# Debiasing Cosmic Gravitational Wave Sirens

Ryan E. Keeley,[1★] Arman Shafieloo[1,2] Benjamin L'Huillier,[1] Eric V. Linder[1,3,4]
[1]*Korea Astronomy and Space Science Institute, Daejeon 34055, Korea*
[2]*University of Science and Technology, Daejeon 34113, Korea*
[3]*Berkeley Center for Cosmological Physics & Berkeley Lab, University of California, Berkeley, CA 94720 USA*
[4]*Energetic Cosmos Laboratory, Nazarbayev University, Nur-Sultan, Kazakhstan 010000*

arXiv:1905.10216v1 [astro-ph.CO] 24 May 2019

Accepted XXX. Received YYY; in original form ZZZ

**ABSTRACT**

Accurate estimation of the Hubble constant, and other cosmological parameters, from distances measured by cosmic gravitational wave sirens requires sufficient allowance for the dark energy evolution. We demonstrate how model independent statistical methods, specifically Gaussian process regression, can remove bias in the reconstruction of $H(z)$, and can be combined model independently with supernova distances. This allows stringent tests of both $H_0$ and ΛCDM, and can detect unrecognized systematics. We also quantify the redshift systematic control necessary for the use of dark sirens, showing that it must approach spectroscopic precision to avoid significant bias.

**Key words:** distance scale – gravitational waves – cosmological parameters – dark energy

## 1 INTRODUCTION

Using general relativity (GR) to model the observed waveform of a gravitational wave (GW), the luminosity distance to a GW source can be measured. This makes GWs from mergers of compact objects into standard sirens and offers a potential way to measure the present value of the expansion rate of the Universe, the Hubble constant $H_0$ (Schutz 1986; Holz & Hughes 2005; Dalal et al. 2006).

In order to do cosmology with these standard sirens, their redshifts must also be measured. The most straightforward way to obtain the redshift is to use GW systems with electromagnetic (EM) counterpart events (e.g. X-ray or optical flashes associated with the merger), where the redshift comes from the EM counterpart. An alternative to obtain the needed redshift information is to cross-correlate GW events with galaxy redshift surveys, as explored in Zhang (2018); Fishbach et al. (2019). Rather than assigning a redshift and luminosity distance to an individual event, this technique would, in a statistical sense, assign an average luminosity distance to a redshift bin. These 'dark sirens' would allow binary black hole mergers to be used as standard sirens. Binary black hole mergers are much louder (compared to binary neutron star mergers) and so are detectable at much higher redshifts and distances, implying many more of them will be seen.

The fact that GR is used to calibrate GW standard sirens makes them particularly useful in mapping the cosmic expansion history. The current standard candles used to map the expansion history are Type Ia supernova (SN). On their own, however, SN only measure ratios of distances and so can only constrain the shape of the Hubble distance-redshift relation, not its absolute scale. Thus they require calibration.

This calibration is currently done with the distance ladder including Cepheid periodic variables and the results of the calibration is summarized as a measurement of $H_0$. This Cepheid measurement of $H_0$ has generated significant interest recently since it is currently discrepant with ΛCDM inferences of Planck measurements of the CMB (Planck Collaboration et al. 2018) at the $4\sigma$ level (Riess et al. 2016, 2019; Joudaki et al. 2018b), potentially pointing to new physics. Since GW do measure an absolute distance scale, they can be used to calibrate the SN distances and estimate $H_0$. Thus GW standard sirens offer a potentially useful cross check on other methods for determining $H_0$ (e.g. Feeney et al. 2019).

However, as we show in Shafieloo et al. (2018), using overly restrictive model dependent techniques to infer $H_0$ from GW datasets runs the risk of yielding substantially biased results. This can arise from assuming the acceleration of the Universe is driven by a cosmological constant (the Λ in ΛCDM) rather than being more general, or appropriately agnostic, about the evolution of the dark energy component. For instance if ΛCDM were assumed, but the dark energy component were truly dynamical (to the extent allowed by current cosmological datasets), then the inferred values of $H_0$ and the matter density $\Omega_m$ could be biased at the $3\sigma$ level.

At low redshifts, $z \lesssim 0.1$, this is less of a problem al-

★ E-mail: rkeeley@kasi.re.kr

© 2019 The Authors

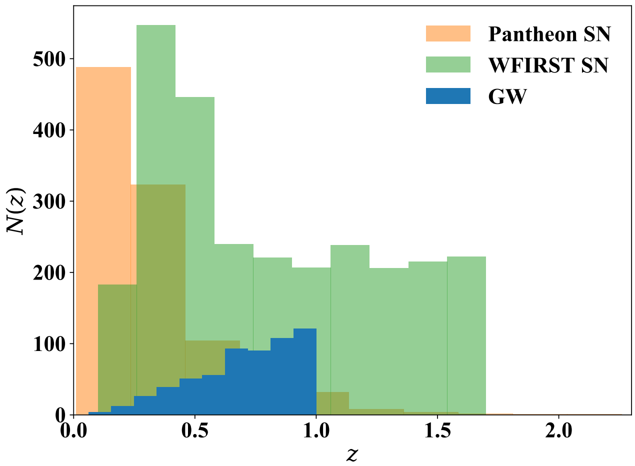


**Figure 1.** The redshift distribution for GW and SN events in our mock datasets. The 'Pantheon-like' dataset is shown in orange, the forecasted WFIRST dataset is in green, and the next next generation GW dataset is in blue.

though a simple linear Hubble law is insufficient. Systematics can also arise due to peculiar velocities, as well as coherent velocity flows (e.g. Mortlock et al. 2018; Cooray & Caldwell 2006; Hui & Greene 2006). At higher redshift, what we call cosmic standard sirens, such systematics are mitigated. Furthermore there is far more volume and a greater number of sources. If the redshifts for these sources could be measured (e.g. by cross-correlation in the absence of EM counterparts) and a robust model independent technique shown to be effective, then cosmology could be tested to much better accuracy and both $H_0$ and the $\Lambda$CDM model could be put to stringent tests.

We quantify the application of model independent statistical techniques to accurately and precisely infer $H_0$, and the expansion history of the Universe, from mock GW and SN datasets. In Sec. 2, we lay out how we construct these mock datasets, aiming for future high precision measurements. We apply Gaussian processes in Sec. 3 as a model independent method and demonstrate its success in reconstructing various expansion histories in an unbiased manner. Section 4 addresses the issue of required control of redshift estimation systematics, quantifying the effects of both additive and multiplicative errors. We conclude in Sec. 5.

## 2 MOCK DATASETS

In order to test how well our model independent methods can recover alternative cosmologies, we generate mock data according to three cosmologies as in Shafieloo et al. (2018). One case is a $\Lambda$CDM cosmology with parameters $H_0 = 69$ km/s/Mpc and $\Omega_\mathrm{m} = 0.3$. The other two are dynamical dark energy cosmologies with $(w_0, w_a)$=$(-0.90, -0.75)$ and $(-1.14, 0.35)$ respectively that are consistent with the current joint cosmological probe analysis at the 68% level in Scolnic et al. (2018). All models have $(H_0,\, \Omega_\mathrm{m}) = (69, 0.3)$.

For each cosmology we generate mock GW datasets, 'Pantheon-like' SN datasets (Scolnic et al. 2018), and 'WFIRST-like' SN datasets. For the GW datasets, we are interested in how well these model independent methods can do compared to model dependent methods in terms of accuracy. To this end, we look at the "Next Next Generation" case from Shafieloo et al. (2018), which has 600 events up to a redshift of $z = 1$. In this optimistic scenario (to test strongly whether bias can be overcome out to the maximum redshift), we assume the measured GW source redshift distribution follows the cosmic volume element,

$$\frac{dN}{dz} = \frac{dN}{dV_c}\frac{dV_c}{dz}\,, \tag{1}$$

and we sample from this distribution.

The redshift distribution for the 'Pantheon-like' SN datasets is the same as the actual Pantheon dataset, which includes 1048 SN in the redshift range $0.01 < z < 2.3$ (Scolnic et al. 2018). The redshift distribution for the WFIRST is taken from the WFIRST-AFTA 2015 Report (Spergel et al. 2015), which forecasts the observation of 2725 SN in the range $0.1 < z < 1.7$. Each of these redshift distributions is shown in Fig. 1.

For the mock GW datasets, the distances are sampled as in Shafieloo et al. (2018) with a standard deviation of 7% in distance. The distance moduli of the 'Pantheon-like' SN dataset are sampled with the covariance matrix of the actual Pantheon dataset. The distance moduli of the 'WFIRST-like' SN dataset are sampled with forecasted errors from the WFIRST-AFTA 2015 Report (Spergel et al. 2015).

## 3 GAUSSIAN PROCESS

To infer the expansion history of the Universe in a model independent manner we use Gaussian process (GP) regression (Holsclaw et al. 2010; Shafieloo et al. 2012). This is a statistical sampling method where instead of sampling a parameter space, the sampling is done over the infinite dimensional space of random realizations of families of curves defined by the GP as informed by the data. In other words, instead of the expansion history being determined by $H_0$, $\Omega_m$, etc. and their uncertainties, it is determined by a family of model independent curves subject to the GP covariance function between data points.

For a more accurate reconstruction of $H(z)$ we control the dynamic range by defining the variable

$$\gamma(z) = \log\left[H_\mathrm{fid}(z)/H(z)\right]\ . \tag{2}$$

We use $1/H(z)$ since this is what distances are linearly proportional to. The fiducial expansion history $H_\mathrm{fid}(z)$ is taken to be the best fit $\Lambda$CDM cosmology for the given input cosmology (e.g. each of our three test models). The log ratio also enables a clear test of $\Lambda$CDM – a deviation from zero points to a deviation from $\Lambda$CDM.

Since the expansion history is expected to be smoothly varying we use the standard squared exponential covariance function

$$\langle\gamma(s_1)\gamma(s_2)\rangle = \sigma_f^2\, e^{-(s_1-s_2)^2/(2\ell^2)}\,, \tag{3}$$

where our redshift variable is $s(z) = \log(1+z)/\log(1+z_\mathrm{max})$, again to control the dynamic range. We take $z_\mathrm{max} = 2.3$, the highest redshift of the Pantheon dataset. The hyperparameters $\sigma_f$ and $\ell$ play important roles for both error control and physical insight, with the first characterizing the amplitude of deviations from the fiducial cosmology and the

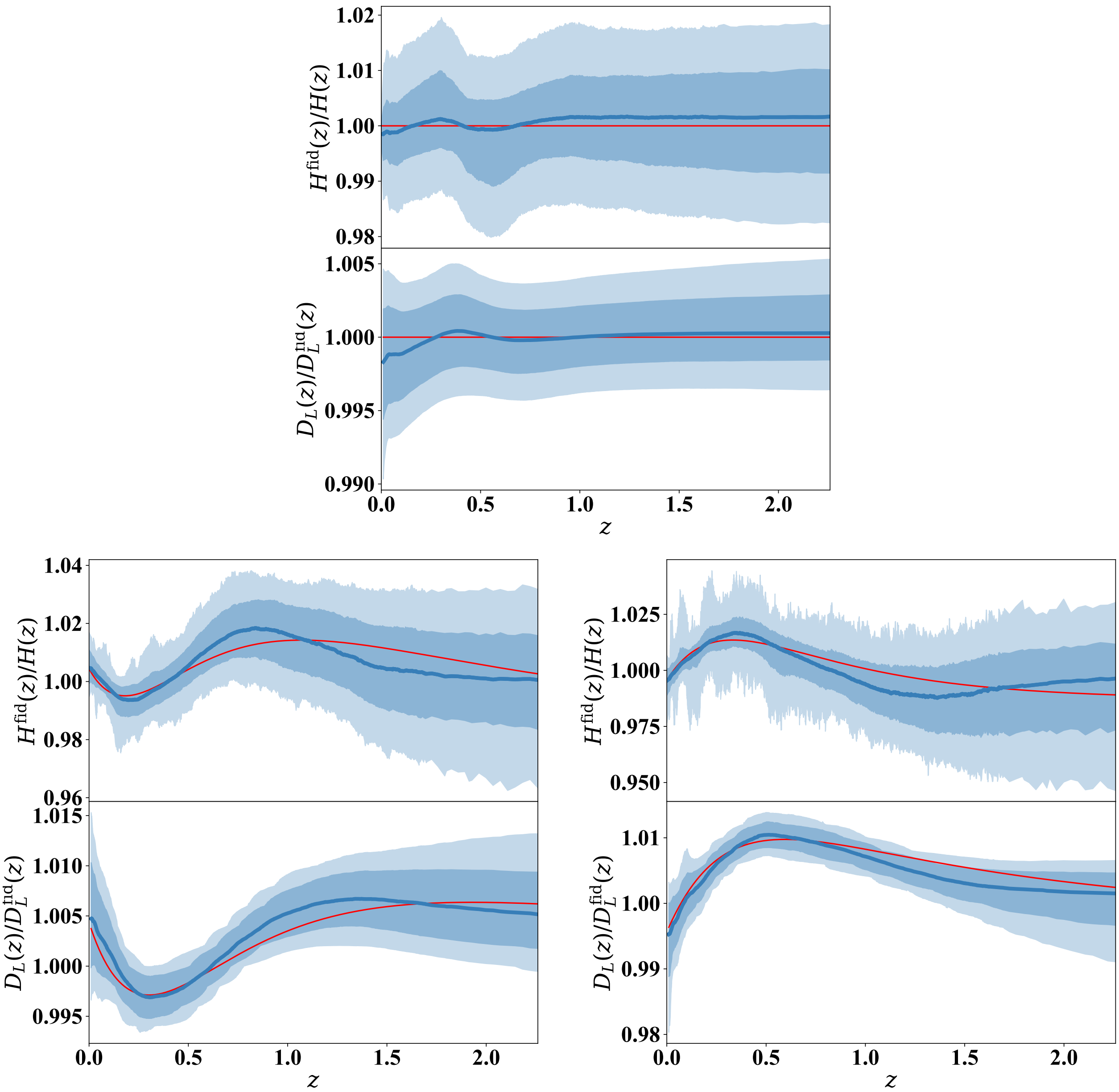


**Figure 2.** GP reconstructions for the different input cosmologies (top: ΛCDM, bottom-left: $(w_0, w_a) = (-0.9, -0.75)$, bottom-right: $(w_0, w_a) = (-1.14, 0.35)$). The top panel in each pair is the reconstructed $1/H(z)$ and the lower panel is the reconstructed $D_L(z)$. In both panels the best fit ΛCDM model is divided out. The thick red curve is the input truth, the thick blue curve is the GP best fit, and the dark and light blue bands are the 68.3% and 95.4% confidence intervals.

second the correlation scale of fluctuations. Since they have physical meaning and impact on the derived cosmology, they cannot be fixed but must be fit for. As such, we impose a scale invariant prior on these hyperparameters. Their posterior probability distribution functions carry important information. If $\sigma_f$ is consistent with zero this means there is no statistically significant evidence for deviations from the fiducial, i.e. ΛCDM cosmology (Shafieloo et al. 2013; Aghamousa et al. 2017). If the correlation length $\ell$ is very small this may mean one is fitting for noise in the data; if it is very large this may mean the data is uninformative about the expansion history.

The GP regression works by randomly generating a family of functions $\gamma(s)$ described by the covariance function, and evaluating the likelihood when comparing to the data. As stated, the fiducial cosmologies ($H_{\rm fid}(z)$) are taken to be the best fit ΛCDM cosmology for each of the three cases. For the resampled ΛCDM input cosmology, this is unsurprisingly the input one, namely $(h, \Omega_m) = (0.69, 0.3)$. For the input $(w_0, w_a) = (-0.9, -0.75)$ cosmology, the best fit ΛCDM cosmology is $(h, \Omega_m) = (0.695, 0.286)$, and for the input $(w_0, w_a) = (-1.14, 0.35)$ cosmology the best fit ΛCDM cosmology is $(h, \Omega_m) = (0.685, 0.295)$.

For each of these expansion histories, we then calculate the corresponding luminosity distances,

$$D_L(z) = (1+z) \int_0^z dz'/H(z') \ . \quad (4)$$

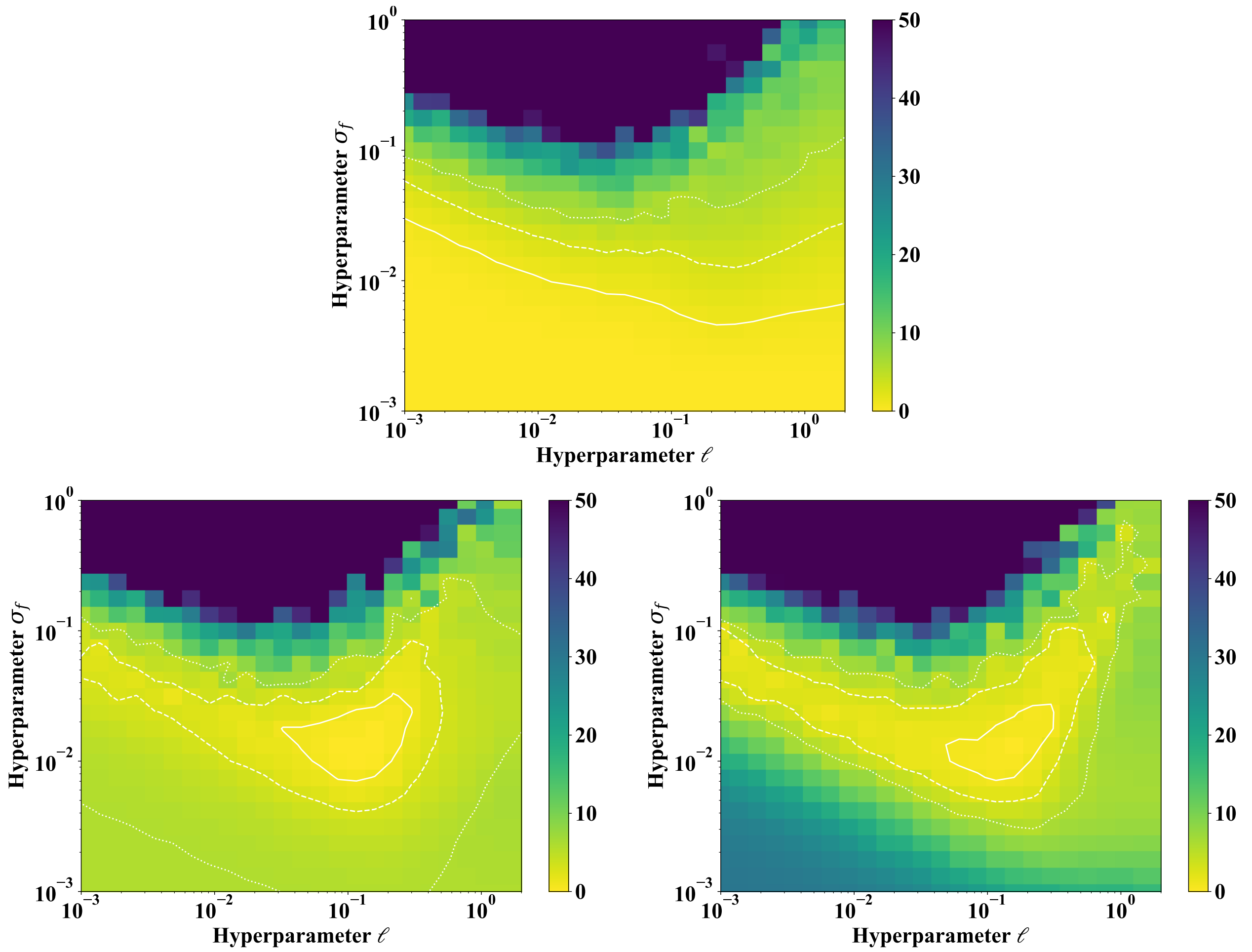


**Figure 3.** Posteriors of the hyperparameters for the different input cosmologies (top: ΛCDM, bottom-left: $(w_0, w_a) = (-0.9, -0.75)$, bottom-right: $(w_0, w_a) = (-1.14, 0.35)$). The white contours are the 68.3%, 95.4%, 99.7% (1, 2, 3 "$\sigma$") confidence levels. The color corresponds to the $-\Delta$log-likelihood.

An advantage of GP is that linear operations (integration or differentiation) on a GP are themselves GP. The regression is done by weighting these expansion histories by how well their distances fit the data, which is equivalent to calculating the posterior. The code used for this calculation was adapted from `GPHist` Kirkby & Keeley (2017), which first appeared in Joudaki et al. (2018a). This modification is located in an open repository[1].

The posteriors, i.e. the reconstructed expansion histories and distance-redshift relations, are shown in Fig. 2. In both ΛCDM and $w_0$–$w_a$ example cosmologies, the median of the GP successfully tracks the input values (despite using ΛCDM as the initial, or fiducial, model). Thus, without needing to make any assumptions about the nature of the true expansion history, it can be recovered accurately using GP regression.

The posteriors of the hyperparameters are shown in Fig. 3. As discussed previously, these posteriors can be used to determine if the reconstruction is meaningfully different than the mean function. Specifically, since for each input cosmology we chose the mean function to be the best-fit flat ΛCDM cosmology for the specific realization of that input cosmology, we can then conclude that if the posterior for the hyperparameters picks out a value for $\sigma_f$ larger than 0, then these forecasted mock datasets contain information disfavoring flat ΛCDM. In such a case this points to needing some additional physics, e.g. dark energy or spatial curvature. For our input ΛCDM cosmology, the posterior for $\sigma_f$ is consistent with 0, indicating the data have no preference for anything beyond the best-fit ΛCDM cosmology.

However the resampled $w_0$–$w_a$ cosmologies do show the need for flexibility beyond the best-fit ΛCDM cosmology. In both cases, the posterior for $\sigma_f$ rules out $\sigma_f = 0$ at the $\gtrsim 3\sigma$ level. In the $(w_0, w_a) = (-0.9, -0.75)$ case, $\sigma_f = 0$ is ruled out at a moderate significance and in the $(w_0, w_a) = (-1.14, 0.35)$, it is ruled out at a more extreme significance. Thus, when the true input cosmology includes a dark energy equation of state beyond $w = -1$, this methodology is able to detect the data's preference for information beyond the best-fit ΛCDM.

Fig. 4 summarizes the results presented in this paper. They agree with the model dependent $w_0$–$w_a$ approach taken in Shafieloo et al. (2018). For each of the different input cosmology cases, the posterior on $H_0$ is shown as calculated from an MCMC sampling assuming ΛCDM (green, as in Shafieloo et al. (2018)), from a GP regression using only

[1] https://github.com/rekeeley/gphist_GW

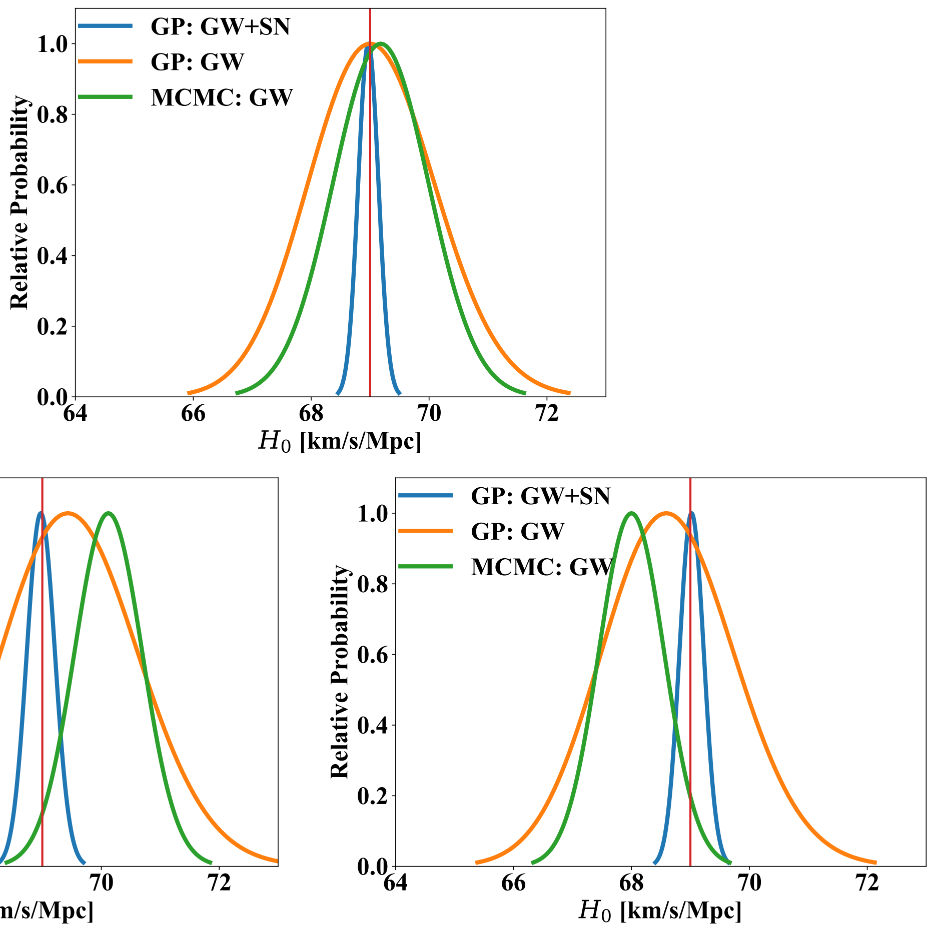


**Figure 4.** Posteriors of $H_0$ for the different input cosmologies (top: ΛCDM, bottom-left: $(w_0, w_a) = (-0.9, -0.75)$, bottom-right: $(w_0, w_a) = (-1.14, 0.35)$). The GP posterior for the combined GW and SN datasets is shown in blue and the GP posterior for the GW dataset alone is shown in orange. The MCMC results that assume ΛCDM (as calculated in Shafieloo et al. (2018)) are shown in green. The true input value is shown as the vertical red line.

GW data (orange), and from a GP regression using GW and SN data (blue). The bias from assuming ΛCDM is seen most clearly in the $H_0$–$\Omega_m$ plane but is seen in just the 1D posterior for $H_0$ as well (green lines).

However, when being more agnostic than assuming ΛCDM, such as using model independent GP regression (orange and blue lines), the bias disappears. We can successfully debias cosmic gravitational wave sirens, even when they are the sole distance probe. While then accurate, even this next next generation dataset will not be more precise than ≈ 2% on $H_0$ (apart from local sirens). If GW are combined with SN datasets – essentially using GW distances instead of the distance ladder to calibrate SN distances – then 1% precision *and accuracy* can be achieved even with an appropriately agnostic model independent method.

## 4 REDSHIFT ERRORS

Dark sirens rely on cross-correlation with large scale structure to estimate the redshift of the GW event that should be associated with the measured GW luminosity distance. We now examine the accuracy needed for the redshift estimation so as not to bias the cosmological parameter determination. In particular, a systematic constant offset could look similar to a shift in Hubble constant, while redshift dependence might propagate into biases on the matter density or dark energy equation of state parameters.

We begin with a simple redshift residual systematics of the form

$$\delta z = d_0 + d_1 z \, , \tag{5}$$

i.e. an additive and a multiplicative systematic such that $z \to (1 + d_1)z + d_0$. Thus the observable $D_L$ is interpreted as $D_L(z)$ but is really $D_L(z + \delta z)$. We can propagate this offset easily into the cosmological parameter estimation through the Fisher bias formalism as (see Eq. 3 of Shafieloo et al. (2018))

$$\delta p_i = \left(F^{-1}\right)_{ij} \sum_k \frac{\partial O_k}{\partial p_j} \frac{1}{\sigma_k^2} \Delta O_k \, , \tag{6}$$

where the observable $O_k = D_L(z_k)$, $\sigma_k$ is its uncertainty, and $\Delta O_k$ is the difference between the distance at the assumed redshift and at the true redshift.

For example, the systematic $(d_0, d_1) = (0.01, 0)$ biases the Hubble constant by $\delta h = -0.011$ or $1.4\sigma$ and the matter density by $\delta\Omega_m = -0.0195$ or $0.8\sigma$ within a ΛCDM model. While neither of these is too severe, the bias is nearly orthogonal to the degeneracy direction of the joint probability contour for $h$–$\Omega_m$, giving a substantial $\Delta\chi^2 = 75$ when fixing to ΛCDM. Note that this is not purely a shift in $H_0$ because $D_L$ is not linearly proportional to redshift for $z \gtrsim 0.05$.

A systematic with some redshift dependence but no low redshift systematic, e.g. $(d_0, d_1) = (0, 0.01)$, biases $\Omega_m$ more substantially, by $2.2\sigma$, and $h$ by $0.6\sigma$, again in a direction such that $\Delta\chi^2 = 44$. Including both systematic contributions, e.g. $(d_0, d_1) = (0.01, 0.01)$, gives a nearly linear additive effect in the parameter biases since they are nearly linear proportional to $\delta z$. However, the $\Delta\chi^2$ reacts more extremely since it is a product of parameter biases and parameter covariances; for example $(0.01, 0.01)$ now gives $\Delta\chi^2$=230.

Figure 5 shows examples of parameter bias in the ΛCDM model space for various $(d_0, d_1)$. Again note that the joint bias in terms of $\Delta\chi^2$ is much larger than individual parameter biases, being 44, 75, and 230 for the three examples, corresponding to well over $5\sigma$.

In the $d_0$ only systematic case, we would require the systematics be controlled to $|d_0| < 0.0018$ to obtain $\Delta\chi^2 < 2.3$ (i.e. $1\sigma$ joint confidence bias). The equivalent for $d_1$ only is $|d_1| < 0.0023$, and for the more general case of both $d_0$ and $d_1$, when they are equal then $|d_i| < 0.001$ is needed. This basically requires spectroscopic redshift precision for GW sirens to be used as an unbiased cosmological probe. Note that the addition of data from other probes, e.g. to constrain $\Omega_m$ does not help. If we add an external prior of $\sigma(\Omega_m) = 0.01$ then the statistical errors shrink, and $\Omega_m$ is less biased, but the bias on $h$ can actually increase due to covariances. We find the $\Delta\chi^2$ are almost unchanged, with the three systematics cases above giving 39, 74, 223 (recall the statistical contour shrinks, so even a smaller bias can give a larger $\Delta\chi^2$).

Returning from Fisher bias to GP regression, we can show how to use GP regression to infer the existence of redshift systematics or other unidentified systematics (see, for example, L'Huillier et al. (2019) for the case of Malmquist bias or source evolution). This is done first by calculating the median of the GP inference for one of the datasets (SN for our case), and using this as the mean function in the GP inference for the other dataset. This allows us to perform the test that if the posterior of the hyperparameters picks out a value for $\sigma_f$ that is significantly above $\sigma_f = 0$, then there is some unaccounted for discrepancy between the two datasets. Since the two datasets are generated from the same Universe, the conclusion would then be that some sort of systematic bias exists in the data.

To perform an example of this test we use the same mock GW distances and SN distance moduli from a ΛCDM cosmology, as in the previous section, but the GW redshifts used in the inference are biased by the following equation, $\delta z = \mathcal{N}(0.01, 0.01)\, z$, where $\mathcal{N}(0.01, 0.01)$ is a normal distribution with mean 0.01 and standard deviation 0.01. (This is a Monte Carlo version of the $d_1$ case above.)

The result for this systematics test from biased redshifts in the GW dataset is shown in Figure 6. The posterior of the GP hyperparameters picks out a value for $\sigma_f$ that is significantly above $\sigma_f = 0$ (at more than 99.9% level). This indicates that the GP regression is able to identify a systematic discrepancy between the GW and SN datasets (it even correctly identifies the order of magnitude of the effect). This test can only identify that some sort of systematic bias exists in the data, not that such offset comes specifically from biased redshift measurements.

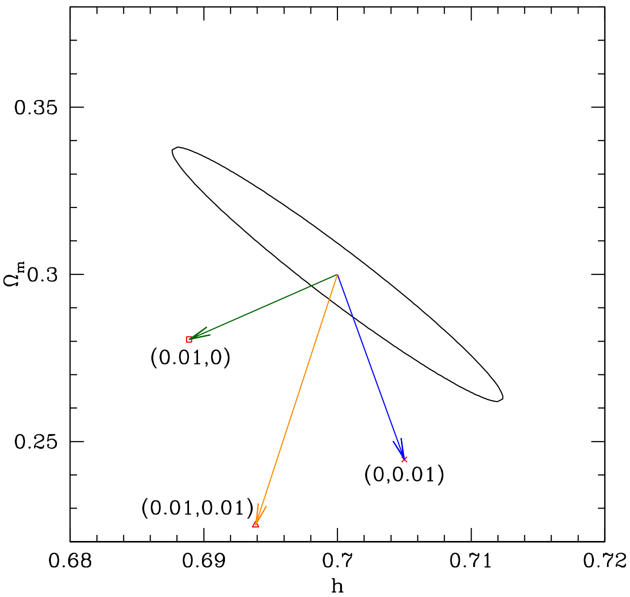


**Figure 5.** Systematic residuals in dark siren redshift estimation can cause substantial bias in the cosmological parameters. Here we show the statistical 68.3% CL joint probability contour in the ΛCDM parameter space, and the bias induced by $\delta z = d_0 + d_1 z$, with the square, triangle, square, and x symbols giving the derived cosmological values, labeled by $(d_0, d_1)$.

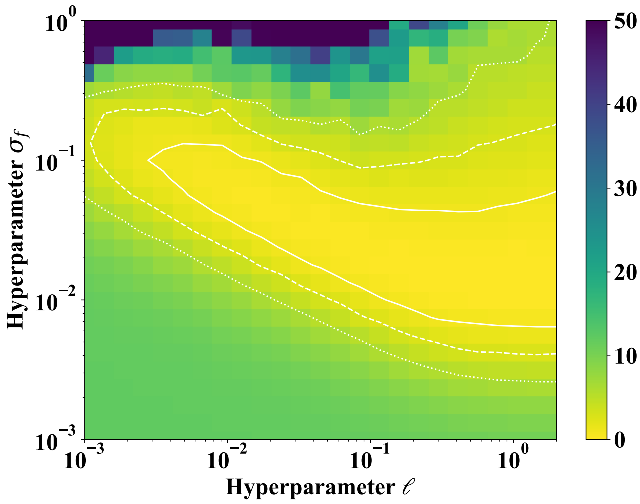


**Figure 6.** GP hyperparameter posterior for GW data where the mean function of the GP is taken from the median GP inference of the SN data but a GW data redshift systematic exists. The offset from $\sigma_f = 0$ indicates discrepancy between the data sets.

## 5 CONCLUSIONS

Gravitational wave sirens are a new distance measure with some unique characteristics. They have the potential to contribute to mapping the expansion history of the universe, including determining the Hubble constant, if appropriately treated within the cosmological context. In particular, assumptions about the background cosmology can significantly bias the Hubble constant and other parameter estimation. However, we demonstrate that a proper model independent method such as Gaussian process regression can debias the estimation and accurate reconstruct the expansion history $H(z)$ including $H_0$.

Furthermore, we illustrate how to use the GP hyperparameters as a test to determine whether the data require a beyond-ΛCDM cosmology. This can be done by fitting a ΛCDM model to the data, then using this best-fit model as a mean function for the GP regression. If the posterior of the hyperparameter $\sigma_f$ prefers values significantly different from 0, then that implies the data requires an explanation beyond the best-fit ΛCDM. For a "Next Next Generation" gravitational wave siren dataset, coupled with "Pantheon-like" and "WFIRST-like" supernovae datasets, GP regression was able to show mock data from reasonable $w_0$–$w_a$ cosmologies was incompatible with the best-fit ΛCDM cosmology, while accurately recovering the best-fit ΛCDM cosmology from ΛCDM-generated mock data.

This could also be used to detect unrecognized systematics in a dataset. Using the best fit expansion history from one data set (e.g. SN) as seed for the GW GP, one can again look for consistency with $\sigma_f = 0$.

A particular example of such a systematic could be redshift inaccuracy through indirect estimation of the dark siren redshift. We derived constraints on additive and multiplicative systematics, showing that even an apparently modest single parameter bias in a model dependent fit can actually lead to quite strong bias in joint parameter confidence contours. To remove the bias requires the additive and multiplicative redshift systematics to be controlled at the spectroscopic precision level.

Cosmic gravitational wave sirens alone, even from next next generation surveys, will only determine $H_0$ to the $\sim 2\%$ accuracy level, using the model independent formalism to debias. However, the Hubble constant and expansion history can potentially be mapped more accurately by using them in conjunction with supernovae and/or local GW sirens, with systematics appropriately controlled.

The GP regression code used for this study is made publicly available.

## ACKNOWLEDGEMENTS

We thank the CosKASI 2019 conference "The Correlated Universe" for providing a collaborative venue, and Tamara Davis for discussions about redshift errors and $H_0$. A.S. would like to acknowledge the support of the National Research Foundation of Korea (NRF- 2016R1C1B2016478). A.S. would like to acknowledge the support of the Korea Institute for Advanced Study (KIAS) grant funded by the Korea government. This work is supported in part by the Energetic Cosmos Laboratory and by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Award DE-SC-0007867 and contract no. DE-AC02-05CH11231.

## REFERENCES

Aghamousa A., Hamann J., Shafieloo A., 2017, Journal of Cosmology and Astro-Particle Physics, 2017, 031
Cooray A., Caldwell R. R., 2006, Phys. Rev. D, 73, 103002
Dalal N., Holz D. E., Hughes S. A., Jain B., 2006, Phys. Rev. D, 74, 063006
Feeney S. M., Peiris H. V., Williamson A. R., Nissanke S. M., Mortlock D. J., Alsing J., Scolnic D., 2019, Physical Review Letters, 122, 061105
Fishbach M., et al., 2019, ApJ, 871, L13
Holsclaw T., Alam U., Sansó B., Lee H., Heitmann K., Habib S., Higdon D., 2010, Phys. Rev. Lett., 105, 241302
Holz D. E., Hughes S. A., 2005, ApJ, 629, 15
Hui L., Greene P. B., 2006, Phys. Rev. D, 73, 123526
Joudaki S., Kaplinghat M., Keeley R., Kirkby D., 2018a, Phys. Rev. D, 97, 123501
Joudaki S., Kaplinghat M., Keeley R., Kirkby D., 2018b, Phys. Rev., D97, 123501
Kirkby D., Keeley R., 2017, Cosmological expansion history inference using Gaussian processes, doi:10.5281/zenodo.999564
L'Huillier B., Shafieloo A., Linder E. V., Kim A. G., 2019, MNRAS, 485, 2783
Mortlock D. J., Feeney S. M., Peiris H. V., Williamson A. R., Nissanke S. M., 2018, preprint, (arXiv:1811.11723)
Planck Collaboration et al., 2018, preprint, (arXiv:1807.06209)
Riess A. G., et al., 2016, ApJ, 826, 56
Riess A. G., Casertano S., Yuan W., Macri L. M., Scolnic D., 2019, ApJ, 876, 85
Schutz B. F., 1986, Nature, 323, 310
Scolnic D. M., et al., 2018, ApJ, 859, 101
Shafieloo A., Kim A. G., Linder E. V., 2012, Phys. Rev. D, 85, 123530
Shafieloo A., Kim A. G., Linder E. V., 2013, Phys. Rev. D, 87, 023520
Shafieloo A., Keeley R. E., Linder E. V., 2018, preprint (arXiv:1812.07775)
Spergel D., et al., 2015, preprint, (arXiv:1503.03757)
Zhang P., 2018, preprint, (arXiv:1811.07136)

This paper has been typeset from a TEX/LATEX file prepared by the author.