VaR and Expected Shortfall in Foreign Exchange Risk Management in Kazakhstan

Various models' forecasting performance evaluation

BY

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Abstract

The thesis paper assesses methodologies, including Historical Simulation, Parametric Approach, ARMA-GARCH, and ARMA-GJR-GARCH, for quantifying Value at Risk and Expected Shortfall in currency pairs involving the Kazakhstani tenge. Both Parametric and GARCH-type models assume Normal Distribution,

Student's t distribution, and Skewed Student's t distribution. Comprehensive analysis revealed no universally superior model; each exhibited distinct strengths and weaknesses, complicating the identification of an optimal risk measurement approach. Additionally, it is observed that the GJR-GARCH model does not consistently outperform the GARCH model, despite its extension to incorporate asymmetric volatility. GARCH-type models that assume skewed Student's t distributions and Student's t distributions for the residuals, demonstrated favorable outcomes for currency pairs originating from developed markets. Additionally, models exhibited improved predictions for currency pairs from developed countries during periods of floating exchange rate adoption. In contrast, more accurate results were observed for currencies of emerging countries in the period spanning 2009 to 2015.

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1 Introduction

This chapter serves as an introductory exploration of the research topic addressed in this thesis. The section begins with a discussion of risk oversight principles and risk metrics, including value at risk (VaR) and expected shortfall (ES). Following this, the next section discusses the characteristics and challenges associated with working with financial time series. The following describes the purpose of the thesis. The final section of the Introduction discusses the literature review in the relevant field, as well as the contribution of the article to the field.

1.1 Risk Oversight and Risk Metrics

Risk management involves identifying and quantifying potential risks to ensure preparedness for unforeseeable events (McNeil et al., 2015, p. 7). Companies typically manage operational, credit, and market risks. Market risk, defined as potential losses in on and off-balance-sheet positions from market price fluctuations (BIS, 2006), includes variations in equity, commodity prices, interest rates and foreign exchange rates(Swami et al., 2015).

This study focuses on foreign exchange risk, wherein a bank's asset or liability values may change due to currency exchange rate fluctuations. Banks are exposed to this risk through buying and selling foreign exchange for their customers. According to Swami et al. (2015), there exist three different of foreign exchange

risk: transaction (commitment), economic (operational, competitive, or cash flow), and translation (accounting). Transaction risk arises when existing obligations lose value due to foreign exchange rate movements, while economic risk results from unexpected volatility affecting equity/income in both domestic and foreign operations, and translation risk is linked to assets or income from offshore activities. In general, risk measures assess the degree of risk associated with a financial stance by connecting it to a quantifiable potential loss (McNeil et al., 2015, p. 61). Mitigating unforeseen losses and restricting risk exposure, their roles encompass the establishment of capital and margin prerequisites for financial institutions and investors. Until the issuance of The Fundamental Review of the Trading Book (FRTB) by the Basel Committee on Banking Supervision (BCBS), Value-at-Risk (VaR) stood as the primary measure for market risk. Released in response to shortcomings revealed during the global financial crisis, the FRTB revised international regulatory standards for banks, introducing new capital requirements for market risk exposures (BIS, 2013). A notable shift in risk assessment occurred as the conventional use of VaR, mandated in the Basel framework for an extended period, was replaced with Expected Shortfall (ES). This paper examines both Value at-Risk (VaR) and Expected Shortfall (ES), placing a greater emphasis on ES.

Expected Shortfall and Value-at-Risk are two examples of distributional risk metrics, which are statistical variables obtained from a loss distribution. One can

associate Value-at-Risk with a certain quantile of the loss distribution. It shows the highest loss that can be anticipated with a pre-specified level of confidence. While Value-at-Risk (VaR) has advantages such as clear interpretation and robust backtesting capabilities, its primary limitation, emphasized in FRTB, lies in its inability to address tail risk. To put it differently, VaR does not provide information on the magnitude of losses once the specified quantile is exceeded. In contrast, Expected Shortfall offers insights into the expected loss beyond a specific quantile in the loss distribution. It not only indicates the likelihood of a significant loss occurring but also estimates the size of the loss when it happens. Expected Shortfall does have certain possible drawbacks, though, mostly in relation to backtesting. In order to achieve comparable precision, backtesting Expected Shortfall is more challenging than Value-at-Risk and needs a bigger sample size.

Various methods utilize historical data for predicting Value-at-Risk (VaR) and Expected Shortfall (ES), encompassing both non-parametric approaches (without distributional assumptions on return series) and parametric approaches (with specific distributional assumptions on return series). In financial time series, common phenomena such as volatility clustering exist. Therefore, variations of the generalized autoregressive conditional heteroscedasticity (GARCH) model are considered due to their ability to address this challenge. This paper analyzes both

parametric approaches, with and without GARCH versions. Additional typical patterns or challenges in financial time series are outlined below.

1.2 Challenges of time series data

Titov (2022) notes that there are consistent statistical characteristics observed across a diverse set of price series, encompassing various assets, markets, and time periods. As highlighted in the preceding section, a notable phenomenon is volatility clustering. It points to a pattern where significant price fluctuations, regardless of their direction, tend to follow or be followed by other substantial price changes. This implies a cyclicality, where phases of low volatility are succeeded by periods of increased volatility, and vice versa. When time series data exhibit these traits, they are labeled as conditionally heteroskedastic, indicating that the conditional variance undergoes changes over time.

Another notable feature in financial time series is the tendency for returns to display heavy tails. This has prompted inquiries into the suitability of modeling return series using the commonly favored normal distribution, given the potential risk of underestimation.

The leverage effect constitutes the third stylized fact in financial time series. It reflects the tendency for past negative shocks to exert a more significant influence on current volatility than equally substantial positive shocks. Essentially, this

suggests that periods of heightened turbulence are generally expected to follow losses compared to gains of similar magnitude (Titov, 2022).

1.3 Purpose

This paper aims to evaluate the predictive accuracy of diverse models in calculating Value at Risk (VaR) and Conditional Value at Risk (ES) metrics across six different currency pairs, encompassing both emerging and developed countries, traded with the Kazakh tenge. The models under consideration encompass the historical simulation (HS), a non-parametric approach, the Variance-Covariance methods, which are parametric methods without employing GARCH in our context, and various ARMA-GARCH(1,1) model variations.

In response to distinct characteristics identified in the distribution of financial returns, we will employ various distributional assumptions. These include the normal distribution, Student's t-distribution, and skewed Student's t-distribution. These distributional assumptions are applied in Variance-Covariance modeling without GARCH and in modeling the standardized residuals when GARCH modeling is involved.

Various models are employed to determine whether utilizing a distributional assumption other than normal or incorporating a GARCH model results in more accurate predictions. The objective behind employing variations of GARCH models

is to assess whether models incorporating additional stylized facts result in more accurate predictions. Two types of variations are utilized in this study. The first is ARMA-GARCH(1,1), a structure designed to capture volatility clustering while omitting the leverage effect. The second is ARMA-GJR-GARCH(1,1), a structure designed to capture both the leverage effect and volatility clustering.

The models mentioned will be utilized to generate one-day-ahead forecasts for VaR and ES, with their predictive accuracy evaluated. The term "backtesting" will be employed to refer to the evaluation process assessing the accuracy of the models' forecasts.

1.4 Literature review

There are numerous studies that focus on the concept of VAR and ES. We found several studies particularly useful for our thesis. The thesis paper on the topic of "Forecasting Exchange Rate

Value-at-Risk and Expected Shortfall: A GARCH-EVT Approach" written by Christoffer Titov in 2022 analyses the variety of Garch models for the goodness of prediction VAR. Useful data analysis tools and methodologies clearly presented in this study and applied in our study as well.

In the paper, Titov conducted a comprehensive analysis of various GARCH models for modeling volatility of the currency pairs, specifically focusing on EUR/USD,

USD/JPY, GBP/USD, AUD/USD, and USD/CAD. Additionally, they referenced other scholarly works that delve into GARCH models and Extreme Value Theory (EVT). Notably, the study by McNeil et al. (2015) is a valuable resource in this context.

The book named Introductory Econometrics for Finance by Chris Brooks also gave a valuable foundation for the concepts and tools that were used in the thesis paper. The book gives introduction to Mathematical and statistical foundations such as probability density function, cumulative density function and degrees of freedom. Then gives insights into univariate time series modeling. It discusses Moving average processes, Autoregressive processes, the partial autocorrelation function and ARMA processes. ARMA processes are the crucial part of the paper, since it is used to forecast the mean for the Value at Risk.

McNeil and his colleagues, in their book, provide a detailed exploration of time series modeling and forecasting. Their work explains essential concepts related to GARCH models, making it a valuable reference for understanding the fundamentals of these models and their application in financial research.

The primary contribution of this thesis lies in its comprehensive examination of international methodologies for Value at Risk (VaR) and Expected Shortfall (ES) calculations, specifically in the context of the Kazakhstani tenge when compared to other foreign currencies. A noteworthy aspect of this research is the pioneering use

of the ARMA-GARCH model for estimating VaR and Conditional Value at Risk (CVaR) for the Kazakhstani tenge in conjunction with various foreign currencies. This marks a significant contribution as it represents the first application of this model to this specific currency pairing.

2.0 Data

This chapter includes a brief discussion of some facts about Kazakhstan's foreign exchange market and the presentation of the data sets that were implemented in the given research. The organization of the section is as follows: first the Kazakhstan's foreign exchange market is overviewed, then the overall data is described. In the next chapter, summary statistics and various visualizations are provided.

2.1 Kazakhstan's forex market

According to the public release of the National Bank of the Republic of Kazakhstan in October of 2022, average daily trading volume on the Kazakhstan stock exchange was 121 mln USD. Composite trading volume over October 2022 was $2.4 bln. From that, Quasi-public sector entities independently sold approximately $528.6 million in foreign exchange earnings through second-tier banks in the foreign exchange market. Additionally, foreign currency sales amounting to $362 million

were conducted in October to facilitate transfers from the National Fund to the republican budget.

According to the latest report from KASE on the exchange market's performance in August 2023, the foreign currency market experienced a notable 21.8% increase in trading volume, reaching 3.7 trillion tenge compared to the previous month, July.

The average daily trading volume amounted to 166.6 billion tenge, with an average of 823 transactions per day and an average transaction volume of 202.4 million tenge.

The table below presents details regarding the trading volume and the proportion of total turnover for primary currencies involved in exchanges with the tenge on KASE.

FX pairs	USD/KZT	EUR/KZT	RUB/KZT	CNY/KZT
Volume (in millions currency units)	3737.3	103.8	31558.0	617.6
Proportion of total turnover	46.2%	1.4%	4.1%	1%

Table1. Most traded currencies with tenge (on KASE). The amount corresponds to the average daily OTC Turnover

The primary currencies involved in exchanges with the tenge include the US dollar, Euro, British pound, Russian rouble, Kyrgyz som, and Chinese yuan. For the further analysis, mentioned currency pairs will be used. Since they cover the majority of the total turnover, this choice of currencies ensures good liquidity, and in turn beneficial in the study of the volatility.

2.2 Data description

The data for the currency pairs is retrieved from the official website of the National Bank of Kazakhstan. The historical price series retrieved from the website is at a daily frequency (highest resolution available). This study takes 6 currency pairs for analysis which are: GBP/KZT, USD/KZT, EUR/KZT, CNY/KZT, RUB/KZT and KGS/KZT.

By using the last mid-quote price of each day, the return series for each currency pair is created. The time period of the data sets starts from the beginning of the 2000s and ends in 2023. Firstly, considering the lengthy time period is helpful for rigorous analysis since it captures the periods of different volatilities. Secondly, this makes back testing approaches more accurate and applicable.(Another benefit of using such extensive data sets is that it makes back testing possible for extreme quantiles of the loss series distribution). The data sets encompass times of global financial turbulence such as the dot-com bubble, the global financial crisis, the covid-19 pandemic, 2022 financial stress, but also calmer periods in between these events.

2.3 Summary statistics and data visualizations

The return series, r_t, are defined by

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \log(P_t) - \log(P_{t-1}) \quad (1)$$

where P_t represents the most recent mid-quote price observed in the exchange rate series on the day t. The plotted figure in Figure 1 illustrates the return series of the complete datasets.

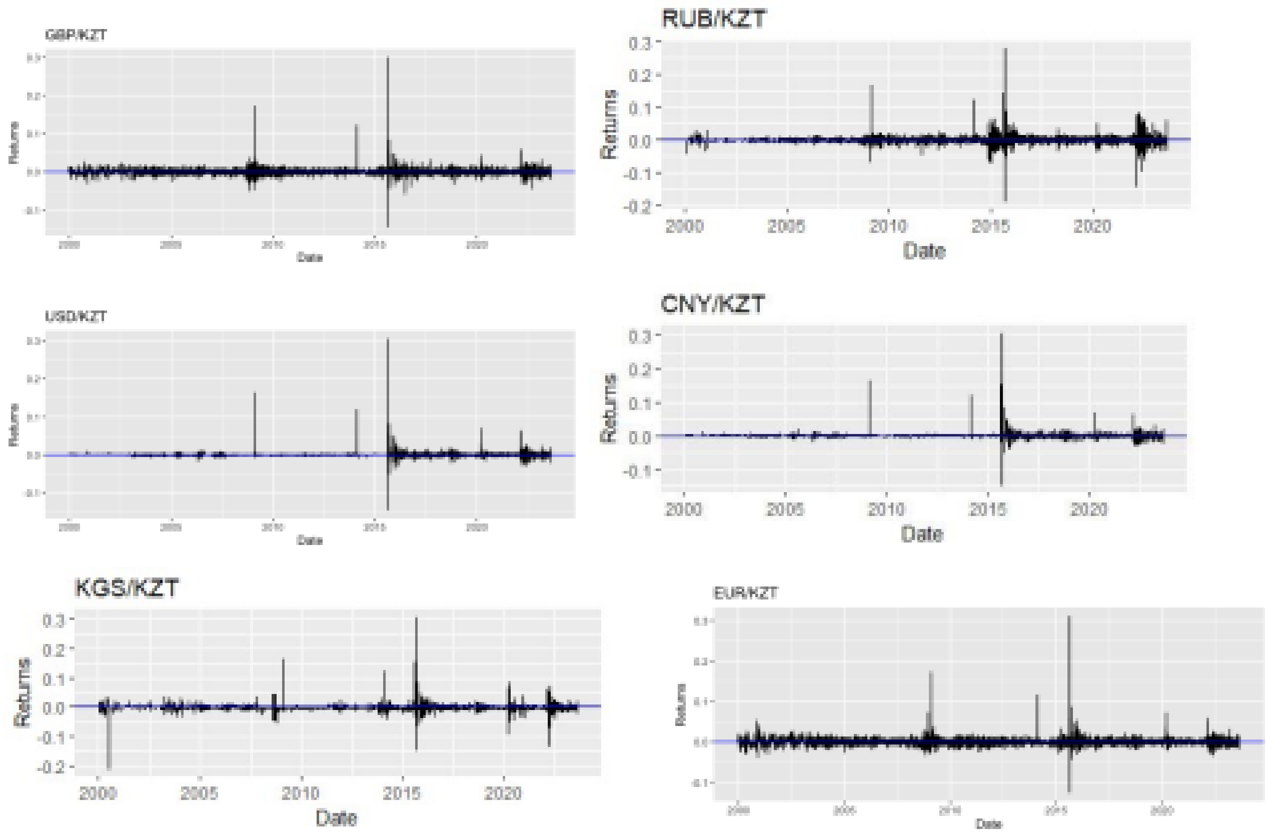


Figure1. A visual representation of the return series

From the figure 1, it is not hard to observe the volatility clustering. There are long periods with high or low volatility, which shows that returns are dependent on the

past observations. The time of global financial turbulence including the 2008 financial crisis, 2014 russian ruble default, and 2020 Covid is also observable from the figure. Kazakhstan's monetary policy change in 2015 regarding the foreign currency regime is also reflected in the graph. (August 20, 2015) .For a more in depth analysis, we have segmented the data into three distinct periods. The initial period spans from January 2000 to February 2009, encompassing the turbulence caused by the 2008 financial crisis (Period 1). Including data from this crisis period allows us to assess the predictive capabilities of our models regarding volatilities based on historical data. The second segment extends from February 2009 to July 2015 (Period 2), preceding the shift from a fixed to a floating currency exchange regime. Notably, this period excludes data from the financial crisis. The third segment covers the period from September 2015 to July 2023 (Period 3), corresponding to the post-regime-change era. The tables below provide summary statistics for the return series across these three periods.

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

RUB/KZT 3306 -0.006% 0.439% -6.470% 16.592% 14.747 642.878 56520731.49 CNY/KZT 3306 0.008% 0.332% -1.681% 16.361% 36.890 1798.390 444776719.35 KGS/KZT 3306 -0.002% 0.630% -20.847% 16.120% -5.882 503.916 34582802.89 GBP/KZT 3314 -0.001% 0.606% -4.996% 17.18% 6.608 204.2555 5617013 USD/KZT 3314 0.002% 0.325% -1.662% 16.30% 38.783 1928.83 512956893 EUR/KZT 3314 0.009% 0.644% -3.783% 17.20% 6.303 165.564 3671084 Period 2

RUB/KZT 2380 -0.015% 0.844% -7.240% 11.990% 1.001 31.878 83097.291 CNY/KZT 2380 0.012% 0.291% -1.801% 11.807% 30.257 1179.795 137693772.966 KGS/KZT 2380 -0.008% 0.433% -3.774% 12.091% 9.132 273.222 7274239.750 GBP/KZT 2372 0.013% 0.555% -2.664% 12.11% 4.671 102.4372 985867.5 USD/KZT 2372 0.01% 0.277% -0.879% 11.84% 35.463 1446.41 206409628.6 EUR/KZT 2372 0.003% 0.595% -2.92% 11.58% 3.298 68.452 427702.2 Period 3

RUB/KZT 2902 0.010% 1.052% -14.102% 8.543% -1.030 37.466 144150.151 CNY/KZT 2902 0.016% 0.585% -4.942% 8.268% 2.357 38.806 157708.789 KGS/KZT 2902 0.011% 0.795% -13.257% 8.349% -1.007 58.842 377542.448 GBP/KZT 2902 0.014% 0.711% -5.763% 8.26% 0.671 19.23760 32098.91
USD/KZT 2902 0.021% 0.565% -5.007% 8.15% 2.69 44.66313 213387.94 EUR/KZT 2902 0.019% 0.650%

-4.308% 8.556% 1.93 27.44394 74049.76

Table 2. Summary statistics of the return series . JB- denotes Jarque-Bera test statistic.

From the table provided, it is evident that normality is rejected across all data partitions and currency pairs. This rejection is primarily attributed to three factors. First, examining skewness reveals a deviation from the normal distribution where the expected value is zero. Remarkably, a majority of the observed skewness values are significantly greater than zero. There are only three instances of negative skewness, arising from KGS/KZT returns in Period 1 and Period 3, as well as from

RUB/KZT in Period 3. Notably, in the case of GBP/KZT in data Period 3, the skewness is 0.671, a value relatively close to zero.

Second, considering kurtosis, which measures the tail heaviness of a distribution, the observed values are markedly higher than the expected value of 3 for a normal distribution. This deviation from the expected kurtosis further supports the rejection of normality across the data sets and currency pairs.

Lastly, the Jarque-Bera test statistic which is consistently nonnegative is employed to assess normality. A significant deviation from zero indicates that the data does not conform to a normal distribution. The obtained test statistics for the data sets are significantly greater than zero, providing additional evidence of non-normality.

Furthermore, upon transitioning to a floating regime in data part 3, notable changes in parameters such as skewness and kurtosis are observed. These changes, characterized by a significant drop in their values, suggest a relatively closer alignment with a normal distribution compared to data parts 1 and 2. However, despite this improvement, the data still falls short of conforming to the expectations of a perfectly normal distribution.

Upon scrutinizing the data, a noteworthy trend emerged in the return series, particularly in the period leading up to the monetary policy change. The identified trends in the data and the corresponding adjustments are outlined below. Analysis of the data uncovered that from the beginning of 2000 until early April 2003, the occurrences of non-zero returns were consistently restricted to 2 per week in most instances. Consequently, we chose to omit this period from our model evaluation and initiated the analysis from April 3, 2003.

Starting from the April 2003 period, we noticed variations in the number of non zero returns, ranging from 3 to 5 per week, potentially influenced by weekends and holidays. Consequently, all instances of zero returns were excluded. Consequently, the original dataset underwent modifications, including changing the starting date to commence in 2003 and excluding all zero returns. As a result, our adjusted datasets begin from April 2003, have only non-zero observations and exhibit varying lengths for each currency pair.

In the upcoming sections, we will distinguish the initial datasets as "Sample 1" and the modified datasets, containing exclusively non-zero observations, as "Sample 2." Subsequent analyses related to the estimation of Value at Risk (VaR) and Expected Shortfall (ES) forecasts will be undertaken for both Sample 1 and Sample 2. The summary statistics for the Sample 2 are presented in the table below.

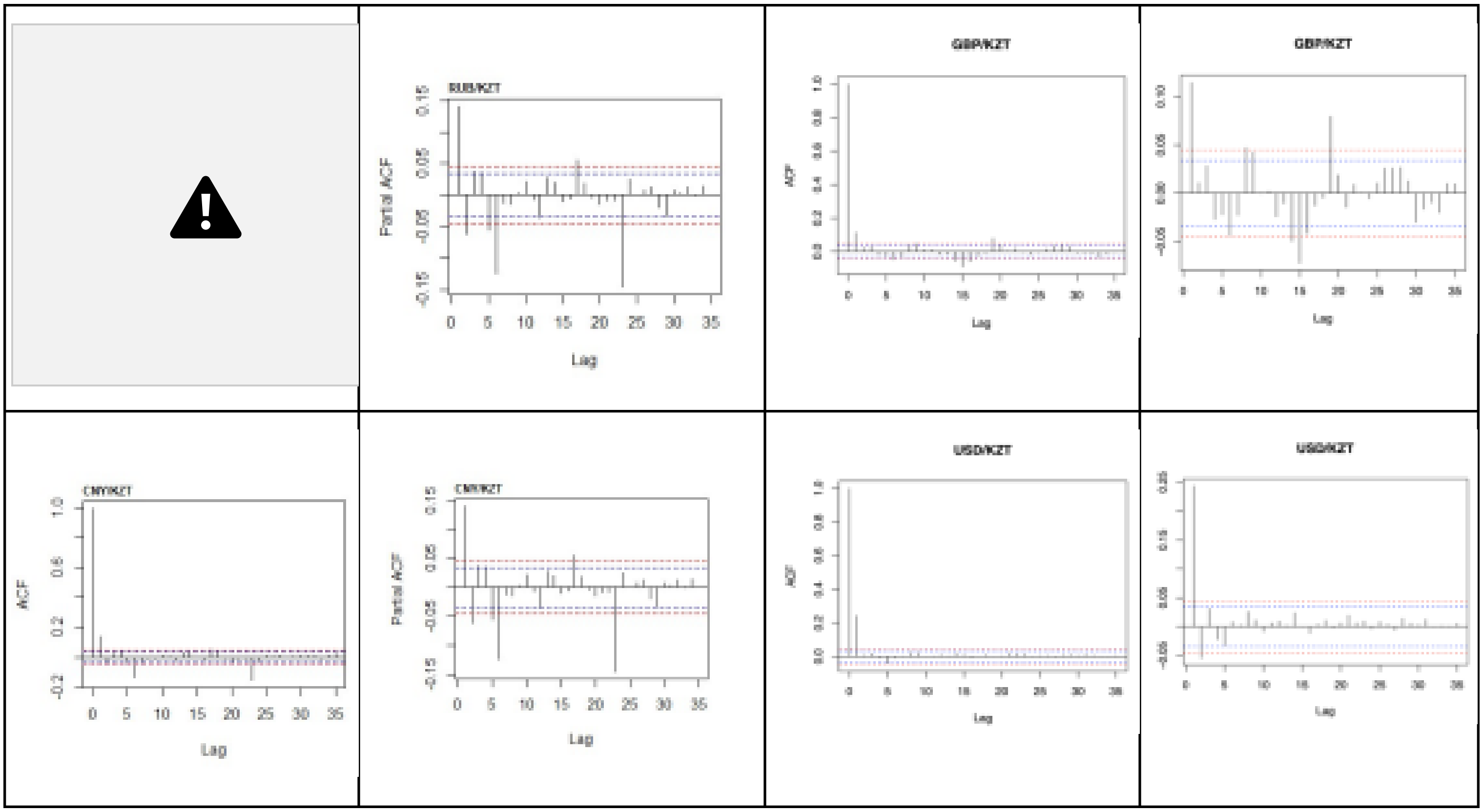
Approaches	Length	mean	sd	min	max	skewness	kurtosis	JB_Statistics
Period 1								
RUB/KZT	996	-0.017%	0.745%	-6.470%	16.592%	10.214	253.621	2697678.863
CNY/KZT	1207	0.015%	0.545%	-1.681%	16.361%	22.685	669.632	22730164.331
KGS/KZT	639	0.011%	1.082%	-5.053%	16.120%	5.088	79.291	171256.606
GBP/KZT	1461	-0.006%	0.834%	-4.996%	17.18%	5.802987	128.6775	969711
USD/KZT	1387	-0.002%	0.498%	-1.662%	16.303%	25.63468	830.704	39744628
EUR/KZT	1458	0.011%	0.832%	-3.36%	17.201%	6.401795	131.3608	1010906
Period 2								
RUB/KZT	1368	-0.027%	1.113%	-7.240%	11.990%	0.789	15.337	13596.913
CNY/KZT	1345	0.022%	0.387%	-1.801%	11.807%	22.694	662.377	24777201.924
KGS/KZT	681	-0.029%	0.809%	-3.774%	12.091%	4.957	75.588	165914.452
GBP/KZT	1596	0.02%	0.677%	-2.66%	12.11%	3.805	68.818	291932.7
USD/KZT	1307	0.018%	0.374%	-0.879%	11.839%	26.30	796.37	34428842
EUR/KZT	1596	0.005%	0.725%	-2.92%	11.576%	2.69876	46.0346	125094
Period 3								
RUB/KZT	1718	0.016%	1.367%	-14.102%	8.543%	-0.806	19.172	26569.735
CNY/KZT	1921	0.025%	0.719%	-4.942%	8.268%	1.883	22.593	42091.332
KGS/KZT	1538	0.021%	1.092%	-13.257%	8.349%	-0.760	28.182	51192.283
GBP/KZT	1948	0.021%	0.868%	-5.763%	8.26%	0.526	12.901	8047.215
USD/KZT	1945	0.031%	0.69%	-5.007%	8.15%	2.159496	29.845	59913.93
EUR/KZT	1948	0.029%	0.793%	-4.308%	8.556%	1.546073	18.36299	19933.14

Table 3. Summary statistics for the return series from Sample 2 . JB- denotes Jarque-Bera test statistics.

In Sample 2, as per Table 3, there is a noticeable deviation from zero in the mean across all currency pairs and data partitions when compared to the means for Sample 1. Furthermore, both skewness and kurtosis parameters experience a substantial reduction, albeit remaining markedly different from the expected values for a normal distribution. Despite this decline, the skewness and kurtosis values still deviate significantly from the normal distribution benchmarks. Additionally, the Jarque Bera test statistic, designed to assess normality, persists in displaying a substantial difference from the expected value. In summary, the removal of zero values has led

to noticeable changes in central tendency and distributional characteristics. However, the data continues to exhibit substantial deviations from normality, as indicated by persistent differences in mean, skewness, kurtosis, and the Jarque-Bera test statistic.

The analysis of the autocorrelation (ACF) and partial autocorrelation functions (PACF) is useful in quantifying the return series dependence. These correlograms provide the information on the existence of the serial correlation and dependence of observations from each other. The ACF and PACF plots of the return series for three periods are separately presented in the figures below. Horizontal dashed blue lines in the plots represent 5% significance level and the red lines represent 1%.



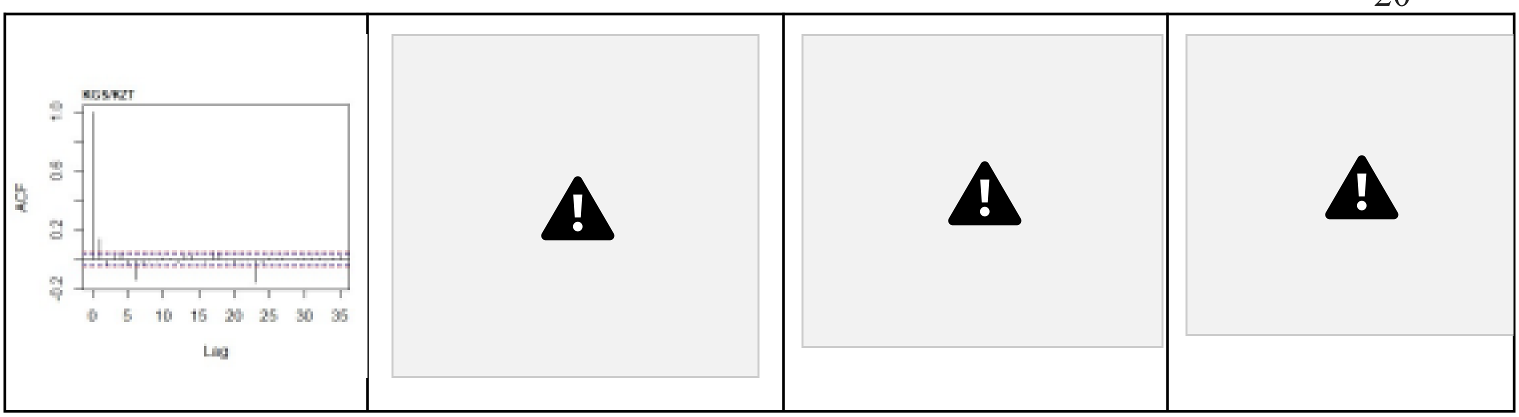


Figure 2. Autocorrelation (left) and partial autocorrelation (right) for the return series (Period 1). The blue dashed horizontal lines in the plots represent 5% significance level and the red lines represent 1% significance level.

As indicated in the table above, the ACF plots reveal a significant serial correlation at lag 1 for all currency pairs, suggesting the presence of a moving average of order 1. Although spikes are observed beyond the 95% and 99% confidence bands, particularly at substantial lags like 30, we conjecture that these spikes may be ascribed to random variability or noise rather than indicating a meaningful pattern in the data.

Upon inspecting the Partial Autocorrelation Function (PACF), significant spikes are evident at lag 6 for currencies from emerging markets. This observation can be attributed to specific characteristics of the data, particularly in data Period 1 spanning the period of 2000-2009. During this timeframe, a pattern emerges where there are six consecutive days with a prevalence of zeros, with only a singular change in price (non-zero return) occurring weekly.

Respective graphs for the currency pairs from developed markets, show that there is a persistent significant spike at lag 1

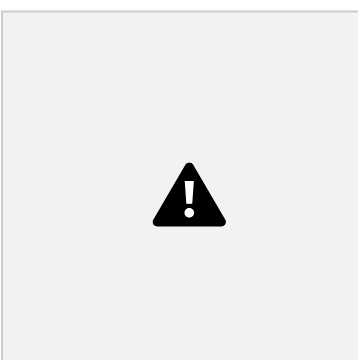
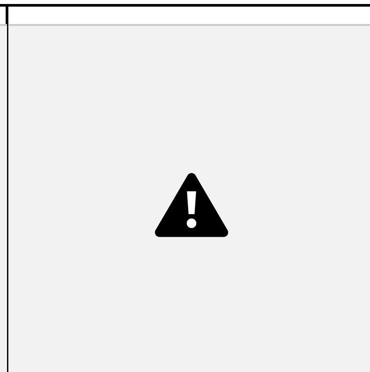
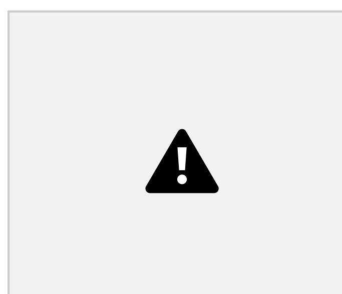
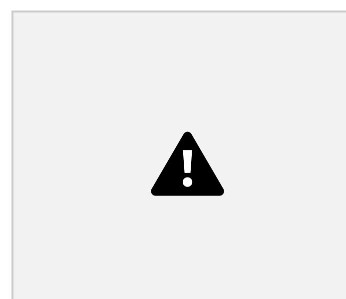
			

Figure 3. Autocorrelation (left) and partial autocorrelation (right) for the return series (Period 2). The blue dashed blue lines in the plots represent 5% significance level and the red lines represent 1%.

In the interval spanning 2009 to 2015 , a consistent pattern emerges across all currency pairs in the ACF table, with a noteworthy spike at lag 1—a trend consistent with the previous graphical representations. Examining the PACF graphs reveals

significant spikes at lag 1 for all currency pairs, and at lag 4 specifically for emerging market currencies. While the PACF graphs exhibit numerous spikes, attributing a clear rationale to them proves challenging. Nevertheless, we postulate that spikes occurring at larger lags may be indicative of errors.

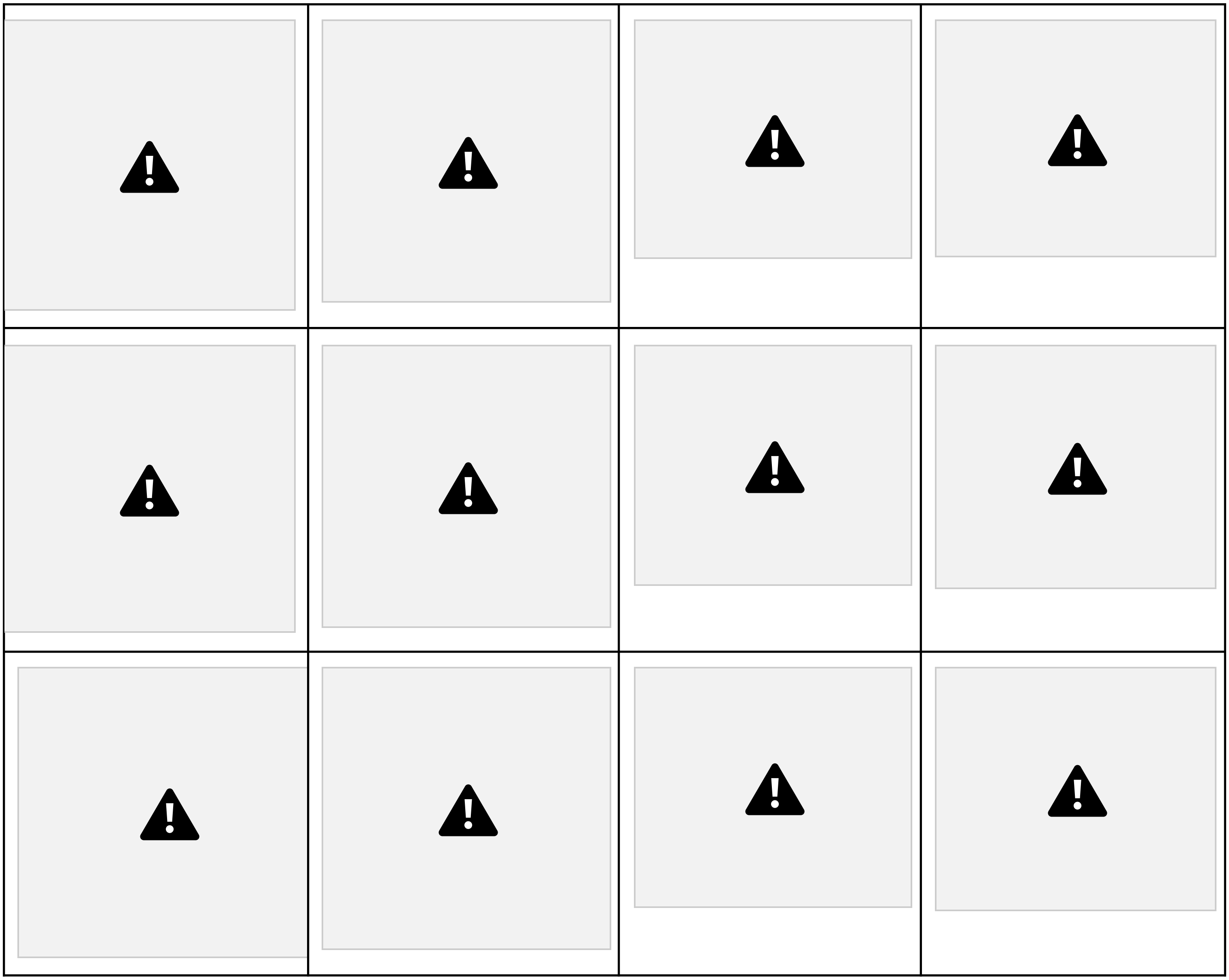


Figure 4. Autocorrelation (left) and partial autocorrelation (right) for the return series (Period 3). The blue dashed blue lines in the plots represent 5% significance level and the red lines represent 1%.

Analyzing the table above, it becomes apparent that for Period 3, encompassing the period from 2015 to August 2023, there appears to be a low-order or non-existent moving average effect. This inference is drawn from the significant spikes observed predominantly at lag 1, often within or below the confidence intervals. Turning to the Partial Autocorrelation Function (PACF) graphs, a noteworthy pattern emerges: at the 95% confidence interval, all currency pairs exhibit substantial positive correlation at lag 6, with the exception of the British pound. Moreover, there are lots of spikes at other lags too, but they might be spurious.

Overall, ACF and PACF show significance at most lags for all return series, indicating a higher order dependency. In the following sections we will use volatility models designed to capture this trend.

3.0 Theoretical background and methodology

This section presents the methodology that will be applied in this paper, along with the fundamental theory supporting our models and predictions. The initial section offers a precise definition of the risk measures and elucidates the methodology for their forecasting. The following section discusses a rationale for adopting a rolling window strategy in out-of-sample forecasting. Following that, details about models discussed.

3.1 Risk measures. Definition and concept

In broad terms, the evolution of risk measurement can be delineated into three phases: initially, the conventional stage involving variance and risk factors as primary indicators; secondly, the contemporary stage characterized by VaR (Value at Risk); and lastly, the stage of risk measurement embodied by Conditional VaR

(CVaR) or Expected Shortfall (ES). This research examines two categories of risk measures, utilizing the rationale outlined by McNeil et al. (2015, pp. 64-72) to elucidate the concept of these risk measures.

VaR (Value at Risk)

The initial risk measure we concentrate on is Value at Risk (������). Defined within a loss distribution, $������_{��}$ represents a q-quintile. Specifically, $������_{��}$ is the minimum loss, denoted as $��_{��}$, where the likelihood of encountering a future loss $��_{��+1}$ is exceeding $��_{��}$ is $1 - ��$:

$$������_{��,��} = ������\{��_{��} \in �� : ��(��_{��+1} > ��_{��}) \leq 1 - ��\} \quad (2)$$

If a random variable ��, characterized by a location parameter �� and scale parameter ��, adheres to a certain location-scale distribution ��, then ������ of �� is defined as: $������_{��}(��) = �� + ����^{-1}(��)$

(3)

In this context, F denotes a normalized cumulative distribution that is adjusted to possess a mean of zero and a variance of one.

Expected Shortfall (ES)

$����_{��}$ represents the anticipated value of the loss variable �� under the condition that the loss exceeds $������_{��}$:

$$����_{��}(��) = ��(��|�� > ������_{��}) \quad (4)$$

If a random variable ��, characterized by a location parameter �� and scale parameter ��, adheres to a certain location-scale distribution ��, then ���� of �� is defined as: ������(��) = �� + ����(��$^{-1}$(��))

$1 -$ ��(5)

In this context, �� denotes the probability density function, and �� represents the standardized cumulative distribution scaled to possess a mean of zero and a variance of one. This probability density can be approximated through integrals.

3.2 Methodology

Out of Sample forecasting

In this study, we have applied an out-of-sample forecasting approach to produce 1-day ahead forecasts of VaR and ES. The forecasts are produced by employing the rolling window technique. A rolling window approach involves systematically analyzing data in sequential, overlapping subsets, known as "windows". The window "rolls" through the dataset, and at each step, an analysis or computation is applied to the data within that window.

The detailed structure of the methodology is as follows: Let's denote the full sample size as "n." The rolling window size, denoted by "w," represents the number of observations used to calibrate the model and remains fixed. The forecast horizon, denoted as "h," signifies the number of days to be forecasted. This study specifically focuses on forecasting volatility for a 1-day ahead period, setting h equal to 1. The process involves using the initial window, comprising observations from the first observation to observation w, to fit the model and generate a forecast for day w+1. Subsequently, the forecast for day w+2 is produced using observations from day 2 to w+1, with the window size remaining constant. This procedure is iteratively repeated until the forecast origin reaches n, resulting in n-w forecasted values of Value at Risk (VaR) and Expected Shortfall (ES). The final output can then be assessed by comparing it against the actual return series through backtesting methodologies.

The most suitable window length is probably influenced by the unique characteristics of the dataset under consideration. In this research, for estimations via variations of Garch models in Sample 1, a window size of 1000 is employed, mirroring the approach taken by McNeil & Frey (2000) and Titov (2015). For Variance-Covariance approaches a window size of 250 has been taken. For Sample 2, window size of 250 is consistently used for all models

3.3 VAR and ES Models

The Historical Simulation method, a non-parametric approach, and parametric methods for Value at Risk (VaR) estimation are elucidated. These include the variance–covariance approaches without GARCH and the variance–covariance approach conditioned on mean and volatility, specifically the ARMA-GARCH(1,1) and ARMA-GJR-GARCH(1,1) models.

3.3.1 Historical Simulation (Non-Parametric)

The Historical Simulation method (HS) is a widely used approach for calculating VaR. This method uses past data to simulate potential future returns at a specified confidence level. Calculation of the VaR by HS involves gathering a historical time series of returns, sorting them in ascending order, and selecting a quantile (confidence level). For instance, if the chosen quantile is 95%, the return corresponding to the 5th percentile of the sorted returns will be considered as VaR estimate. This value represents the anticipated maximum expected loss at the specified confidence level.

The HS approach is advantageous in terms of capturing the empirical distribution of historical returns, including non-normalities and extreme events. However, this method relies on the assumption that past market conditions are indicative of future conditions, which in turn makes the accuracy of predictions contingent on the quality and length of past data. More advanced approaches offer improved accuracy, yet the

HS method continues to be widely used for daily VaR estimation by banks (Swami et.al, 2016).

3.3.2 Variance-Covariance Approach (without GARCH)

VaR and ES estimation through the variance–covariance approach, based solely on distributional assumptions, can be accomplished using expressions 3 and 5, respectively. This study employs three distinct distributional assumptions: the normal distribution, student-t distribution, and skewed student-t distribution. The density functions for these distributions are available in Section 6.1 (Appendix).

3.3.3 Conditional Variance Models

This section outlines the GARCH-type models utilized in this paper. Parameter estimation is performed using the maximum likelihood method.

Basic structure (loss modeling)

In Section 2.3, we have denoted the return series as ����(actual loss), a logarithmic return derived from observed mid-quote prices. Several steps need to be taken to model the loss process as accurately as possible. We assume that return series dynamics can be described by the stochastic process.

���� = ���� + ���� (6)

���� = √����^{2}���� ���� ~��(*0,1*) ��. ��. ��. (7)

Here, ����, referred to as innovations, represent random variables originating from a pure white noise process with a zero mean and unit variance, derived from a marginal distribution ��.

We assume that conditional mean, ����, and the conditional variance, ����^{2}values can be obtained by using the information about the loss process up to time t-1 , denoted by ��, such that

$$���� = ��(����|����_{−1}) \quad (8)$$

$$����^2 = ������(����| ����_{−1}) = ������(����| ����_{−1}) \quad (9)$$

To effectively represent the loss process, it is also essential to explicitly define the equation for the conditional mean. In the preceding section, by analyzing the ACF and PACF plots, we have identified the dependency within the return series and identified lower-order serial correlation in certain FX pairs. In order to accommodate potential serial correlation in the loss series, it is necessary to articulate a model for the conditional mean. As a result, the GARCH models can be estimated on the process adjusted for the mean, denoted as ����.

Conditional mean and model selection

Assuming that the conditional mean adheres to a stationary autoregressive-moving average (ARMA) process, it is characterized by the following equation: $���� = \phi_0 + \sum^{��}_{��=1} \phi_{��} ��_{��−��} + \sum^{��}_{��=1} ��_{��} ��_{��−��}$ (10)

In this context, p represents the lag order of an autoregressive (AR) process, and q denotes the lag of a moving average (MA) process. This representation enables the forecasting of the conditional mean. The forecast for one step ahead is acquired through the following procedure:

$$r_{t+1} = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i+1} + \sum_{j=1}^{q} \phi_j \varepsilon_{t-j+1} \quad (11)$$

Determining whether the process conforms to an autoregressive (AR) and/or moving average (MA) model can be achieved through correlograms. An autoregressive process of order p exhibits an exponentially declining autocorrelation function (ACF) and p distinct spikes in the partial autocorrelation function (PACF). Conversely, a moving average process of order q is marked by an exponentially declining PACF and q distinct spikes in the ACF.

Utilizing the in-sample period or the initial window is a conventional method for ascertaining the suitable model for the conditional mean. However, this approach may prove inadequately comprehensive as the dynamics of the return series may evolve during the progressive rolling of the window. Conducting manual assessments of correlograms for each window is impractical for inference purposes. Consequently, we will employ an algorithmic approach to systematically select an appropriate autoregressive-moving average (ARMA) model for the conditional mean in each window. The selection is based on different information criteria including Akaike Information Criterion (AIC) and Bayesian Information Criterion

(BIC). This methodology aims to effectively capture potential structural changes within the return series.

To assess the effectiveness of model selection using information criteria in the context of our GARCH modeling objectives, we conducted the Ljung-Box test on the standardized residuals derived from all models, each based on the identified ARMA process within its respective window. The findings, detailed in Table 26 and Table 27 in the Appendix, indicate that the standardized residuals of all rolling windows from ARMA models exhibit no autocorrelation up to lag 5 for developed countries' currencies. This suggests that the conditional mean is accurately specified.

Standard GARCH (1,1)

The GARCH model, or the Generalized Autoregressive Conditional Heteroskedasticity model that allows the conditional variance to rely on its own previous lags was introduced by Bollerslev(1986). This model is designed to capture the conditional volatility of a time series, particularly in the context of financial asset returns. The model focuses on the conditional variance, meaning it accounts for how the variance (volatility) of a time series changes over time. It assumes that this volatility is not constant but rather depends on past observations.

The conditional variance equation in the case GARCH(1,1) is

$$��_{��}^{2} = ��_{0} + ��_{1}��_{��-1}^{2} + ��_{1}��_{��-1}^{2} \quad (12)$$

$��_{��}^{2}$ is referred to as the conditional variance because it serves as a forecast for the variance one period ahead, computed with consideration to past information deemed relevant. With the GARCH model, the present fitted variance, $��_{��}^{2}$, can be interpreted as a weighted combination of a long-term average value (dependent on ��$_0$), volatility information from the preceding period (��$_1$����$_{-1}$^{2}), and the fitted variance from the model in the previous period (β ����^{2}).

The GARCH(1,1) model has the potential for extension into a GARCH (p,q) framework, wherein the current conditional variance is parameterized to be influenced by q lags of the squared error and p lags of the conditional variance.

����^{2} = ��$_0$ + $\sum$�� ��$_{=1}$�������� ��−��^{2} + $\sum$�� ��$_{=1}$�������� ��−��^{2} (13)

Here, ��$_0$ > *0,* ���� > *0,* ���� > *0,* for *i* = 1,…,*q* and *j* = 1 ,…,*p*, denoting the lagged values of the residuals and conditional variances, respectively. To guarantee a non negative conditional variance the initial conditions are necessary. Having a requirement that $\sum$�� ��$_{=1}$���� + $\sum$�� ��$_{=1}$���� < *1*, the process is variance-covariance stationary.

The extent to which a shock today influences the volatility in the next period is quantified by ��, while �� gauges the persistence level of past observations. When �� significantly outweighs ��, substantial past conditional variances lead to elevated values of ����^{2}, and conversely, fostering a clustering effect. Conversely, if �� is notably larger than ��, the conditional variance responds more swiftly to shocks, giving rise to more abrupt volatility processes.

For GARCH models utilized in this thesis, we will fix the parameters for the lagged values of the residuals and conditional variances at 1, aligning with the most commonly chosen modeling approach in the literature.

GJR-GARCH (1,1)

The GJR model introduced by Glosten et al. (1993) extends the GARCH framework by introducing an extra term to accommodate the leverage effect. This effect signifies that past negative shocks exert a more significant influence on current volatility than equally substantial positive shocks. This model is characterized by the following representation for the conditional variance.

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i I_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (14)$$

where $I_{t-i} = 1$ if $\varepsilon_{t-i} < 0$ and $I_{t-i} = 0$ otherwise. For a leverage effect, the condition is $\gamma_i > 0$. Taking a value of one when a positive (or negative) return shock occurs, the indicator variable enables the model to discern between positive and negative shocks. γ_i, on the other hand, gauges the magnitude of the leverage effect's impact on the volatility process.

It is important to observe that the requirement for non-negativity is $\omega_0 > 0$, $\alpha_i > 0$, $\beta_j > 0$, and $\alpha_i + \gamma_i \geq 0$ for $i = 1,\ldots,q$ and $j = 1,\ldots,p$, denoting the lagged values of the residuals and conditional variances, respectively.. Thus, the model remains valid even if $\gamma_i < 0$, as long as $\alpha_i + \gamma_i \geq 0$.

The conditional variance equation in the case GJR-GARCH(1,1) is

$$\sigma_t^2 = \omega_0 + (\alpha_1 + \gamma\, I_{t})\varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (15)$$

Conditional Distribution

The final step in specifying GARCH models involves determining the distributional assumption for standardized residuals z_t. Parameters are estimated using maximum likelihood functions, dependent on the parametric structure of innovation distribution. Three distributions—normal, student's t, and skewed student's t—are considered for standardized residuals, applied universally across models. Density functions for these distributions can be found in Section 6.1 (Appendix). Residual distributions are scaled to zero mean and unit variance to emulate ��$_{��}$ behavior; for instance, the standardized student's t-distribution is scaled with √$^{(��-2)}$

$_{��}$, where �� is

degrees of freedom.

3.4 Evaluating accuracy of the models

Backtesting is conducted to assess the predictive accuracy of our models. In evaluating Value at Risk (VaR), we will employ two tests. The initial test involves a proportion assessment: by considering the 5% quintile, our anticipation is that VaR breaches will occur in 5% of instances. A violation is identified when actual returns are more negatively pronounced than the forecasted VaR.

To validate the accuracy of Expected Shortfall (ES) estimates, instances where Value at Risk (VaR) surpassed returns were examined. In addition the unconditional coverage tests of Christoffersen (1998) were performed for VaR backtesting. The ES estimates for these instances were gathered and compared with the actual returns

on corresponding dates. The disparities between the ES estimates and actual returns in these cases were calculated, and an assessment was conducted to ascertain whether these differences exhibited a distribution centered around zero.

3.5 Implementation

We have used R programming language and Excel to make all required calculations for our research. To fit GARCH models in R, a rugarch package that was developed by Ghalanos in 2022 was used. As it was mentioned in the previous sections, the ARMA orders were calculated for each rolling window. This was done by using the auto.arima function from the forecast package in R created by Hyndman et al. in 2022.

4. Results

This section is split into two segments: one focusing on the analysis of Sample 1, and the other on the examination of data with zeros omitted, Sample 2. Comparable methodologies and procedures were employed for both samples. Examining Historical Simulation, Variance-Covariance approach, and S-GARCH (1;1) and GJR-GARCH(1;1) models with three distributional assumptions, we assess a total of 10 models. The denomination Variance-Covariance approach used to identify parametric models solely based on distributional assumption, without volatility modeling. For clarity, acronyms representing these models are provided

in the table below.

Acronym	Model description
��������	Historical simulation
������	Variance-Covariance with normal distribution assumption
������	Variance-Covariance with t-distribution assumption
��������	Variance-Covariance with skewed t-distribution assumption
������	Standard GARCH(1,1) with normally distributed standardized residuals
������	Standard GARCH(1,1) with t-distributed standardized residuals

����$_{����}$ Standard GARCH(1,1) with skewed t-distributed standardized residuals ����$_{��}$ GJR GARCH(1,1) with normally distributed standardized residuals ����$_{��}$ GJR GARCH(1,1) with t-distributed standardized residuals ����$_{����}$ GJR GARCH(1,1) with skewed t-distributed standardized residuals

Table 4 . Acronyms of each model

Before delving into the analysis, it is noteworthy to mention that the coefficients for both GARCH and GJR-GARCH models exhibit positivity and statistical significance. This aligns with the intrinsic non-negativity of volatility. We have incorporated the coefficients from the analysis of USD/KZT returns and RUB/KZT returns into the tables in the Appendix section.

4.1 Results: Sample 1 analysis

4.1.1 Value at Risk backtesting

To initiate our analysis, we will examine the outcomes of the backtesting procedures for Value-at-Risk (VaR). Table below presents the proportion of VaR exceedances, VaR violations, for each model across currency pairs and data periods. In this context, VaR violations are the cases when actual returns are more negatively pronounced than the VaR forecasts. Notably, the primary distinguishing factor among models is the assumed distributional characteristics. The analysis is

conducted over three distinct periods: 2000-2009 (Period 1), 2009-2015 (Period 2), and the post-2015 era (Period 3).

FX Pair �������� ����$_{��}$ ����$_{��}$ ����$_{����}$ ����$_{��}$ ����$_{��}$ ����$_{����}$ ����$_{��}$ ����$_{��}$ ����$_{����}$ Period 1

RUB/KZT 0.071 0.055 0.095 0.148 0.060 0.121 0.117 0.057 0.101 0.100 CNY/KZT 0.055 0.0410 0.066 0.078 0.063 0.112 0.107 0.061 0.089 0.089 KGS/KZT 0.056 0.029 0.112 0.345 0.042 0.137 0.143 0.041 0.124 0.134 GBP/KZT 0.062 0.057 0.089 0.191 0.067 0.1270 0.134 0.069 0.123 0.133 USD/KZT 0.168

0.042 0.068 0.081 0.0687 0.12 0.111 0.067 0.099 0.092 EUR/KZT 0.062 0.058 0.086 0.145 0.063 0.132 0.139 0.065 0.125 0.132 Period 2
RUB/KZT 0.065 0.055 0.094 0.089 0.067 0.106 0.107 0.068 0.105 0.100 CNY/KZT 0.054 0.045 0.065 0.112 0.046 0.065 0.078 0.047 0.064 0.076 KGS/KZT 0.077 0.043 0.141 0.141 0.059 0.168 0.171 0.058 0.168 0.173 GBP/KZT 0.075 0.044 0.066 0.067 0.051 0.067 0.071 0.055 0.069 0.069 USD/KZT 0.268 0.041 0.053 0.122 0.036 0.050 0.052 0.035 0.051 0.051 EUR/KZT 0.057 0.056 0.083 0.079 0.050 0.0736 0.074 0.054 0.073 0.074 Period 3
RUB/KZT 0.051 0.048 0.079 0.085 0.067 0.096 0.095 0.062 0.095 0.092 CNY/KZT 0.053 0.047 0.078 0.086 0.052 0.080 0.089 0.055 0.081 0.090 KGS/KZT 0.047 0.037 0.077 0.078 0.050 0.083 0.088 0.050 0.082 0.089 GBP/KZT 0.041 0.043 0.063 0.070 0.053 0.072 0.079 0.055 0.068 0.073 USD/KZT 0.088 0.045 0.081 0.090 0.055 0.088 0.091 0.055 0.089 0.093 EUR/KZT 0.055 0.050 0.075 0.076 0.053 0.0757 0.077 0.055 0.080 0.085

Table 5: The proportion of Value-At-Risk exceedances of each model for Sample 1 (q = 0.95)

For the selected quantile, q=0.95, adhering to the definition of Value at Risk (VaR), the expected proportion of VaR violations is 5%. Table 4 suggests that the proportion of VaR violations in models assuming a normal distribution tends to be more closely aligned with the expected 5% than in models assuming student-t and skewed student

t distributions. The Variance-Covariance approach without volatility modeling appeared to overestimate risk, while all other models underestimate risk for most

currency pairs. In general, models based on the student-t and skewed student-t distribution assumptions exhibit a tendency to underestimate VaR to a greater degree.

Examining the results in Table 5 below, we will assess whether the observed VaR violations in the models correspond to the expected number of exceedances through the unconditional coverage test.

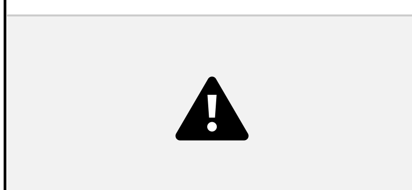
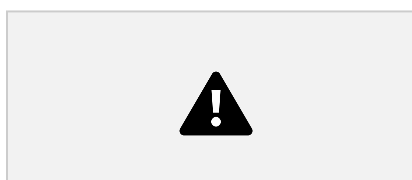
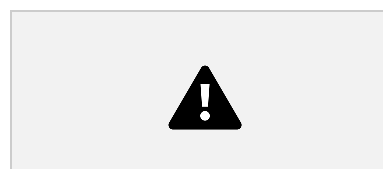
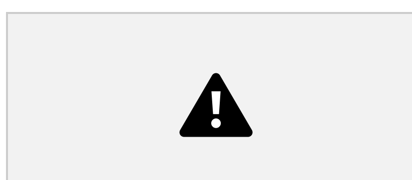
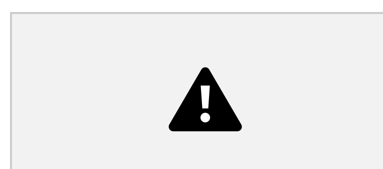
FX Pair	��������	������	������	���������	������	������	��������	������	������	��������
Period 1										
RUB/KZT	0.000	0.260	0.000	0.000	0.028	0.000	0.000	0.118	0.000	0.000
CNY/KZT	0.197	0.019	0.000	0.000	0.006	0.000	0.000	0.017	0.000	0.000
KGS/KZT	0.147	0.000	0.000	0.000	0.058	0.000	0.000	0.046	0.000	0.000
GBP/KZT	0.004	0.090	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
USD/KZT	0.000	0.049	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EUR/KZT	0.004	0.054	0.000	0.000	0.005	0.000	0.000	0.001	0.000	0.000
Period 2										
RUB/KZT	0.002	0.304	0.000	0.000	0.007	0.000	0.000	0.003	0.000	0.000
CNY/KZT	0.404	0.289	0.003	0.000	0.452	0.013	0.000	0.618	0.024	0.000
KGS/KZT	0.000	0.140	0.000	0.000	0.149	0.000	0.000	0.093	0.000	0.000
GBP/KZT	0.000	0.220	0.000	0.000	0.860	0.006	0.000	0.430	0.001	0.002
USD/KZT	0.000	0.040	0.496	0.000	0.010	0.960	0.680	0.010	0.862	0.860
EUR/KZT	0.175	0.210	0.000	0.000	0.960	0.000	0.000	0.490	0.000	0.000
Period 3										
RUB/KZT	0.831	0.680	0.000	0.000	0.001	0.000	0.000	0.026	0.000	0.000
CNY/KZT	0.407	0.439	0.000	0.000	0.609	0.000	0.000	0.305	0.000	0.000
KGS/KZT	0.494	0.001	0.000	0.000	0.992	0.000	0.000	0.842	0.000	0.000
GBP/KZT	0.024	0.110	0.002	0.000	0.605	0.000	0.000	0.300	0.000	0.000
USD/KZT	0.000	0.250	0.000	0.000	0.310	0.000	0.000	0.300	0.000	0.000
EUR/KZT	0.207	0.960	0.000	0.000	0.610	0.000	0.000	0.290	0.000	0.000

Table 5: The p-values of each model from the unconditional coverage test. The model is rejected if the p value is less than 0.05.

Table 5 affirms several trends noted in Table 4. Evaluating p values at q = 0.95, it becomes evident that models assuming a normal distribution exhibit a subtle superiority over those assuming a t-distribution or skewed t-distribution. It is unclear whether one model should be favored over another or whether there is a general trend among these models. For example, in Data Part 2 for USD/KZT returns, GARCH and GJR-GARCH models assuming student's t and skewed student's t distributions captured the risks better, as the p-value is much greater than 0.05. Whereas, for GBP/KZT and EUR/KZT returns, GARCH and GJR-GARCH models assuming normal distributions managed to capture the risk.

4.1.2 Expected Shortfall forecasts and backtesting

The analysis of the accuracy of the ES forecasts can be initiated with the visual inspection of the forecasts and actual losses. In the tables below, we present the ES forecasts derived from 10 considered models, juxtaposed with the actual losses incurred. The first row encompasses results from the historical simulation approach and the Variance-Covariance approach without GARCH, under three distributional assumptions. In the second row, forecasts from standard S-GARCH and GJR GARCH modeling, under three distributional assumptions, a total of 6 trends, are presented alongside actual losses.

RUB/KZT	Period 1	Period 2	Period 3
Hist and VC forecasts			
GARCH type Models' forecasts			

ES forecasts from different models and actual losses for RUB/KZT (Sample1)

The figure shows that the variation of GARCH models tends to yield more accurate results compared to forecasts generated from historical simulation and variance-covariance without GARCH components. Notably, in most instances, the Student’s t distribution and the Skewed Student’s t distribution demonstrate superiority over the normal distribution in terms of forecast accuracy.

The accuracy of Expected Shortfall (ES) predictions was evaluated in instances of VaR violations. The tables presented below showcase summary statistics and histograms, revealing the differences between Expected Shortfall (ES) and Actual Losses under different assumptions regarding return distributions, including normal, Student’s t, and skewed Student’s t.

The general idea is to assess whether the Discrepancies in Expected Shortfall (ES) that are estimated via different approaches versus actual losses are distributed around zero. The analysis is segmented into two distinct parts. The first part focuses on

emerging market currencies traded with KZT, specifically RUB/KZT, CNY/KZT, and KGS/KZT. In the second part, attention shifts to additional emerging market currencies exchanged with KZT, namely GBP/KZT, USD/KZT, and EUR/KZT.

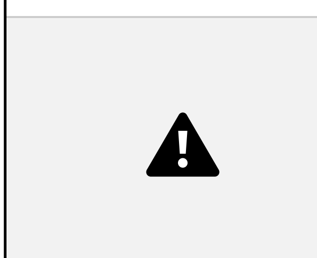
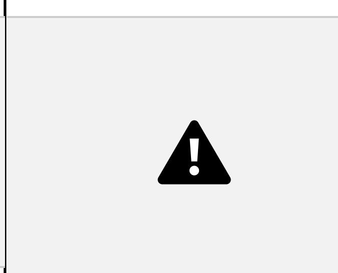
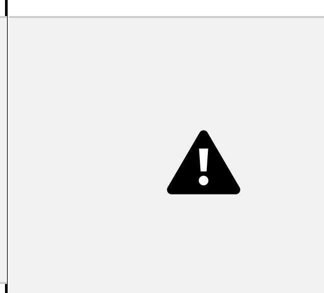
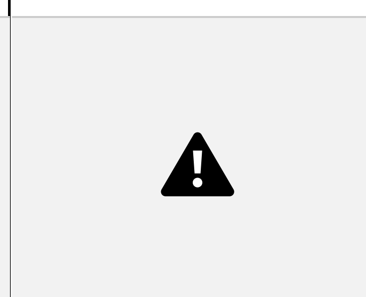
Emerging:

To commence our examination, let's delve into the efficacy of ES forecasts concerning emerging market currencies:

1) Historical Simulation and Variance-Covariance approach without GARCH

2)

The outcomes of backtesting for ES in the parametric approach, relying exclusively on distributional assumptions, are displayed in the tables and figures below.

	Historical	Normal dist.	Student's t distribution	Skewed Student's t dist.
Period 1				

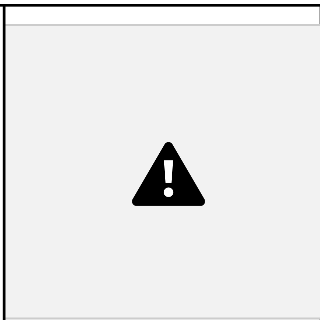
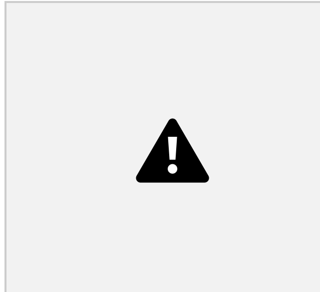
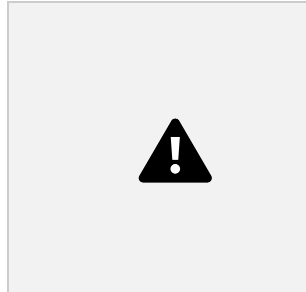
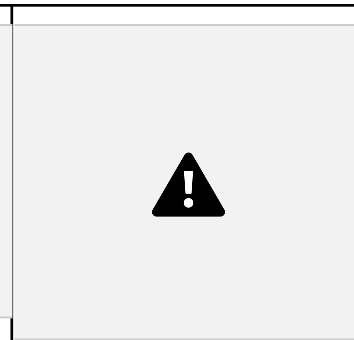
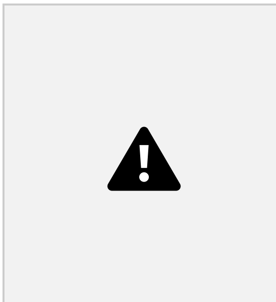
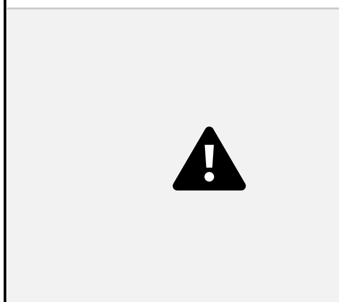
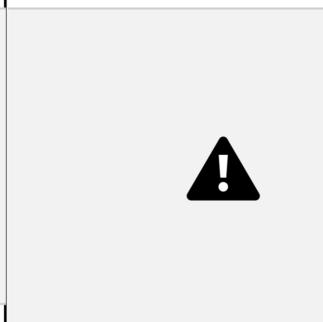
Period 2				
Period 3				

Figure 5. Discrepancies in Expected Shortfall (ES) estimated via Variance-Covariance Approach versus actual losses for emerging market fx currencies relative to KZT.

Approaches Length Mean sd min max skewness kurtosis JB_Statistics Period 1

Hist 490 0.000 0.006 -0.022 0.052 2.478 17.305 6677.312 Norm 338 0.002 0.005 -0.004 0.033 3.593 15.203 4035.311 Std 826 0.002 0.006 -0.007 0.051 3.913 23.104 20587.552 Sstd 1728 0.001 0.004 -0.008 0.051 5.859 50.917 197028.136 Period 2

Hist 384 0.001 0.006 -0.012 0.041 3.388 17.427 5659.583 Norm 291 0.003 0.006 -0.005 0.045 3.845 19.105 5221.121 Std 639 0.001 0.006 -0.020 0.050 2.232 12.059 4435.270 Sstd 730 0.001 0.006 -0.024 0.040 1.827 11.031 4134.692 Period 3

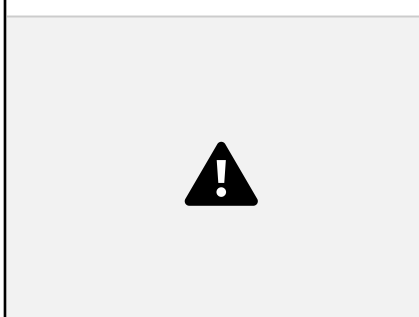
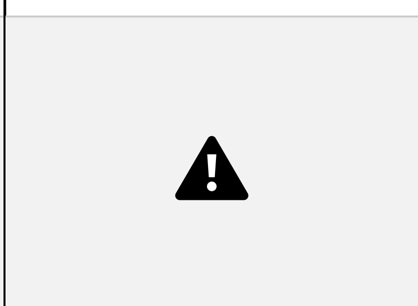
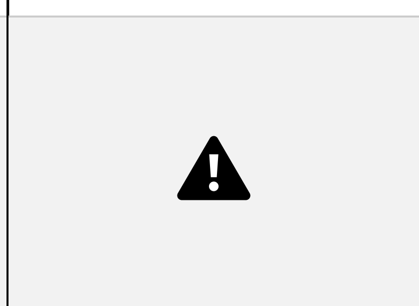
Hist 336 0.001 0.016 -0.032 0.127 4.887 33.445 17213.975 Norm 301 0.007 0.015 0.000 0.124 5.573 36.953 18947.558 Std 621 -0.001 0.013 -0.024 0.126 5.518 42.991 51320.672 Sstd 658 -0.000 0.013

-0.026 0.126 5.576 44.027 56915.315Table 6: Summary of discrepancies in Expected Shortfall (ES) estimated via var-covar Approach versus
actual losses for emerging market fx currencies relative to KZT.

After examining the summary statistics and histogram of the difference between Expected Shortfall and Actual Shortfall following conclusions are drawn: For the Period 1 and Period 3, Historical Simulation approach results appeared to be the most accurately distributed around zero. For Period 2, Variance Covariance approach that assumes Skewed student's t distribution showed superior performance.

3) S-GARCH (1;1)

The outcomes of backtesting for ES in the parametric approach using s-GARCH are displayed in the tables and histograms below

	Normal dist.	Student's t distribution	Skewed Student's t dist.
Period 1			

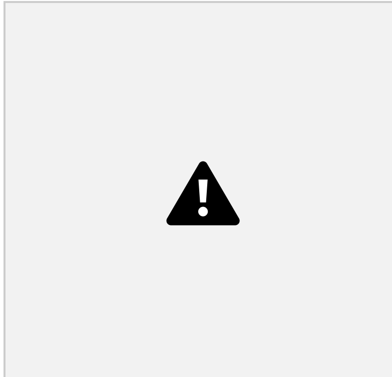
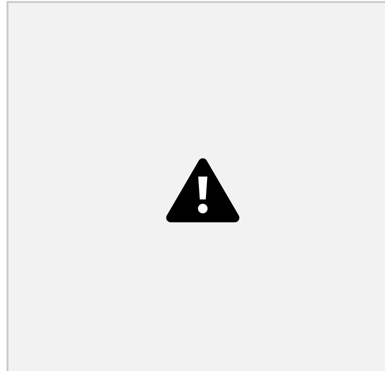
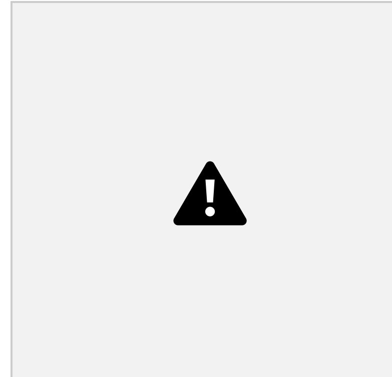
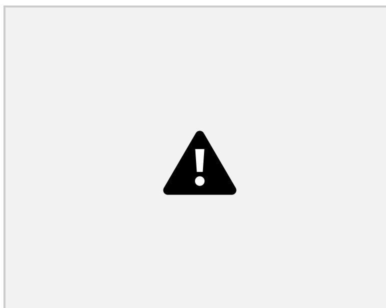
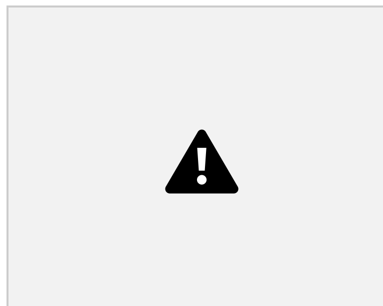
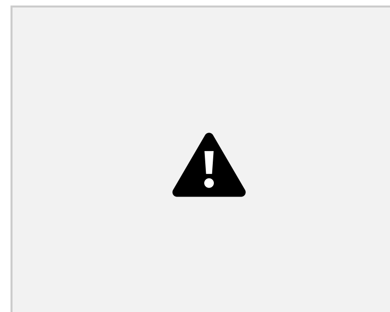
Period 2			
Period 3			

Figure 6. Discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for emerging market fx currencies relative to KZT.

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Norm 342 0.002 0.005 -0.002 0.055 5.765 45.122 31288.665 Std 853 0.001 0.005 -0.012 0.055 5.119 42.015 66794.821 Sstd 845 0.002 0.005 -0.011 0.055 4.897 36.131 49588.379 Period 2

Norm 226 0.002 0.005 -0.008 0.035 3.318 15.672 2782.499 Std 467 0.001 0.006 -0.015 0.039 1.985 8.658 1784.361 Sstd 489 0.001 0.006 -0.017 0.038 1.649 7.138 1273.164 Period 3

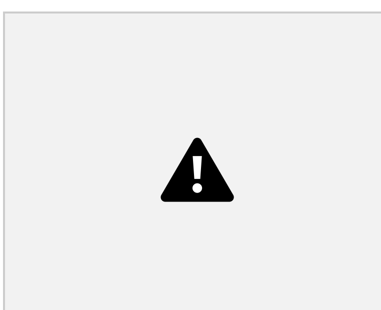
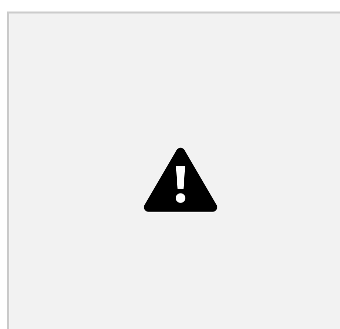
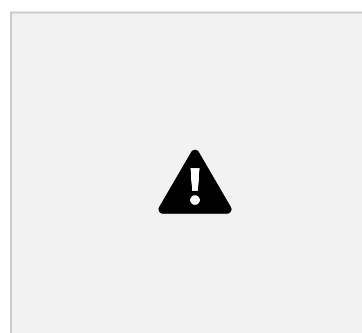
Norm 310 0.003 0.011 -0.011 0.112 6.405 51.426 36770.790 Std 493 -0.001 0.011 -0.047 0.113 5.297 48.693 51446.324 Sstd 516 -0.001 0.011 -0.049 0.114 5.268 49.534 55589.534

Table7: Discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for emerging market fx currencies relative to KZT.

The table implies that ARMA-S-GARCH approach with innovations under Skewed student's t distribution and student's t distribution assumption showed similar performance, that is superior compared to results under normal distribution assumption .

4) GJR-GARCH (1;1)

The outcomes of backtesting for ES in the parametric approach using GJR-GARCH are displayed in the tables and histograms below.

	Normal dist.	Student's t distribution	Skewed Student's t dist.
Period 1			

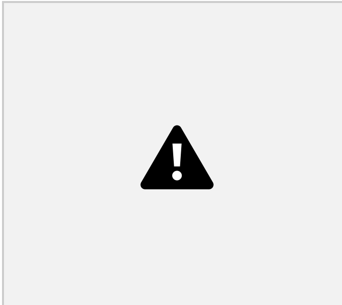
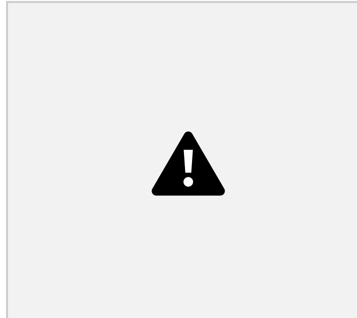
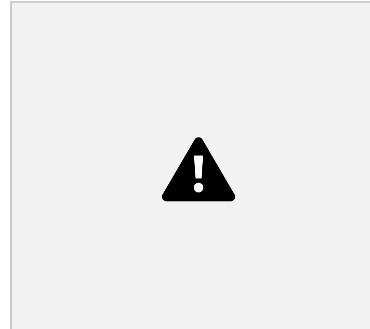
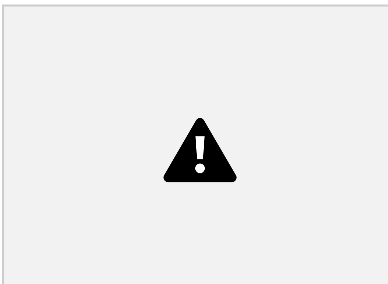
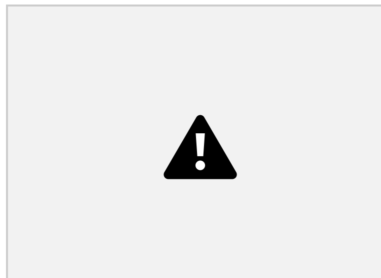
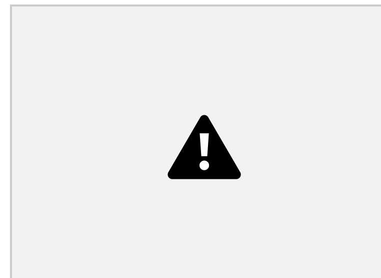
Period 2			
Period 3			

Figure 7. Discrepancies in Expected Shortfall (ES) estimated via gjr-Garch Approach versus actual losses for emerging market fx currencies relative to KZT

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Norm 346 0.002 0.005 -0.003 0.054 5.351 39.957 24970.633 Std 687 0.001 0.005 -0.021 0.053 4.913 41.163 51581.655 Sstd 705 0.001 0.005 -0.006 0.053 4.899 35.665 40427.670 Period 2

Norm 210 0.001 0.005 -0.005 0.033 3.469 16.132 2756.501 Std 388 -0.000 0.005 -0.019 0.028 0.935 5.530 559.309 Sstd 401 0.000 0.005 -0.019 0.029 1.170 5.764 655.753 Period 3

Norm 312 0.003 0.010 -0.014 0.113 6.671 57.763 46300.674 Std 491 -0.001 0.011 -0.060 0.113 5.143 50.871 55580.004 Sstd 513 -0.001 0.011 -0.071 0.113 4.782 51.809 59817.055

Discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for emerging market fx currencies relative to KZT.

The table implies that ARMA-GJR-GARCH approach with innovations under Skewed student's t distribution and student's t distribution assumption showed similar performance, that is superior compared to results under normal distribution assumption. The results are similar with ones from the ARMA-S-GARCH approach. Overall, analyzing ES estimates for the returns of emerging market currencies traded with KZT in Sample 1, several key conclusions can be drawn. Notably, ES forecasts derived from the Student's t and Skewed Student's t distributions demonstrate superiority. All three distributional assumptions produced more accurate results in Period 2 compared to other periods. ES forecasts appear to slightly underestimate risk in both Period 1 and Period 2, with positive mean values across all approaches. In Period 3, there is evidence of a slight overestimation, attributed to negative means

when assuming Student's t and Skewed Student's t distributions. Comparing between models, the Variance-Covariance without GARCH method performed well in Period 1. For Periods 2 and 3, the GJR-GARCH model exhibited superior performance based on mean and skewness parameters.

Developed

Analyzing the currencies within the developed market, the following results have been derived:

1) Historical Simulation and Variance-Covariance approach without GARCH

The outcomes of backtesting for ES in the parametric approach, relying exclusively on distributional assumptions, are displayed in the tables and histograms below.

	Historical	Normal dist.	Student’s t distribution	Skewed Student’s t dist.
Period 1				
Period 2				
Period 3				

Figure 8. Discrepancies in Expected Shortfall (ES) estimated via Variance-Covariance Approach versus actual losses for developed market fx currencies relative to KZT.

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1 Historical 1666 0.0015 0.004 -0.009 0.0375 2.74 13.85 10260.53 Norm 481 0.002 0.004 -0.004 0.036 2.83 14.687 3381.177 Std 745 0.0007 0.006 -0.01 0.0378 1.512 6.597 685.71 Sstd 1277 0.0004 0.005 -0.013 0.037 1.95 11.04 4253.06 Period 2

Historical 889 0.003 0.004 -0.0044 0.028 1.45 5.82 606.05 Norm 299 0.001 0.003 -0.002 0.021 2.34 11.16 1104.818 Std 431 -0.002 0.0038 -0.011 0.021 0.7186 5.90 188.61 Sstd 569 -0.0016 0.003 -0.012

0.0178 0.0006 4.949 90.044 Period 3

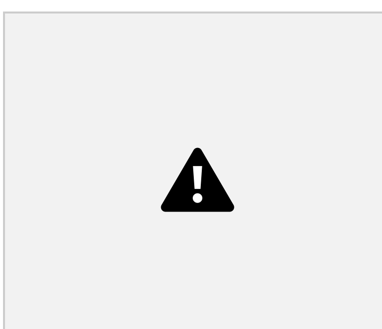
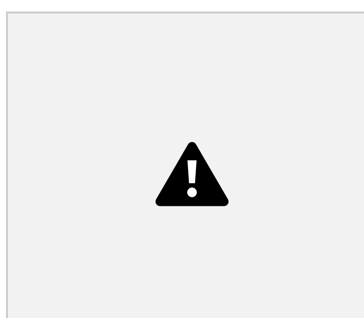
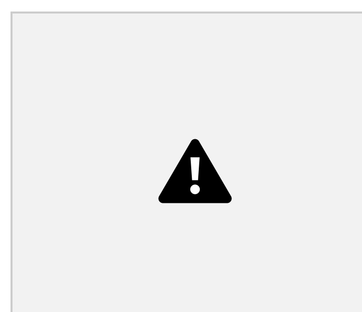
Historical 489 0.0008 0.0045 -0.009 0.0203 1.398 5.678 305.6415 Norm 366 0.0018 0.004 -0.004 0.0269 2.22 8.876 829.07 Std 580 -0.0024 0.0057 -0.016 0.022 1.65 6.849 620.96 Sstd 627 -0.002 0.0056 -0.014 0.032 1.97 8.956 1333.004

Table 9: Discrepancies in Expected Shortfall (ES) estimated via var-covar Approach versus actual losses for developed market fx currencies relative to KZT.

Among widely used approaches globally, the Variance-Covariance approach assuming a Student's t distribution exhibited superior results for the Period 1. In Period 2, the Skewed Student's t distribution outperformed, while for Period 3, the Historical Simulation method proved to be superior.

2) S-GARCH (1;1)

The outcomes of backtesting for ES in the parametric approach using s-GARCH are displayed in the tables and histograms below.

	Normal dist.	Student's t distribution	Skewed Student's t dist.
Period 1			

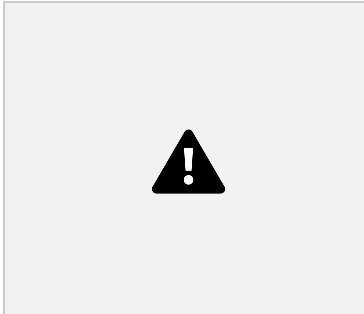
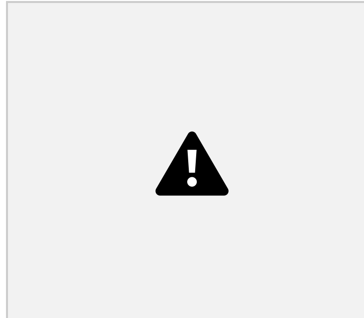
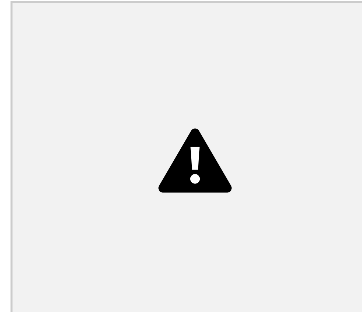
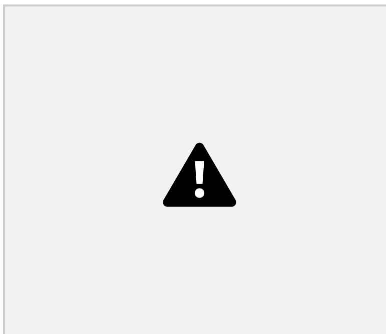
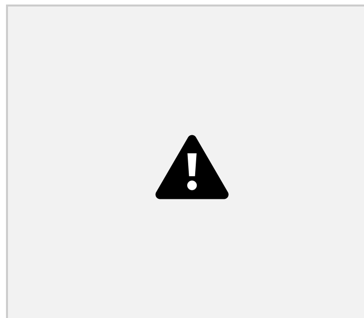
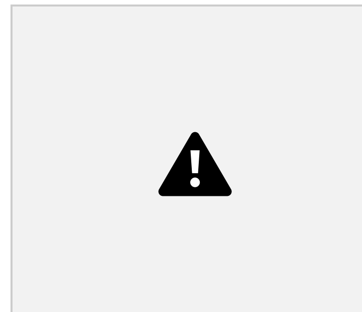
Period 2			
Period 3			

Figure 9. Discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for developed market fx currencies relative to KZT.

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Norm 460 0.0012 0.0035 -0.005 0.027 3.187 17.62 4876.27 Std 877 0.0012 0.005 -0.0097 0.0255 1.186 5.784 489.18 Sstd 891 0.0013 0.005 -0.010 0.025 1.01 5.239 338.09 Period 2

Norm 188 0.0012 0.0031 -0.0019 0.0198 2.585 12.16 866.97 Std 262 -0.001 0.003 -0.0068 0.0192 1.811 9.701 633.41 Sstd 271 -0.001 0.0033 -0.0066 0.0188 1.62 8.76 493.8 Period 3

Norm 304 0.0016 0.0038 -0.004 0.0298 2.569 14.62 2044.976 Std 446 -0.0018 0.004 -0.012 0.0155 1.23 5.44 223.69 Sstd 469 -0.0017 0.004 -0.014 0.0159 1.30 5.746 279.595

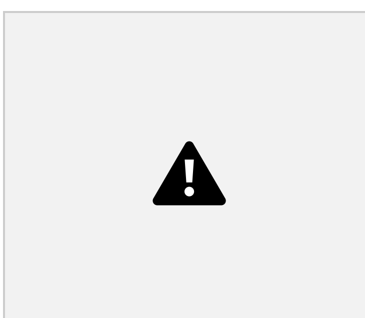
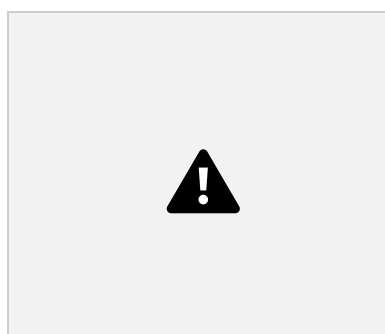
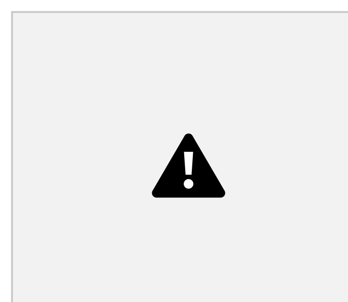
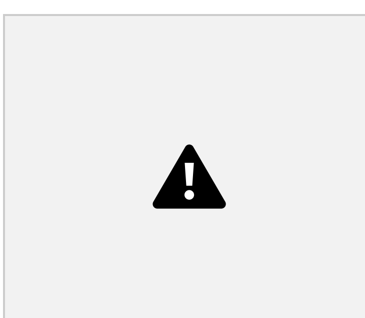
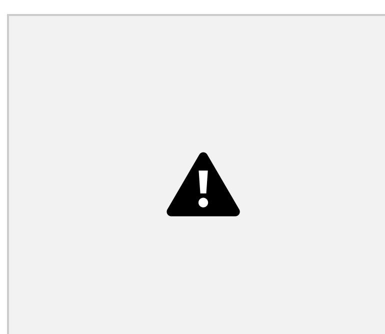
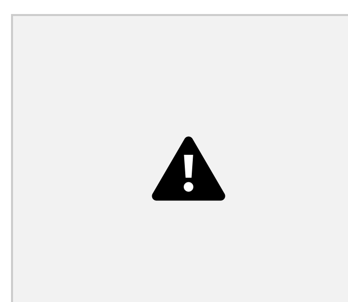
Table 10: Discrepancies in Expected Shortfall (ES) estimated via s-Garch Approach versus actual losses for developed market fx currencies relative to KZT.

Observing the figures above, models assuming skewed Student's t distributions for residuals demonstrated superior performance in Period 1 and Period 2. Conversely, in Period 3, while the Student's t distribution showed slightly better results, the parametric indicators for both models remained closely comparable.

3) GJR-GARCH (1;1)

The outcomes of backtesting for ES in the parametric approach using GJR-GARCH are displayed in the tables and histograms below.

Initially, let's examine the effectiveness of Expected Shortfall (ES) forecasts for emerging market currencies.

Period 1	Normal dist.	Student's t distribution	Skewed Student's t dist.
			
Period 2			

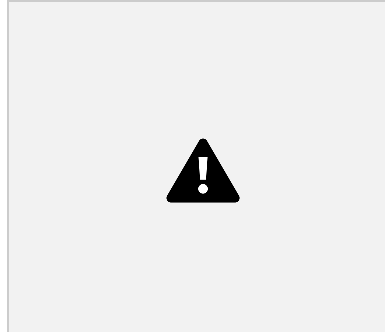
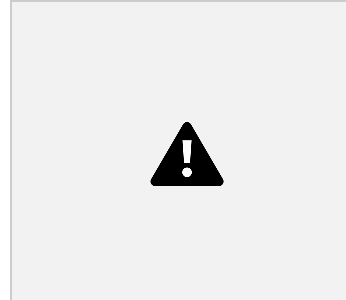
Period 3			

Figure 10. Discrepancies in Expected Shortfall (ES) estimated via gjr-Garch Approach versus actual losses for developed market fx currencies relative to KZT.

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1
Norm 464 0.0013 0.003 -0.005 0.025 3.04 16.07 4018.064 Std 765 0.0007 0.005 -0.011 0.0258 1.386 6.92 733.69 Sstd 783 0.0011 0.005 -0.011 0.0255 1.13 5.888 438.46 Period 2
Norm 198 0.0012 0.003 -0.0017 0.019 2.522 11.428 795.95 Std 266 -0.001 0.003 -0.007 0.0185 1.63 9.052 524.6295 Sstd 266 -0.0013 0.003 -0.007 0.018 1.52 8.41 428.356 Period 3
Norm 314 0.0017 0.0036 -0.0038 0.0187 1.77 6.938 366.98 Std 452 -0.0017 0.004 -0.013 0.016 1.279 5.965 288.86 Sstd 476 -0.0017 0.0039 -0.0127 0.017 1.20 6.016 294.88

Table 11: Discrepancies in Expected Shortfall (ES) estimated via s-Garch Approach versus actual losses for developed market fx currencies relative to KZT.

The figures above reveal that the results for ARMA-GJR-GARCH models are identical to those discussed for ARMA-GARCH models just above.

Overall, analyzing ES estimates for the returns of developed market currencies traded with KZT in Sample 1, several key conclusions can be drawn. ES, forecasts derived from the Student’s t and Skewed Student’s t distributions demonstrated superiority. Particularly, in Data Part 1, the ARMA-GARCH model with a skewed Student's t distribution demonstrated superior performance. Moving to Data Part 2, the Variance-Covariance approach assuming a skewed Student's t distribution exhibited enhanced results. In the case of Data Part 3, both the ARMA-GARCH and ARMA-GJR-GARCH models, when assuming both Student's t and skewed Student's t distributions, presented superior outcomes. Most of the models performed

comparatively better in Period 3. During Period 1 and consistently across all periods under the assumption of a normal distribution, the Expected Shortfall (ES) forecasts suggest a tendency to slightly underestimate risk, as evidenced by positive mean values across various approaches. While for Period 2 and Period 3, there is evidence of a slight overestimation, attributed to negative means when assuming Student’s t and Skewed Student’s t distributions.

When comparing results between emerging market currencies and developed market currencies traded with KZT for Sample 1, several distinctions become apparent. Notably, for developed countries' currencies, most models exhibited comparatively better performance during Period 3, whereas, for emerging markets, their efficacy was more pronounced during Period 2. Additionally, Expected Shortfall (ES) forecasts indicated signs of overestimation for Period 2 in the case of developed market currencies, while for emerging markets, the forecasts tended to be underestimated. Interestingly, for both market types, forecasts were underestimated in Period 1 and overestimated in Period 3. In terms of model selection, during Period 3, the GJR-GARCH model provided accurate forecasts for all currencies under consideration. Regarding distributional assumptions, both the Student's t and Skewed Student's t distributions demonstrated comparably similar and superior results.

4.2 Results: Sample 2 analysis

In accordance with the details outlined in section 2.3, the original dataset underwent alterations involving adjustments to the initial time period and the omission of zeros. The subsequent section is devoted to showcasing the outcomes of a comparable analysis performed on the modified data (Sample 2). Furthermore, two distinct quantiles considered in this section: q = 0.95 and q = 0.99.

4.2.1 Value at Risk backtesting

The outcomes of the backtesting procedures for Value-at-Risk (VaR) are examined. Table below presents the proportion of VaR exceedances for each model across currency pairs and data periods. A violation is identified when actual returns are more negatively pronounced than the forecasted VaR. The analysis is conducted over three distinct periods: 2003-2009 (Period 1), 2009-2015 (Period 2), and the post-2015 era (Period 3).

FX Pair ��������� ������ ������ �������� ������ ������ �������� ������ ������ �������� Period 1 (q= 0.95)

RUB/KZT 0.086 0.077 0.089 0.055 0.062 0.060 0.044 0.071 0.071 0.046 CNY/KZT 0.052 0.037 0.044 0.062 0.046 0.043 0.059 0.047 0.044 0.067 KGS/KZT 0.085 0.054 0.062 0.089 0.060 0.058 0.081 0.049 0.049 0.068 GBP/KZT 0.0636 0.0586 0.0685 0.052 0.0562 0.0603 0.0520 0.0595 0.0619 0.0532

USD/KZT 0.0510 0.036 0.045 0.048 0.0501 0.0572 0.0563 0.0528 0.0572 0.0607 EUR/KZT 0.067 0.061 0.065 0.0596 0.0579 0.0597 0.0621 0.0604 0.0604 0.0637 Period 1 (q= 0.99)

RUB/KZT 0.0335 0.0295 0.0281 0.0187 0.0148 0.0242 0.0201 0.0215 0.0228 0.0161 CNY/KZT 0.0157 0.0167 0.0125 0.0146 0.0010 0.0105 0.0167 0.0136 0.0104 0.0167 KGS/KZT 0.0283 0.0308 0.0282 0.0386 0.0366 0.0183 0.0314 0.0105 0.0131 0.0289 GBP/KZT 0.016 0.019 0.016 0.012 0.013 0.013 0.011 0.013 0.013 0.011

USD/KZT 0.013 0.019 0.011 0.011 0.013 0.011 0.016 0.016 0.011 0.015 EUR/KZT 0.022 0.025 0.017 0.011 0.015 0.011 0.012 0.017 0.013 0.012 Period 2 (q= 0.95)

RUB/KZT 0.079 0.054 0.073 0.073 0.054 0.063 0.073 0.058 0.062 0.074 CNY/KZT 0.050 0.035 0.048 0.045 0.049 0.054 0.056 0.039 0.054 0.055 KGS/KZT 0.074 0.033 0.053 0.114 0.039 0.052 0.218 0.041 0.060 0.236 GBP/KZT 0.0636 0.0586 0.0685 0.052 0.0394 0.0461 0.0446 0.0453 0.0475 0.0438

USD/KZT 0.0510 0.036 0.045 0.048 0.0435 0.0577 0.0539 0.0539 0.0577 0.0634 EUR/KZT 0.067 0.061 0.065 0.0596 0.0587 0.0676 0.0349 0.0617 0.0661 0.0379 Period 2 (q= 0.99)

RUB/KZT 0.024 0.018 0.015 0.023 0.015 0.0136 0.0245 00172 0.0136 0.0244 CNY/KZT 0.018 0.015 0.014 0.010 0.020 0.0146 0.0110 0.0119 0.0101 0.0083 KGS/KZT 0.021 0.019 0.023 0.037 0.012 0.0117 0.0634 0.0120 0.0144 0.0961 GBP/KZT 0.015 0.007 0.007 0.006 0.009 0.009 0.009 0.014 0.008 0.008

USD/KZT 0.017 0.016 0.013 0.051 0.019 0.013 0.016 0.021 0.013 0.019 EUR/KZT 0.013 0.014 0.012 0.007 0.015 0.008 0.006 0.018 0.011 0.006 Period 3 (q= 0.95)

RUB/KZT 0.059 0.044 0.055 0.045 0.054 0.060 0.035 0.062 0.064 0.036 CNY/KZT 0.054 0.042 0.052 0.056 0.057 0.063 0.065 0.064 0.063 0.073 KGS/KZT 0.048 0.029 0.048 0.043 0.046 0.051 0.044 0.049 0.057 0.046 GBP/KZT 0.0536 0.039 0.0453 0.052 0.0501 0.0483 0.0589 0.0536 0.0571 0.0677

USD/KZT 0.050 0.035 0.0549 0.044 0.056 0.0608 0.0513 0.0732 0.0673 0.0572 EUR/KZT 0.052 0.040 0.053 0.054 0.0548 0.0648 0.066 0.0673 0.0684 0.0778 Period 3 (q= 0.99)

RUB/KZT 0.0143 0.0169 0.0152 0.0116 0.0264 0.0214 0.0099 0.0252 0.0215 0.0111 CNY/KZT 0.0136 0.0136 0.006 0.0073 0.0146 0.0094 0.0105 0.0230 0.0136 0.0136 KGS/KZT 0.0116 0.0139 0.0148 0.0225 0.0171 0.0148 0.0156 0.0196 0.0172 0.0181 GBP/KZT 0.012 0.013 0.011 0.013 0.017 0.012 0.015 0.019 0.010 0.014

USD/KZT 0.016 0.013 0.013 0.011 0.022 0.015 0.007 0.032 0.019 0.008 EUR/KZT 0.013 0.012 0.010 0.012 0.011 0.010 0.013 0.019 0.009 0.016Table 12: The proportion of Value-At-Risk exceedances of each model for Sample 2

For the selected quantiles, q=0.95 and q = 0.99, adhering to the definition of Value at Risk (VaR), the expected proportions of VaR violations are 5% and 1% respectively.

Table 9 presents mixed results, making it challenging to identify a single model or distributional assumption consistently providing accurate outcomes across all currency pairs.

For Period 1, a key observation is that the historical simulation method did not consistently emerge as the optimal model. Instead, a dynamic pattern emerged wherein alternative models exhibited superior performance depending on the specific currency pair and confidence level under consideration.

Similar to Period 1, in Period 2, the historical method was not the optimal model. Instead, models like Parametric, ARMA-GARCH, and ARMA-GJR-GARCH demonstrated superior performance for specific currency pairs and confidence levels.

In Period 3, a notable finding emerged when considering the 99% confidence level of Value at Risk for the KGS/KZT pair. Despite a violation proportion exceeding 1%, Historical Simulation yielded the best results, contrary to the expectation of it being below 1%. Additionally, an intriguing observation surfaced regarding currencies from developed markets: at the 95% confidence level, a parametric

approach assuming normal distribution outperformed, while at the 99% confidence level, ARMA-GJR-GARCH and ARMA-GARCH models assuming Student's t and skewed Student's t distributions displayed superior performance.

4.2.2 Expected Shortfall forecasts and backtesting

To start examination of the accuracy of Expected Shortfall (ES) forecasts, we begin with visual inspection of forecasts and actual losses. The tables below present ES forecasts from 10 models. The first row includes results from historical simulation and Variance-Covariance without GARCH, under three distributional assumptions. The second row features forecasts from standard S-GARCH and GJR-GARCH modeling, considering three distributional assumptions alongside actual losses.

RUB/KZT	Period 1	Period 2	Period 3
Hist and VC			

Garch type Models			

ES forecasts from different models and actual losses for RUB/KZT (Sample 2)

The figure suggests that the variation of GARCH models tends to yield more accurate results compared to forecasts generated from historical simulation and variance-covariance without GARCH components. Notably, in most instances, the Student's t distribution and the Skewed Student's t distribution demonstrate superiority over the normal distribution in terms of forecast accuracy.

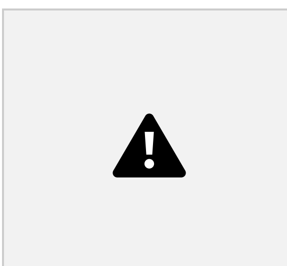
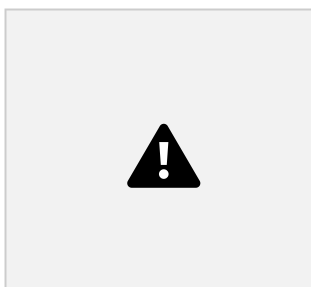
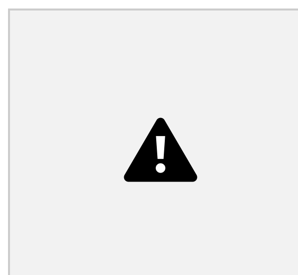
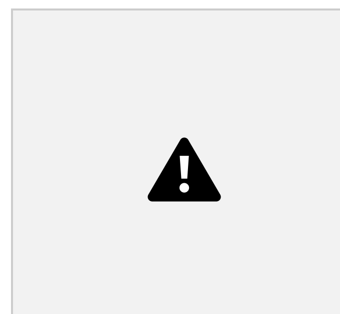
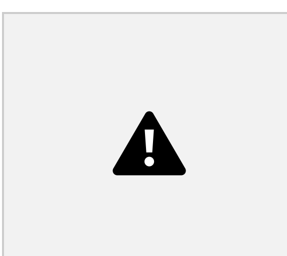
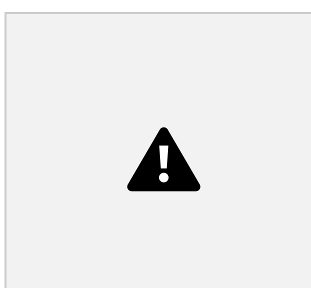
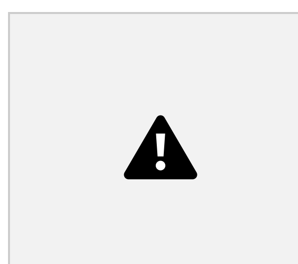
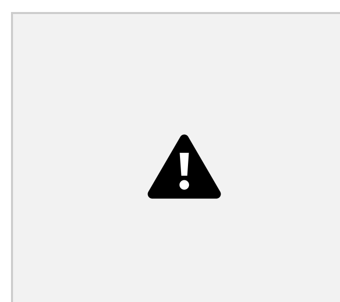
The accuracy of Expected Shortfall (ES) predictions was evaluated in instances of VaR violations. The tables presented below showcase summary statistics and histograms, revealing the differences between Expected Shortfall (ES) and Actual Losses under different assumptions regarding return distributions, including normal, Student's t, and skewed Student's t.

The following analysis is segmented into two distinct parts. The first part focuses on emerging market currencies traded with KZT, specifically RUB/KZT, CNY/KZT, and KGS/KZT. In the second part, attention shifts to additional emerging market currencies exchanged with KZT, namely GBP/KZT, USD/KZT, and EUR/KZT.

Emerging

1) Historical Simulation and Variance-Covariance approach without GARCH

The outcomes of backtesting for ES in the parametric approach, relying exclusively on distributional assumptions, are displayed in the tables and histograms below. The q = 0.95 and q = 0.99 quantiles are considered:

	Historical	Normal dist.	Student's t distribution	Skewed Student's t dist.
Period 1				
Period 2				

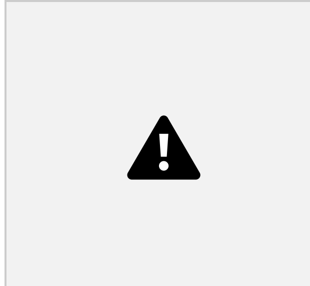
Period 3				

Figure 11. Discrepancies in Expected Shortfall (ES) estimated via Variance-Covariance Approach versus actual losses for emerging market fx currencies relative to KZT (q = 0.95).

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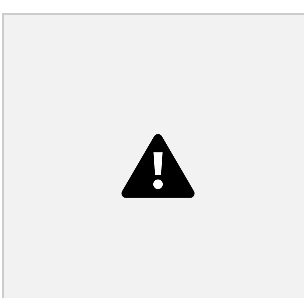
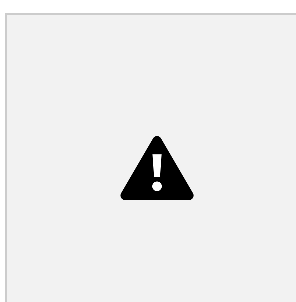
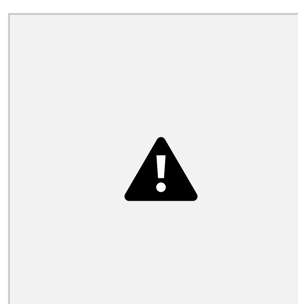
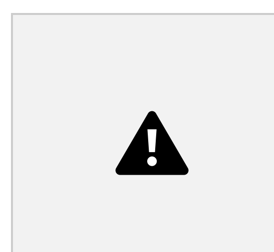
Approaches Length Mean sd min max skewness kurtosis JB_Statistics Period 1

Hist 145 0.002 0.008 -0.012 0.052 3.288 14.462 1573.163 Norm 112 0.003 0.008 -0.002 0.053 3.348 12.905 1027.152 Std 132 0.002 0.008 -0.009 0.052 3.464 14.772 1515.040 Sstd 135 0.002 0.007 -0.018 0.047 3.383 14.626 1510.594

Hist 175 0.001 0.008 -0.012 0.041 2.694 10.398 1027.161 Norm 117 0.002 0.008 -0.006 0.039 3.002 10.036 693.612 Std 158 0.000 0.008 -0.024 0.036 1.477 6.699 365.053 Sstd 180 0.001 0.008 -0.020 0.042 1.987 7.752 585.256 Period 3

Hist 235 0.002 0.018 -0.032 0.118 4.084 23.309 6084.874 Norm 169 0.006 0.018 -0.007 0.119 4.462 22.442 4213.929 Std 229 0.003 0.018 -0.033 0.121 4.257 23.366 6014.026 Sstd 214 0.003 0.018 -0.036 0.122 3.784 20.285 4266.786

Table 13: Summary of discrepancies in Expected Shortfall (ES) estimated via var-covar Approach versus actual losses for emerging market fx currencies relative to KZT.

	Historical	Normal	Std	Stdd
Period 1				

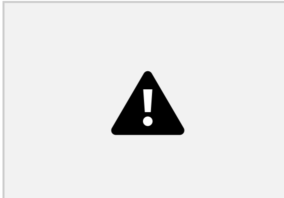
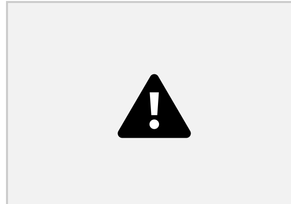
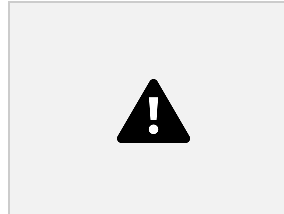
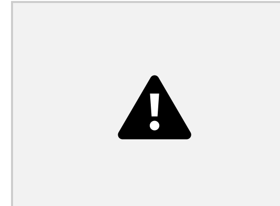
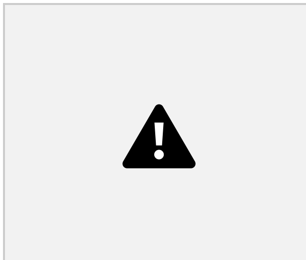
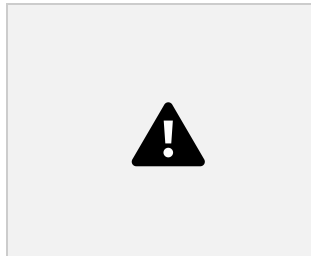
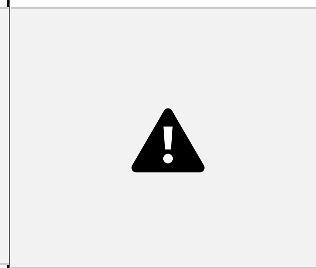
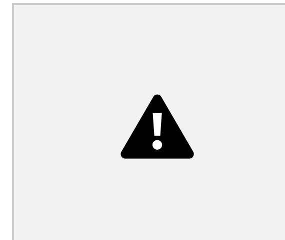
Period 2				
Period 3				

Figure 12. Discrepancies in Expected Shortfall (ES) estimated via Variance-Covariance Approach versus actual losses for emerging market fx currencies relative to KZT (q = 0.95).

Approaches Length Mean sd min max skewness kurtosis JB_Statistics Period 1
Hist 51 0.003 0.008 -0.004 0.048 3.382 14.286 580.297 Norm 49 0.005 0.01 -0.002 0.049 2.37 6.211 137.717 Std 44 0.003 0.009 -0.01 0.047 2.829 10.158 275.81 Sstd 43 0.002 0.012 -0.047 0.039 -0.608 7.255 110.449 Period 2

Hist 56 0.003 0.008 -0.008 0.036 2.244 5.453 127.107 Norm 44 0.004 0.009 -0.003 0.030 1.918 2.552 43.408 Std 42 -0.001 0.011 -0.038 0.025 -0.582 2.773 18.917 Sstd 53 -0.001 0.010 -0.031 0.031 0.068 1.984 10.514 Period 3

Hist 44 0.008 0.019 -0.009 0.102 3.403 12.574 415.611 Norm 50 0.005 0.023 -0.046 0.111 2.058 8.763 215.673 Std 42 0.007 0.023 -0.034 0.106 2.108 7.559 147.691 Sstd 49 0.001 0.023 -0.026 0.112 2.629 9.986 286.553

Table 14: Summary of discrepancies in Expected Shortfall (ES) estimated via var-covar Approach versus actual losses for emerging market fx currencies relative to KZT.

Upon analyzing the summary statistics and histogram depicting the variance between Expected Shortfall and Actual Shortfall for both q = 0.95 and q= 0.99 quantiles, it becomes apparent that historical simulations consistently manifest outlier behavior. This is evident in the higher skewness and kurtosis parameters compared to alternative methods, along with the JB_statistic surpassing corresponding values of other approaches.

Analyzing the mean, skewness, kurtosis, and Jarque-Bera (JB) statistics across different distributional assumptions leads to the following observations:

Examining the quintile q = 0.95, the Variance Covariance approach under the assumption of a Skewed Student's t distribution exhibits superior performance for

both Period 1 and Period 3. In contrast,In Period 2, the Student's t distribution emerges as the superior choice. Shifting focus to the quintile q = 0.99, the Skewed Student's t distribution consistently demonstrates superior results across all periods.

S_GARCH (1;1)

The outcomes of backtesting for ES in the parametric approach using s-GARCH are displayed in the tables and histograms below. The cases for quantiles q = 0.95 and q = 0.99 are considered:

	Normal distribution	Student's t distribution	Skewed Student's t dist.
Period 1			
Period 2			

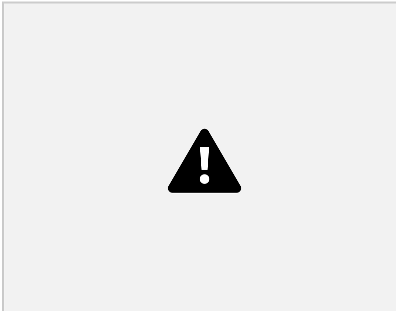
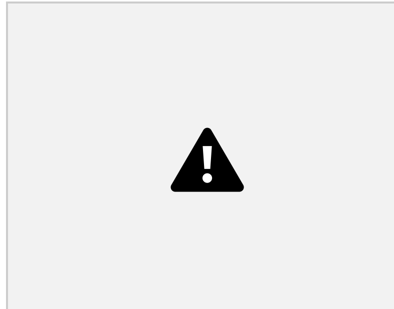
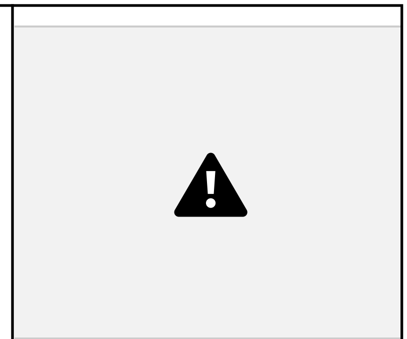
Period 3			

Figure 13. Summary of discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for emerging market fx currencies relative to KZT (q = 0.95).

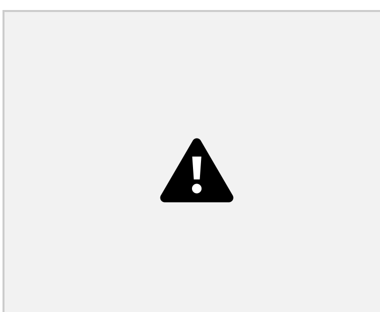
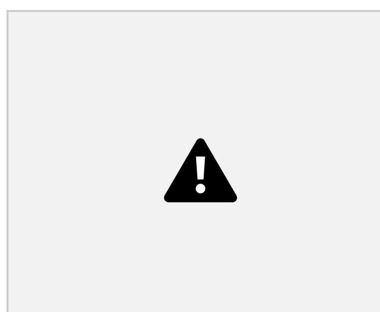
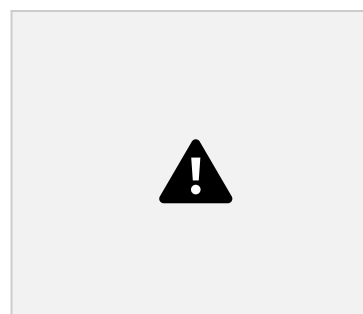
Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Norm 111 0.002 0.006 -0.006 0.048 5.035 31.465 5245.700 Std 110 0.002 0.006 -0.007 0.048 4.600 26.891 3850.379 Sstd 138 0.001 0.004 -0.002 0.041 5.900 46.697 13750.957 Period 2

Norm 129 0.001 0.005 -0.009 0.029 3.119 13.105 1173.162 Std 150 0.000 0.006 -0.014 0.029 2.776 12.368 1184.832 Sstd 235 0.001 0.006 -0.016 0.038 3.034 14.972 2605.389 Period 3

Norm 228 0.003 0.012 -0.014 0.107 6.321 48.643 24440.490 Std 255 0.001 0.011 -0.018 0.106 5.811 44.513 22860.651 Sstd 215 0.001 0.011 -0.017 0.103 7.040 60.904 35686.265

Table 15: Summary of discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for emerging market fx currencies relative to KZT. (q = 0.95)

	Normal dist.	Student’s t distribution	Skewed Student’s t dist.
Period 1			

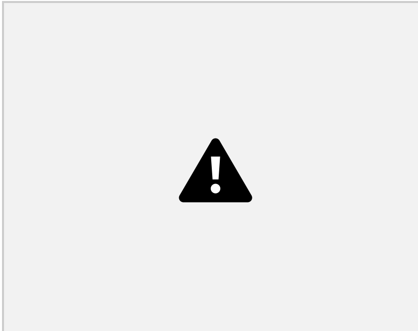
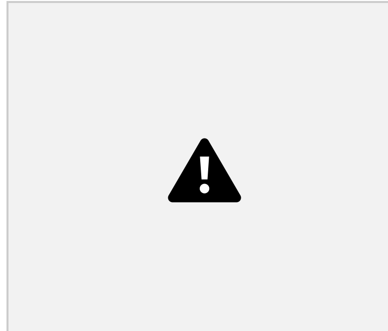
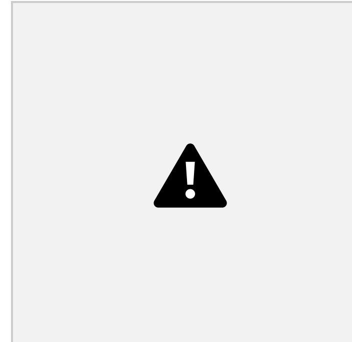
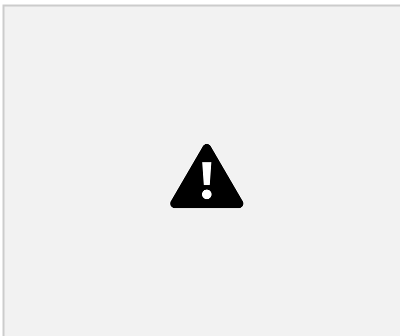
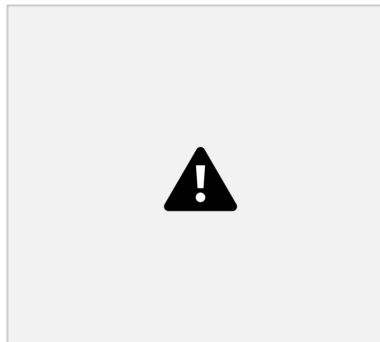
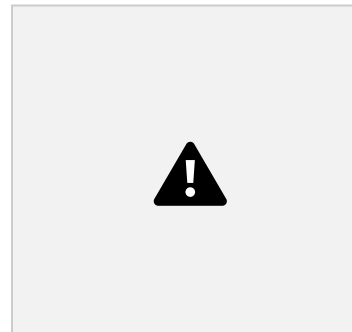
Period 2			
Period 3			

Figure 14. Discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for emerging market fx currencies relative to KZT. (q = 0.99)

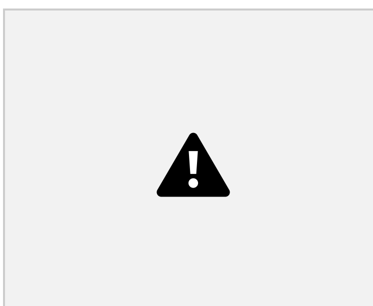
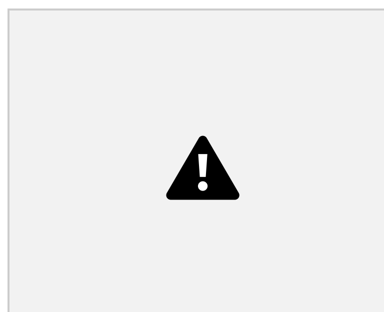
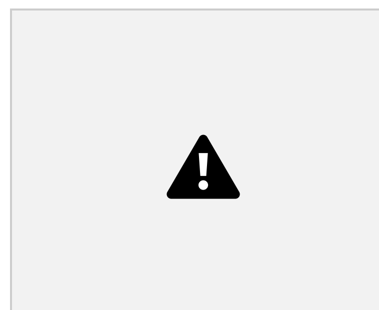
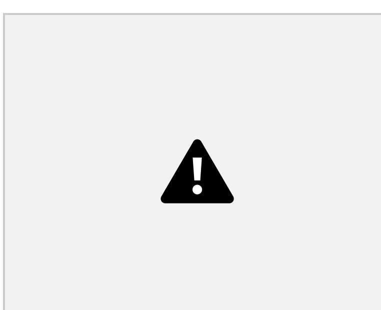
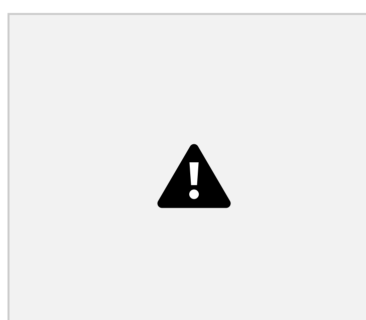
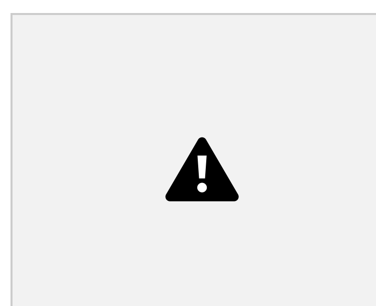
Approaches	Length	mean	sd	min	max	skewness	kurtosis	JB_Statistics
Period 1								
Norm	26	0.011	0.012	-0.001	0.040	0.968	-0.434	4.623
Std	35	0.003	0.008	-0.003	0.042	3.248	11.841	303.354
Sstd	43	0.001	0.006	-0.012	0.034	3.278	16.359	618.356
Period 2								
Norm	44	0.002	0.005	-0.003	0.026	2.588	7.326	164.520
Std	36	0.001	0.006	-0.004	0.025	2.516	6.639	119.059
Sstd	66	0.001	0.006	-0.015	0.026	1.802	4.667	103.547
Period 3								
Norm	73	0.006	0.018	-0.016	0.102	4.097	18.049	1269.452
Std	58	0.005	0.016	-0.026	0.091	3.335	14.447	661.726
Sstd	44	0.005	0.019	-0.022	0.085	3.043	10.419	296.616

Table 16: Summary of discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for emerging market fx currencies relative to KZT. (q = 0.99)

Examining the quintile q = 0.95, the standard ARMA-GARCH approach with innovations assumed under the assumption of a student's t distribution exhibits superior performance for all three periods. Shifting focus to the quintile q = 0.99, models with innovations assumed under a Student's t distribution and a skewed Student's t distribution demonstrate similarly superior results for all data periods.

2) GJR-GARCH(1;1)

The outcomes of backtesting for ES in the parametric approach using GJR-GARCH are displayed in the tables and histograms below. The cases for quantiles q = 0.95 and q = 0.99 are considered:

	Normal dist.	Student's t distribution	Skewed Student's t dist.
Period 1			
Period 2			

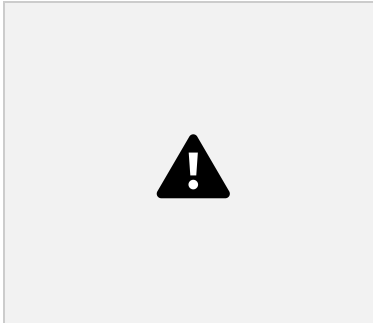
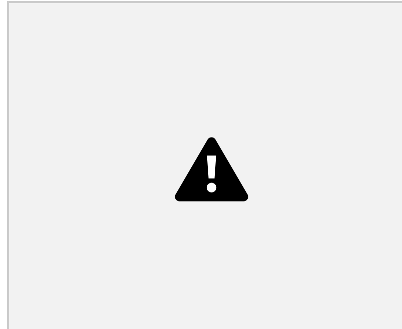
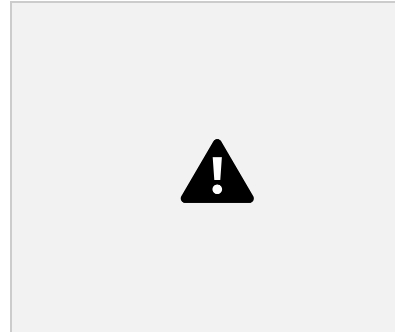
Period 3			

Figure 15 . Discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for emerging market fx currencies relative to KZT. (q = 0.95)

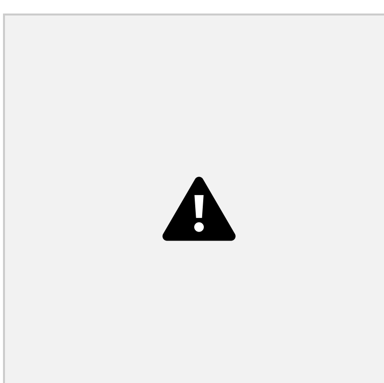
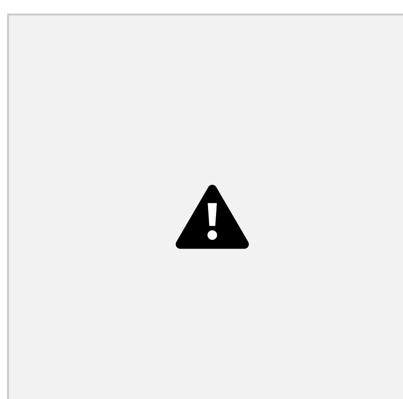
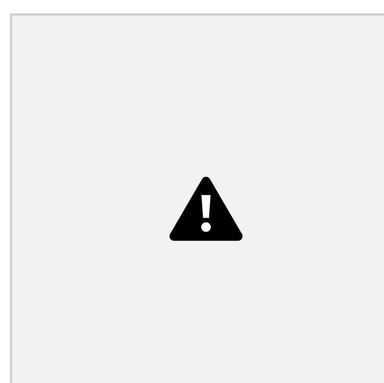
Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Norm 116 0.001 0.006 -0.006 0.048 5.375 34.580 6574.312 Std 114 0.001 0.006 -0.011 0.048 4.727 28.647 4488.587 Sstd 124 0.001 0.005 -0.004 0.041 5.679 42.867 10511.833 Period 2

Norm 123 0.001 0.005 -0.009 0.031 3.315 14.303 1321.448 Std 152 -0.000 0.005 -0.015 0.031 2.616 13.916 1443.267 Sstd 240 0.001 0.005 -0.014 0.032 2.434 9.779 1217.188 Period 3

Norm 251 0.003 0.011 -0.007 0.107 6.416 51.262 29693.751 Std 263 0.002 0.011 -0.017 0.107 6.041 49.285 28669.947 Sstd 228 0.001 0.011 -0.041 0.098 5.727 49.730 25198.593

Table 15: Summary of discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for emerging market fx currencies relative to KZT (q = 0.95).

	Normal dist.	Student’s t distribution	Skewed Student’s t dist.
Period 1			

Period 2			
Period 3			

Figure 16 . Discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for emerging market fx currencies relative to KZT. (q = 0.99)

Approaches	Length	mean	sd	min	max	skewness	kurtosis	JB_Statistics
Period 1								
Norm	33	0.004	0.009	-0.002	0.044	3.045	9.329	196.499
Std	32	0.003	0.009	-0.002	0.043	3.324	11.881	285.214
Sstd	39	0.001	0.006	-0.014	0.033	2.760	13.443	386.722
Period 2								
Norm	37	0.002	0.006	-0.002	0.027	2.702	7.719	155.679
Std	32	-0.000	0.007	-0.011	0.027	2.138	6.893	102.667
Sstd	76	0.000	0.005	-0.008	0.020	1.775	3.772	91.025
Period 3								
Norm	81	0.007	0.016	-0.005	0.095	4.090	18.566	1466.718
Std	64	0.005	0.015	-0.038	0.092	3.030	15.879	827.177
Sstd	51	0.005	0.016	-0.022	0.081	3.256	12.305	450.527

Table 19: Summary of discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for emerging market fx currencies relative to KZT (q = 0.99).

For q = 0.95, ARMA-GJR-GARCH approach incorporating innovations assumed to follow a student's t distribution has shown most accurate results in Period 1. In Period 2 and Period 3, innovations assumed to follow skewed student's t distribution showed superior results. Analysis of the quintile q = 0.99 gave similar results.

Overall, analyzing ES estimates for the returns of emerging market currencies traded with KZT in Sample 2, several observations can be highlighted.

The information from tables above suggest that there is a tendency for the averages of the distributions to hover around zero. Specifically, when considering a quantile (q) value of 0.95, the mean exhibits a positive trend across most of the approaches and data periods, with consistent positive skewness in every instance. These patterns imply that Expected Shortfall (ES) forecasts consistently lean towards

underestimating risk. Upon examining the scenario with a higher quantile, q = 0.99, it becomes apparent that the mean deviates further from zero, especially for data Period 1 and data Period 3. This suggests that the distributions are less centered around zero compared to the q = 0.95 case.

All three distributional assumptions produced more accurate results in Period 2 compared to other periods both for q = 0.95 and q = 0.99 cases. Notably, the tables indicate that, for q = 0.95, both the students' t-distribution and the skewed student's t-distributional assumption, and for q = 0.99, the skewed student's t-distributional assumption, yield relatively superior results. This superiority can be attributed to the presence of relatively lower skewness and kurtosis parameters associated with these particular assumptions. Comparing models forecasting performances across data periods, it is evident that all models exhibit superior forecasting accuracy for Period 2, which corresponds to the period following the 2008 financial crisis until 2015, marked by a relatively lower level of volatility due to a fixed regime change.

In terms of inter-model comparisons, for a quantile (q) value of 0.95, the GJR GARCH model outperformed others in forecasting accuracy for Period 1, while the S-GARCH model demonstrated superior accuracy for Period 3. Remarkably, for Period 2, all models yielded similarly accurate results. However, when considering

a higher quantile, q = 0.99, the performance of the Variance-Covariance approach without GARCH was commendable across all data partitions.

Developed

1) Historical Simulation and Variance-Covariance approach without GARCH

The outcomes of backtesting for ES from Variance-Covariance approach without GARCH are displayed in the tables and histograms below. The q = 0.95 and q =

0.99 quantiles are considered:

	Historical	Normal dist.	Student’s t distribution	Skewed Student’s t dist.
Period 1				
Period 2				
Period 3				

Figure 17. Discrepancies in Expected Shortfall (ES) estimated via Variance-Covariance Approach versus actual losses for developed market fx currencies relative to KZT (q = 0.95).

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Hist 216 0.001 0.004 -0.009 0.032 2.770 15.740 1737.390 Norm 186 0.002 0.005 -0.005 0.034 2.950 15.780 1538.823 Std 213 0.000 0.005 -0.012 0.032 2.017 12.920 1018.040 Sstd 190 0.000 0.004 -0.013 0.029 1.830 12.620 839.560

Period 2

Hist 195 0.0005 0.002 -0.004 0.018 2.120 10.120 559.420 Norm 146 0.0007 0.0029 -0.003 0.012 1.757 6.157 135.810 Std 204 -0.0003 0.003 -0.007 0.015 1.540 7.850 281.530 Sstd 292 -0.0002

0.002 -0.007 0.013 1.280 9.740 634.130 Period 3

Hist 265 0.001 0.0056 -0.009 0.021 1.400 4.750 121.560 Norm 195 0.002 0.005 -0.004 0.021 1.587 5.042 115.770 Std 260 -0.001 0.006 -0.015 0.018 0.700 4.160 36.280 Sstd 255 -0.000 0.005 -0.013 0.020 0.880 4.180 47.910

Table 20: Summary of discrepancies in Expected Shortfall (ES) estimated via var covar Approach versus actual losses for developed market fx currencies relative to KZT (q = 0.95).

	Historical	Normal	Std	SSTD
Period 1				
Period 2				
Period 3				

Figure 18. Discrepancies in Expected Shortfall (ES) estimated via var covar Approach versus actual losses for developed market fx currencies relative to KZT (q=0.99)

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1
Hist 62 0.002 0.005 -0.008 0.023 1.771 8.045 98.169 Norm 77 0.003 0.005 -0.002 0.030 3.017 13.796 490.771 Std 54 -0.001 0.005 -0.014 0.021 0.648 5.923 23.010 Sstd 40 0.000 0.005 -0.016 0.015 -0.123 5.703 12.285 Period 2
Hist 57 0.001 0.003 -0.003 0.015 1.734 7.520 77.123 Norm 46 0.001 0.002 -0.002 0.009 1.184 3.829 12.061 Std 40 -0.000 0.003 -0.008 0.005 -0.537 3.480 2.310 Sstd 73 -0.000 0.002 -0.011 0.005 -1.677 10.902 224.177 Period 3

Hist 72 0.003 0.005 -0.007 0.015 0.470 3.057 2.668 Norm 65 0.004 0.004 -0.003 0.015 0.697 2.713 5.494 Std 58 -0.003 0.007 -0.021 0.012 -0.677 3.340 4.722 Sstd 64 -0.003 0.008 -0.030 0.015 -1.035 4.246 15.574

Table 21: Summary of discrepancies in Expected Shortfall (ES) estimated via var covar Approach versus actual losses for developed market fx currencies relative to KZT. (q=0.99)

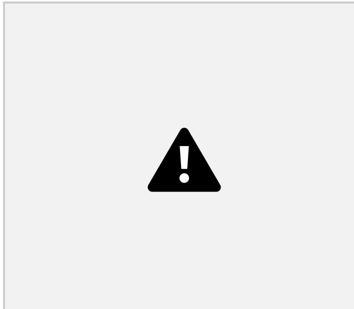
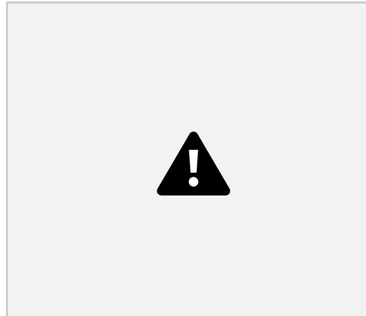
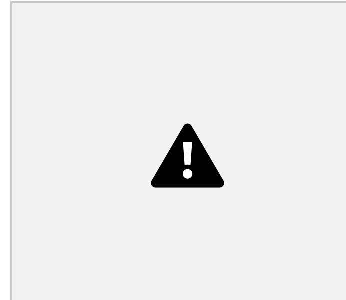
Upon analyzing the summary statistics and histogram depicting the variance between Expected Shortfall and Actual Shortfall for q = 0.95 quantiles, it becomes

apparent that historical simulations always showed the worst or second worst results. For the Period 3, student’s t distribution, and for the Period, skewed Student’s t distribution showed superior results.

Surprisingly for q = 0.99, in Period 3 the performance of the HS was superior. Analyzing 99% Expected Shortfall, we found that forecasts are most accurate for Period 3. The skewness, kurtosis, and Jarque-Bera stats consistently decreased across periods for each model. For instance, Parametric (student’s t assumption) approach showed decreasing kurtosis: 5.923 (Period 1), 3.480 (Period 2) and then 3.340 (Period 3). Despite this pattern, the optimal model varied for each period: Skewed Student’s t distribution in Period 1, Student’s t distribution in Period 2, and a choice between normal and student’s t distribution in Period 3.

2) S_GARCH (1;1)

The outcomes of backtesting for ES estimates from the parametric approach using s-GARCH are displayed in the tables and histograms below. The q = 0.95 and q = 0.99 quantiles are considered:

	Normal	Std	SSTD
Period 1			

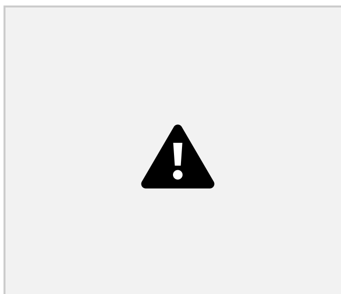
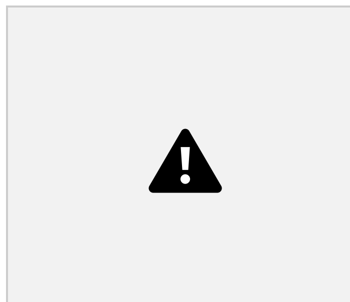
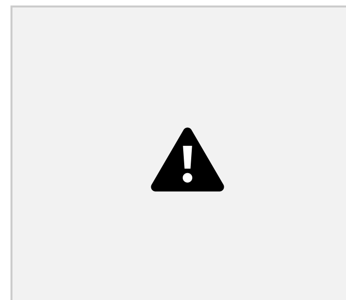
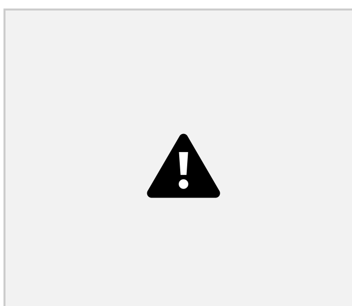
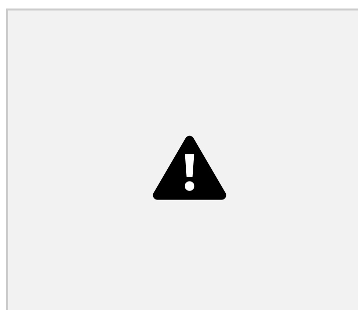
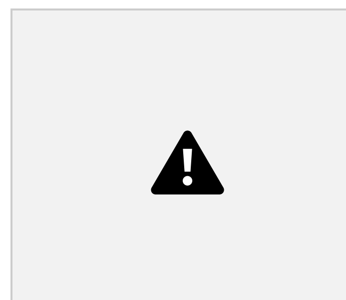
Period 2			
Period 3			

Figure 19. Discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for developed market fx currencies relative to KZT (q = 0.95).

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1
Norm 195 0.001 0.004 -0.007 0.018 2.224 9.7313 528.99 Std 210 0.000 0.004 -0.011 0.018 -1.510
9.235 436.63 Sstd 202 0.000 0.004 -0.014 0.019 1.470 10.660 567.96 Period 2
Norm 178 0.001 0.003 -0.003 0.015 2.190 8.865 397.590 Std 214 -0.000 0.003 -0.006 0.016 1.711
8.841 408.710 Sstd 164 0.000 0.003 -0.006 0.013 1.540 7.960 233.060 Period 3
Norm 273 0.0008 0.0035 -0.005 0.019 2.407 11.608 1106.566 Std 295 -0.0002 0.0037 -0.011 0.020
1.720 10.096 766.070 Sstd 299 -0.0001 0.0039 -0.011 0.021 1.900 9.986 788.540

Table 22: Summary of discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for developed market fx currencies relative to KZT (q = 0.95).

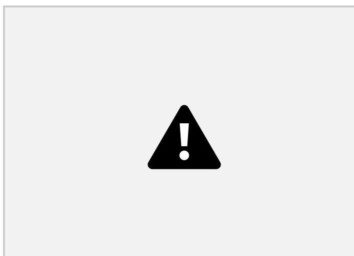
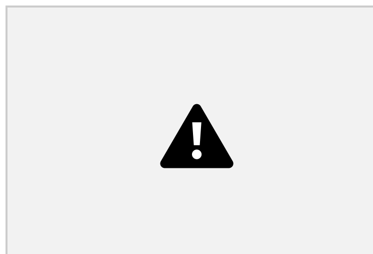
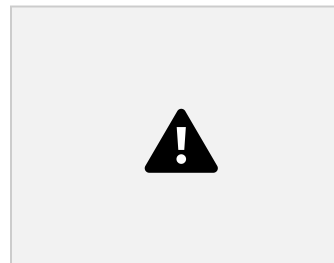
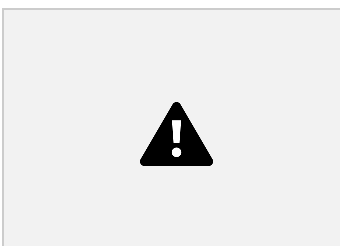
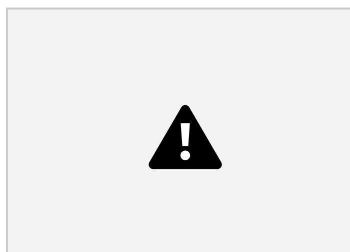
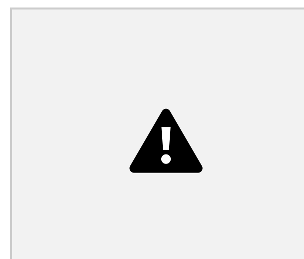
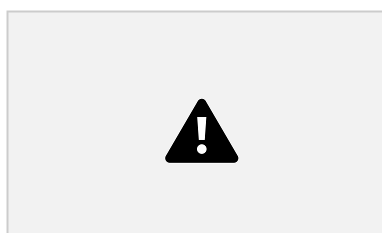
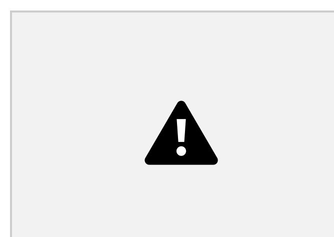
	Normal	Std	SSTD
Period 1			
Period 2			
Period 3			

Figure 20. Discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for developed market fx currencies relative to KZT. (q=0.99)

Approaches Length mean sd min max skewness kurtosis JB_Statistics Period 1

Norm 49 0.002 0.003 -0.003 0.015 1.467 5.428 29.625 Std 43 0.001 0.003 -0.007 0.014 1.446 6.853 41.609 Sstd 46 0.001 0.004 -0.004 0.015 1.867 7.028 57.839 Period 2

Norm 53 0.002 0.003 -0.002 0.012 1.318 4.788 22.418 Std 38 0.000 0.003 -0.009 0.009 -0.362 4.624 5.007 Sstd 39 -0.000 0.003 -0.008 0.008 -0.194 3.541 0.722 Period 3

Norm 87 0.002 0.003 -0.002 0.017 2.264 8.952 202.774 Std 63 -0.001 0.004 -0.014 0.011 0.091

5.226 13.099 Sstd 61 -0.000 0.004 -0.013 0.014 0.768 4.992 16.091

Table 23: Summary of discrepancies in Expected Shortfall (ES) estimated via sGarch Approach versus actual losses for developed market fx currencies relative to KZT. (q=0.99)

For both confidence levels, in Period 2, Skewed Student's t distribution consistently outperformed. In Period 3, Student's and Skewed Student's distributions exhibited a trade-off, with one showing superior kurtosis and the other displaying better skewness. Period 1 witnessed similar performance across all three assumptions.

3) GJR-GARCH(1;1)

The outcomes of backtesting for ES estimates from the parametric approach using GJR-GARCH are displayed in the tables and histograms below. The q = 0.95 and q = 0.99 quantiles are considered:

	Normal	Std	Sstd
Period 1			
Period 2			
Period 3			

Figure 21 . Discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for developed market fx currencies relative to KZT. (q=0.95)

Approaches	Length	mean	sd	min	max	skewness	kurtosis	JB_Statistics
Period 1								
Norm	205	0.001	0.003	-0.004	0.019	2.441	10.255	653.35
Std	213	0.000	0.003	-0.008	0.017	1.926	9.232	476.576
Sstd	210	0.000	0.003	-0.012	0.020	2.302	12.91	1045.300
Period 2								
Norm	201	0.001	0.003	-0.0034	0.0161	2.250	9.550	530.703
Std	214	0.000	0.003	-0.007	0.0155	1.870	9.220	471.320
Sstd	176	-0.000	0.0029	-0.0071	0.012	1.185	6.920	153.950
Period 3								
Norm	329	0.0011	0.0038	-0.0068	0.022	2.396	10.710	1131.72
Std	327	-0.0002	0.0037	-0.009	0.022	2.240	11.670	1299.200
Sstd	344	-0.0001	0.004	-0.0098	0.023	1.900	9.990	908.170

Table 24: Discrepancies in Expected Shortfall (ES) estimated via gjrGarch Approach versus actual losses for developed market fx currencies relative to KZT. (q=0.95)