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# Polynomial Observer-Based Controller Synthesis and Fault-Tolerant Control for Tracking Optimal Power of Wind Energy Conversion Systems

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**ABSTRACT** This article proposes a new approach to design a fault-tolerant control (FTC) scheme for tracking the optimal power of wind energy conversion systems (WECSs). In this article, the considered fault will not only impact on actuator but also sensors. As the fault severely affects the performance of WECSs, the FTC are required to be worked accurately and effectively. The polynomial observer, as a part of the proposed FTC system, is synthesized to estimate the aerodynamic torque, electromagnetic torque, and fault simultaneously without using sensors to measure. The information of these parameters is sent back to the LQR (Linear Quadratic Regular) controller of WECSs. Both fault and aerodynamic torque in this study are unnecessary to fulfil any constraint. It should be noted that WECSs is reconstructed to a new form based on the descriptor technique, then the observer will design for this new form instead of the original system. Based on Lyapunov methodology and with the aid of SOS (Sum-Of-Square) technique, the conditions for polynomial observer design are derived in the main theorems. Finally, the simulation results have proved the effectiveness and merit of the proposed FTC method.

**INDEX TERMS** WECSs, fault-tolerant control, observer-based controller, SOS.

## I. INTRODUCTION

Wind energy has been increasingly utilized for electric power generation as one of promising renewable energy resources to replace fossil-fuel energies. However, wind energy conversion systems (WECSs) are nonlinear systems with parameter uncertainties, un-modeling errors, disturbances, and noise. Moreover, the wind speed information is required real-time feedback to controllers but it cannot be measured precisely by anemometers. Therefore, wind speed estimation techniques are preferable in WECSs. There are two approaches to estimate the wind speed: the first one is direct estimation [1]–[7] and the second is indirect estimation [8]–[12]. In the first approach, the wind speed is estimated directly by physical estimation methods [1], [2], Gray models and Kalman filter [3]–[6], genetic algorithm [7]. In the second approach, the wind speed is calculated via estimating aerodynamic torque. i.e, the aerodynamic torque is estimated instead of the wind speed, then the wind speed is calculated accordingly from the estimated aerodynamic torque [8]–[18]. Via this approach, both aerodynamic torque and wind speed are estimated.

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Because Wind Turbine always works under the harsh environment; therefore, the existence of the fault in the WECS system is inevitable. The fault will cause WECSs to work with low capacity and even make some parts of the system be broken down. Thus, an important issue related to control design is fault ride-through capability of control systems. In order to deal with the challenging of fault, many papers have been studied in recent years [19]–[21]. For instance, a fault-tolerant control strategy has been synthesized for a strict-feedback nonlinear system in [21]. Furthermore, regarding problem of fault in the WECS, in [22], a fuzzy fault-tolerant control (FTC) method was introduced for WECSs in cases of sensor faults. In [23] and [24], FTCs was presented to deal with the fault in the electronic valves of inverters. Besides, the influences of the faults on the current sensor have been investigated in [25]. Although these methods can effectively deal with the fault in WECSs, only one type of faults was taken into consideration in each paper.

It should be noted that, in practice, there are plenty of the physical parameters which are unable/hard to obtain by using sensors. In addition, using the sensors will increase the cost and some types of sensors are very sensitive to noise that causes measurement error. Owing to these reasons, a lot of studies focusing on observer design for control systems [26]–[30] has been published in recent years. In addition, another format of observer called disturbance observer has been studied to estimate the disturbance [31], [32]. For instance, an observer was designed to estimate both unmeasurable states and disturbances of the polynomial system in [31]. Applications of disturbance observer for WECSs have been investigated in [12], and [13] in which the wind speed was estimated by observer instead of using wind speed measuring instruments. However, a study in [12] exists a limitation is that the disturbance must be slow time-varying, otherwise, the methods in these papers were failed to synthesize the observer. Moreover, the existence of the fault was not taken into consideration in either [12] or [13].

Recently, an extended framework of a linear system called polynomial linear system was studied in many papers [33]–[37]. The polynomial linear systems have the same format with the linear system but the elements of the system matrices were with the polynomial forms instead of the constant values. There have been several papers designed the polynomial controller [33], [35]–[37] and filter [25] for the polynomial system. The advantage of modeling the nonlinear system under framework of the polynomial method is that some nonlinear parts of the nonlinear system are put inside the system matrices, the number of linearization terms are significantly reduced. This leads to reducing the modeling errors.

With the aforementioned discussions, it motivates us to develop a new approach to synthesize a controller for tracking the optimal power as well as being tolerant to faults of WECSs. In this article, the disturbance observer is synthesized to estimate the unknown states, aerodynamic torque, and fault. In addition, in order to obtain the maximum power, the LQR controller are designed to WECSs for tracking the optimum reference of turbine rotor speed. The optimum reference of turbine rotor speed is computed via estimating the aerodynamic torque. The novels of this study are emphasized according to the following aspects:

i) Unlike the methods in [22]–[25] which merely consider one type of fault (actuator or sensor fault), the WECSs in this article will be simultaneously impacted by both actuator and sensor fault.

ii) With the influence of faults in both sensor and actuator, the previous methods in [19]–[25] are not able to apply for this case. To overcome this challenge, the WECSs is modified to the polynomial system framework, and the descriptor technique is employed to reconstruct the WECS models to a new form. After that, the observer is designed for this system. The modelling WECS under the framework of the polynomial system assists to reduce the modelling error between the WECS model and the original nonlinear model.

iii) A polynomial disturbance observer is synthesized for WECSs to estimate aerodynamic torque, electromagnetic torque, and fault simultaneously without using sensors. The information of these parameters is feed-backed to the controller which is constructed on the basis of the LQR technique for FTC and tracking the optimal power point of WECSs.

iv) The aerodynamic torque and fault in this article are arbitrary and unnecessary to satisfy any constraint that is mandatory in [8]–[12].

The rest of paper is organized as follows. The system model of WECS is introduced in Section 2. The procedure for controller design based on the LQR technique is showed in Section 3. The polynomial observer design for WECSs is presented in Section 4. The closed-loop stability analysis will be found in Section 5. The simulation results are shown in Section 6 to prove the effectiveness of the proposed method. Finally, several conclusions are drawn in Section 7.

*Notations:* $\Theta^T$ and $\Theta^{-1}$ stand for transpose and inverse of matrix $\Theta$, respectively. $\Theta > 0$ ($\Theta < 0$) indicates the positive(negative) definite matrix. $I$ denotes an identity matrix.

## II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

### A. SYSTEM DESCRIPTION

First, let us consider power captured by the wind turbine (WT), expressed as follows,

$$P_a = \frac{1}{2}\rho\pi R^2 C_p(\lambda, \beta) v^3 \tag{1}$$

where $\rho$ is the air density, $v$ is denoted the wind speed, $R$ is expressed the WT rotor radius, and $C_p(\lambda, \beta)$ is for the power coefficient. In (1), $C_p$ is a nonlinear function of the tip-speed ratio $\lambda$ and of the pitch angle $\beta$ of the blades. Typically, its dynamic characteristic is experimentally provided by the manufacturer. Fig. 1 shows the characteristics of a typical power coefficient $C_p(\lambda, \beta)$ for different values of $\lambda$ and $\beta$.

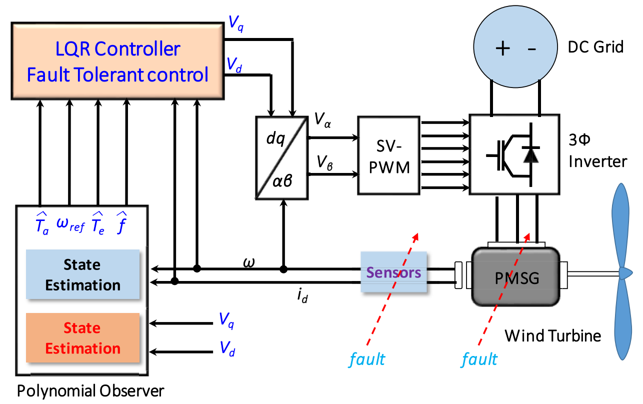


**FIGURE 1.** **Structure of the WECS with observer-based FTC scheme.**

It is easy to observe that at each angle of $\beta$, $C_p$ achieved its maximum value at a specific value of $\lambda$. And, the tip-speed ratio $\lambda$ is defined as

$$\lambda = \frac{\omega_t R}{v} \tag{2}$$

where $\omega_t$ is the rotor speed of the WT.

On the other hand, from (1) and (2), the aerodynamic torque $T_a$ is given by

$$T_a = \frac{P_a}{\omega_t} = \frac{1}{2}\rho\pi R^3 C_q(\lambda, \beta)\, v^2 \tag{3}$$

where $C_q(\lambda, \beta) = C_p(\lambda, \beta)/\lambda$ is called the torque coefficient.

According to (1), the captured power $P_a$ is promotional to $C_p$, which is maximized at a valued named optimal tip-speed ratio, $\lambda_{opt}$. This optimal ratio is constant at a fixed blade angle. Hence, by referring to (2), the maximum captured power is achieved by tracking the optimum reference of turbine rotor speed given by,

$$\omega_{t,ref} = \frac{\lambda_{opt}}{R} v \tag{4}$$

Note that the relations between speed and torque of the turbine side and generator side are expressed as

$$n_{gb} = \frac{\omega}{\omega_t} = \frac{T_a}{T_{gs}} \tag{5}$$

where $n_{gb}$ is the gearbox ratio, $\omega$ is the mechanical angular speed of the generator, and $T_{gs}$ is the equivalent aerodynamic torque applied to the generator.

The dynamics of the WTs are just presented. Next, the dynamic of the generator will be considered. Referring to [28], the dynamic model of a permanent magnet synchronous generator (PMSG) is stated as,

$$\begin{cases} J\dfrac{d\omega}{dt} = -B_v\omega + T_{gs} - T_e \\ \dfrac{di_q}{dt} = -\dfrac{R_s}{L}i_q - P\omega i_d - \dfrac{\psi_m P}{L}\omega + \dfrac{1}{L}v_q \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + P\omega i_q + \dfrac{1}{L}v_d \end{cases} \tag{6}$$

where $T_e$ is the electromagnetic torque, $i_q$ and $i_d$ are the stator currents in $q$-axis and $d$-axis, respectively; $v_q$ and $v_d$ are the stator voltages in $q$-axis and $d$-axis, respectively; $B_v$ is the equivalent viscous friction coefficient; $J$ denotes the equivalent rotor inertia; $P$ is the number of pole pairs; $R_s$ is the nominal stator resistance; $L$ is the nominal stator inductances; and $\psi_m$ is the magnet flux linkage. In (6), the electromagnetic torque can be considered by,

$$T_e = Ki_q \tag{7}$$

where $K = 3/2\ \psi_m P$.

Therefore, the dynamic equations of a PMSG can be rewritten as follows,

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{B_v}{J}\omega - \dfrac{1}{J}T_e + \dfrac{1}{J.n_{gb}}T_a \\ \dfrac{dT_e}{dt} = -\dfrac{R_s}{L}T_e - PK\omega i_d - \dfrac{\psi_m PK}{L}\omega + \dfrac{K}{L}v_q \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + \dfrac{P}{K}\omega T_e + \dfrac{1}{L}v_d \end{cases} \tag{8}$$

Suppose that during operation, the WECSs is affected by the faults, then (8) is modified as

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{B_v}{J}\omega - \dfrac{1}{J}T_e + \dfrac{1}{J.n_{gb}}T_a \\ \dfrac{dT_e}{dt} = -\dfrac{R_s}{L}T_e - \dfrac{\psi_m PK}{L}\omega - PK\omega i_d + \alpha f(t) + \dfrac{K}{L}v_q \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + \dfrac{P}{K}\omega T_e + \dfrac{1}{L}v_d + \beta f(t) \end{cases} \tag{9}$$

where $f$ is the fault and $\alpha$ and $\beta$ are coefficients.

*Assumption 1:*

1) $\omega$ and $i_d$ are available.

2) $i_q$ is not known, i.e., $T_e$ is unknown either.

3) Wind speed $v$ and aerodynamic torque $T_a$ are unknown

### B. PROBLEM STATEMENT

We suppose that the system (9) is affected by the fault and $i_q$, $T_e$, wind speed $v$, and aerodynamic torque $T_a$ are not available. However, these parameters are necessary for design the controller to track the optimal power for WECSs. If we use the sensors to collect the information of these parameters, it will increase the cost and the sensor may be sensitive to noise that will cause the measurement errors. In addition, the existence of the fault in the system (9) will degrade the WECS's performance. Owing to these reasons, in this article, we proposed a new approach to design a polynomial observer for estimating both the unknown states and faults. An observer-based LQR controller is synthesized for fault-tolerant and tracking the optimal power point of WECS.

## III. CONTROLLER DESIGN

To facilitate the controller design and stability analysis, (9) can be transformed to following error dynamics,

$$\begin{cases} \dfrac{d\tilde{\omega}}{dt} = -\dfrac{B_v}{J}\tilde{\omega} - \dfrac{1}{J}\tilde{T}_e \\ \dfrac{d\tilde{T}_e}{dt} = -\dfrac{R_s}{L}\tilde{T}_e - \dfrac{\psi_m PK}{L}\tilde{\omega} + \dfrac{K}{L}\left(v_q - u_{qc}\right) \\ \dfrac{di_d}{dt} = -\dfrac{R_s}{L}i_d + \dfrac{1}{L}\left(v_d - u_{dc}\right) \end{cases} \tag{10}$$

where $\tilde{\omega}$ is the tracking error of rotor speed, $\tilde{T}_e$ is the tracking error of the electromagnetic torque of the generator, and $u_{qc}$ and $u_{dc}$ are the compensating terms of control inputs, respectively. They are defined as follows,

$$\begin{aligned}
\tilde{\omega} &= \omega - \omega_{ref}; \quad \omega_{ref} = \omega_{t,ref}.n_{gb} = \frac{\lambda_{opt}}{R} v.n_{gb} \\
\tilde{T}_e &= T_e - T_{e,ref}; \quad T_{e,ref} = \frac{1}{n_{gb}} T_a - B_v \omega_{ref} - J\dot{\omega}_{ref} \\
u_{qc} &= \frac{R_s}{K} T_{e,ref} + \frac{L}{K}\dot{T}_{e,ref} + \psi_m P \omega_{ref} - \frac{\alpha L}{K} f + PL\omega i_d \\
u_{dc} &= -\beta L f - \frac{PL}{K}\omega T_e. \qquad (11)
\end{aligned}$$

where $\omega_{ref}$ is the speed reference of the generator, $T_{e,ref}$ is the reference of electromagnetic torque. It should be noted that, since wind speed and aerodynamic torque are not known, speed reference and electromagnetic torque reference are also unknown. In the next sections, a polynomial observer will be designed for estimating the aerodynamic torque, wind speed, $T_e$ and fault $f$.

The system (10) can be rewritten in the following state-space form,

$$\dot{x} = Ax + B(u - u_c) \qquad (12)$$

where

$$x = \begin{bmatrix} \tilde{\omega} & \tilde{T}_e & i_d \end{bmatrix}^T, \quad u = \begin{bmatrix} v_q & v_d \end{bmatrix}^T, \; u_c = \begin{bmatrix} u_{qc} & u_{dc} \end{bmatrix}^T,$$

$$A = \begin{bmatrix} -\frac{B_v}{J} & \frac{1}{J} & 0 \\ -\frac{\psi_m PK}{L} & -\frac{R_s}{L} & 0 \\ 0 & 0 & -\frac{R_s}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{K}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}.$$

Consider the following performance index:

$$J(x,u) = \int_0^{\infty} \left(x^T Q_c x + u^T T_c u\right) \qquad (13)$$

where $Q_c \in \mathbb{R}^{3\times 3}$ and $T_c \in \mathbb{R}^{2\times 2}$ are the positive matrices. In optimal control theories, by minimizing this quadratic performance index with a specific selection of the pair $(Q_c, T_c)$, the optimal control gain matrix is calculated.

The optimal control law for (12) is

$$u = u_c + K_u x \qquad (14)$$

where $K_u = -T^{-1}B^T P_u$ is the gain matrix of the controller, in which $P_u$ is a positive-definite solution of the following algebraic Riccati Equation:

$$P_u A + A^T P_u - P_u B T_c{}^{-1} B^T P_u + Q_c = 0 \qquad (15)$$

*Remark 1:* For a large value of $Q_c$, the tracking performance will be improved; however, this would require a large control effort. On the other hand, a large value of $T$ will cause a large tracking error with low control effort. Considering these facts, a tradeoff between control performance and energy consumption should be taken into account when selecting these parameters.

The matrices $Q_c$ and $T_c$ are typically selected as the following diagonal matrices:

$$Q_c = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}, \quad T_c = \tau \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \qquad (16)$$

with

$$q_i = \frac{1}{t_{si}\left(x_{imax}\right)^2}, \quad t_i = \frac{1}{\left(u_{imax}\right)^2}, \; \rho > 0 \qquad (17)$$

where $t_{si}$ is the preferred settling time of $x_i$, $\tau$ is a control parameter, and $x_{imax}$ and $u_{imax}$ are constraints on $|x_i|$ and $|u_i|$, respectively. Generally, the initial value of the weighting matrices is considered as identity matrix ($q_i = t_i = \tau = 1$). Then, $Q_c$ and $T_c$ can be tuned via extensive simulation studies until the performance is satisfied.

*Theorem 1:* With the control law in (14), the state vector $x$ in the system (12) exponentially converges to zero.

*Proof:* Define the Lyapunov function as follows:

$$V(x) = x^T P_u x \qquad (18)$$

It is obvious that $V(x)$ is positive-definite and radially unbounded. From (12), (14), and (15), its time derivative satisfies,

$$\begin{aligned}
\dot{V}(x) &= \frac{d}{dt} x^T P_u x = 2x^T P_u (A + BK_u) x \\
&= 2x^T P_u \left(A - BT^{-1}B^T P_u\right) \\
&= x^T P_u \left(P_u A + A^T P_u - 2P_u B T_c^{-1} B^T P_u\right) x \leq -x^T Q_c x
\end{aligned} \qquad (19)$$

Equation (19) shows that the time-derivative of $V$ is negative for all non-zero values of $x$.

From (18) and (19), it can be inferred that $x$ exponentially converges to zero.

## IV. OBSEVER SYNTHESIS FOR UNKNOWN STATES/FAULT ESTIMATION

### A. RECONSTRUCT WECS SYSTEM

Let us consider the wind energy conversion system with the influence of faults as follows:

$$\begin{cases}
\frac{d\omega}{dt} = -\frac{B_v}{J}\omega - \frac{1}{J}T_e + \frac{1}{J.n_{gb}} T_a \\
\frac{dT_e}{dt} = -\frac{R_s}{L}T_e - PK\omega i_d - \frac{\psi_m PK}{L}\omega + \frac{K}{L} v_q + \alpha f \\
\frac{di_d}{dt} = -\frac{R_s}{L} i_d + \frac{P}{K}\omega T_e + \frac{1}{L} v_d + \beta f
\end{cases} \qquad (20)$$

Reformulating the system (20) results in

$$\begin{bmatrix}\dot{\omega}\\ \dot{T}_e\\ \dot{i}_d\end{bmatrix}=\begin{bmatrix}-\frac{B_v}{J} & -\frac{1}{J} & 0\\ -\frac{\psi_m PK}{L} & -\frac{R_s}{L} & PK\omega\\ 0 & \frac{P}{K}\omega & -\frac{R_s}{L}\end{bmatrix}\begin{bmatrix}\omega\\ T_e\\ i_d\end{bmatrix}+\begin{bmatrix}0 & 0\\ \frac{K}{L} & 0\\ 0 & \frac{1}{L}\end{bmatrix}\begin{bmatrix}v_q\\ v_d\end{bmatrix}+\begin{bmatrix}\frac{1}{Jn_{gb}}\\ 0\\ 0\end{bmatrix}T_a+\begin{bmatrix}0\\ \alpha\\ \beta\end{bmatrix} \tag{21}$$

Suppose that the fault also affects to the sensors, and based on Assumption 1, the output of the system (20) is formulated by

$$y=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\omega\\ T_e\\ i_d\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}f \tag{22}$$

Let us define

$$\tilde{x}=\begin{bmatrix}\omega & T_e & i_d\end{bmatrix}^T,\quad u=\begin{bmatrix}v_q & v_d\end{bmatrix}^T$$

$$A(\omega)=\begin{bmatrix}-\frac{B_v}{J} & -\frac{1}{J} & 0\\ -\frac{\psi_m PK}{L} & -\frac{R_s}{L} & PK\omega\\ 0 & \frac{P}{K}\omega & -\frac{R_s}{L}\end{bmatrix},\quad B=\begin{bmatrix}0 & 0\\ \frac{K}{L} & 0\\ 0 & \frac{1}{L}\end{bmatrix}.$$

$$C=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\quad D=\begin{bmatrix}\frac{1}{Jn_{gb}} & 0 & 0\end{bmatrix}^T,\ E=\begin{bmatrix}0\\ \alpha\\ \beta\end{bmatrix},$$

$$T=\begin{bmatrix}0\\ 1\end{bmatrix}.$$

From (21) and (22), the WECS is reformulated under the framework of the polynomial system

$$\begin{cases}\dot{\tilde{x}}=A(\omega)\tilde{x}+Bu+DT_a+Ef\\ y=C\tilde{x}+Tf.\end{cases} \tag{23}$$

It should be noted that because of the existence of a fault, the method in [13] is unable to apply to design observer for the system (23). In order to overcome this difficulty, in this article, the argument technique is employed to is reconstructed the system (23) in the following form

$$\begin{cases}\begin{bmatrix}\dot{\tilde{x}}\\ \dot{f}\end{bmatrix}=\begin{bmatrix}A(\omega) & E\\ 0 & 0\end{bmatrix}\begin{bmatrix}\tilde{x}\\ f\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}u+\begin{bmatrix}D & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}T_a\\ \dot{f}\end{bmatrix}\\ y=[C\quad T]\begin{bmatrix}\tilde{x}\\ f\end{bmatrix}.\end{cases} \tag{24}$$

Denote

$$\dot{\overline{x}}=\begin{bmatrix}\dot{\tilde{x}}\\ \dot{f}\end{bmatrix},\quad \overline{x}=\begin{bmatrix}\tilde{x}\\ f\end{bmatrix},\ \overline{\omega}(t)=\begin{bmatrix}T_a\\ \dot{f}\end{bmatrix},\ \overline{A}(\omega)=\begin{bmatrix}A(\omega) & E\\ 0 & 0\end{bmatrix},$$

$$\overline{B}=\begin{bmatrix}B\\ 0\end{bmatrix},\ \overline{D}=\begin{bmatrix}D & 0\\ 0 & I\end{bmatrix},\quad \text{and } \overline{C}=[C\quad T],$$

then (24) is rewritten as follows

$$\begin{cases}\dot{\overline{x}}=\overline{A}(\omega)\overline{x}+\overline{B}u+\overline{D}\overline{\omega}(t) & (25a)\\ y=\overline{C}\overline{x}. & (25b)\end{cases}$$

### B. POLYNOMIAL OBSERVER SYNTHESIS

From now on, the observer will be designed for the system (25) instead of the original system (20) to estimate the unknown state $T_e$, aerodynamic torque $T_a$, and reconstruct the fault $f(t)$.

Taken into account the polynomial observer framework for the system (25) as follows

$$\begin{cases}\dot{\hat{\overline{x}}}=E(\omega)\hat{\overline{x}}+Qu+H(\omega)y+Z\dot{y} & (26a)\\ \hat{y}=C\hat{\overline{x}} & (26b)\\ \hat{\overline{\omega}}(t)=(\overline{CD})^{-1}\dot{y}-W(\omega)\hat{\overline{x}}-Ru. & (26c)\end{cases}$$

where $\hat{\overline{x}}\in\mathbb{R}^4$, and $\hat{\overline{\omega}}(t)$ are the estimation of $\overline{x}$, and $\overline{\omega}(t)$, respectively. $E(\omega)\in\mathbb{R}^{3\times 3}$, $\in\mathbb{R}^{3\times 1}$, $H(\omega)\in\mathbb{R}^{3\times 1}$, $Z\in\mathbb{R}^{3\times 1}$, $W(\omega)\in\mathbb{R}^{1\times 3}$, and $R\in\mathbb{R}^{1\times 2}$ are the parameters of the observer (26) which will be calculated in the next section.

Then the estimation $\hat{\tilde{x}}$, $\hat{T}_a$, and fault $\hat{f}$ are calculated by the following formula

$$\hat{\tilde{x}}=\begin{bmatrix}I & 0\end{bmatrix}\hat{\overline{x}} \tag{27}$$

$$\hat{f}=\begin{bmatrix}0 & I\end{bmatrix}\hat{\overline{x}} \tag{28}$$

$$\hat{T}_a=\begin{bmatrix}I & 0\end{bmatrix}\hat{\overline{\omega}}(t) \tag{29}$$

In this article, the Sum-Of-Square (SOS) tool is employed to synthesize the polynomial observer (26), therefore, two following propositions are necessary to explain the concept of the SOS technique.

*Proposition 1 [38]:* Suppose that the function $g(x(t))$ can be expressed as the form $g(x(t))=\sum_{i=1}^{n}[e_i(x(t))]^2$, where $e_i(x(t))$ is a polynomial in $x(t)$, then $g(x(t))$ is called a SOS. When $g(x(t))$ is a SOS, it means that $g(x(t))\geq 0$, however, the converse is not guaranteed.

*Proposition 2 [38]:* Taken into consideration a polynomial symmetric matrix $\Lambda(x)$ in $x$ with dimension $n\times n$ and a vector $v\in\Re^n$ independent on $x$, if $v^T\Lambda(x)v$ is an SOS then $\Lambda(x)\geq 0$ for all $x$.

*Theorem 2:* The estimation error between estimated states and real states with the observer (26) approaches zero asymptotically if there exist matrices $E(\omega)$, $Q$, $H(\omega)$, $Z$, $W(\omega)$, $R$ and a symmetric matrix $P$ such that the following conditions hold

$$\overline{A}(\omega)-Z\overline{CA}(\omega)-H(\omega)\overline{C}-E(\omega)=0 \tag{30}$$

$$\overline{B}-Z\overline{CB}-Q=0 \tag{31}$$

$$\overline{D}-Z\overline{CD}=0 \tag{32}$$

$$W(\omega)=(\overline{CD})^{-1}C\overline{A}(\omega) \tag{33}$$

$$R=(\overline{CD})^{-1}C\overline{B} \tag{34}$$

$$v_1^T(P-\varepsilon_1 I)v_1 \text{ is SOS} \tag{35}$$

$$-v_2^T\left(E^T(\omega)P+PE(\omega)-\varepsilon_2(\omega)I\right)v_2 \text{ is SOS} \tag{36}$$

$v_1$, $v_2$ are two vectors which are independent on $\omega$, $\varepsilon_1$ is positive constant and $\varepsilon_2(\omega)$ is positive and $\varepsilon_2(\omega) \neq 0$ with $\omega) \neq 0$.

*Proof:* The estimation errors are defined

$$e(t) = \bar{x} - \hat{\bar{x}} \tag{37}$$

From (37), it yields

$$\dot{e}(t) = \dot{\bar{x}} - \dot{\hat{\bar{x}}} \tag{38}$$

Combining (25a), (26a), and (38) obtains

$$\dot{e}(t) = \overline{A}(\omega)\overline{x} + \overline{B}u + \overline{D}\overline{\varpi}(t) - E(\omega)\hat{\overline{x}} - Qu - H(\omega)y - Z\dot{y} \tag{39}$$

From (25b), it infers that

$$\dot{y} = \overline{C}\dot{\overline{x}} = \overline{CA}(\omega)\bar{x} + \overline{CB}u + \overline{CD}\overline{\varpi}(t) \tag{40}$$

Combining (39) and (40) obtains

$$\begin{aligned}\dot{e}(t) &= \overline{A}(\omega)\overline{x} + \overline{B}u + \overline{D}\overline{\varpi}(t) - E(\omega)\hat{\overline{x}} - Qu - H(\omega)y \\ &\quad - Z[\overline{CA}(\omega)\bar{x} + \overline{CB}u + \overline{CD}\overline{\varpi}(t)] \\ &= E(\omega)\left(\overline{x} - \hat{\overline{x}}\right) + \left[\overline{A}(\omega) - Z\overline{CA}(\omega) - H(\omega)\overline{C} \right.\\ &\quad \left. - E(\omega)\right]\overline{x} + \left[\overline{B} - Z\overline{CB} - Q\right]u + [\overline{D} - Z\overline{CD}]\overline{\varpi}(t)\end{aligned} \tag{41}$$

If the conditions (30)-(32) of Theorem 2 satisfy then (41) becomes

$$\dot{e}(t) = E(\omega)e(t) \tag{42}$$

The estimation error between $\overline{\varpi}(t)$ and its estimation is defined

$$e_{\bar{\varpi}}(t) = \hat{\overline{\varpi}}(t) - \overline{\varpi}(t) \tag{43}$$

Substituting (26c) and (37) into (42) yields

$$\begin{aligned}&e_{\bar{\varpi}}(t) \\ &= \left[\left(\overline{CD}\right)^{-1}\dot{y} - W(\omega)\hat{\bar{x}} - Ru\right] - \overline{\varpi}(t) \\ &= \left[\left(\overline{CD}\right)^{-1}\bar{C}\left(\overline{A}(\omega)\overline{x} + \overline{B}u + \overline{D}\overline{\varpi}(t)\right) - W(\omega)\hat{\bar{x}} - Ru\right] - \overline{\varpi}(t) \\ &= (\overline{CD})^{-1}C\overline{A}(\omega)\overline{x} + (\overline{CD})^{-1}C\overline{B}u + \overline{\varpi}(t) - W(\omega)\hat{\bar{x}} - Ru - \overline{\varpi}(t) \\ &= W(\omega)(\bar{x} - \hat{\bar{x}}) + ((\overline{CD})^{-1}C\overline{A}(\omega) - W(\omega))\overline{x} \\ &\quad + ((\overline{CD})^{-1}C\overline{B} - R)u.\end{aligned} \tag{44}$$

If the conditions (33) and (34) hold, then (40) becomes

$$e_{\bar{\varpi}}(t) = W(\omega)e(t). \tag{45}$$

It is noted that, from (45), it is easily seen that when $e(t) \to \infty$, $e_{\bar{\varpi}}(t) \to \infty$. Therefore, we merely design observer to make $e(t) \to \infty$ then $e_{\bar{\varpi}}(t) \to \infty$ automatically.

Select the Lyapunov function as follows

$$V(e(t)) = e^T(t)Pe(t) \tag{46}$$

The condition (35) of Theorem 1 implies that the matrix $P$ is positive define. Therefore

$$V(e(t)) > 0.$$

Taking the derivative for Lyapunov function (46) yields

$$\dot{V}(e(t)) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \tag{47}$$

Substituting (42) into (46) obtains

$$\dot{V}(e(t)) = e^T(t)[E^T(\omega)P + PE(\omega)]e(t) \tag{48}$$

It is obvious that if the condition (36) of Theorem 2 holds, then $E^T(\omega)P + PE(\omega) < 0$, combining with (48), it can conclude that $\dot{V}(e(t)) < 0$. Hence, the estimation errors $e(t)$ and $e_{T_a}(t)$ approach zero asymptotically. The proof is completed.

However, because both $P$ and $E(\omega)$ are variables, the condition (36) is a Polynomial Bilinear Matrix Inequality (BPMI) which is difficult to solve by SOS Tool in MATLAB. Due to this reason, it is essential to transform (36) into Polynomial Linear Matrix Inequality (PLMI) in the following sections.

*Theorem 3:* The estimation error of the states $e(t)$ and estimation error $e_{T_a}(t)$ with the observer (26) converge to zero asymptotically if there exist the matrices $E(\omega)$, $J$, $H(\omega)$, $Z$, $W(\omega)$, $R$, and symmetric matrix $P$ such that the following conditions are satisfied

$$v_1^T(P - \varepsilon_1 I)v_1 \text{ is SOS} \tag{49}$$

$$-v_2{}^T(\Phi(\omega) + \varepsilon_2(\omega)I)v_2 \text{ is SOS} \tag{50}$$

where

$$\begin{aligned}\Phi(\omega) &= [(I - Z\overline{C})\overline{A}(\omega)]^T - (\overline{C})^T[S(\omega)]^T \\ &\quad + P[(I - Z\overline{C})\overline{A}(\omega)] - S(\omega)\overline{C}\end{aligned} \tag{51}$$

$$S(\omega) = PH(\omega) \tag{52}$$

$v_1$ and $v_2$ are the vectors that are independent on $\omega$. $\varepsilon_1$ is the positive scalar and $\varepsilon_2(\omega) > 0$ with $\omega \neq 0$.

And the observer gains are calculated

$$Z = \overline{D}(\overline{CD})^{-1} \tag{53}$$

$$E(\omega) = \overline{A}(\omega) - Z\overline{CA}(\omega) - H(\omega)\overline{C} \tag{54}$$

$$H(\omega) = P^{-1}S(\omega) \tag{55}$$

$$Q = \overline{B} - M\overline{B} \tag{56}$$

*Proof:* From (32), we have

$$Z = \overline{D}(\overline{CD})^{-1} \tag{57}$$

From (30), one obtains

$$E(\omega) = \overline{A}(\omega) - Z\overline{CA}(\omega) - H(\omega)\overline{C} \tag{58}$$

From (36), it implies that

$$E^T(\omega)P + PE(\omega) < 0 \tag{59}$$

Substituting (58) into (59) yields

$$\begin{aligned}&[\overline{A}(\omega) - Z\overline{CA}(\omega) - H(\omega)\overline{C}]^T P \\ &\quad + P[\overline{A}(\omega) - Z\overline{CA}(\omega) - H(\omega)\overline{C}] \\ &= [(I - Z\overline{C})\overline{A}(\omega)]^T P - (\overline{C})^T[H(\omega)P]^T \\ &\quad + P[(I - Z\overline{C})\overline{A}(\omega)] - PH(\omega)\overline{C} < 0\end{aligned} \tag{60}$$

Let us denote

$$S(\omega) = PH(\omega) \tag{61}$$

From (60) and (61), one obtains

$$[(I - Z\overline{C})\overline{A}(\omega)]^T P - (\overline{C})^T [S(\omega)]^T + P[(I - Z\overline{C})\overline{A}(\omega)] - S(\omega)\overline{C} < 0 \tag{62}$$

It is easily seen that the (62) and (50) are the same, and they are PLMIs. Hence, the BMI condition (36) of Theorem 2 is successfully converted into LMI (50) of Theorem 3. The proof is completed.

Procedure for synthesizing observer is briefly summarized as follows

***Step 1***: Compute $Z$ from condition (53).

***Step 2***: Substitute the obtained results $Z$ from *step 1* into the condition (50), then solve (49) and (50) by SOS tool in Matlab to get $S(\omega)$ and $P$.

***Step 3***: Observer gains are calculated from (53)-(56).

***Step 4:*** With obtained observer gains, the observer (26) is constructed and the $\hat{\tilde{x}}$, $\hat{T}_a$, and $\hat{f}$ are computed as follows

$$\hat{\tilde{x}} = \begin{bmatrix} 0 & I \end{bmatrix} \hat{\bar{x}}$$
$$\hat{f} = \begin{bmatrix} 0 & I \end{bmatrix} \hat{\bar{x}}$$
$$\hat{T}_a = \begin{bmatrix} I & 0 \end{bmatrix} \hat{\bar{\omega}}(t)$$

## V. OBSERVER-BASED CONTROLLER AND FAULT TOLERANT CONTROL DESIGN

### A. OBSERVER-BASED CONTROLLER AND FAULT TOLERANT CONTROL DESIGN

On the basis of the estimated information of speed reference, aerodynamic torque, and fault, the tracking error and compensators are expressed in the following forms

$$\begin{aligned}
\hat{\tilde{\omega}} &= \omega - \hat{\omega}_{ref}; \quad \hat{\omega}_{ref} = \sqrt{\frac{\hat{T}_a}{k_{opt}}} \\
\hat{\tilde{T}}_e &= \hat{T}_e - \hat{T}_{e,ref}; \quad \hat{T}_{e,ref} = \frac{1}{n_{gb}}\hat{T}_a - B_v\hat{\omega}_{ref} - J\dot{\hat{\omega}}_{ref} \\
\hat{u}_{qc} &= \frac{R_s}{K}\hat{T}_{e,ref} + \frac{L}{K}\dot{\hat{T}}_{e,ref} + \psi_m P\hat{\omega}_{ref} - \frac{\alpha L}{K}\hat{f} + PL\omega i_d \\
\hat{u}_{dc} &= -\beta L\hat{f} - \frac{PL}{K}\omega\hat{T}_e.
\end{aligned} \tag{63}$$

The observer-based controller is described under the following framework

$$u = \hat{u}_c + K_u\hat{x} \tag{64}$$

in which

$$\hat{x} = \begin{bmatrix} \hat{\tilde{\omega}} & \hat{\tilde{T}}_e & i_d \end{bmatrix}^T, \quad \hat{u}_c = \begin{bmatrix} \hat{u}_{qc} & \hat{u}_{dc} \end{bmatrix}^T.$$

From (63), $\hat{x}$ and $\hat{u}_c$ are determined as follows

$$\begin{aligned}
\hat{x} &= x + Hx_e \\
\hat{u}_c &= u_c + Mx_e
\end{aligned} \tag{65}$$

where

$$x_e = \begin{bmatrix} e_{T_a} & \dot{e}_{T_a} & \ddot{e}_{T_a} & e_{T_e} & e_f \end{bmatrix}^T$$

$$H = \begin{bmatrix} h_1 & 0 & 0 & 0 & 0 \\ h_2 & h_3 & 0 & h_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} m_1 & m_2 & m_3 & 0 & m_4 \\ 0 & 0 & 0 & m_5 & m_6 \end{bmatrix},$$

$$h_1 = \frac{1}{\sqrt{k_{opt}}\left(\sqrt{T_a} + \sqrt{\hat{T}_a}\right)}, \quad h_2 = \frac{1}{n_{gb}} - B_v h_1 + Jl_2,$$

$$h_3 = -Jl_1, \quad h_4 = -1; \; l_1 = \frac{1}{2\sqrt{k_{opt}T_a}}, \; l_2 = \frac{\dot{\hat{T}}_a}{2\sqrt{T_a\hat{T}_a}}h_1;$$

$$l_3 = \frac{l_1}{2T_a}$$

$$l_4 = \left(\frac{T_a + \hat{T}_a + \sqrt{T_a\hat{T}_a}}{2T_a\hat{T}_a} + \frac{\ddot{\hat{T}}_a}{\dot{\hat{T}}_a}\right) l_2 h_1,$$

$$m_1 = \psi_m P h_1 + \frac{L}{K}l_3 + \frac{R_s}{K}h_2, \quad m_2 = h_3 + \frac{L}{K}l_4,$$

$$m_3 = \frac{L}{K}l_1, \quad m_4 = \frac{\alpha L}{K}, \; m_5 = \frac{PL}{K}\omega, \; m_6 = \beta L,$$

$e_{T_a} = T_a - \hat{T}_a$, $e_{T_e} = T_e - \hat{T}_e$, and $e_f = f - \hat{f}$.

### B. STABILITY ANALYSIS

Combining (65) and (64), it yields

$$u = u_c + K_u x + N x_e \tag{66}$$

where $N = M + KH$,

Substituting (65) into (12) yields

$$\dot{x} = (A + BK_u)x + BNx_e \tag{67}$$

The system (67) is a closed-loop system of the observer-based controller.

*Lemma 1 [39]:* Let us consider a system

$$\begin{cases} \dot{z} = f(z, y) \\ \dot{y} = s(y) \end{cases} \tag{68}$$

in which $\dot{y} = s(y)$ has an asymptotically stable equilibrium at $y = 0$. If $\dot{z} = f(z, 0)$ has an asymptotically stable equilibrium at $z = 0$, then (68) has an asymptotically stable equilibrium at $(z, y) = (0, 0)$

*Theorem 4:* The tracking error $x$ and estimation error $x_e$ with observer (26) and controller (64) asymptotically approach zero.

*Proof:* If the observer gains of the observer (26) fulfill Theorem 2 and 3 then observer (26) successfully estimates the state $T_e$, aerodynamic torque $T_a$, and reconstruct the fault $f(t)$. It implies that $x_e \to 0$ when $t \to \infty$. Thus, (67) becomes

$$\dot{x} = (A + BK_u)x \tag{69}$$

According to Theorem 1, system (69) is asymptotically stable equilibrium at zero. Relied on Lemma 1, it is obvious that tracking error $x$ and estimation error $x_e$ in (35) are asymptotically stable at zero.

## VI. RESULTS AND DISCUSSION

In this section, we will carry out the simulating the WECS system with the parameters shown in Table 1. Fig. 1 shows the structure of the proposed FTC scheme. Suppose that the WECS is affected by the fault with the formula in (70) and demonstrated in Fig. 2. The wind speed has the waveform in Fig. 3. It should be noted that the fault impacts on both actuator and sensor of WECS.

$$f(t) = \begin{cases} 2 & 10 \le t \le 40 \\ 2\sin(t) & 50 \le t \\ 0 & otherwise \end{cases} \tag{70}$$

**TABLE 1.** **WECS parameters.**

| Symbols | Parameters | Values | Unit |
|---|---|---|---|
| $P_{rated}$ | Rated power | 5 | kW |
| $P$ | Pole pairs | 14 | - |
| $R_s$ | Stator resistance | 0.3676 | Ω |
| $L$ | Stator inductance | 3.55 | mH |
| $\psi_m$ | Magnet flux linkage | 0.2867 | V· s/rad |
| $J$ | Mechanical inertia | 7.856 | kg· m$^2$ |
| $B$ | Viscous friction coefficient | 0.002 | kg· m$^2$/s |
| $R$ | Rotor radius | 1.84 | m |
| $\rho$ | Air density | 1.25 | kg/m$^3$ |
| $\alpha$ | Fault coefficient | 3 | - |
| $\beta$ | Fault coefficient | 2 | - |

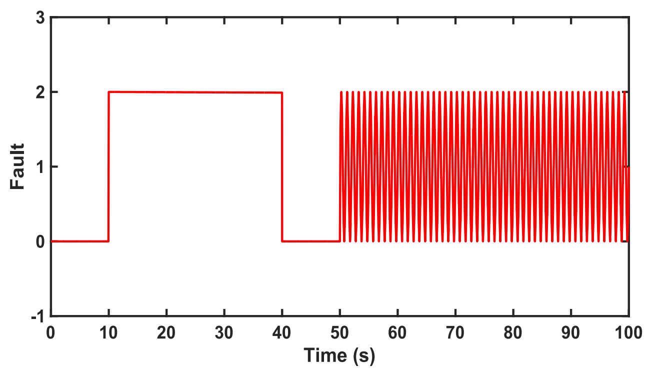


**FIGURE 2.** **Real fault.**

On the basis of the LQR method in Theorem 1, the matrix $Q_c$ and $T_c$ are selected

$$Q_c = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad T_c = 10^{-2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

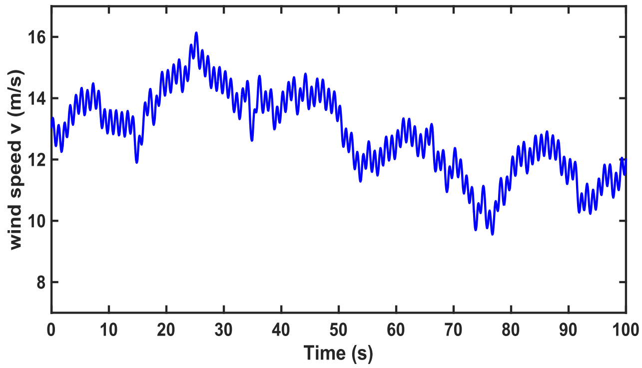


**FIGURE 3.** **Wind speed waveform.**

Then the controller gains are obtained as follows:

$$K_u = \begin{bmatrix} -35.8703 & 9.9394 & 0.0000 \\ 0.000 & -0.000 & 9.6392 \end{bmatrix}.$$

Solving the conditions in Theorem 3 yields the observer gains of the observer (26)

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad Q = 10^3\begin{bmatrix} 0 & 0 \\ 1.696 & 0 \\ 0 & 0.2817 \\ 0 & -0.2817 \end{bmatrix};$$

$$W(\omega) = \begin{bmatrix} -1/500 & -1 & 0 & 0 \\ 0 & 2.325\omega & 103.55 & 2 \end{bmatrix};$$

$$R = \begin{bmatrix} 0 & 0 \\ 0 & 281.69 \end{bmatrix},$$

$$E(\omega) = \begin{bmatrix} E_{11}(\omega) & 0 & E_{13}(\omega) & E_{14}(\omega) \\ E_{21}(\omega) & -7352/71 & E_{23}(\omega) & E_{24}(\omega) \\ E_{31}(\omega) & (20000\omega)/8601 & E_{33}(\omega) & E_{34}(\omega) \\ E_{41}(\omega) & ,-(20000*\text{y}1)/8601 & E_{43}(\omega) & E_{44}(\omega) \end{bmatrix},$$

where

$$\begin{aligned}
E_{11}(\omega) &= -0.587\omega^2 + 0.192 * 10^{-5}\omega - 0.669, \\
E_{12}(\omega) &= -0.344 * 10^{-5}\omega^2 - 0.002\omega - 0.001, \\
E_{14}(\omega) &= -0.345 * 10^{-6}\omega^2 - 0.001\omega - 0.001, \\
E_{21}(\omega) &= 0.113\omega^2 - 0.118\omega - 88.226, \\
E_{23}(\omega) &= -0.00029 * \omega^2 - 41.307 * \omega - 1.47, \\
E_{24}(\omega) &= -0.00029\omega^2 + 42.982\omega + 1.529, \\
E_{31}(\omega) &= 0.0008\omega^2 + 0.352\omega + 0.055, \\
E_{33}(\omega) &= -0.305\omega^2 - 0.004\omega - 67.92, \\
E_{34}(\omega) &= -0.305\omega^2 - 0.0046\omega + 37.627, \\
E_{41}(\omega) &= -0.0008\omega^2 - 0.35\omega - 0.0527, \\
E_{43}(\omega) &= -0.288\omega^2 + 0.0046\omega + 67.32, \\
E_{44}(\omega) &= -0.288\omega^2 + 0.0046\omega - 38.228.
\end{aligned}$$

Simulating the WECSs with proposed observer and controller in MATLAB/Simulink, the simulation results are obtained in Figs. 2-8.

In these simulations, we assume that the system is affected by the fault in Fig. 2 and the wind with speed wave form

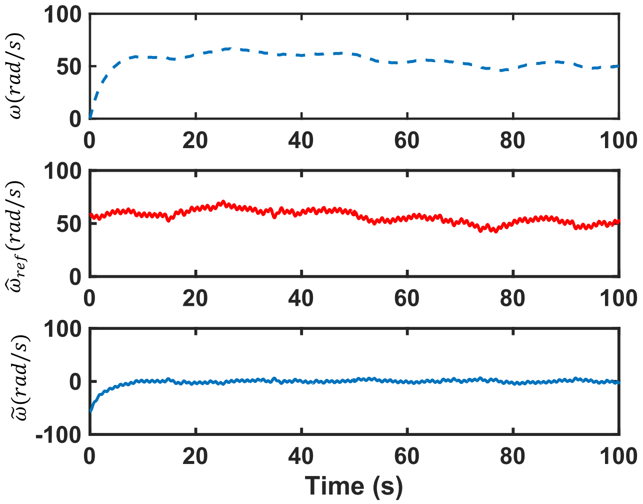


**FIGURE 4.** Speed tracking performance.

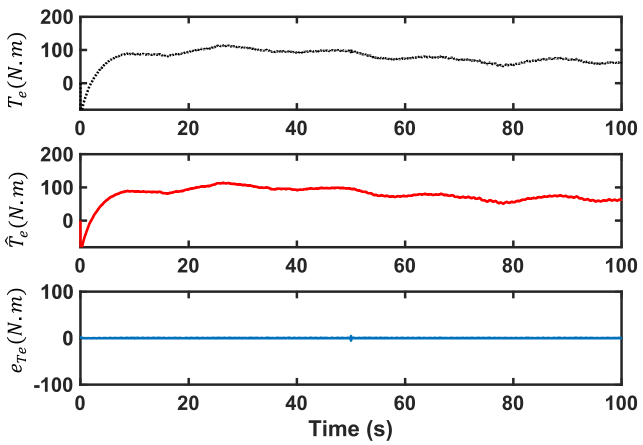


**FIGURE 5.** Estimation performance of the electromagnetic torque $T_e$.

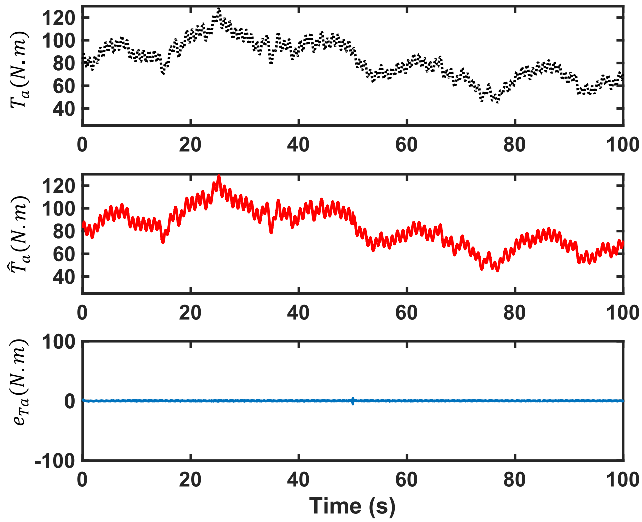


**FIGURE 6.** Estimation performance of aerodynamic torque $T_a$.

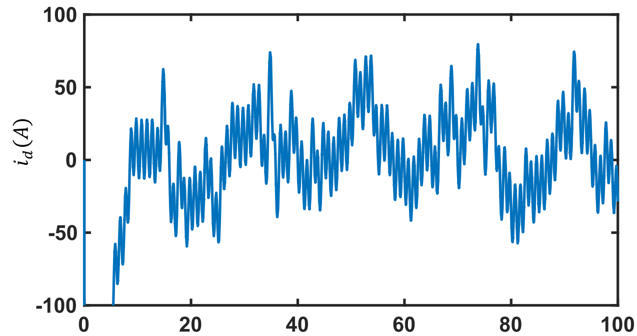


**FIGURE 7.** The stator currents $i_d$ in $d$-axis.

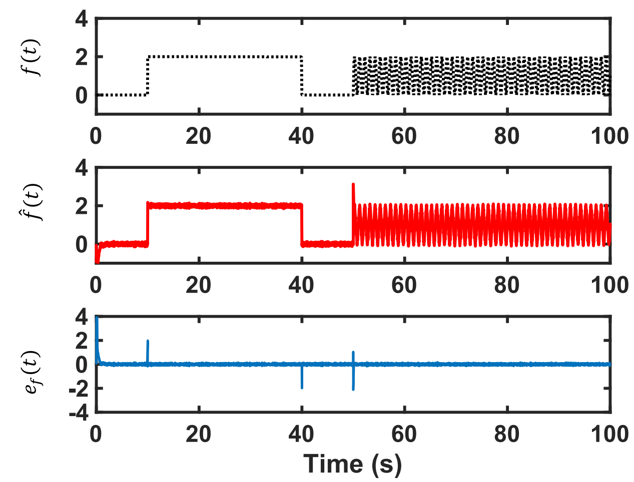


**FIGURE 8.** Estimation performance of fault.

in Fig. 3. Fig. 4 illustrates the speed tracking performance including the responded speed $\omega$, the speed reference $\hat{\omega}_{ref}$ and the speed tracking error $\hat{\tilde{\omega}} = \omega - \hat{\omega}_{ref}$. The Fig. 5 and Fig. 6 show the electromagnetic torque $T_e$, the estmated electromagnetic torque $\hat{T}_e$ and the estimation error $e_{Te} = T_e - \hat{T}_e$, the aerodynamic torque $T_a$, the estimated aerodynamic torque $\hat{T}_a$, and estimation error $e_{Ta} = T_a - \hat{T}_a$, respectively. The real fault $f(t)$, estimated fault $\hat{f}(t)$, and estimation error $e_f(t) = f(t) - \hat{f}(t)$ are demonstrated in Fig. 8 and the the stator currents $i_d$ in $d$-axis is shown in Fig. 7. From Figs. 5, 6, 8, it is seen that the estimations of $\hat{T}_e$, $\hat{T}_a$ and $\hat{f}(t)$ approach the real $T_e$, $T_a$, and $f(t)$, and the all estimation errors converge to zero asympttically, it means that electromagnetic torque $T_e$, aerodynamic torque $T_a$, and fault $f(t)$ are accurately estimated. Addiontionally, from Fig. 4, it is obvious that by applying the observer-based controller, the mechanical angular speed of the generator $\omega$ can precisely track the optimum reference of turbine rotor speed $\hat{\omega}_{ref}$ to obtain the maximum power.

*Discussion 1:* in this study, the fault and $T_a$, are arbitrary and unnecessary to satisfy any constraint. In this simulation, the fault and the Wind speed waveform are shown in Fig. 2 and Fig, 3 respectively. From the simulation results in Figs. 3-8, it is seen that even with arbitray fault and wind speed wave form, the disturbance observer and LQR controller still operated well.

## VII. CONCLUSION

This article has presented a new approach to synthesize a polynomial observer-based FTC system for WECSs. The descriptor technique is employed to transform the original WECSs system into a new framework which allows us to estimate not only aerodynamic torque, the stator currents but also fault instead of using sensors. The estimations of aerodynamic torque, the stator currents and faults which are obtained by from the observer are transmitted to the LQR controller to control WECS and eliminate the impact of faults on WECS. The designed LQR controller can control the WECS for tracking the optimum reference of the turbine rotor speed precisely to obtain the optimal power. The conditions for the controller and polynomial observer synthesis are proposed in Theorems 1-4. The simulation results show the success of the proposed method of this article. In this work, we assume that the data transmission is perfect. However, in practice, the data transmission between the output signal and controller is interrupted and delayed from time to time. Therefore, inspired from studies [40]–[42], the problem of packet dropout and delay time will be taken into account in our future work. In addition, recently, the problem of "trigger-event" have been studied for many systems such as in paper [43]. Hence, trigger-event is also a new research direction for WECS in our future work.

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