or trapezoidal, can also be used. Activation functions for the objectives, in Figure 1, are shown using subscripts 'a' and 'b'. Later, a decision is made on the selection of number of AFs and the parameters deciding their shapes and positions. The number of AFs, to be associated with an objective function, can be two or more depending on the accuracy of results required. However, other parameters, such as minimum fuzziness points (or center points of AFs shown by A, B, C & D in Figure 2) and standard deviation (σ) of AFs are automatically computed during execution of the algorithm using eq. (1-3). Once the objective function values from the initial population of solutions are available,

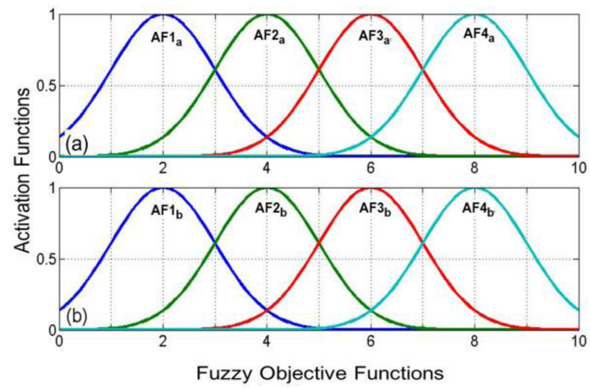

FIGURE 1. Activation functions and their arrangements for two example objectives function (a) & function (b).

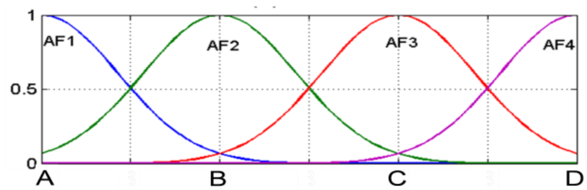

FIGURE 2. Automatic generation of AF parameters.

their extreme values (i.e. minimum and maximum) are used to calculate parameters of AFs (such as positions A, B, C, D and range etc.) using following relations (1-3).

$$A_i = 0.75 * \min(f_i); \quad B_i = A_i + \frac{R_i}{(M_i - 1)};$$
$$C_i = B_i + \frac{R_i}{(M_i - 1)}; \quad D_i = 1.25 * \max(f_i); \tag{1}$$
$$\sigma_i = R_i/(2M_i - 1) \tag{2}$$
$$R_i = (1.25 * \max(f_i) - 0.75 * \min(f_i)) \tag{3}$$

Here R_i stands for the range of objective function values (f_i), and the standard deviation (σ_i) is carefully chosen so that the AFs encompasses the entire range of the objective function values. Total number of AFs for i^{th} objective function is given by M_i. The universe of discourse (or the range) of the objectives is dynamically updated in each iteration and the fuzzy parameters are also altered accordingly.

B. FUZZY DOMINANCE VERSUS NON-DOMINANCE

In the prevailing non-dominated sorting approach, during selection stage, competing solutions providing 'better' or 'not worse' objective values are normally selected. Since 'better' and 'worse' are qualitative variables, it may not possible to decide the extent of these qualitative variables (how much better or worse) using real numbers. Apparently, numerical comparison of objectives may result in an irresolute state, adversely affecting the discrimination capabilities of an algorithm. This apprehension can be further explained using Table 1. Evidently, Sol.5 should not be considered as not-dominated only because its first objective value (2.599)

TABLE 2. Activation scores of linguistic variables.

	Linguistic Variables			
	AF1	AF2	AF3	AF4
Activation Score (*AS*)	0	1	2	3

is slightly better than the first objective value (2.60) of Sol.1. As explained later in this Section, the concept or the definition of dominance of a solution over others should be analyzed qualitatively rather than quantitatively. The fuzzy based sorting, therefore, should rightly replace the conventional notion of non-dominated sorting.

The argument being made here is that when the extent of solution accuracy is not available (which is always true in the real life applications), a qualitative comparison of the objective function values is more justifiable than the numerical/quantitative comparison.

Moreover, the definition of non-dominance is also not free from ambiguity; since we are not looking for solutions which are better than others, rather we select solutions which are not dominated by other solutions. Due to this imprecision in the definition of the non-dominance criterion, more and more solutions get accepted as non-dominated solutions especially while dealing with many-objectives. As explained in the following Section, the proposed fuzzy dominance criterion, contrary to the non-dominance, is simple, wherein dominating solutions, in terms of the fuzzy activation scores of objective functions, are selected and placed in their respective fuzzy dominant fronts.

C. FUZZY DOMINANT FRONTS

Subsequent to the fuzzification of objectives, fuzzy dominant fronts are obtained in the objective space. In order to do that, AFs of objectives are given activation score (AS). In general the activation score for m^{th} activation function can be given by following relation.

$$AS\ (m) = m - 1 \tag{4}$$

Therefore, activation scores for AF1, AF2, AF3, and AF4 (Figure 2), are assigned as 0, 1, 2, and 3 (Table 2).

Next, the entire objective space is divided into *finite number* of oblique *fuzzy dominant fronts* [72, Fig. 3] and this number is calculated using number of objective functions and their AFs. A general expression (5) is devised to give the number of fuzzy dominant fronts (F_f) for a MOP with 'N' objectives and 'M_j' AFs for j^{th} objective function.

$$F_f = 1 + \sum_{j=1}^{N} (M_j - 1) \tag{5}$$

Computation for number of fuzzy dominant fronts has been further explained with the help of Table 3. A two objective MOP is considered here whereby both the objectives are fuzzified using two AFs and later by three AFs (first column of Table 3). Apparently, for two objective functions, with

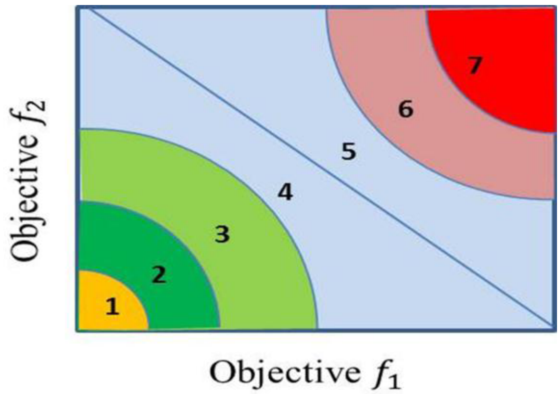

FIGURE 3. EFSGA Fuzzy fronts and their placement in the objective space.

2 AFs for each of the objectives, there shall be 3 fuzzy dominant fronts. Similarly, for a problem involving two objectives, described by 3 AFs each, there will be 5 fuzzy dominant fronts. Therefore, by increasing the number of AFs, the number of fuzzy fronts, dividing the objective space, can be increased.

Formation of fuzzy fronts has been also explained in illustrations 3&4. Once again, an example MOP with two objectives has been considered wherein each of the objectives has been fuzzified using four AFs. According to (5), the total number of fuzzy fronts for this problem shall be seven (i.e. $1 + 3 + 3 = 7$). Oblique placement of these seven fuzzy fronts is explained in Figure 3, whereas a spatial depiction of the fuzzy fronts, is given in Figure 4 [72]. The seven fuzzy fronts have been illustrated in three different figures for enhanced clarity. First three fronts are shown in Fig. 4a, whereas subsequent fronts are shown in Fig. 4b & 4c. Solutions falling in the red color regions clearly belong to the corresponding front, whereas solutions falling in the yellow, green or blue regions may share two or more fronts. As explained later in this section using eq. (11), an operator (*floor*) is used in order to place solutions to their closest red region while deciding their front numbers.

Interestingly, since in the proposed approach, number of fronts (in the solution space) for a MOP with assumed number of AFs (for objectives) is *definite*, the end user can assertively gauge the quality of solutions obtained after each epoch. For instance, when solutions belonging to the first front are obtained, the user can confidently make a decision on termination of the algorithm. Since, all the solutions in the first front shall have acceptable values for the objectives, defined by the user, for a *minimization* goal.

D. EQUITABLE FUZZY SORTING

The next step, after fuzzification of objectives and obtaining fuzzy dominant fronts, is to evaluate a candidate solution by computing a collective fuzzy index for all of its objective function values. The mechanism, used to compute this index, works through a rule-base. A rule-base in fuzzy systems

TABLE 3. **Activation scores of linguistic variables.**

Number of AFs	Activation Functions	Activation Scores for AFs	Possible combinations of fuzzy objective values for two objectives (or fuzzy rule-base)	Combined Activation Scores (AS)	Fuzzy dominant front index (AS+1)
2	$AF1_a$ & $AF2_a$	0 & 1	$AF1_a$ & $AF1_b$	0	1
	$AF1_b$ & $AF2_b$	0 & 1	$AF1_a$ & $AF2_b$	1	2
			$AF2_a$ & $AF1_b$	1	2
			$AF2_a$ & $AF2_b$	2	3
3	$AF1_a$, $AF2_a$, $AF3_a$	0, 1 & 2	$AF1_a$ & $AF1_b$	0	1
	$AF1_b$, $AF2_b$, $AF3_b$	0, 1 & 2	$AF1_a$ & $AF2_b$	1	2
			$AF1_a$ & $AF3_b$	2	3
			$AF2_a$ & $AF1_b$	1	2
			$AF2_a$ & $AF2_b$	2	3
			$AF2_a$ & $AF3_b$	3	4
			$AF3_a$ & $AF1_b$	2	3
			$AF3_a$ & $AF2_b$	3	4
			$AF3_a$ & $AF3_b$	4	5

TABLE 4. **Fuzzy dominant front index or OAS computed for a candidate solution giving objective function values as 4 & 7.**

y_l=(AS+1) Eq. (7)	AF_f1 Eq. (8)	AF_f2 Eq. (8)	W_l Eq. (9)	X=(w_l)/sum(w_l) Eq. (10)	Y=(X)*(y_l) Eq. (10)	Y^* Eq. (11)
1	0.062777	3.06E-08	1.92E-09	1.70E-09	4.4971	4
2	1	0.001973	0.000124	0.00010968		
3	0.062777	0.500553	0.031423	0.02783194		
4	1.55E-05	0.500553	0.031423	0.02783194		
2			3.06E-08	2.71E-08		
3			0.001973	0.00174721		
4			0.500553	0.44334592		
5			0.500553	0.44334592		
3			1.92E-09	1.70E-09		
4			0.000124	0.00010968		
5			0.031423	0.02783194		
6			0.031423	0.02783194		
4			4.76E-13	4.21E-13		
5			3.06E-08	2.71E-08		
6			7.77E-06	6.89E-06		
7			7.77E-06	6.89E-06		

is a collection of "*if and then*" statements connecting the *antecedent* (input) and *consequent variables* (output). General structure of a rule-base is given as follows.

$$If\ f_1\ is\ AF_{i1}\ and, \ldots\ldots\ldots and\ f_N\ is\ AF_{im}\ then\ AS_i\ is\ y_i$$

Here $f_1 \ldots f_N$ are objectives as inputs to the fuzzy system, $AF_{i1}, \ldots, AF_{im}$ are AFs corresponding to the objectives, AS_i is the activation score (which is also *consequent*for the fuzzy rule base) for i^{th} rule and its numerical value is y_i. Total number of rules N_R is derived from the number of AFs and the number of objectives which is given by following relation (6) [73].

$$N_R = \prod_{j=1}^{N} M_j \tag{6}$$

Here j is the index for the objective function, N stands for number of objectives, and M_j is the total number of AFs used for j^{th} objective function. Thus, when two objectives are represented using four AFs (as shown in Figure 1), a total of 4^2 i.e. 16 rules shall be formed. Similarly, if the two objectives are fuzzified using 2 or 3 AFs, they will have 4 or 9 rules respectively as shown in Table 3. These rules are the combinations of all possible arrangements of AFs for the two objectives.

While evaluating a candidate solution, its numerical objective function values are fired through all the rules in the rule-base. The overall activation score (OAS), for input objective values (coming from a candidate solution), is the weighted average of all the rule outputs.

Computation of the output of the fuzzy system for a set of input objectives is based on a weighted average defuzzifier [74]. This procedure has also been explained in Table 4, wherein outputs from respective equations for a hypothetical input are shown in appropriate columns. Initially, outputs from individual rules (y_i) are calculated which are basically the sum of activation scores (AS) of their antecedents ($\sum_{j=1}^{N}[AS(m_{ij})]$) added with unity to avoid null values as outputs (7).

$$y_i = 1 + \sum_{j=1}^{N} [AS(m_{ij})] \tag{7}$$

Here i represents the rule index, N is the number of objectives and $AS(m_{ij})$ is the AS for j^{th} objective function in i^{th} rule. Later, weighing (9) of objectives for each rule is computed considering the product of all the applicable AF values (8).

$$AF_{ij}(f_j, \bar{f}_{ij}, \sigma_{ij}) = ae^{-\frac{(f_j-\bar{f}_{ij})^2}{2\sigma_{ij}}} \tag{8}$$

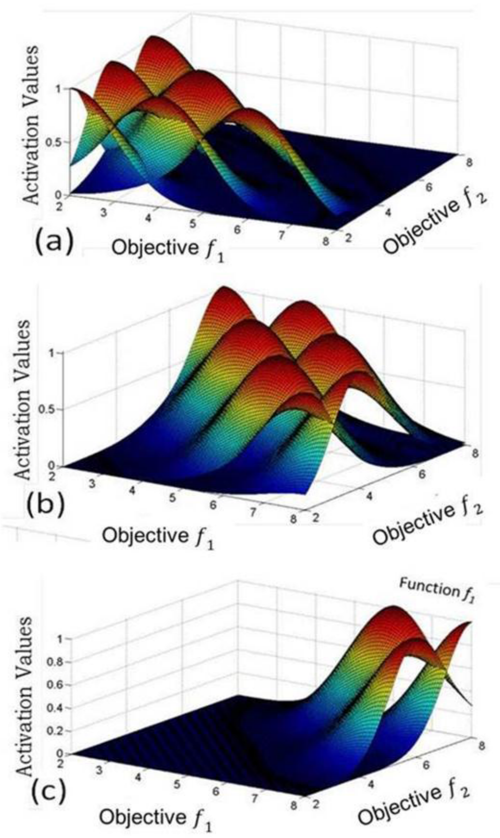

FIGURE 4. Activation functions and their placement in the objective space depicting fuzzy fronts (a) 1, 2 & 3, (b) 4 &5, (c) 6&7.

$$w_i = \prod_{j=1}^{N} AF_{ij} \tag{9}$$

Here f_j represent the input objective values, whereas $\bar{f}_{ij}$ and σ_{ij} are the mean and standard deviations of the respective AFs. Parameters, such as mean and standard, deviation are updated during successive iterations based on the limiting values (f_{min}, f_{max}) of objectives, using equations (1-3). The constant 'a' in (8) is given by $\frac{1}{\sigma\sqrt{2\pi}}$. Real number output from the fuzzy system or the OAS of a solution is the weighted average of all the individual rule consequents. The OAS can be computed using (10). It is important to mention here that the OAS, being a real number, is a unique score for all the competing solutions.

$$Y = \frac{\sum_{i=1}^{N_R} (w_i y_i)}{\sum_{i=1}^{N_R} w_i} \tag{10}$$

Finally, an operator (11) is used to calculate the fuzzy dominant front number Y^*, for a candidate solution. This operator approximates the real output from (10) to the next smaller integer which is less than or equal to its argument.

$$Y^* = floor(Y) \tag{11}$$

The fuzzy dominance criterion can now be defined as below. This definition is similar to the definition of dominance criterion [2]; however, in the present definition the OAS is considered in place of individual objective values.

Definition (Fuzzy Dominance Criterion): Solution x_1 will dominate another solution x_2 provided one of the following conditions is true:

- *The overall activation score of solution x_1 is greater than the overall activation score of x_2.*
- *If the two competing solutions have same overall activation score but solution x_1 has better crowding distance than solution x_2.*

E. HAND CALCULATION

The entire scheme of fuzzy dominance sorting is being explained by demonstrating its implementation numerically. An example, involving two objectives (Fig.1), is discussed here with requisite details of implementation. The limiting values of the two objectives are assumed to be 0 and 10. For the sake of simplicity we assume that all the AFs have exactly the same standard deviation which is calculated using (2) as (10/7) or 1.43 units. Positions or the minimum fuzziness points of all the AFs are also assumed as shown in the Figure 1. Let us consider a candidate solution which has two objective function values (f_1 and f_2) as '4' and '7' units and it is required to find the fuzzy dominant front index corresponding to this solution. Outputs from each of the steps have been displayed in Table 4, explaining the proposed method with proper reference to the corresponding equation numbers.

The first column of the Table 4 has outputs from (7) which gives all the rule outputs and shall be same for both the objectives irrespective of their values. Column 2 and 3 contain AF's values for the given input objective values '4' & '7' respectively. Apparently, objective value '4' has complete activation (which is '1') in AF2, whereas activation for objective value '7' is equally shared by AF3 & AF4. Column 4&5 display 16 values obtained from (9&10). Finally in column 7 the *floored* value of the fuzzy front index is displayed. Therefore, a solution providing objectives values as 4 & 7, shall be placed in the fourth fuzzy front out of the total 7 fronts shown in Figures 3&4.

Shapes of AFs and their placement in the objective function space are also explicable with illustrations in Figures 3&4. These illustrations show the placement of oblique equitable fuzzy fronts wherein the activation values for combination of fuzzy functions are equal. Column 1 of the Table 4 lists the number of AFs in each of the seven fuzzy fronts. According to this, fronts 1 and 7 shall each have a single AF (their frequency of occurrence); whereas 2 & 6 fronts have two AFs in each of the fronts (frequency of occurrence is 2). Remaining fronts 3, 4 and 5 shall each have 3, 4 and 3 AFs respectively.

The distribution of OAS has also been shown in Figure 3 (oblique fronts with numbers), wherein the first front is most desirable for a minimization goal. The *floor* function applied on the outputs (Y), helps to maintain diversity in the solutions. If the output for fuzzy front index is not converted to an integer, all the solutions are likely to have a unique real

Algorithm 1 Framework of the proposed EFSGA

$P_O \leftarrow$ Initialize Population (N)
$k \leftarrow 0$
$R \leftarrow$ Initialize Rank with some integer>1
while $floor(R) > 1$ **do**
  Evaluate objectives for P_k and store their limiting values, std. dev and means
  Fuzzification of objectives
  Fuzzy dominant fronts based on objective fuzzification
  $Q_k \leftarrow$ Generate Offspring Population from P_k
  $T_k \leftarrow P_k \cup Q_k$
  $S_k = \{T_k, r_k\} \leftarrow$ Fuzzy dominant sorting and ranking (r_k)
  $R = min\{r_k\}$
  Sort solutions based on ranks
  $P_{k+1} \leftarrow 0$
  $i \leftarrow 1$
    while $|P_{k+1}| + |S_i| \leq N$ **do**
      $P_{k+1} \leftarrow P_{k+1} \cup S_i$
      $i \leftarrow i + 1$
    end while
    $k \leftarrow k + 1$
end while

number. Consequently, there shall never be a tie between the two solutions and the crowded distance operator shall never be invoked. Eventually, distantly placed solutions shall become extinct over the successive iterations, resulting into a convergence to the local optima.

While performing simultaneous optimization of a multitude of solutions, the issue of convergence is important and should be discussed. According to the explanation given in Section IIC, the fuzzy based selection scheme is applied to the population comprising of the parent and their offspring. Therefore, in successive iterations, a parent in the population will continue to exist until a dominating offspring (with higher fuzzy dominance rank) replaces it. This is a necessary condition for achieving convergence of the entire population to the Pareto optimality. Reference can also be drawn from the study proposed by Rudolph [75], wherein, by means of homogeneous finite Markov chain analysis, it has been shown that an algorithm which always maintains the best solutions in the population will converge to the global optimum.

III. SIGNIFICANT BENEFITS OF FUZZY DOMINANCE CRITERION

A. ENHANCED DISCRIMINATION BETWEEN SOLUTIONS

It has been emphasized in the previous Section that following fuzzy dominance criterion, the objective solution space can be discretely divided into fuzzy dominant fronts of known quality. To further increase discrimination among the solutions, number of these fronts can be easily increased by increasing number of AFs while defining fuzzy objective functions. However, that will also increase the number of fuzzy rules and will affect the computational efficiency of the algorithm. Therefore, a trade-off can be established between the number of AFs or the desired fuzzy fronts and the visibility required into objective values. In order to further explain the improved discrimination abilities of the proposed sorting scheme, Table 1 has been reproduced here as Table 5. Apart from ND front indices, fuzzy front indices have also been given here, which are obviously different for solution declared as non-dominated solutions otherwise. A total of 19 fuzzy fronts shall be obtained if four objective functions (shown in Table 1) are defined using four AFs each. Fuzzy front indices displayed in the Table 5 are obtained following the method discussed in the previous Section. Apparently, the fuzzy front indices for given solutions are more realistic compared to the non-dominated front indices. This clearly shows that the proposed fuzzy dominance method can provide better discrimination between solutions.

TABLE 5. Improved discrimination of EFSGA over NSGAII.

Solutions	Objectives				ND Front indices	Fuzzy dominant Front indices
	f_1	f_2	f_3	f_4		
Sol. 1	3.600	3.50	3.00	3.50	1	**3**
Sol. 2	4.000	3.49	3.00	4.00	1	**4**
Sol. 3	6.000	6.00	6.00	3.49	1	**11**
Sol. 4	8.000	3.48	6.00	9.00	1	**12**
Sol. 5	3.599	9.00	9.00	9.00	1	**17**

B. EXPLICIT TERMINATION CRITERION

In the proposed fuzzy dominance the algorithm shall terminate when either the solutions in the desired front are achieved (as decided by the user) or when the front index remains unchanged over few successive iterations. It is important to note here that, in order to carry out front index calculation for termination criterion, the extreme values of objectives may be obtained through experiments, if not known priori.

C. FINAL SOLUTION FROM THE PF

As a result of the optimization, a PF with equally good solutions is obtained. However, the end user is always interested in finding a singular solution to be finally used. Normally, the user makes a decision based on his/her preferred objective values and picks up a solution. This approach is quite subjective and at the same time purely intuitive. In order to help user in making an objective decision, it is proposed to use the OAS index obtained from equation (10). All the solutions in the fuzzy PF have a unique real number for this OAS (before using *floor* function) and the solution with minimum *OAS* obviously has best compromised combination of all the objectives when the goal of optimization is minimization and vice versa.

The proposed algorithm is quite useful for real world optimization problems with many objectives across disciplines. However, it can be also be successfully used, for problems wherein, extents (universe of discourse) of objectives are not available, extent of accuracy for objective values is undefined and where the qualitative description of objectives, over quantitative treatment is preferred.

IV. BENCHMARK TEST PROBLEMS AND PERFORMANCE INDICES

A. BENCHMARK TEST PROBLEMS

Normally when an alternate optimization method is proposed, a visual description of the Pareto optimal curve or hyper-surface formed with solutions is sufficient to review its adequate functioning. However, in the case of EAs, it becomes imperative to evaluate their performance on various benchmark test problems. Although, variety of test problems exist in the literature, some of them are complex and do not exhibit perceptible shapes and positions of the Pareto-optimal front [76]. Further, it is required that the test problems should be scalable both in terms of objectives as well as decision variables and its implementation should be simple.

Since the aim is to achieve better convergence to the Pareto-front besides good distribution of solutions, the proposed EFSGA should be tested for both these attributes. Normally, different sets of test problems are required to test the convergence of solutions to PF and the diversity of solutions. Moreover, to check the diversity of solutions, the PF provided by the test problems should be non-convex, discrete and may have varying density of solutions along the PF. Several scalable test problems have been given by Deb *et al.* [53] and these problems along with others have been used earlier to investigate MOEAs [49]. In the present research, we have used test problems DTLZ1 to DTLZ7 from [53] to evaluate the proposed EFSGA. Further, in order to test the proposed algorithm on other benchmark problems we have also included 13 unconstrained test problems presented in CEC'09 [54].

The set of test problems, i.e. DTLZ1 to DTLZ7, is used to test the convergence to the PF using the proposed EFSGA. However, additional set of problems (CEC 2009) is used as a case study reflecting real-life problems. All the objectives, in these test instances, are considered to be minimized.

In order to provide a quick access to the nature of these benchmark problems, an example test problem DTLZ2 consisting of 3-objectives is discussed here. Reader should note that all these test problems are expandable in terms of number of objectives and variables. The 3-objective test problem (DTLZ2) is illustrated in Figure 5 and is also described below.

Considering the first quadrant of a sphere of radius $[1 + g(X_M)]$, any two points on the surface of this sphere shall be non-dominating to each other for a minimization goal. Three objectives in the optimization problem (DTLZ2) are formulated as follows:

$$\text{Minimize } f_1(x) = (1 + g(X_M))\cos(x_1\frac{\pi}{2})\cos(x_2\frac{\pi}{2})$$
$$\text{Minimize } f_2(x) = (1 + g(X_M))\cos(x_1\frac{\pi}{2})\sin(x_2\frac{\pi}{2})$$

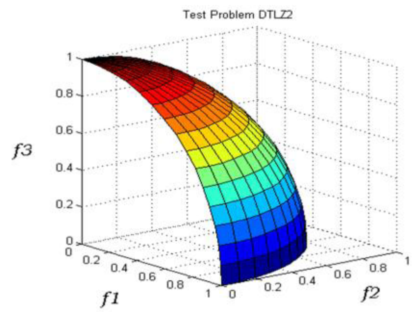

FIGURE 5. Pareto optimal front/curve for test problem DTLZ2 for three objectives.

$$\text{Minimize } f_3(x) = (1 + g(X_M))\sin(x_1\frac{\pi}{2})$$
$$\text{Subject to } 0 \le x_i \le 1 \text{ for } i = 1, 2, 3, \ldots, n,$$
$$\text{and } \sum_{i=1}^{3} (f_i)^2 = 1$$

Here $g(X_M) = \sum_{x_i \in X_M} (x_i - 0.5)^2$, M is the total number of objectives, x_i are variables and $|X_M| = 10$. For a problem involving three objectives, the total number of variables is given by $(|X_M + M - 1|)$ as 12. Further, out of these 12 variables, two variables determine the objective functions, whereas, the rest ten variables are used to decide the radius of the PF which is denoted by $[1 + g(X_M)]$. The final PF solutions should lie on the surface shown in Figure 5 and they should correspond to $X_M = 0.5$. Quality of the PF solutions obtained from any MOEA implementation on DTLZ2 can be assessed against the deterministic PF displayed in Figure 5.

B. PERFORMANCE INDICES

In order to assess the capabilities of the proposed method, three existing EAs, namely, NSGAIII [23], [24], MOEA/D [49] and HypE [8] have also been implemented on the DTLZ test problems suits [53], [77]. These EAs are representative of three different classes/approaches of evolutionary optimization and presenting a comparison with these approaches is an attempt to explain that the proposed algorithm has wide applicability. Final set of solutions (Q) obtained from these methods, including the proposed one, have been further analyzed.

Since in order to evaluate a MOEA, we are attempting to check aspects such as *exactness* and the *extent* or *diversity of solutions*, it is obvious that a single performance index will not suffice the task. There are many performance indices mentioned in [2] which fall under three categories, namely, indices checking closeness to PF, indices evaluating diversity of population and indices evaluating both the closeness and the population diversity. In the present work, we have chosen one performance index from each of the categories to evaluate the quality of solutions obtained from various MOEAs. These are *GD-metric* value [49], Spacing (*S-metric* value) [2], [78] and Normalized Hyper-volume (*H-metric* value) [2], [8] of solutions. A brief description of these metrics has been provided here for the convenience of readers.

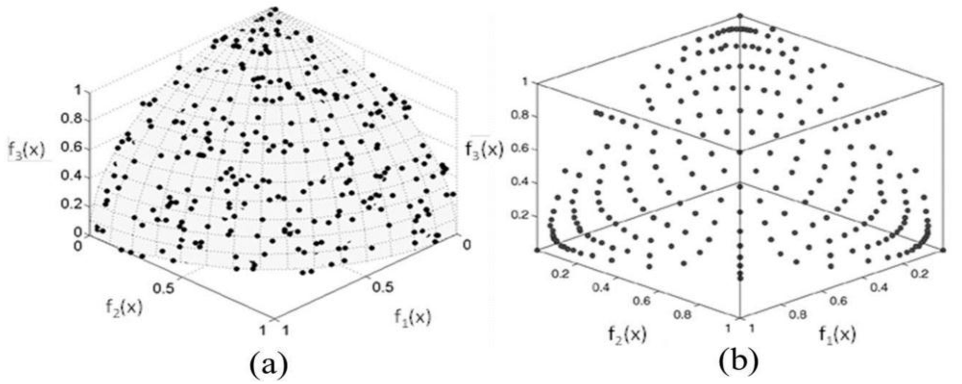

FIGURE 6. PF with lowest D-metric value obtained from (a) EFSGA & (b) MOEA/D for 3-objective test problem DTLZ2.

The first metric used is the *GD -metric* which determines an average distance of solutions in Q from P* (chosen Pareto optimal set) as explained below.

$$GD = \frac{\left(\sum_{i=1}^{|Q|} d_i^2\right)^{1/2}}{|Q|} \tag{12}$$

Here d_i is the Euclidean distance in the objective space between the objective solutions $i \in Q$ and their corresponding nearest member of P*. The Euclidean distance further is given as (13). The notation f_m^{*k} is used for the mth objective function value of the kth member of P*.

$$d_i = \min_{k=1}^{|P^*|} \sqrt{\sum_{m=1}^{M} (f_m^i - f_m^{*k})^2} \tag{13}$$

Another metric, called the spacing or the S-metric, is the measure of the relative distance between consecutive solutions in the final set of solutions (Q).

$$S = \sqrt{\frac{1}{|Q|} \sum_{i=1}^{|Q|} (d_i - \bar{d})^2} \tag{14}$$

Here $d_i = min_{k \in Q \wedge k \neq i} \sum_{m=1}^{M} \left|f_m^i - f_m^k\right|$ and $\bar{d}$ stands for the mean of d_i's, which is given by $\bar{d} = \sum_{i=1}^{|Q|} d_i / |Q|$. The measure of distance (d_i) is basically the minimum value of the sum of the absolute differences in the objective function values between the i^{th} solution and other solutions in the obtained solution set (Q). Thereby, the S-metric measures the standard deviation of d_i values calculated for all the points on the achieved Pareto set (Q). When the spacing between the solutions is nearly uniform, their S-metric values will be less which in turn indicates better diversity among solutions.

Finally, the H-metric provides the volume covered in the objective space (area in case of two objectives) by the solutions Q. In order to find this, a hypercube v_i is constructed for each solution $i \in Q$, whereby a reference point W and the solution i are considered as the diagonal corners of this hypercube. The reference point W is found by constructing a vector of worst objective function values. Later, a union of all the hypercubes is obtained and the Hypervolume is defined as (15).

$$HV = volume(\cup_{i=1}^{|Q|} v_i) \tag{15}$$

However, in order to avoid the arbitration in the scaling of objectives, the normalized value of H-metric is used in the present research which is the ratio of hypervolumes of Q and P*.

$$H = \frac{HV(Q)}{HV(P^*)} \tag{16}$$

When all the objectives are required to be minimized, the best (maximum) value of H is one (for Q = P*). Normalized Hyper-volume is targeted to unity whereas smaller Spacing and GD-metric values show superiority of the MOEA being assessed.

V. CONDUCT OF EXPERIMENTS

Rigorous experiments were carried out while optimizing the sets of benchmark problems (DTLZ and CEC'09) mentioned in the previous Section. Parameter selection, apart from the conduct of experiments has been discussed in the following sub-sections.

A. EXPERIMENTS WITH DTLZ TEST PROBLEMS

The popular test problems suits of DTLZ were optimized using four different approaches including the proposed EFSGA. During all these experiments, following simulation parameters were considered for NSGA III [23], [24] and EFSGA.

Population size: 1000; Crossover prob.: 0.95; Real-parameter mutation prob.:0.05; Distribution index for crossover: 10; Distribution index for mutation: 50

Consequent to the simulation runs for two hundred iterations on the test problems, PFs with the lowest GD-metric value were obtained and analysed. For the sake of visual demonstration, PF solutions, following EFSGA & MOEA/D [48], [49] on 3-objective test problem DTLZ2, are displayed in Figure 6.

GD-metric values of the final set of solutions obtained using EFSGA & MOEA/D are found to be 0.0111 and 0.0417

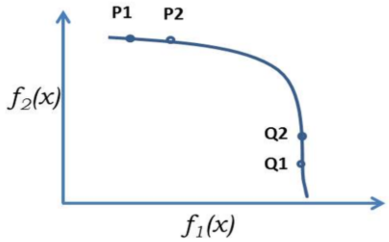

FIGURE 7. Solutions in objective space not likely accessible using MOEA/D.

respectively, which shows that improved convergence can be achieved following EFSGA [49]. Interestingly, results obtained using MOEA/D, showed patterns which further strengthens our apprehension (Section IC) that following MOEA/D, complete objective solution space may not be explored. More specifically, when the solution space is not convex, MOEA/D may not be able to access points which lie in the perpendicular directions (points P1, P2 and Q1, Q2) to the objective axes, as shown in Figure 7. Additionally, in order to explore the complete solution space, MOEA/D shall require infinite aggregations of the objective functions with different weight vectors, which may not be feasible.

While conducting experiments on DTLZ problems, two hundred iterations were carried out for each of the experiments. Further, every experiment was carried out five times and average values of the performance indices from these five experiments were recorded. Although, it is recommended to use varied number of iterations for different problems, we have used same number of iterations for all the problems in order to conduct statistical evaluation of all four EAs. For each of the DTLZ problems, numbers of objectives were increased in steps from 2 to 20.

Experiments were initialized with test problem DTLZ1 which provides a linear Pareto-optimal front for two objectives and a triangular plane PF for three objectives. DTLZ2, on the other hand is a generic sphere problem as discussed in the previous Section. While test problems such as, DTLZ1, DTLZ2 & DTLZ4 are used to test the diversity of solutions obtained from a particular EA, test problem DTLZ3 investigates convergence ability of EA for a global PF. Test problem DTLZ5 also examines ability of EA to converge to a curve and is a good problem to visually demonstrate EA's effectiveness. Test Problem DTLZ6 is obtained by a modification of 'g' function in DTLZ5 problem and is also a complicated one. NSGA-III does not quite converge to the true Pareto curve of DTLZ6, rather gives a pseudo PF. As a departure from previous problems, DTLZ7 gives a disconnected set of Pareto-optimal regions and therefore, tests ability of an EA to maintain subpopulation in different Pareto-optimal regions.

B. RESULTS FROM EXPERIMENTS WITH DTLZ TEST PROBLEMS

Populations of multiple solutions obtained after each of the simulation were further investigated by calculating three performance indices namely, GD-metric, S-metric and H-metric. Results from all the experiments have been displayed in Table 6 for DTLZ test problems. From a brief analysis of the experimental results, it was found that when the number of objectives is more than five, the proposed EFSGA more or less excels other methods. It has been emphasized all through this manuscript that the proposed method provides better discrimination between competing solutions, especially when the number of objectives is large. This fact can also be immaculately observed through the results in Table 6. Considering H metric in particular (which should be close to unity), it is found that EFSGA surpasses all other approaches for almost all the test problems from DTLZ test suits.

Experimental findings (DTLZ problems) suggested that the Hypervolume index for the solutions obtained through HypE was not very encouraging and found to be as low as 0.6154. Further, considering the minimum S metric, EFSGA (with $S_{min} = 0.0016$) is found to be performing better than HypE ($S_{min} = 0.0404$) and MOEA/D ($S_{min} = 0.0025$) for various problems whilst its performance was comparable to that of NSGAIII ($S_{min} = 0.0003$). In terms of GD metrics again both NSGAIII ($GD_{min} = 0.0052$) and EFSGA ($GD_{min} = 0.0066$) have fared well whereas the other two methods, HypE ($GD_{min} = 0.1264$) and MOEA/D ($GD_{min} = 0.0317$) have not performed to the expectations.

However, results of *GD-metric* from the proposed EFSGA are found to be comparable to NSGAIII results surpassing the other two methods. Analyzing the performance indices obtained from different approaches, it can be stated that the fuzzy based sorting method, proposed through EFSGA in the present work, has proven its effectiveness.

C. THE WILCOXON SIGNED RANKS TEST

In order to further evaluate the performance of the proposed EFSGA method, it is required to examine that the differences between the solutions obtained from EFSGA and other approaches are non-random. Therefore, a non-parametric statistical test, namely, the *Wilcoxon signed rank test*, has been conducted during the present research [79]. This is a non-parametric statistical test which aims to check whether the difference between two sample means is significant, or in other words, the difference between two samples represent two different populations. The Wilcoxon signed rank test does not depend on the form of population distribution and its parameters. Therefore, this test is often used, for the instances, where the population may not be normally distributed.

The test has been briefly explained here, however, readers are referred to read [79] for further details.

While comparing population of solutions from two algorithms, let's assume that the difference between i^{th} performance metric of the two populations on j^{th} out of n problems is Δ_i^j. Absolute values of these differences (Δ_i^j) are ranked and the sum of ranks (R_i^+) for which the proposed algorithm (EFSGA) performs better is calculated. Similarly, the sum of ranks (R_i^-), for which performance of the proposed algorithm has been poor, is also calculated. In order to avoid

TABLE 6. Mean values of performance indices from the populations of PF solutions obtained from NSGAII, MOEA/D, HypE and EFSGA implemented on DTLZ problems known as artificial landscapes.

No. of Objectives		NSGA III			MOEA/D			HypE			EFSGA		
		GD	S	H	GD	S	H	GD	S	H	GD	S	H
DTLZ1	2	0.0196	0.0003	0.9689	0.0317	0.0157	0.2414	0.1264	0.1440	0.8231	0.1142	0.0035	0.9257
	3	0.0498	0.0009	0.9513	0.0654	0.0170	0.2452	0.2696	0.2395	0.8150	0.1868	0.0016	0.7641
	5	0.0244	0.0086	0.9589	0.8860	0.0183	0.2528	0.2360	0.2691	0.6154	0.3629	0.0071	0.9031
	10	1.0756	0.0920	0.8733	1.5731	0.1149	0.6234	1.4954	0.4018	0.8021	1.4565	0.0885	0.8554
	20	1.0845	0.1425	0.8866	1.8489	0.6663	0.9202	1.2831	0.5078	0.8224	1.4547	0.1349	0.9441
DTLZ2	2	0.0052	0.0007	0.9865	0.0389	0.0234	0.8508	0.1319	0.0404	0.7440	0.0066	0.0073	0.9982
	3	0.0147	0.0088	0.9745	0.0417	0.0230	0.8301	1.2773	0.1403	0.8640	0.0111	0.0032	0.8452
	5	0.0096	0.0348	0.8206	1.0967	0.0241	0.84225	1.1611	0.2609	0.8080	1.0731	0.0901	0.7203
	10	0.0961	0.1890	0.7826	1.1272	0.0245	0.8453	1.0959	0.4024	0.8450	1.3902	0.2244	0.8709
	20	0.7950	0.2280	0.8287	1.3637	0.0254	0.8624	1.0598	0.6354	0.8240	1.3927	0.4297	0.8989
DTLZ3	2	0.2080	0.0046	0.8744	0.0752	0.0235	0.8623	1.7815	0.4064	0.8348	1.0159	0.0044	0.8987
	3	0.0641	0.0117	0.7492	10.670	0.6645	0.8445	3.0434	0.1245	0.8333	0.8758	0.0722	0.8347
	5	0.6184	0.0689	0.8511	10.655	1.1029	0.7931	7.6523	1.0009	0.9978	1.2311	0.0027	0.9675
	10	10.421	1.0471	0.9251	9.5083	1.3281	0.8492	8.8912	2.2921	0.9100	9.8646	1.1968	0.99
	20	11.545	1.9205	0.8863	9.2960	2.2081	0.7331	10.462	0.957	0.9090	11.3774	2.2603	0.98
DTLZ4	2	0.0058	0.0012	0.9475	0.6857	0.0066	0.8718	1.2343	0.1067	0.8770	0.0536	0.0107	0.8800
	3	0.0246	0.0094	0.8899	0.3070	0.0379	0.8775	1.0437	0.1189	0.8170	0.2860	0.0140	0.9306
	5	0.0658	0.0801	0.8790	0.9782	0.0025	0.813	1.1420	0.2219	0.8380	1.1315	0.0579	0.8626
	10	1.1515	0.2922	0.8526	1.0697	0.0203	0.8244	1.1318	0.5860	0.8270	1.2814	0.2639	0.9183
	20	1.9536	0.4172	0.8284	1.3007	0.0453	0.8105	1.9118	0.6979	0.8013	1.4819	0.3139	0.9567
DTLZ5	2	0.0664	0.0056	0.9785	0.7686	0.0108	0.8080	1.3427	0.0639	0.7980	0.7898	0.0110	0.8721
	3	0.5734	0.0541	0.9741	0.8639	0.0076	0.8230	1.3767	0.1141	0.7670	0.9537	0.0377	0.9260
	5	1.3850	0.0866	0.9261	1.1098	0.1325	0.9162	1.2664	0.2086	0.7270	1.3452	0.1047	0.9704
	10	1.4294	0.1351	0.9700	1.1883	0.1446	0.9524	1.5176	0.1848	0.7830	1.6274	0.1907	0.9997
	20	2.1141	0.2338	0.8284	1.8824	0.0101	0.8483	1.4370	0.2020	0.7825	1.8573	0.2210	0.9996
DTLZ6	2	5.2144	0.0368	0.9602	7.8628	0.0593	0.9987	8.5931	0.2795	0.8700	6.0179	0.0249	0.9987
	3	6.5176	0.0885	0.9644	8.079	0.3976	0.9992	8.9072	0.6491	0.8790	6.2622	0.1917	0.9988
	5	8.6903	0.2230	0.9636	8.9104	1.2419	0.9990	8.9802	1.0097	0.8126	7.8974	0.1774	0.9918
	10	8.7532	1.2421	0.8900	9.1532	1.8075	0.9870	9.0173	1.8722	0.8422	8.1360	1.3672	0.9969
	20	9.2134	1.8482	0.8766	8.9248	2.5721	0.9761	9.1900	2.6980	0.8068	8.9059	1.6131	0.8918
DTLZ7	2	1.1166	0.0019	0.9447	8.2671	0.0220	0.9232	11.487	0.0514	0.7033	2.9452	0.0077	0.7901
	3	5.4186	0.0178	0.8636	12.3103	0.0897	0.9279	17.984	0.1064	0.7535	7.0973	0.0679	0.8018
	5	18.5081	0.0720	0.8854	27.262	0.3264	0.8823	27.982	0.3040	0.7677	23.802	0.0650	0.8932
	10	58.6788	0.1678	0.8602	57.850	0.7150	0.9615	59.652	0.8295	0.7434	56.9858	0.0145	0.9562
	20	115.547	0.3977	0.8447	115.26	1.9394	0.9890	115.85	1.8859	0.7707	112.627	1.4947	0.9899
Av. Values		**7.78432**	**0.2619**	**0.8984**	**9.2089**	**0.4533**	**0.8224**	**9.4575**	**0.5718**	**0.8118**	**8.0942**	**0.3078**	**0.9149**

a bias due to different ranges of these performance metrics, these are normalized prior to the ranking.

The null hypothesis being tested here is that the two populations of solutions, which are being compared, have equal means. Let S_i be the smaller of the two sums $(R_i^+ \& R_i^-)$ for the i^{th} performance metric. Results from the Wilcoxon signed ranks test (Table 7) shows that EFSGA outperforms NSGA III (GD-metric), MOEA/D (S & H-metrics) and HypE with a level of significance of $\alpha = 0.01$, NSGA III (S & H-metrics), MOEA/D (GD-metric) with a level of significance of $\alpha = 0.05$. Further, the null hypothesis of equality of means is rejected since S_i is less than or equal to the critical values for the Wilcoxon distribution for n degrees of freedom. This further means that the proposed algorithm outperforms the other algorithm with the associated p-value.

D. EXPERIMENTS WITH CEC'09 TEST PROBLEMS SUIT

The proposed EFSGA algorithm is later implemented on all the 13 unconstrained test problems provided during CEC 2009 [54]. The parameters settings adopted were as follows.

Population size: 1000; Crossover prob.: 0.95; Real-parameter mutation prob.:0.05; Distribution index for crossover: 10; Distribution index for mutation: 50; Number of independent runs: 30 times for each test problem.

Parameters were not altered during the experiments with different test problems. Each experiment was carried out independently for 30 times and the GD-metric and S-metric for the resulting population of solutions were calculated. Expected values and the standard deviations of GD-metric from these experiments have been provided in Table 8.

Briefly analyzing the tabulated results (Table 8) one can observe that the proposed algorithm finds good approximations to the true PF for test instances such as UF01, UF02, UF6 and UF07 amongst the seven (UF01 to UF07) two objective problems. For test problems UF03 to UF05, however, the performance in terms of GD-metric has not been equally good. Further, for problems with three objectives (UF08 to UF10), EFSGA provides lower GD-metric values for UF08 & UF09 as compared to test instance UF10. Amongst the remaining five objective test problems,

TABLE 7. Results from the Wilcoxon signed ranks test shows that EFSGA outperforms NSGA III (GD-metric), MOEA/D (S & H-metrics) and HypE with a level of significance of $\alpha = 0.01$, NSGA III (S & H-metrics), MOEA/D (GD-metric) with a level of significance of $\alpha = 0.05$.

	EFSGA versus NSGA III			EFSGA versus MOEA/D			EFSGA versus HypE		
	GD-metric	S-metric	H-metric	GD-metric	S-metric	H-metric	GD-metric	S-metric	H-metric
p-Values	< 0.001	0.0497	0.0174	0.0458	0.0018	< 0.001	0.0022	< 0.001	< 0.001
R^+	1054	1081	1165	1164	1339	1404	1702	1767	1744
R^-	342	753	856	666	491	426	128	63	86

TABLE 8. Performance indices from the populations of PF solutions obtained after implementation of EFSGA on CEC'09 problem suit.

Test Problems	GD-metric Mean (Std. Dev.)	S-metric Mean (Std. Dev.)
UF01	0.0041 (0.00129)	0.0059 (0.00176)
UF02	0.0052 (0.00239)	0.0014 (0.00377)
UF03	0.0104 (0.00379)	0.0059 (0.00466)
UF04	0.0776 (0.00381)	0.0101 (0.00257)
UF05	0.1022 (0.03010)	0.0088 (0.03524)
UF06	0.0061 (0.00189)	0.0084 (0.00344)
UF07	0.0079 (0.00270)	0.0099 (0.00332)
UF08	0.0731 (0.00310)	0.0165 (0.00685)
UF09	0.0438 (0.03543)	0.0126 (0.06315)
UF010	0.1581 (0.07254)	0.0364 (0.06449)
R2DTLZ2M5	0.0847 (0.00627)	0.0653 (0.00730)
R2DTLZ3M5	130.658 (34.1896)	1.8652 (1.06031)
WFG1M5	1.7265 (0.01671)	0.1482 (0.01525)

it appears that, EFSGA could not find a good approximation for R2DTLZ3M5 problem. On the other hand, spacing (S-metric) between the consecutive solutions on the PF obtained following EFSGA was found to be good for all the test instances barring a couple of problems such as R2DTLZ3M5 & WFG1M5.

VI. DISCUSSION ON RESULTS

There are few aspects of EFSGA which are worth reiterating. Firstly, the solution space can be divided into known discrete fuzzy fronts while initializing the optimization. It has been demonstrated well that, by following fuzzy based sorting, it is possible to place solutions in their respective fuzzy fronts unambiguously. Consequently, a better population of offspring is available for implementation of various EA operators in the process of further evolution. Finally, in the proposed EFSGA, the termination criterion is quite explicit and once a solution in the first front is obtained, the user can terminate the optimization process fully content with the solution quality. Final solution from the group of Pareto optimal solutions can also be selected based on OAS, which is calculated using (10) and is a unique score for all the competing solutions. Therefore, EFSGA can be a potential tool for real life optimization problems. Nevertheless, the proposed method has also been found performing well during its implementation on the benchmark problems.

Normally the quality or the exactness (closeness to the PF) of any population of solutions is acceptable if the S and GD-metric values for the population are small and the hyper-volume metric H approaches to unity. For most of the test problems (Tables 6&7) it has been found that following EFSGA method, improved average values for these metrics were obtained. Average values of indices have been provided at the end of the Table 6. Further, results from the *Wilcoxon signed ranks test* also show that EFSGA performs better compared to other algorithms. Pairwise comparisons of EFSGA with other approaches, in terms of R_i^+, R_i^- and p-values, have been provided in Table 7. The associated p-values for all the comparisons all found lower than the usual threshold of 0.05. Apparently, EFSGA outperforms NSGA III (GD-metric), MOEA/D (S & H-metrics) and HypE with a level of significance of $\alpha = 0.01$, NSGA III (S & H-metrics), MOEA/D (GD-metric) with a significance level of $\alpha = 0.05$.

In the present research, the emphasis has been on MOP with large number of objectives and therefore experiments with as many as twenty objectives were conducted on DTLZ test problems. The proposed method seems to be performing better especially when the objectives are five or more. While performance of NSGA III has been found comparable to EFSGA, MOEA/D has problems when the solutions space is not convex. As illustrated in Figure 7, MOEA/D is likely to miss the solution space points which lie in the perpendicular directions to the objective axes. HypE approach, on the other hand has also not produced promising results compared to other contemporary methods. Amongst the four methods used, HypE gave us no better results in terms of the performance indices.

The proposed algorithm also performed well while solving the CEC'09 problem suit. Lower GD & S-metrics were obtained for almost all the problems apart from a few namely, UF05, UF10, R2DTLZ3M5 & WFG1M5. The very nature of these test instances is such that achieving better PF estimation is difficult. Although, the results given here are only indicative, it can be observed that once the number of objectives is large, EFSGA performs better for various test problems discussed in the present research.

VII. CONCLUSION

Multi-objective problem can be efficiently solved by simultaneously optimizing a set of solutions using NSGA-III or other EAs. However, when the number of objectives increases, the discriminating capabilities of the inherent sorting schemes of EAs diminishes which adversely affects the selection of better amongst good solutions. Following few iteration, more and more solutions become non-dominated undesirably. As a result, a pseudo PF is obtained giving false impression that the optimization process has converged or completed. Ambiguity in the termination criterion and selection of the final best solution from the set of Pareto optimal solutions are couple of other difficulties which motivated the present research. Opportunities for improvement in the existing EAs were pointed out and the same were attempted by proposing a fuzzy based sorting approach called, *Equitable Fuzzy Sorting Genetic Algorithm (EFSGA)*.

To investigate various aspects of the proposed algorithm and assess the quality of resulting solutions, bench mark test problems, variants of DTLZ, have been implemented and optimized. Three other popular methods, namely, NSGAIII, MOEA/D and HypE have also been implemented on the same set of problems. In order to evaluate and compare the quality and accuracy of population of solutions from various approaches, three performance indices, namely, *GD-metric*, *S-metric* and *H-metric* have been calculated, compared and analyzed. Further, the proposed method has also been implemented on test problems presented in CEC'09, which closely represent the complicated real-life optimization problems.

It was found that the proposed algorithm fared well for all the test instances from DTLZ problem suits, in terms of all the performance indices when compared with other optimization methods. A good approximation for the true PF was also obtained on all the 13 unconstrained test problems from CEC'09. More specifically EFSGA performed better when the number of objectives was large. So far in the present research, improved discriminating power of the fuzzy based sorting scheme has been well exhibited.

During the present research it was found that there are two other important issues with EAs which need urgent attention. In most of the real world problems pertaining to MOP's, users are expected to have varied priorities concerning the objectives. Accordingly, solutions biased towards a particular objective may be preferred during the process of optimization. Therefore, user-preference should be inducted prior to commencing the optimization process. From the preliminary experiments conducted, it has been found that the user preference can be inducted in the proposed method by altering the mean and the standard deviation of the Gaussian AFs used to represent objective functions. As a result, the width of fuzzy fronts in the objective space can be reduced which in turns may penalise the preferred objective function values and bias the optimization process. Nevertheless, it will be interesting to see how user preference can be incorporated on priori basis in the proposed fuzzy sorting genetic algorithm. The other challenge posed by the contemporary EAs is the exploration of the complete PF solutions. It has been reported [9] that the extreme end solutions of the PF are difficult to obtain following existing EAs. Therefore, future work in this research will focus around exploration of extreme end solutions on the Pareto front and induction of the user preferences.

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