

Variations on Barbalat's lemma

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Abstract

It is not hard to prove that a uniformly continuous real function, whose integral up to infinity exists, vanishes at infinity, and it is probably little known that this statement runs under the name "Barbalat's lemma." In fact, the latter name is frequently used in control theory, where the lemma is used to obtain Lyapunov-like stability theorems for nonlinear and nonautonomous systems. Barbalat's lemma is qualitative in the sense that it asserts that a function has certain properties, here convergence to zero. Such qualitative statements can typically be proved by "soft analysis," such as indirect proofs. Indeed, in the original 1959 paper by Barbalat, the lemma was proved by contradiction, and this proof prevails in the control theory textbooks. In this short note, we first give a direct, "hard analysis" proof of the lemma, yielding quantitative results, i.e., rates of convergence to zero. This proof allows also for immediate generalizations. Finally, we unify three different versions that recently appeared and discuss their relation to the original lemma.

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