Capstone Project

Meirzhan Kurmanov

April 5, 2026

1 BACKGROUND INFORMATION

In the real world, the formation of networks—whether political alliances or conflict coalitions—is not something that emerges purely at random. They arise from the strategic choices of individual actors (agents) who often weigh the costs and benefits of forming or dissolving ties.

There are two main aspects that define this perspective:

1. Modeling of costs and benefits
2. Prediction of outcomes based on the incentives

The very first step is to define what the *network* is. We start with a set $N = \{1, 2, ..., n\}$ of n nodes (agents). The canonical and simplest network structure is an *undirected* and *unweighted graph*, in which two nodes are either connected or not. A *graph* is a tuple (N, g) consisting of a set of nodes N and a set of edges between the agents denoted by g. An edge ij represents the link connecting nodes (agents) i and j. We write $ij \in g$ to indicate that i and j are linked under the network g.

Such a network relationship can be represented by an *adjacency matrix* of size $n \times n$, where the entry $g_{ij} = 1$ means that $ij \in g$ and $g_{ij} = 0$ otherwise. The notation $g + ij$ represents the network obtained by adding link ij to an existing network g, similarly, let $g - ij$ represent the network obtained by deleting the link ij from the network.

In addition, there is also a *signed* graph (network) where the entries of the adjacency matrix can take values in $\{-1, 0, 1\}$, where -1 represents a negative relationship (e.g., enmity), 0 - no relationship, and 1 - positive relationship (e.g., friendship).

Let $G(N)$ denote the set of all undirected and unweighted networks on the agent set N. Each agent in such a network has its own preferences and utility associated with the network structure. The overall benefit that a player receives from a network is modeled by a *utility function* or *payoff function*. That is, the payoff to player i is represented by a function $u_i : G(N) \to \mathbb{R}$, where $u_i(g)$ represents the net benefit that i receives from the network g. We define a *payoff structure for network g* as the vector-valued function $u : G(N) \to \mathbb{R}^n$, $g \mapsto (u_i(g))_{i \in N}$.

Now that we have defined what the network and utility functions are, we can proceed to the study of the properties of networks that arise from the strategic behavior of agents.

The first very important property is *pairwise stability* (Jackson, 2008)—a network g is *pairwise stable* if

I. for all $ij \in g,\ u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$

II. for all $ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$

i.e. given the network g no agent wants to form or sever a link within that network, because it won't increase their utility.

This begs the question—if a human society achieves pairwise stability (could be a local community), why would it soon dissolve as we have seen usually happens in the real world (even under the assumption that no catastrophic events or death of the participants happen)? That's where the notion of network efficiency comes into play. A network g is *efficient* (Jackson, 2008) relative to a profile of utility functions $(u_1, ...u_n)$ if for all $g' \in G(N)$

$$\sum_i u_i(g) \geq \sum_i u_i(g')$$

The main problem is that the stability of the network doesn't imply its efficiency and vice versa. For example, just because every participant of the network feels "satisfied" with the current state of affairs (i.e., nobody wants to either add or remove the link), the network could still be inefficient, meaning that the total sum of agents' utilities could still be maximized. And that's a huge problem, because it might so happen that a stable network doesn't benefit the community as much as it could, so it may naturally leave its stable state in the pursuit of maximizing its efficiency. One illustrative example can clearly show this, but for that, we need to add the definitions of the *Pareto efficient* network and *Pareto domination*. A network g is *Pareto efficient* relative to $(u_1, ..., u_n)$ if there doesn't exist $g' \in G(N)$ such that $u_i(g') \geq u_i(g)$ for all i with strict inequality for some i (Jackson, 2008). One network *Pareto dominates* another if it leads to a weakly higher payoff for all individuals (agents) (Jackson, 2008). Now we can show how one network can first be efficient but then evolve into a stable one, and then be neither of them.

It's easy to see that throughout the evolution of the network, it attained efficiency and stability at different stages of its life cycle. However, the concept of Pareto efficiency and pairwise stability alone can't give us a detailed picture of the real world, because human relations are much more complicated than that. We can add the notion of the distance-based utility function to account for how the creation of the indirect connection can benefit/harm the agent.

Let $b : \{1, ..., n-1\} \rightarrow \mathbb{R}$ denote the benefit that the player gets from (indirect) connections as a function of the distance between the players. The *distance-based utility model* (DBUM) is one where a player's utility can be written as

$$u_i(g) = \sum_{j \neq i : j \in N^{n-1}(g)} [b(l_{ij}(g))] - d_i(g)c$$

Here, the $l_{ij}(g)$ is the shortest path length between i and j and $d_i(g)$ is the number of links that i has formed (it's degree centrality, essentially). Let $b(k) > b(k+1) > 0$ for any k and $c \geq 0$. This embodies the idea that a player sees higher benefits for having a lower distance to other players (Jackson, 2008).

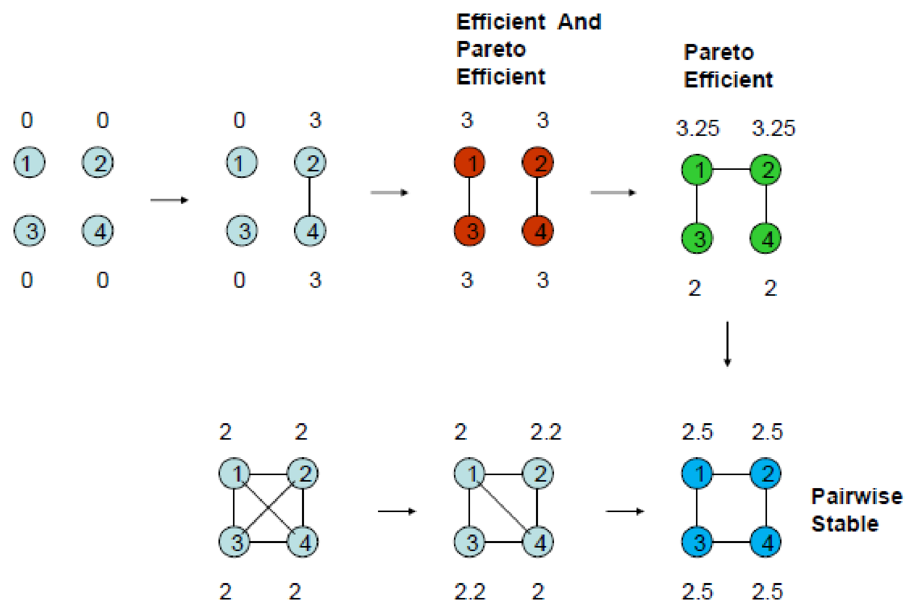

Figure 1: An Example of efficient and Pareto efficient, Pareto efficient and pairwise stable network (Jackson, 2008)

In Jackson (2008), it is then proven that for some parameters of the distance-based utility model, the complete network is efficient, but not stable. For others, who have higher costs, the star network might be stable but not efficient, and sometimes, no efficient network is stable. So, we come to the conclusion that even when agents behave rationally, the equilibrium structure of the network can be inefficient.

We can elaborate further and provide one of the common structures of benefit function ($b(x) = \delta^x$) that includes the decay parameter $\delta \in (0, 1)$. We thus obtain a revised form of DBUM:

$$u_i(g) = \sum_{j \neq i} \delta^{l_{ij}(g)} - cd_i(g)$$

Based on reasoning from Jackson, 2008, we can outline the following table of examples:

Parameters c, δ	**Efficient network**	**Stable**
Low c, high δ	Complete	Complete
Moderate c	Star	Could be star, could be empty
High c	Empty	Empty

But in many intermediate cases, as we vary c and δ, the efficient network does not coincide with the stable one.

So far, we understood the difference between stable and efficient networks, but to quantify the degree of inefficiency that stable networks may cause in the setup of pre-defined agents and utility functions, one must appeal to the concept of *price of anarchy*. If we define the total welfare of the network $W(g)$ as $W(g) = \sum_i u_i(g)$ and denote the stable network with least efficiency as g_s and network structure that maximizes the total welfare as g^* then the *price of anarchy* (PoA) is defined as

$$\text{PoA} = \frac{W(g_s)}{W(g^*)},\ 0 \leq \text{PoA} \leq 1$$

Using that concept, one can define an upper bound of how much welfare is lost due to the "selfish" actions of agents, who try to achieve a stable network as is usually opposed to the efficient one.

Now we can take it even further and consider a situation where an agent might be against his neighbor forming a link, because it would lead to a negative externality (i.e. it would decrease the agent's utility, assuming negative values in b are allowed). For example, if your friend has befriended a person you don't like, that would be considered a negative externality for you. You would probably try to persuade your friend to not form a link while your competitor would try to convince your friend to do the opposite. Such local and conflicting incentives can't be captured in a static analytical framework and that's the part where *co-author based model* would come in handy. Jackson, 2008

In the *co-author based model* (CABM), one can think of the links (edges) as the "collaboration" between the agents - a collaboration that has certain utility value that might as well be negative. The utility function of the agent is now represented in the form:

$$u_i(g) = \sum_{j:ij \in g} f(p_i, p_j) - c_i \cdot d_i(g),$$

where

- $f(p_i, p_j)$ - output or benefit from a collaboration between i and j, depending on their respective qualities p_i, p_j
- c_i - cost for collaboration for agent i
- $d_i(g)$ - number of links the agent i has
- g - a network (co-authorship network, if you will)

> Remark: You can think of CABM as a generalization of DBUM, because you can just set $f(p_i, p_j) = \delta^{d_{ij}(g)}$ and homogenize $c_i = c$ to then obtain $u_i(g) = \sum_{j \neq i} \delta^{l_{ij}(g)} - cd_i(g)$ - something we have already seen before.

Being a generalization, it accounts for more nuances that might pop up in the network formation. Namely, the assumptions of homogeneity, global & network-wide interaction, and only positive externalities are lifted, which makes this type of model much more "powerful" in a sense that it can deliver more realistic insights. However, we can already see that due to the locality of the interaction and the absence of homogeneity, we lose the analytical tractability of the network, as we can't derive a closed-form expression for key outcomes or reason about them using simple analysis.

In fact, there's a sort of "intermediate" solution in the Islands-Connections Model (ICM) (Jackson, 2008), which can be viewed as a structured restriction of the CABM. In that model, the heterogeneity is grouped, meaning that agents that are in the same group (island) share identical linking costs and benefit functions, while cross-island links have different parameters. Here, you can solve the problem analytically by imposing a homogeneity assumption on each separate group and analyzing them as separate networks. You can then possibly use the result to analyze the inter-group interaction. The overall utility of a player i in network g then becomes

$$u_i(g) = \sum_{j \neq i : l(i,j) \leq D} \delta^{l(i,j)} - \sum_{j : ij \in g} c_{ij}$$

Where $c_{ij} = c$ if i and j are on the same island and C otherwise. Here, $C > c > 0$ and D is the diameter of the group. It means that it is more costly for an agent to form a link outside of its island.

The tension between efficiency and stability remains an issue. As a solution, Jackson (2008) proposes a few strategies. In very brief statements, these are

1. Subsidies/taxes on links: Adjust the cost or benefits associated with individual links so that you "fine-tune" the network until its stable configuration gets sufficiently close to the efficient one.
2. Component balance: transfers should occur only within connected components, which means that no net transfers happen across separate groups of players. This ensures that the total value created by the component is distributed among its components.
3. Equal treatment of equals: If two agents are in symmetric positions, then they should receive the same allocation.

2 AGENT-BASED MODELING

Agent-Based Modeling is a simulation technique in which a system is represented by a collection of individual agents (denote the set of agents by $N = \{1, \ldots, n\}$) — each following a set of behavioral rules and interacting with other agents and their environment — to study how local interactions give rise to emergent, system-level patterns and dynamics.

To better study the implementation of agent-based modeling in the context of network formation, we first need to define some key concepts that will be used throughout the rest of the section.

First, let $\mathcal{W}$ denote the set of all possible states that the system can attain. Each state $w(t) \in \mathcal{W}$ represents a specific configuration that the system can be in at a given point in time t. Consequently, let $\mathcal{W}_i$ denote the set of all possible states that agent i can attain and $w_i(t) \in \mathcal{W}_i$ represent the specific configuration of agent i at time t.

Due to limited information, an agent can only partially observe the state of the system, denote the agent's knowledge at time t as $o_i(t) \in O_i$, where O_i is the set of all possible observations that agent i can make about the system.

Remark: by defining observation projector $\pi_i : \mathcal{W} \to O_i$, you can map the system's state to the agent's observation such that $o_i(t) = \pi_i(w(t))$

Then, by denoting the set of mixed actions available to agent i as $\Delta(A_i)$, one can construct a decision function $f_i : O_i \to \Delta(A_i)$ that maps the agent's observations to a probability distribution over its possible actions. On top of that you can define a transition function $\tau_i : \mathcal{W}_i \times A_i \to \mathcal{W}_i$ that describes how the agent's state evolves over time based on its current state and the actions taken by the agent, where A_i is the set of all possible actions that agent i can take.

With all these constructions in mind, one can think of an agent in the network as an *automaton (or machine)* that follows a set of pre-defined rules to make decisions regarding its connections with other agents in the network. An *automaton*, in that case, is a tuple $(\mathcal{W}_i, w_i^0, f_i, \tau_i)$ which consists of a set of states $\mathcal{W}_i$, an initial state $w_i^0 = w_i(0)$, a decision function f_i, and a transition function τ_i.

For convenience, it's useful to generalize the notion of automaton to the entire network of agents. To accomplish that, we represent the set of states of the entire system as $\mathcal{W} = \mathcal{G} \times \prod_{i \in N} \mathcal{W}_i$ and the state of the system at time t as $w(t) = (G_t, (w_i(t))_{i \in N}) \in \mathcal{W}$. The decision function will then be defined as $f : \mathcal{W} \to \prod_{i \in N} \Delta(A_i)$ and the transition function as $\tau : \mathcal{W} \times A \to \mathcal{W}$, where $A = \prod_{j \in N} A_j$ denotes the set of joint action profiles.

In combination, these generalized notions allow us to model the behavior of the entire network of agents as a single automaton $(\mathcal{W}, w(t), f, \tau)$.

The use of ABM is particularly useful in the context of this project, because conflict networks form through decentralized, heterogeneous, and information-limited actors whose local interactions generate emergent coalition/link patterns that are hard to derive in closed form as in Jackson, 2008. In particular, it allows one to lift several restrictive assumptions that are usually imposed in analytical models of network formation, such as homogeneity of agents, global interaction, and perfect information (Axtell and Farmer, 2022). Furthermore, macro patterns of network formation are usually imposed in analytical models, while in agent-based modeling, they can emerge endogenously from the local interactions of agents. By observing the emergence of such networks, one can derive insights about the dynamics of network formation that are hard to obtain through traditional analytical methods.

However, it's important to note that ABM also has its limitations; the two most important ones are *curse of dimensionality* and *forecasting/initialization* problems (Axtell and Farmer, 2022). It is often the case that ABM has more parameters than analytical models. However, as the size of the parameter space grows, so does the cost of its exploration, as it does so exponentially (Axtell and Farmer, 2022) So, the challenge then becomes how to balance the complexity of the model with the feasibility of its exploration. Furthermore, to have any predictive power, ABM requires, at the very least, the detailed data about the initial state of the system and the behavior of individual agents, which may not always be available or accurate (Axtell and Farmer, 2022). This particular limitation implies that more time and effort must be spent on the data collection and processing, if we aim to derive empirically meaningful insights from the model.

3 PROJECT DESCRIPTION

Our broader goal is to analyze known game-theoretic network formation models with ABM and explore what different or new insights we can derive from it by fully or partially lifting restrictive assumptions discussed before. Then, we aim to use it for the study of conflict network formation. In particular, we are interested in understanding how conflict networks form and evolve over time, given the local interactions of agents with limited information and heterogeneous characteristics. One particular application of such models within the scope of our study is the conflict in Sudan from 2003 to 2011. By using empirical data from the ACLED database, we aim to construct an agent-based model that captures the dynamics of conflict network formation in Sudan during that period.

4 METHODOLOGY

4.1 Computational Experiment: Agent-Based Network Formation under Distance-Based Utility

This section presents the first computational experiment designed to validate the analytical predictions of the distance-based utility model through agent-based simulation. The goal of the experiment is to examine how network structures emerge endogenously as agents locally optimize their utilities under varying decay, cost parameters, and myopic radii.

4.1.1 Model Setup

We consider a population of $n = 10$ agents represented as nodes in the network formation g, initially empty. Agents form or sever links over time according to a myopic, utility-increasing rule. Time is discrete, and at each step a single agent is selected uniformly at random to evaluate a potential local network modification. Agent preferences are represented by a distance-based utility function, where benefits decay exponentially with graph distance and each direct link has a fixed cost. Agents are assumed to only evaluate marginal changes to their current network position.

4.1.2 Distance-Based Utility Function

For agent i, utility is defined as

$$u_i(g) = \sum_{j \neq i,\ d(i,j) \leq r} \delta^{d(i,j)} - c \cdot \deg(i),$$

where $d(i,j)$ is the shortest-path distance between agents i and j, $\delta \in (0,1)$ is the decay parameter, $c > 0$ is the per-link maintenance cost, and $\deg(i)$ is the degree centrality of agent i. Agent's utility is evaluated within a fixed radius r, ensuring myopic behavior. Only nodes with $d(i,j) \leq r$ contribute to the benefit term.

4.1.3 Agent-Based Dynamics

At each iteration, the algorithm performs the following steps:

1. Select an agent i uniformly at random.
2. With probability p, attempt to add a link; otherwise, attempt to remove an existing link.
3. If adding a link, select a non-neighbor j uniformly at random.

4. If removing a link, select an existing neighbor j uniformly at random.
5. Compute the marginal utility change induced by the proposed action.
6. Implement the action if it strictly increases utility (for both agents in the case of addition).

This process induces a stochastic best-response dynamic that converges to pairwise stable networks, meaning that the resultant network will eventually satisfy the conditions of pairwise stability.

4.1.4 Pseudocode Description

Algorithm 1 Distance-Based Utility Network Formation

```
Initialize graph (N, g) with n isolated nodes
for t = 1 to T do
    Select agent i ~ Uniform({1, ..., n})
    if random draw (from 0 to 1) < p then                ▷ Attempt link addition
        Select j uniformly from non-neighbors of i
        if Δu_i^{add}(i, j) > 0 then
            Add edge (i, j) to g
        end if
    else                                                 ▷ Attempt link removal
        Select j uniformly from neighbors of i
        if Δu_i^{remove}(i, j) > 0 then
            Remove edge (i, j) from g
        end if
    end if
end for
```

4.1.5 Parameter Grid and Experimental Design

To test the analytical predictions, simulations are run over a grid of (δ, c, r) parameter triples that include low, intermediate, and high cost-benefit settings. For each parameter setting:

- The network is reset to an empty graph
- A fixed number of update steps is performed
- The resulting network structure is recorded

4.1.6 Visualization and Structural Classification

Rather than tracking individual edges, the experiment focuses on degree distributions as a sufficient statistic for network structure. At each time step, a bar chart displays the degree of each node. To rigorously define the types of networks that emerge, we are going to use the following statistics about our network structure:

Let n denote the number of agents in network,

$d_{max} = max_i(d_i)$ - maximum degree centrality observed in the network

$d_{min} = min_i(d_i)$ - minimum degree centrality observed in the network

m = median(d) - median degree centrality

$|E|$ - number of edges in network

Networks are classified into the following structures based on their degree sequences:

- **Empty network**: $|E| = 0$
- **Complete network**: $|E| = \binom{n}{2}$
- **Star network**: $d_{max} = n - 1$ and all other agents have degree 1
- **Regular network**: $d_{max} - d_{min} = 0$
- **Near-regular network**: $1 \leq d_{max} - d_{min} \leq 2$
- **Hub-dominated network (star-like)**: $m > 0$ and $\frac{d_{max}}{m} \geq \rho$ with $\rho = 1.5$ (weak) and $\rho = 3.3$ (strong)

This allows for a direct comparison between simulated outcomes and the analytical predictions summarized in table (Jackson, 2008):

Parameters c, δ	**Efficient network**	**Stable**
Low c, high δ	Complete	Complete
Moderate c	Star	Could be star, could be empty
High c	Empty	Empty

We evaluate the model over a structured grid of decay parameters, costs and radii. Specifically, the decay parameter δ is fixed at {0.35, 0.65, 0.95}, the cost parameter c is at $\{0.1, 0.2, 0.3, ..., 0.9, 1.0\}$, and radii are at {3, 5, 8, 10} accounting for low, intermediate, and high values. In total, we obtain $3 \times 10 \times 4 = 120$ parameter combinations which are then categorized and presented in the form of table.

We categorize values of δ into three groups:

- **Low**: $\delta = 0.35$
- **Intermediate**: $\delta = 0.65$
- **High**: $\delta = 0.95$

Similarly, we categorize values of c into three groups:

- **Low cost**: $c \in \{0.1, 0.2, 0.3\}$
- **Intermediate cost**: $c \in \{0.4, 0.5, 0.6\}$

- **High cost**: $c \in \{0.7, 0.8, 0.9, 1.0\}$

5 RESULTS

5.1 Computational Experiment I

5.1.1 Tabular presentation of results

We first compare the distribution of network structures that emerge under different values of myopic radius R.

Table 1: Emerging network structures across different myopic radii

Myopic Radius	**Complete**	**Star-like**	**Weakly Star-like**	**Regular**	**Near-regular**	**Empty**	**Other**
3	4	2	6	0	7	12	2
5	4	2	9	0	7	12	1
8	4	3	8	0	7	12	1
10	4	4	6	0	5	12	4

Now we observe the same distribution for different values of decay parameter δ and cost parameter c.

Table 2: Emerging network structures across different decay parameters δ

Decay parameter δ	**Complete**	**Star-like**	**Weakly Star-like**	**Regular**	**Near-regular**	**Empty**	**Other**
High	0	11	21	0	16	4	2
Intermediate	8	0	5	0	7	16	6
Low	8	0	3	0	3	28	0

Table 3: Emerging network structures across different link costs c

Cost c	**Complete**	**Star-like**	**Weakly Star-like**	**Regular**	**Near-regular**	**Empty**	**Other**
High	0	4	7	0	5	36	0
Intermediate	0	5	10	0	10	12	6
Low	16	2	12	0	11	0	2

6 DISCUSSION

In the following section we discuss how change of parameters of the model affects the dynamic of network formation and its resulting topology.

6.1 Change of radius

It can be observed from the data that distribution of network formations across different radii is very stable—Complete (always 4), Near-regular (5-7), and Empty (always 12). The only trend that appears is in Star-like cluster: as r increases from 3 to 10, the number of Star-like networks monotonically increases from 2 to 4, while Weakly Star-like fluctuates without a clear trend. It can point to the fact that the relevant strategic signals are contained within a small neighborhood of agents, meaning that having global information in DBUM is not a defining factor.

6.2 Change of decay parameter δ

High decay parameter means that the indirect connections decay slowly, meaning that the distant nodes are still valuable. This makes it rational for agents to invest in connections through intermediate actors which strongly favors hub-dominated structures where a single node provides short paths to everyone at low individual cost.

The complete absence of empty networks could be explained by the fact that under high δ, the indirect connections between the nodes are valuable to the point where marginal benefit of a direct link over an indirect path is insufficient to justify its cost.

There appears to be a critical δ^* in between intermediate and high values of decay, below which the incentive to form hub structures collapses entirely. It could explain why switching from high to intermediate decay value decreases the number of Star-like networks from 11 to 0, and Weakly Star-like networks from 21 to 5.

It is worth noting that the number of complete networks remains at 8 under intermediate and low decay parameters while it is 0 for high δ. It could be explained by the fact that for lower decay values, the benefit of intermediate connections decreases thus making direct connections the main source of utility.

6.3 Change of link cost c

The complete elimination of empty networks under low cost (0 out of 36) and complete elimination of complete networks under high cost (0 out of 36) indicates that cost is the primary factor that affects connectivity and isolation of the network.

As expected, for higher costs, if the cost of forming a single link is higher than the benefit of forming a link then the network structure won't be able to escape the empty formation.

Across all cost levels, regular networks never emerged. It could imply that regular networks, despite being theoretically appealing as symmetric equilibria, are rarely stable under best-response dynamic with distance-based utility model.

7 CONCLUSION

In conclusion, this project examined the formation of networks through the lens of game-theoretic models and agent-based simulations. We began by first establishing the foundational concepts of network theory and introduced the distance-based utility model as a tractable framework for demonstrating how agents weigh the costs and benefits of both direct and indirect connections.

The computational experiment demonstrated that agent-based simulation can successfully reproduce and extend analytical predictions of the distance-based utility model. The decay parameter δ emerged as the primary determinant of network architecture. Meanwhile, link cost c determined whether agents formed sparse or no connections at all, with low cost being a necessary condition for complete networks and high cost blocking formation of any links. The myopic radius r had a negligible effect on the distribution of emerging network topologies—it suggests that strategically relevant information is concentrated locally even in the distance-based setting.

Across all parameters, regular networks never emerged, which points to their instability under sequential best-response dynamics. This particular finding carries a broader insight: even under simple and well-understood models, the endogenous dynamics of network formation can produce outcomes that are difficult to anticipate from static equilibrium analysis alone.

The broader goal of this project is to carry this methodology into empirical study of the conflict network formation in Sudan, where heterogeneity of actors, local interactions and incomplete information make agent-based modeling an appropriate tool. Future work will explore other, more complex utility models and empirical data from ACLED to derive insights about the dynamics of conflict network formation.