

Direct and indirect methods in determination of water retention curve of residual soils

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ABSTRACT: Many steep slopes are associated with deep groundwater table so they are located within unsaturated zone. Therefore, it is important to incorporate the unsaturated soil properties in the analysis of slope and design of slope preventive measures. The main property of the unsaturated soil is Water Retention Curve (WRC) describing the relationship between suction and water content. Many researchers developed technologies and methods in the determination of WRC which are associated with certain theory and assumption. The establishment of WRC can be performed using direct and indirect method. This paper focuses on the comparison of three different laboratory testing methods as direct methods to generate WRC. In addition, the paper summarizes the advantages and disadvantages of different methods in the determination of the air-entry value which is the important variable of WRC. The estimation of WRC using indirect method is presented as the practical approach for rapid determination of WRC.

Keywords: Rainfall-induced landslides, unsaturated soil, water retention curve, air-entry value

1 INTRODUCTION

In classical soil mechanics (e.g., excavation and foundation), geotechnical problems have been solved based on the saturated shear strength principles from Terzaghi (1936). This is admissible because of its conservative approach. Saturated condition can only be found below the groundwater table. For residual soils, unsaturated conditions need to be considered (Satyanaga and Rahardjo, 2020). Unlike saturated soil, an unsaturated soil has negative pore water pressures (Rahardjo et al., 2016). Therefore, understanding on unsaturated soil mechanics is required in solving geotechnical issues.

The water retention curve (WRC) describes the relation between the water content and the suction of the unsaturated soil (e.g., Satyanaga et al., 2017). The determination of WRC is merely done in the laboratory whereby the mathematical modelling is of huge importance to reduce the efforts in the laboratory. The WRC is highly depended on the soil type, but it has usually the form of a sigmoid function. The water content can be displayed as degree of saturation, gravimetric water content or volumetric water content. The soil suction has a range between nearly 0 and 10^6 kPa (Rahardjo et al., 2019). The boundary effect zone is the zone between zero suction and the air-entry value (AEV) (Satyanaga and Rahardjo, 2019). Basically, this zone represents the capillary fringe of groundwater in an unsaturated soil. The

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transition zone is the zone between the AEV and the residual condition. The air-entry value marks the suction where air begins to penetrate into the largest pores. The determination of the AVE can be done deterministic (Zhai & Rahardjo, 2012) or graphically (Zhai et al., 2020). On the WRC, the AEV is the interception of the tangent at the point of the maximal water content and the tangent at the point of the maximum gradient called inflection point.

This paper focuses on the comparison of different laboratory test methods as direct methods and different estimation methods to generate WRC. The scope of works includes the WRC testing in laboratory using Tempe cell, pressure plate, Fredlund cell, modified triaxial cell.

2 APPLICABLE THEORY

Laboratory experiments deliver a series of data for defining the WRC. For a practical usage, simple mathematical description of the WRC is needed. This description are empirical formulations, which can be fitted at most exactly to the measured data. Over the past century, investigators presented a whole variety from simple to sophisticated mathematical models. Statistical assessment from Sillers and Fredlund (2001) with the Akaike Information Criterion (AIC) and Leong and Rahardjo (1997) with a residual analysis concluded that Fredlund and Xing (1994) model as the best model in best fitting WRC data. Therefore, the model from Fredlund and Xing (1994) (Equation 1) was used in this study.

$$\theta(\psi, a, n, m) = C(\psi) * \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad (1)$$

whereas a, n, m are unknown fitting parameters, ψ is the matric suction ($u_a - u_w$), θ_s is the saturated water content, e is the natural number (≈ 2.72) and $C(\psi)$ is a correction factor defined as follow:

$$C(\psi) = \frac{\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\left[\ln 1 + \left(\frac{1'000'000}{\psi_r} \right) \right]} + 1 \quad (2)$$

The parameter ψ_r is a fitting parameter related to the residual suction. However, Leong and Rahardjo (1997) favour to use $C(\psi) = 1$ and this was applied in this study as well. The fitting parameters a, n and m are empirical in nature and they can be fitted with the method of the least squares whereas the parameters need to be reasonable (Fredlund & Xing, 1994). Parameter a is related to the AEV and basically defines the “position” of the curve. Parameter n expresses the ratio of drain of a soil and could be regarded as slope of the SWCC. Parameter m is related to the residual stage of the curve and is an indicator for the logarithmic symmetry of the curve. Zhai and Rahardjo (2012) proposed deterministic approach (Equations 3 and 4) to calculate AEV.

$$\psi_{AEV} = a * 0.1 \frac{3.72 * 1.31^{m+1} * \left(1 - e^{-\frac{m}{3.67}} \right)}{n * m * \ln(10)} \quad (3)$$

for correction factor = 1

$$\psi_{AEV} = a * 10^{\frac{\theta_i - \theta_s}{s_1}} \quad (4)$$

for correction factor is not equal to 1

where the parameters a, m and n are the fitting parameters from D. G. Fredlund and Xing (1994). The parameter s_1 is the slope, θ_i the volumetric water content at the inflection point a_f and θ_s the saturated residual water content.

To avoid time consuming and expensive laboratory experiments, some researchers came up with estimations for the SWCC. Such estimations can also be useful for planning the laboratory experiments (i.e., define the suction to be measured). M. D. Fredlund et al., (2002)

presented an estimation based on the grain-size distribution. This method is implemented in the software SoilVision and it is rather difficult to calculate by hand but it delivers reasonably reliable results for sands and silts. However, it has some weaknesses for clays, tills and loams.

A straight forward estimation for plastic soil is proposed by Zapata (1999) and further developed by Perera et al., (2005). This method is based on the grain-size distribution and the plasticity index (PI). The method predicts WRC based on a weighed Plasticity index, wPI for non-plastic soils (Equation 5), respectively. Equations 6 to 8 can be used to determine WRC of plastic soils.

$$wPI = PI * P_{200} \text{ (expressed as a decimal)} \quad (5)$$

$$a = 32.835\{\ln(wPI)\} + 32.438 \quad (6)$$

$$n = 1.421(wPI)^{-0.3185} \quad (7)$$

$$m = -0.2154\{\ln(wPI)\} + 0.7145 \quad (8)$$

The parameter a , n , m and ψ_r , representing the fitting parameters from the Fredlund & Xing equation in Equation 3 and 4 where $\psi_r = 500$.

3 APPLICABLE THEORY RESEARCH METHODOLOGY

The soil used in the laboratory test was obtained from a borehole at Orchard Boulevard site located at the central of Singapore which is a part of residual soil from Bukit Timah Granite. In this study undisturbed samples from a depth between 6 to 7 meters were used. The index properties tests were performed based on ASTM standard. The grain-size distribution was determined based on ASTM D422-63. The tests to obtain Atterberg limits were performed based on ASTM D4318-00. The specific gravity was carried out based on ASTM D854-02.

Three sets of WRC tests were determined in the laboratory with three different methods (Satyanaga et al., 2019): (i) Tempe cell and pressure plate; (ii) Fredlund cell and (iii) modified Triaxial device. The Tempe cell in Figure 1 was used for measurement of WRC at suction < 100 kPa. A 1 bar high entry disk was used in this experiment. A schematic diagram of a cross sectional view of Tempe cell is shown in Figure 2. The pressure plate (Figure 3) was measurement of WRC at suction > 100 kPa. A 5-bar high entry disk was applied in this experiment. The image of the open pressure plate is shown in Figure 4. A cross sectional view of pressure plate is shown in Figure 5.

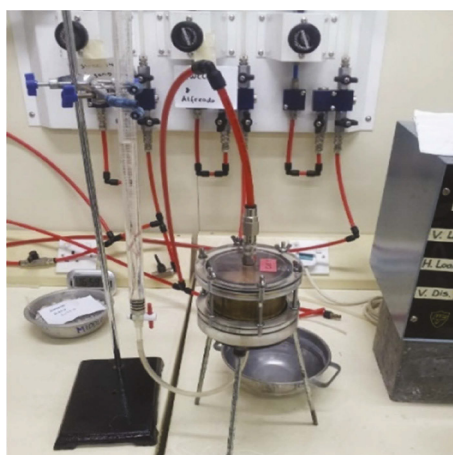


Figure 1. Tempe cell set up during WRC testing.

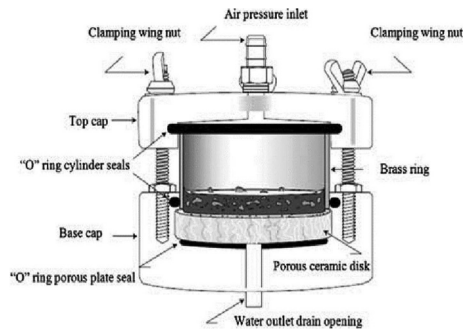


Figure 2. Cross sectional view tempe cell from Satyanaga et al. (2021a).

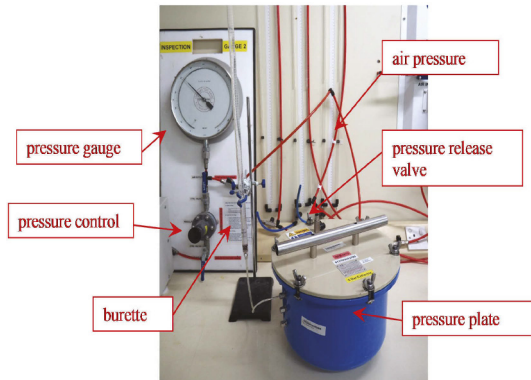


Figure 3. Pressure plate set up during WRC testing.

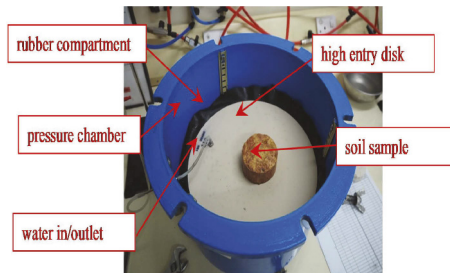


Figure 4. Open pressure plate during WRC testing.

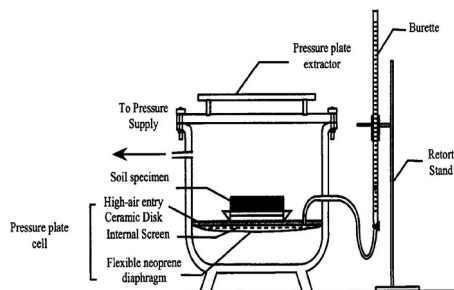


Figure 5. Cross sectional view pressure plate from Satyanaga et al. (2021b).

The previous WRC estimation from M. D. Fredlund et al. (2002) was used to define the applied matric suction, based on the grain-size distribution. The following suction stages were applied in the measurement of WRC: 1 kPa, 2 kPa, 5 kPa, 10 kPa, 20 kPa, 25 kPa, 50 kPa, 75 kPa, 90 kPa for Tempe cell testing and 200 kPa, 300 kPa, 400 kPa for pressure plate testing. The high entry disc (or porous ceramic disc) was connected via a tube to a burette filled with de-aired and distilled water. The purpose of this connection was to keep the disc during the experiment saturated. Air pressure was applied from the top of the cell. The matric suction resulted from the difference between the applied air pressure and the piezometric head in the burette. The equalization was controlled by measuring the mass of the sample by weighting the closed cell.

The Fredlund cell set up during the WRC testing is shown in Figure 6. A 5-bar high entry disk was applied in the experiment. The set-up of the open Fredlund cell is shown in Figure 7. A schematic diagram of a cross sectional view is shown in Figure 8.

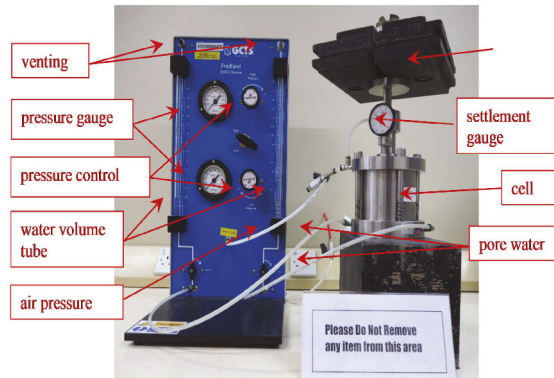


Figure 6. Fredlund cell set up during WRC testing.

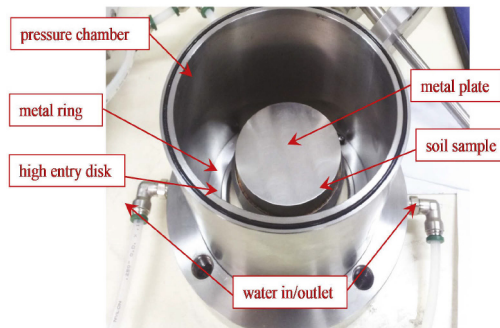


Figure 7. Open Fredlund cell before WRC testing is started.

The different suction stages were estimated according to the findings of the first suction measurements of the Tempe cell. The following suction stages were used in the measurement of WRC: 5 kPa, 10 kPa, 25 kPa, 50 kPa, 75 kPa, 100 kPa, 150 kPa, 200 kPa, 300 kPa, 400 kPa and 500 kPa. Before the testing, the water volume tube was calibrated for determining the following correction factors. They are the ratio between mass water and the height in the tube, and the volume change in the tube which can directly be converted into water mass or water volume. Air pressure was applied from the top of the cell. The matric suction resulted from the difference between the applied air pressure and the piezometric head in the volume tube. The equalization was controlled by reading the volume change.

The modified triaxial device for the WRC testing is shown in Figure 9. A 5-bar high entry disk was applied in this experiment. The image of open cell is shown in Figure 10.

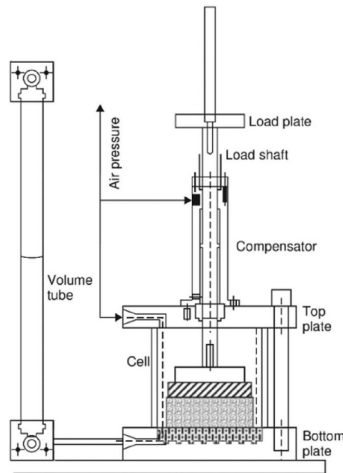


Figure 8. Cross sectional view Fredlund cell (after Fredlund et al., 2012).

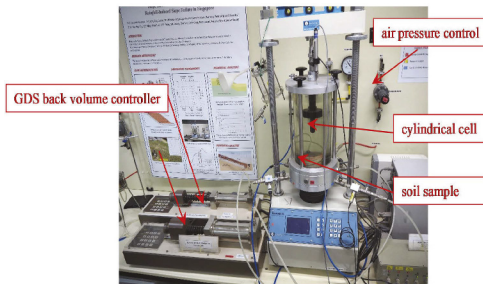


Figure 9. Modified triaxial apparatus set up during WRC testing.

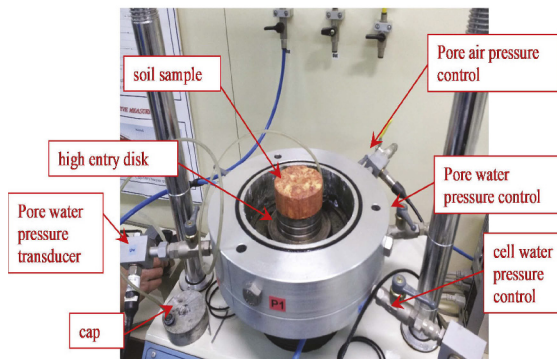


Figure 10. Open triaxial apparatus prior to WRC testing.

The different suction stages were estimated according to the findings of the suction measurements of the Tempe cell and Fredlund cell. The following suction stages were used in the measurement: 10 kPa, 25 kPa, 50 kPa, 75 kPa, 100 kPa, 150 kPa, 200 kPa, 300 kPa, 400 kPa and 500 kPa. Soil suction could be applied to the sample by reducing the pore water back pressure and keeping the air pressure on 490 kPa. As soon as no significant change in the back pressure volume change was observed, the sample was regarded as equalized, and the soil suction could be increased. The different readings were done automatically by transducers attached to the air pressure control, pore water pressure and cell pressure control. Since the

specimen was from a depth of 6 to 7m, confining pressure of 90 kPa was applied to the specimen to simulate the conditions in such depth.

4 RESULTS AND DISCUSSIONS

The WRC from direct and indirect method are displayed in Figure 11. The duration of WRC test using Tempe cell and pressure plate are around 5 weeks. Totally, 175 measurements were made at 8 different matric suctions. The curve was fitted with Fredlund and Xing (1994) by the sum of the least square with the correction factor $C = 1$ on the degree of saturation points. In order to get reasonable results, the fitting parameter were limited with the values $a = 500$, $n \leq 6$ and $m \leq 6$. Following parameter were determined $a = 500$, $n = 0.716$, $m = 2.475$, $\theta_s = 49\%$. The duration of WRC test using Fredlund cell are around 6 weeks. Totally, 152 measurements were made at 11 different matric suctions. $a = 500$, $n = 1.048$, $m = 0.974$, $\theta_s = 51\%$. The duration of WRC test using modified Triaxial cell test are around 5 weeks. Totally, 6 different matric suctions were measured. $a = 500$, $n = 1.030$, $m = 1.472$, $\theta_s = 50\%$. The estimation based on Zapata resulted in following SWCC. $a = 131$, $n = 0.548$, $m = 0.070$, $\theta_s = 50\%$, $C_r = 500$. The estimation based on Fredlund & Wilson was done with the software SoilVision and resulted in following SWCC. $a = 583$, $n = 0.722$, $m = 0.988$, $\theta_s = 50\%$, $C_r = 500$.

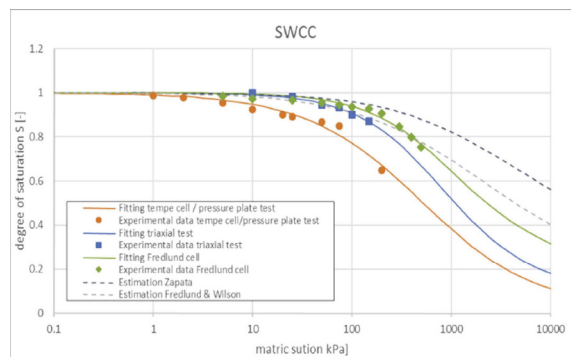


Figure 11. Overview of determined and estimated SWCC.

Figure 11 shows that different methods result in different curves. It shows that the application of confining pressure has a significant influence on the SWCC. The water content is higher on the same suction when confining pressure is applied. The two estimations perform up to 200 kPa quite well compared with the experimental data from the Fredlund Cell and the triaxial test. It seems that estimation based on Perera et al. (2005) overestimates the data. However, both curves seem to be too flat. This has a significant influence on the bending point and therefore on the AEV and on higher suctions.

As the determination of the WRC with the Fredlund cell could be done without any problems, this WRC is assumed to be the most accurate. Therefore, the air entry value (114 kPa) was determined based on Zhai and Rahardjo (2012). In order to show the variability and uncertainties, which comes with the AEV determining with a fitting by Fredlund and Xing (1994) and $C=1$, the AEV of different methods and different parameters were analyzed. Then, the comparisons of AEV were made with the following methods: (i) Determination (Zhai & Rahardjo, 2012)/ graphically estimation (Vanapalli et al., 1998); (ii) Fitting methods: least square / R^2 / least absolute deviation/sum of weighted least squares; (iii) SWCC determination/estimation: Tempe cell & Pressure plate / Fredlund cell / Triaxial device / Zapata estimation / Fredlund & Wilson estimation; (iv) Fitting degree of saturation / volumetric water content; (v) Fitting limits. The obtained value above (114 kPa) and the applied approach (i.e., determination from the Fredlund & Xing fitted saturated water content based on a least square fitting with given limits) were regarded as standard. The AEVs were analyzed by

varying one element. The AEV was determined mathematically. The other, more classical, approach is a graphical estimation and is described by Vanapalli et al. (1998). The AEV is the interception of the tangent at the point of the maximal water content (i.e., $S=1$) and the tangent at the inflection point (a_f). The method is illustrated in Figure 12, whereby the dotted line is the function of the slope. The determined *AEV* from graphical method is 143 kPa. All the different AEV are summarized in Figure 13.

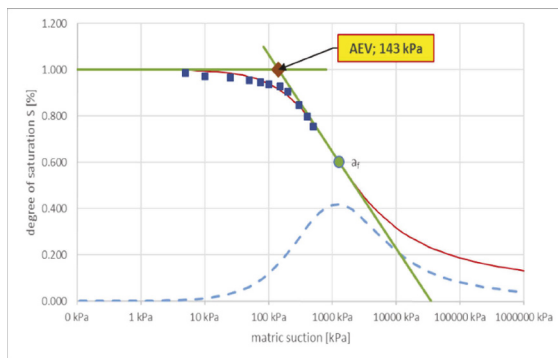


Figure 12. Determination of AEV based on graphical method.

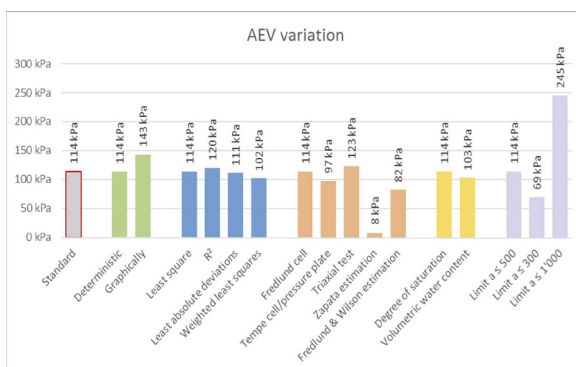


Figure 13. AEV from different method.

5 CONCLUSIONS

The following conclusions can be deduced from this study

- The biggest influence on the *AEV* seems to have a variation of the limits, whereby as described above, the variation is much higher if the *AEV* is determined mathematically.
- There is a significant difference between determining the *AEV* mathematically and graphically.
- The variation due to different fitting methods is the smallest and does not have an influence on the shape of the SWCC.
- With different laboratory methods, the *AEV* varies. Whereby an applied confining pressure results in a higher *AEV*.
- The estimation methods in this study underestimated the *AEV*. The shape of the SWCC estimated by Perera et al. (2005) cannot lead to a reliable estimation of the *AEV*.
- The *AEV* determined based on a volumetric water content fitting is smaller than the *AEV* obtained from a degree of saturation fitting.

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