

SOME BASIC BVPS FOR COMPLEX PDES AND METHOD OF REFLECTION ON EXAMPLE OF TWO IRREGULAR DOMAINS

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Introduction. The theory of BVPs for Complex PDEs were developed by many famous mathematicians such as B.Riemann [1], D.Hilbert [2], F.D.Gakhov [3], I.N.Muskhelishvili [4], I.Vekua [5] etc. The explicit solutions were found for some particular domains and have applications in engineering, mathematical physics, fluid dynamics etc. The basic BVPs include the Schwarz, the Neumann and the Dirichlet problems for the Cauchy-Riemann (first order) and the Poisson (second order) equations.

Methods. The main way to solve such problems requires obtaining the Schwarz kernel and the Green and Neumann functions. The method of reflection or "parqueting" can be successfully applied to certain domains, which are simply connected e.g. unit disc [6,7,9] or multiply connected e.g. concentric ring [10], regular or irregular domains [8, 11]. The method includes the reflection of the given domain at all parts of the boundary. Continuously repeated procedure leads to the covered complex plane [13]. The main tool for treating BVPs for the Cauchy-Riemann equation is the Cauchy-Pompeiu representation formula; for the Poisson equation the Green and Neumann representation formulas are used. For the inhomogeneous cases the Pompeiu operator should be treated as to satisfy the boundary condition.

Results and discussion. The advantage of the method described is the fact that solution of the problems can be obtained analytically and presented in explicit form. This fact will be presented on example of two different irregular domains such as quarter ring and half hexagon [11,12]. Here the the Schwarz, the Dirichlet and the Neumann problems are solved, that is, the relative Schwarz kernel, the harmonic Green and Neumann functions were obtained according to the method of "parqueting".

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