Financial Intermediation and Information Production in a New Keynesian Model

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Abstract

This study develops a novel New Keynesian framework incorporating information-producing mutual funds and market segmentation to investigate how passive investment trends may interact with monetary policy transmission. The model highlights fundamental tensions between the fund's information production incentives and its intermediation role. Analysis reveals these tensions manifest in contrasting steady-state corner solutions for fund holdings (depending on model specification) alongside a consistent tendency towards minimal fund share allocation. While these structural complexities lead to significant computational challenges, including violations of standard stability conditions that preclude robust dynamic analysis, the steady-state findings offer valuable insights. They suggest that intermediaries may optimize towards extreme allocations, potentially leading to regime-dependent market behavior rather than smooth adjustments, which carries implications for central banks operating within evolving financial landscapes and identifies avenues for future research.

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1. Introduction

The investment landscape has changed dramatically over the past decade, exhibiting a significant shift in the balance between active and passive investment funds. Passive funds reached approximately 37% of global market share by 2022, a substantial increase from just 19% in 2013 (Kerzérho, 2023). This development was not merely a temporary trend but represented a clear acceleration, since, for the first time in history, by the end of 2023, U.S. passive funds overtook active funds in terms of market capitalization (Morningstar Direct data, 2024).

This structural transformation raises critical questions about broader macroeconomic effects, particularly concerning market efficiency, which has traditionally depended upon active investment strategies. Jaquart et al. (2023) highlight the complex dynamics involved; their research demonstrates that higher proportions of active investment improve fundamental market efficiency, although exhibiting diminishing returns in markets already dominated by active investors. What is perhaps more concerning is their finding that markets with large shares of passive investors can unintentionally fuel technical price bubbles, potentially culminating in market failure.

This situation creates a paradox within financial markets: higher ratios of active to passive investment enhance market efficiency but simultaneously reduce active investors' returns by rendering passive instruments more attractive. Conversely, lower market efficiency tends to improve active investors' potential returns but makes passive funds—an increasingly popular choice—less profitable. While passive funds hold appeal for retail investors (who constitute most of the investor base), these same passive funds may act to undermine the market efficiency benefiting all participants. Active funds, hence, remain crucial for maintaining healthy stock market functioning, creating a scenario where actions benefiting individual investors could ultimately prove detrimental to the market itself.

These dynamics suggest a cyclical pattern should emerge: as passive assets gain dominance, market efficiency would decline, which makes it more likely for active investors to outperform the market. Naturally, this outperformance ought to increase the share of active assets, leading towards improvements in market efficiency and eventually rendering passive investments appealing once more, primarily due to their lower costs. However, contrary to what intuition might predict, empirical evidence indicates that passive fund market share continues to grow regardless of prevailing market conditions.

This paper undertakes an examination of this phenomenon's implications for monetary policy transmission. Despite the growing importance associated with passive investment, limited research currently exists concerning how this structural change within financial markets affects the capacity of central banks to influence the real economy. To investigate this relationship, this paper aims to develop a novel New Keynesian model featuring a financial sector engaged in information production. The framework developed herein incorporates a representative mutual fund that adheres to a three-stage optimization process: information production, premium setting, and portfolio allocation. Through this mechanism, the fund's information production activities enhance the precision of firms' observed productivity signals. This reduction in productivity noise, as a result, diminishes the variance of realized marginal costs, thereby improving the efficiency of firms' contemporaneous production decisions and establishing a direct channel linking financial market structure to real economy.

The framework presented here is designed to facilitate the addressing of critical policy questions: Does the rise of passive investment uniformly weaken monetary policy transmission, as intuition might suggest? How might the balance between active and passive investment influence the strength of various transmission channels? Could a threshold exist beyond which passive investment concentration might undermine central bank effectiveness? While the model is structured to investigate these quantitative relationships, the computational challenges encountered in the analysis, particularly the violation of stability conditions, prevented the derivation of robust dynamic results. Consequently, definitive conclusions regarding the precise impact of passive investment share on policy effectiveness cannot be drawn from the current simulations, though the theoretical structure itself highlights potential mechanisms and complexities relevant to these questions. Notably, this study represents an important first step towards analyzing contemporary financial phenomena, specifically the rise of passive investment, through the lens of standard macroeconomic modeling tools.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature pertaining to passive investment trends and market efficiency, information production in financial markets, and the integration of financial markets within macroeconomic models. Section 3 presents the model setup, providing detailed descriptions of firms, households, and the mutual fund, alongside the government sector and market clearing conditions. Section 4 presents the model solution. Section 5 discusses the equilibrium characterization, computational challenges, the simulation approach adopted, and the resulting steady-state and dynamic properties. Section 6 discusses the key results and their economic implications. Finally, Section 7 concludes the paper.

2. Literature Review

2.1 Passive Investment Trends and Market Efficiency

Jaquart et al. (2023) built a simulated financial market to analyze how different levels of active and passive investments impact market efficiency. Their findings—especially about how active investment improves fundamental market efficiency while passive investment might fuel price bubbles—are a key reference for understanding the market dynamics modeled in this paper.

Kerzérho (2023) provides a detailed breakdown of assets under management, market shares, and flows of money into passive and active retail funds, highlighting the huge growth of passive funds globally. By documenting how investor preferences have shifted toward passive funds because of their higher returns, predictability, and transparency, Kerzérho's paper gives essential statistics on market shares and expense ratios that support the rationale behind the analysis of this paper.

Weissensteiner (2017) examines how correlated noise might actually help passive investments improve market efficiency under certain conditions, offering a counterpoint to the common view that passive investment always reduces market efficiency.

Providing recent industry context, Von Moltke and Sløk (2024) review the impacts of passive investing, arguing that its rise has contributed to increased market volatility, reduced liquidity, and heightened concentration, particularly in large-cap stocks like the "Magnificent Seven." They synthesize arguments concerning the mechanisms, including reduced price elasticity and potential macroeconomic inefficiencies stemming from passive flows. This analysis highlights the contemporary concerns and observed market phenomena that motivate the theoretical investigation undertaken in this thesis regarding the structural effects of passive investment.

Brough et al. (2018) examine the observed cyclicality in active equity manager performance and identify significant headwinds post-GFC (Global Financial Crisis), including the substantial shift of assets into passive strategies. They note that large passive inflows can create feedback loops influencing market-cap weightings and potentially disadvantaging active managers with different portfolio construction approaches (e.g., more equal-weighted). This paper offers further empirical motivation for studying the active-passive dynamic and its potential structural consequences for market functioning, which is a central theme of this thesis.

2.2 Information Production in Financial Markets

The Grossman-Stiglitz Paradox (Grossman & Stiglitz, 1980) establishes a fundamental tension between market efficiency and the incentives to acquire information. It suggests that if markets were perfectly efficient, nobody would bother to acquire costly information, yet without this information acquisition, markets can't become efficient in the first place. This paradox is central to understanding the dynamics between passive and active investing that I explore in my model.

Vives (2014) investigates whether markets can be informationally efficient, providing theoretical frameworks that help contextualize the analysis of this paper regarding the information production when passive investment funds are present.

The literature exploring the "feedback effect," surveyed by Goldstein (2022), examines how information aggregated in financial market prices influences decisions in the real economy. This research distinguishes between the market's ability to forecast future outcomes (Forecasting Price Efficiency) and its ability to reveal new information useful for corporate decisions (Revelatory Price Efficiency), highlighting potential conflicts between the two. This conceptual framework is directly relevant for analyzing the mechanism central to this thesis, where the intermediary's information production activities potentially guide real economic activity, and for assessing the overall efficiency implications.

2.3 Macroeconomic Models and Financial Markets

Gomes and Michaelides (2008) study asset pricing and portfolio choice in a life-cycle model featuring heterogeneous agents, incomplete markets, and endogenously determined limited stock market participation arising from a fixed entry cost. They successfully match both aggregate asset pricing moments and key features of household portfolio data, notably finding that endogenous non-participation by poorer households has a negligible impact on the aggregate equity premium. Their work serves as a key reference regarding the implications of agent heterogeneity and market segmentation for asset pricing and participation, providing a contrasting mechanism (endogenous fixed cost) to the market access segmentation employed in this thesis.

Matsuyama (2004) demonstrates how financial market globalization interacting with pre-existing credit market imperfections can lead to "symmetry breaking," where initially identical countries endogenously diverge into stable rich and poor equilibria. In this framework, integration destabilizes the symmetric steady state, leading to a polarized world where borrowing constraints bind only for the poor nations. This perspective on how market structure and financial frictions can generate endogenous segmentation and multiple stable equilibria offers a relevant theoretical backdrop for considering the potential regime-switching dynamics suggested by the contrasting steady states found in this thesis.

Sims and Wu (2019) develop a tractable four-equation New Keynesian model incorporating financial intermediation through the addition of segmented households, long-term bonds, and channels for quantitative easing (QE) and credit shocks. Their framework demonstrates how such financial frictions break the Divine Coincidence, creating distinct roles for conventional policy and QE even outside the zero lower bound. This work provides a valuable benchmark for incorporating financial sector details and unconventional policy tools within a simplified NK structure, similar in spirit to the objective of this thesis, albeit through different specific friction and intermediation mechanisms.

3. Model Setup

3.1 Overview of the Model

This paper develops an extension of the standard New Keynesian macroeconomic framework by adding a financial sector in the form of a mutual fund engaged in information production. Unlike the classical model, this framework incorporates a stock market dimension with information asymmetries and market segmentation. The model features three main types of economic agents: two types of households (wealthy and poor), a representative mutual fund with information production capabilities, and firms at both the final and intermediate levels.

The key difference between the household types lies in their initial endowment, labor productivity, and critically, their ability to access financial markets. Wealthy households can participate directly in stock markets, while poor households must invest exclusively via mutual funds, which creates a natural captive clientele for mutual funds and ensures a well-defined role for financial intermediation.

Within each period, events unfold in a structured sequence. The mutual fund first determines its information production effort X_t, which directly enhances the precision of productivity signals through reducing the variance $\sigma_\eta^2(X_t) = \frac{\bar{\sigma}^2}{1+\phi X_t}$ of the noise in firm productivity. Subsequently, in accordance with the Calvo (1983) pricing framework, a fraction of intermediate firms - specifically $1-\theta_H$ of high-productivity firms and $1-\theta_L$ of low-productivity firms - reset their prices based on their productivity expectations. The fund then sets its premium ω_t^* by taking household demand elasticity as given, maximizing premium revenue based on the downward-sloping demand curve it faces. Based on this premium and the information quality achieved, the fund makes its portfolio allocation decisions, optimizing its holdings of each stock type ($\bar{S}_t^{H*}$, $\bar{S}_t^{L*}$) and determining what fraction of each (α_t^{H*}, α_t^{L*}) to sell as fund shares. Households then make their consumption, labor supply, and investment decisions in response to market prices and fund offerings.

The central innovation of this model is introducing a mutual fund that actively produces information about firm productivity. When firms make production decisions, they face uncertainty about their true productivity due to the noise component $\eta_t(f)$. The fund's information production activities help reduce this uncertainty by decreasing the variance of productivity noise. This creates a direct link between financial market activities and production efficiency in the real economy.

This mechanism effectively captures an important feature of modern financial markets: the dual role of institutional investors in both allocating capital and producing information that helps guide real economic decisions. The framework presented in this paper incorporates this mechanism directly in order to study how the rise of passive investment might affect productivity, price-setting behavior and, consequently, monetary policy transmission.

A key feature of the model specification developed in this paper involves structuring the fund's decision-making across three sequential stages. This particular approach was implemented primarily as a means by which to bypass indeterminacy issues encountered in earlier iterations of this framework. The intended purpose of adopting such a sequential methodology was to foster a well-functioning stock market mechanism that possesses clear equilibrium properties, thereby allowing meaningful comparative statics and policy analysis. While this staged approach represents a necessary step towards analytical tractability and significant progress from earlier iterations, the subsequent analysis reveals that critical computational challenges remain, ultimately limiting the model's capacity for such standard policy evaluation in its current form.

3.2 Production

3.2.1 Final good firm

The production side of economy consists of two sectors: a representative final good firm and a continuum of intermediate firms of two types. For easier notation, the types can be denoted as $I \in \{\text{High-Productivity (H)}, \text{Low-Productivity (L)}\}$. High-productivity firms are indexed by $f \in [0, \varsigma]$ and low-productivity firms by $f \in (\varsigma, 1]$, where ς is the measure of high-productivity firms. The final good firm combines intermediate goods using CES technology with the production function:

$$Y_t = \left(\int_0^{\varsigma} Y_t^H(f)^{\frac{\epsilon-1}{\epsilon}} df + \int_{\varsigma}^1 Y_t^L(f)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where $\epsilon > 1$ is the elasticity of substitution between varieties, and $Y_t^H(f)$ and $Y_t^L(f)$ are inputs from high and low-productivity firms, respectively.

The final good firm chooses input quantities $Y_t^H(f)$ for $f \in [0, \varsigma]$ and $Y_t^L(f)$ for $f \in (\varsigma, 1]$ to maximize profits:

$$\max_{Y_t^H(f), Y_t^L(f)} P_t \left(\int_0^{\varsigma} Y_t^H(f)^{\frac{\epsilon-1}{\epsilon}} df + \int_{\varsigma}^1 Y_t^L(f)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^{\varsigma} P_t^H(f) Y_t^H(f) df - \int_{\varsigma}^1 P_t^L(f) Y_t^L(f) df \tag{2}$$

Due to perfect competition and constant returns to scale, the firm earns zero economic profits in equilibrium.

3.2.2 Intermediary good firms

The model features two types of intermediate firms with noisy productivity. Each firm's productivity includes both a deterministic component based on firm type and a stochastic noise component:

$$A_t^H(f) = A^H(1 + \eta_t(f)) \quad \text{for} \quad f \in [0, \varsigma] \tag{3}$$

$$A_t^L(f) = A^L(1 + \eta_t(f)) \quad \text{for} \quad f \in (\varsigma, 1] \tag{4}$$

where $A^H > A^L$ captures the baseline productivity differential between firm types, and $\eta_t(f) \sim N(0, \sigma_\eta^2(X_t))$ is a noise term with variance $\sigma_\eta^2(X_t) = \frac{\bar{\sigma}^2}{1+\phi X_t}$ that depends on the mutual fund's information production level X_t. The parameter $\phi > 0$ determines how quickly signal variance decreases with the fund's information investment. The productivity parameter $A_t^I(f)$ represents labor productivity - the amount of output that can be produced per unit of labor input.

This productivity noise creates uncertainty about the precise productivity level for all market participants. When observing productivity $A_t^I(f)$, firms know their underlying type $I \in \{H, L\}$ but cannot perfectly discern the magnitude of the noise component $\eta_t(f)$. This noise affects short-term production decisions while firms recognize its transitory nature when making forward-looking price-setting decisions.

To address the viability of low-productivity firms in equilibrium, the model introduces a form of labor heterogeneity. Each firm type requires a specific labor input that is most productive with it:

$$Y_t^H(f) = A_t^H(f) L_t^H(f) \quad \text{for} \quad f \in [0, \varsigma] \tag{5}$$

$$Y_t^L(f) = A_t^L(f) L_t^L(f) \quad \text{for} \quad f \in (\varsigma, 1] \tag{6}$$

Households supply both types of labor according to:

$$L_t^J = L_t^{J,H} + L_t^{J,L} \quad \text{for} \quad J \in \{W, P\} \tag{7}$$

Where $L_t^{J,I}$ represents the amount of type-I labor supplied by household type J. In this economy, there is a measure λ of wealthy households and a measure $(1-\lambda)$ of poor households, as will be discussed in Section 1.3.

Each labor type earns its own wage, W_t^H and W_t^L, with $W_t^H > W_t^L$ in equilibrium due to higher productivity of H-type labor. Importantly, the wage differential is not large enough to make low-productivity firms unprofitable or to perfectly reveal firm types, due to the productivity noise.

Each intermediate firm has a monopoly on its good and faces demand from the final good firm. Each firm issues one unit of stock, entitling the stockholder to receive the firm's profits as dividends:

$$S_t^H(f) = 1 \quad \text{for each} \quad f \in [0, \varsigma] \tag{8}$$

$$S_t^L(f) = 1 \quad \text{for each} \quad f \in (\varsigma, 1] \tag{9}$$

The total measure of stocks equals the measure of firms of each type:

$$\int_0^\varsigma S_t^H(f) df = \varsigma \quad \text{and} \quad \int_\varsigma^1 S_t^L(f) df = 1 - \varsigma \tag{10}$$

These stocks can be owned by both the fund and wealthy households.

Intermediate firms follow Calvo price setting with type-specific parameters $\theta_H < \theta_L$, reflecting more frequent price adjustments by high-productivity firms. While price adjustment faces Calvo frictions, firms can optimize production decisions every period.

The intermediate goods firm's problem is solved in two stages. The sequence within each period unfolds as follows: First, firms observe their current productivity $A_t^I(f) = A^I(1 + \eta_t(f))$. Then, a fraction $(1 - \theta_I)$ of firms get the opportunity to reset prices. These firms recognize that the noise component $\eta_t(f)$ is transitory, so they set prices optimally based on their permanent productivity component A^I as will be shown in Section 4. Finally, all firms produce to meet demand at their current prices using their current productivity, which includes the noise component. For expositional purposes, the presentation separates the production decision (Stage 1), applicable to all firms each period, from the price-setting decision (Stage 2), applicable only to the resetting fraction $(1 - \theta_I)$. Within the model's actual timing per period t, however, production decisions by all firms occur concurrently with price-setting decisions by the eligible subset.

Stage 1: Production Decision (Every Period)

Note that firms know their general type (H or L) but face uncertainty about their precise productivity due to the noise component $\eta_t(f)$. In Stage 1, given its price $P_t^I(f)$ and observed productivity $A_t^I(f)$, each firm meets the demand for its product by hiring the appropriate amount of labor. This demand-meeting behavior follows from the firm's monopolistic position - with a preset price, the optimal strategy is to produce the quantity demanded at that price. So, given its current price $P_t^I(f)$ and observed productivity $A_t^I(f)$, each firm f of type $I \in \{H, L\}$ must hire enough labor to meet the demand:

$$L_t^I(f) = \frac{Y_t^I(f)}{A_t^I(f)} \tag{11}$$

where $Y_t^I(f)$ is a standard demand function obtained from the solution of the final goods firm's problem:

$$Y_t^I(f) = Y_t \left(\frac{P_t^I(f)}{P_t} \right)^{-\epsilon} \tag{12}$$

The intermediate good firm's profit is:

$$\Pi_t^I(f) = P_t^I(f) Y_t^I(f) - W_t^I L_t^I(f) \tag{13}$$

Substituting the expressions for the output and labor demands of the final good firm into the profit function allows to obtain the expression for the marginal cost of the intermediate goods firm:

$$MC_t^I(f) = \frac{W_t^I}{A_t^I(f)} = \frac{W_t^I}{A^I(1 + \eta_t(f))} \tag{14}$$

Importantly, each firm knows which labor market it participates in based on its type, so it faces the appropriate wage W_t^I. The price $P_t^I(f)$ is either the result of previous optimization decisions or, for firms that can reset prices in the current period, determined through the optimization described in Stage 2.

Stage 2: Price-Setting Decision (With Probability $1 - \theta_I$)

With probability $(1 - \theta_I)$, a firm of type I can reset its price. When setting prices, firms recognize that the noise component $\eta_t(f)$ is transitory and provides no information about future productivity. Therefore, it can be shown that firms base their price-setting decisions on their base productivity level A^I rather than their current noisy productivity $A_t^I(f)$. When allowed to reset, the firm chooses optimal reset price $\hat{P}_t^I(f)$ to maximize the expected present value of profits:

$$\max_{\hat{P}_t^I(f)} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_I)^k \left[\hat{P}_t^I(f) Y_{t+k}^I(f) - W_{t+k}^I L_{t+k}^I(f)\right] \tag{15}$$

subject to the production and demand functions:

$$Y_{t+k}^I(f) = A_{t+k}^I(f) L_{t+k}^I(f) \tag{16}$$

$$Y_{t+k}^I(f) = Y_{t+k} \left(\frac{\hat{P}_t^I(f)}{P_{t+k}}\right)^{-\epsilon} \tag{17}$$

When setting prices, firms face two key sources of uncertainty: they don't know future aggregate demand with certainty and must form expectations about Y_{t+k}, and while they observe their current productivity $A_t^I(f) = A^I(1 + \eta_t(f))$, they recognize that the noise component $\eta_t(f)$ is purely transitory. Since it can be shown that this noise provides no information about future productivity, firms rationally form expectations about future variables and base their forward-looking price decisions on their permanent productivity component A^I rather than their current noisy productivity. These expectations are captured in the optimization problem in equation (15) through the expectation operator $\mathbb{E}_t$ applied to the entire future profit stream.

Solving this optimization problem yields the optimal reset price, which will be discussed in Section 4.

For periods when a firm cannot adjust its price (with probability θ_I), the price remains fixed at the previous level:

$$P_t^I(f) = P_{t-1}^I(f) \tag{18}$$

All operating profits are distributed as dividends to shareholders:

$$D_t^I(f) = P_t^I(f) Y_t^I(f) - W_t^I L_t^I(f) \tag{19}$$

With aggregate dividends for each stock type:

$$D_t^H = \int_0^{\varsigma} D_t^H(f)df \tag{20}$$

$$D_t^L = \int_{\varsigma}^{1} D_t^L(f)df \tag{21}$$

Note that it is possible to write the dividends in terms of the marginal cost:

$$D_t^I(f) = Y_t \left(\frac{P_t^I(f)}{P_t}\right)^{-\epsilon} \left[P_t^I(f) - \frac{W_t^I}{A_t^I(f)}\right] \tag{22}$$

where the term $[P_t^I(f) - \frac{W_t^I}{A_t^I(f)}]$ represents the profit margin per unit of output. Due to monopolistic competition, firms charge a markup over marginal cost, making this term positive in general. Market clearing is ensured because higher prices reduce demand according to equations (11) and (12), placing natural bounds on labor demand.

3.3 Households

In order to capture important distributional aspects of financial markets, this model distinguishes between two types of households. Wealthy households, with higher initial endowment, can analyze firms and participate in the stock market directly, while poor households, in contrast, have no direct access to the stock market. This limitation may represent information constraints or regulatory barriers occasionally arising in real financial markets. As a result, poor households are forced to invest exclusively through mutual funds making them a captive clientele for mutual funds and creating a well-defined role for financial intermediation in this economy.

The model contains households of types $J \in \{\text{Wealthy (W)}, \text{Poor (P)}\}$ with measure λ of wealthy households and $1-\lambda$ of poor households. Both types receive initial endowment in the form of one-period government-issued bonds, but wealthy households start with higher endowment ($B_0^W > B_0^P$).

Each household supplies two types of labor: $L_t^{J,H}$ (labor suitable for high-productivity firms) and $L_t^{J,L}$ (labor suitable for low-productivity firms). The total labor supply is:

$$L_t^J = L_t^{J,H} + L_t^{J,L} \quad \text{for} \quad J \in \{W, P\} \tag{23}$$

Each period, wealthy households make decisions about consumption C_t^W, labor supply allocation $\{L_t^{W,H}, L_t^{W,L}\}$, and their investment portfolio, including direct stock holdings ($\tilde{S}_t^{H,W}$ and $\tilde{S}_t^{L,W}$), mutual fund shares $S_t^{F,W}$, and bonds B_t^W. Poor households choose consumption C_t^P, labor supply allocation $\{L_t^{P,H}, L_t^{P,L}\}$, mutual fund shares $S_t^{F,P}$, and bonds B_t^P, but cannot directly purchase stocks.

Wealthy households face uncertainty when investing directly in the stock market. For stocks they identify as high-productivity, with probability p_0 they correctly identify H-type stocks and with probability $(1-p_0)$ they mistakenly identify L-type stocks as H-type. In both cases, they pay the market price of H-type stocks Q_t^H. Similarly, for stocks they identify as low-productivity, they pay Q_t^L regardless of whether they've correctly identified the stock type.

At the beginning of period t, each household's portfolio consists of bonds B_{t-1}^J and previous investments. For wealthy households, this includes stock holdings $\{\tilde{S}_{t-1}^{H,W}, \tilde{S}_{t-1}^{L,W}\}$ and fund shares $S_{t-1}^{F,W}$, while poor households only hold fund shares $S_{t-1}^{F,P}$.

To solve the household's problem, a two-stage approach is employed as a solution method, not as sequential decisions in time. This approach is mathematically equivalent to solving the complete problem in equations (25-26) directly:

Stage 1: Solving for consumption and labor supply decisions, deriving the standard Euler equations.

Stage 2: Given optimal consumption and labor decisions, as well as Euler equations with respect to portfolio allocation variables, solving for optimal portfolio allocation across all available assets.

This reformulation allows to derive tractable asset demand functions presented in Section 4'.

3.3.1 Household's Optimization Problem: Stage 1

In the household utility function, the parameter $\eta > 0$ represents the inverse Frisch elasticity of labor supply, not to be confused with the noise term $\eta_t(f)$ in firm productivity. The general household optimization problem is:

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^J)^{1-\sigma} - 1}{1-\sigma} - \frac{(L_t^{J,H})^{1+\eta} + (L_t^{J,L})^{1+\eta}}{1+\eta} \right] \tag{24}$$

Wealthy households maximize with respect to $\{C_t^W, L_t^{W,H}, L_t^{W,L}, \tilde{S}_t^{H,W}, \tilde{S}_t^{L,W}, S_t^{F,W}, B_t^W\}$, while poor households maximize with respect to $\{C_t^P, L_t^{P,H}, L_t^{P,L}, S_t^{F,P}, B_t^P\}$.

The optimization is subject to:

1. Budget constraint for wealthy households:

$$\begin{aligned} P_t C_t^W + B_t^W + Q_t^H \tilde{S}_t^{H,W} + Q_t^L \tilde{S}_t^{L,W} + Q_t^F S_t^{F,W} \\ \leq W_t^H L_t^{W,H} + W_t^L L_t^{W,L} + (1+r_t) B_{t-1}^W \\ + [p_0(Q_t^H + D_t^H) + (1-p_0)(Q_t^L + D_t^L)] \tilde{S}_{t-1}^{H,W} \\ + [p_0(Q_t^L + D_t^L) + (1-p_0)(Q_t^H + D_t^H)] \tilde{S}_{t-1}^{L,W} \\ + (Q_t^F + D_t^F) S_{t-1}^{F,W} \end{aligned} \tag{25}$$

The left side shows that households pay Q_t^H for all stocks they identify as H-type and Q_t^L for all stocks they identify as L-type. The right side shows that the returns they receive depend on whether they correctly identified the stock types, with probability p_0 of correct identification.

2. Budget constraint for poor households:

$$P_t C_t^P + B_t^P + Q_t^F S_t^{F,P} \leq W_t^H L_t^{P,H} + W_t^L L_t^{P,L} + (1 + r_t) B_{t-1}^P + (Q_t^F + D_t^F) S_{t-1}^{F,P} \quad (26)$$

Solving this optimization problem yields standard Euler equations for both household types.

3.3.2 Household's Optimization Problem: Stage 2

Having obtained optimality conditions for labor and consumption, as well as Euler equations for investment portfolio variables, it is now possible to solve for optimal portfolio allocation using the mean-variance framework. Note that this two-stage approach is mathematically equivalent to solving the complete lifetime utility maximization problem directly, but offers greater analytical tractability.

In Stage 1, the optimization yields standard Euler equations for each available asset. These Euler equations relate current asset prices to expected discounted future returns and implicitly define the optimal portfolio allocation, but do not provide closed-form solutions. To derive a more tractable formulation of the portfolio problem, the mean-variance framework is employed, which emerges from the Euler equations under certain assumptions discussed in Section 4 and allows to reformulate the portfolio optimization problem in terms of expected log returns and their variances.

In this reformulated portfolio problem, households optimize over portfolio shares rather than absolute quantities of assets. This requires the definition of financial wealth that will be allocated across different assets. The household's financial wealth $W_t^{F,J}$ represents the portion of total wealth available for investment after consumption expenditure has been subtracted:

$$W_t^{F,J} = W_t^{Tot,J} - P_t C_t^J \quad (27)$$

Total wealth differs by household type. For wealthy households, total wealth consists of labor income from both types of labor, returns on previous bond holdings, and returns on previous stock and fund investments:

$$\begin{aligned} W_t^{Tot,W} &= W_t^H L_t^{W,H} + W_t^L L_t^{W,L} + (1 + r_t) B_{t-1}^W \\ &+ [p_0(Q_t^H + D_t^H) + (1 - p_0)(Q_t^L + D_t^L)] \tilde{S}_{t-1}^{H,W} \\ &+ [p_0(Q_t^L + D_t^L) + (1 - p_0)(Q_t^H + D_t^H)] \tilde{S}_{t-1}^{L,W} \\ &+ (Q_t^F + D_t^F) S_{t-1}^{F,W} \end{aligned} \quad (28)$$

For poor households, total wealth consists only of labor income, bond returns, and fund share returns:

$$W_t^{Tot,P} = W_t^H L_t^{P,H} + W_t^L L_t^{P,L} + (1 + r_t)B_{t-1}^P + (Q_t^F + D_t^F)S_{t-1}^{F,P} \tag{29}$$

The financial wealth concept is critical for connecting the consumption and investment decisions in the two-stage approach. Once consumption is chosen optimally in Stage 1, the remaining financial wealth must be allocated across available assets to maximize expected utility of returns. This allocation is expressed in terms of portfolio shares in Stage 2.

For wealthy households, the portfolio allocation problem involves four assets: H-type stocks, L-type stocks, fund shares, and bonds. The fraction of financial wealth allocated to each asset is denoted as $a_t^{H,W}$, $a_t^{L,W}$, $a_t^{F,W}$, and $a_t^{B,W}$, with the constraint that $a_t^{H,W} + a_t^{L,W} + a_t^{F,W} + a_t^{B,W} = 1$.

The portfolio optimization problem is therefore formulated as:

$$\max_{a_t^{H,W}, a_t^{L,W}, a_t^{F,W}, a_t^{B,W}} \mathbb{E}_t \left[\log(R_{t+1}^P)\right] - \frac{\gamma^W}{2} \mathrm{Var}_t \left[\log(R_{t+1}^P)\right] \tag{30}$$

$$\text{subject to } a_t^{H,W} + a_t^{L,W} + a_t^{F,W} + a_t^{B,W} = 1 \tag{31}$$

The gross return on the portfolio is calculated as the weighted sum of returns on individual assets:

$$R_{t+1}^P = a_t^{H,W} \cdot R_{t+1}^{H,W} + a_t^{L,W} \cdot R_{t+1}^{L,W} + a_t^{F,W} \cdot R_{t+1}^F + a_t^{B,W} \cdot (1 + r_{t+1}) \tag{32}$$

The returns on each asset type are:

$$R_{t+1}^{H,W} = \frac{p_0(Q_{t+1}^H + D_{t+1}^H) + (1 - p_0)(Q_{t+1}^L + D_{t+1}^L)}{Q_t^H} \tag{33}$$

$$R_{t+1}^{L,W} = \frac{p_0(Q_{t+1}^L + D_{t+1}^L) + (1 - p_0)(Q_{t+1}^H + D_{t+1}^H)}{Q_t^L} \tag{34}$$

$$R_{t+1}^F = \frac{Q_{t+1}^F + D_{t+1}^F}{Q_t^F} \tag{35}$$

The return expressions reflect the uncertainty in stock identification: when a wealthy household purchases what it believes to be an H-type stock, it correctly identifies it with probability p_0 and incorrectly identifies it with probability $(1 - p_0)$.

Under the log-normality assumption, the expected log portfolio return can be expressed as:

$$\mathbb{E}_t[\log(R_{t+1}^P)] = a_t^{H,W}(\log(1+r_{t+1}) + \pi_t^{H,W}) + a_t^{L,W}(\log(1+r_{t+1}) + \pi_t^{L,W}) \\ + a_t^{F,W}(\log(1+r_{t+1}) + \pi_t^{F,W}) + a_t^{B,W}\log(1+r_{t+1}) \tag{36}$$

where the term $\pi_t^{I,J}$ represents the risk premium for asset type I as perceived by household type J, defined as:

$$\pi_t^{I,J} = \mathbb{E}_t\left[\log\left(\frac{R_{t+1}^{I,J}}{Q_t^I}\right)\right] - \log(1+r_{t+1}) \tag{37}$$

This risk premium is derived from the Stage 1 Euler equations directly and incorporates both expected excess returns and a risk adjustment term. Moreover, the risk premium depends on the variance of returns and their covariance with the household's stochastic discount factor, capturing both volatility risk and intertemporal hedging motives.

The variance of log portfolio returns is:

$$\text{Var}_t[\log(R_{t+1}^P)] = \begin{pmatrix} a_t^{H,W} & a_t^{L,W} & a_t^{F,W} \end{pmatrix} \Sigma_t \begin{pmatrix} a_t^{H,W} \\ a_t^{L,W} \\ a_t^{F,W} \end{pmatrix} \tag{38}$$

where Σ_t is the variance-covariance matrix of log excess returns:

$$\Sigma_t = \begin{pmatrix} \sigma_H^2 & \sigma_{HL} & \sigma_{HF} \\ \sigma_{HL} & \sigma_L^2 & \sigma_{LF} \\ \sigma_{HF} & \sigma_{LF} & \sigma_F^2 \end{pmatrix} \tag{39}$$

Each element of this matrix represents either a variance (σ_I^2) of a specific asset's log excess return or a covariance (σ_{IJ}) between two assets' log excess returns. The values of these variances and covariances depend on the fund's information production level X_t through its effects on both the identification probability $p(X_t)$, discussed in the following section, and the productivity noise variance $\sigma_\eta^2(X_t)$.

For poor households, who can only invest in bonds and fund shares, the portfolio problem is structurally similar but significantly simpler:

$$\max_{a_t^{F,P}, a_t^{B,P}} \mathbb{E}_t\left[\log(R_{t+1}^P)\right] - \frac{\gamma^P}{2}\text{Var}_t\left[\log(R_{t+1}^P)\right] \tag{40}$$

$$\text{subject to } a_t^{F,P} + a_t^{B,P} = 1 \tag{41}$$

The fractions $a_t^{F,P}$ and $a_t^{B,P}$ represent the allocation of financial wealth to fund shares and bonds.

The expected log portfolio return for poor households is:

$$\mathbb{E}_t[\log(R^P_{t+1})] = a^{F,P}_t(\log(1+r_{t+1}) + \pi^{F,P}_t) + a^{B,P}_t \log(1+r_{t+1}) \tag{42}$$

The variance of log portfolio returns is:

$$\text{Var}_t[\log(R^P_{t+1})] = (a^{F,P}_t)^2 \cdot \sigma^2_{F,t} \tag{43}$$

where $\sigma^2_{F,t}$ is the variance of log excess returns on fund shares, which is the $(3,3)$ element of the variance-covariance matrix Σ_t in (39).

While the mean-variance framework provides a more tractable formulation of the portfolio problem, it does not yield independent closed-form solutions for portfolio allocations. The expressions for optimal portfolio weights involve circular dependencies with other endogenous variables in the model. The variance-covariance matrix elements themselves are functions of endogenous variables, particularly the fund's information production level. The risk premia also depend on equilibrium asset prices and returns. Consequently, the resulting system of expressions must be solved simultaneously with the fund's optimization problem and market clearing conditions.

3.4 Mutual Fund

A novel feature of this model is a representative mutual fund that engages in stock selection and information production. The fund can invest in both high-productivity (H) and low-productivity (L) firms but faces uncertainty when identifying firm types due to the noise in firms' productivity. Through information production, the fund may improve its ability to correctly identify these firms.

The mutual fund serves two distinct but related functions. First, it acts as an information producer, investing resources to reduce uncertainty about firm types. This information production benefits the whole economy by improving the precision of firms' productivity signals. Second, it serves as an investment intermediary, allowing households (particularly poor households who can't access the stock market directly) to gain diversified stock market exposure.

As a mean of compensation for these services, the fund may charge a premium ω_t over the fundamental value of its portfolio. This premium, along with capital gains and dividends from proprietary trading, serves as an important incentive for the fund to engage in costly information production.

Each period, the fund makes decisions in a sequential manner:

1. *Stage 1:* The fund chooses its information production effort (X_t) to improve the

probability of correctly identifying firm types.

2. *Stage 2:* The fund sets its premium (ω_t) based on the demand for fund shares.

3. *Stage 3:* The fund makes portfolio allocation decisions, determining how many stocks to purchase of each type ($\bar{S}_t^I$) and what fraction of each type to sell as fund shares ($\alpha_t^I \in [0, 1]$).

The fund's information production improves its probability of correctly identifying stock types from a baseline p_0 according to $p(X_t) = p_0 + (1 - p_0)(1 - e^{-\psi X_t})$. This identification probability affects both the price at which the fund purchases stocks and the value of fund shares it sells.

The mutual fund's decision-making process occurs in three sequential stages that reflect different optimization horizons and information sets.

3.4.1 Mutual Fund's Optimization Problem: Stage 1

In the first stage, the fund optimizes its information production level (or effort), taking into account how this decision will influence subsequent stages. This decision reveals important economic tensions captured by the model's structure. The fund must weigh the benefits of more accurate firm identification for proprietary trading against the costs of information production. At the same time, it must consider how information production affects the value proposition of the fund shares it offers to clients.

Information production in this model creates value through both a private channel (improved investment returns) and a social channel (reduced productivity noise in the broader economy). The fund can only appropriate returns from the private channel, which creates a potential wedge between socially optimal and privately optimal levels of information production. This one-sided appropriability may contribute to the corner solutions observed in the model's equilibrium in upcoming sections, as the fund does not directly benefit from the improved firms' productivity identification.

The fund chooses its information production effort X_t to maximize the expected profit across all subsequent decisions:

$$\max_{X_t} \mathbb{E}_t \Pi_t^F(X_t) - \gamma X_t^{\xi} \tag{44}$$

Subject to the resource constraint:

$$\gamma X_t^{\xi} \leq \kappa Y_t \tag{45}$$

where $\Pi_t^F(X_t)$ is the expected profit from subsequent stages given the chosen information production level, and γX_t^{ξ} is the cost of information production. The parameter κ limits the fund's information production as a fraction of total output, while γ and $\xi > 1$ determine the cost structure.

3.4.2 Mutual Fund's Optimization Problem: Stage 2

In the second stage, having already chosen its information production level X_t from Stage 1, the fund sets its premium ω_t to maximize premium revenue. This decision reveals important strategic considerations in the fund's behavior as a financial intermediary. The premium directly influences both the fund's revenue per share and the quantity of shares demanded by households. The fund's problem in Stage 2 can be expressed as:

$$\max_{\omega_t} \omega_t \cdot V_t^F(\omega_t, X_t) \cdot S_t^{F,D}(\omega_t, p(X_t)) \tag{46}$$

Where:

- $V_t^F(\omega_t, X_t)$ is the fundamental value of the fund's portfolio per share, which depends on the premium through its effect on the optimal portfolio allocation variables from Stage 3
- $S_t^{F,D}(\omega_t, p(X_t))$ is the total quantity of fund shares demanded by households

The aggregate demand for fund shares combines demand from both household types:

$$S_t^{F,D}(\omega_t, p(X_t)) = \lambda S_t^{F,W}(\omega_t, p(X_t)) + (1 - \lambda) S_t^{F,P}(\omega_t, p(X_t)) \tag{47}$$

The fundamental value V_t^F depends on the optimal portfolio allocation:

$$V_t^F = \frac{\alpha_t^H \bar{S}_t^H}{\alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L} \cdot Q_t^H + \frac{\alpha_t^L \bar{S}_t^L}{\alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L} \cdot Q_t^L \tag{48}$$

This can be expressed more compactly by defining $Z_t = \alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L$, which represents the total quantity of fund shares issued:

$$V_t^F = \frac{\alpha_t^H \bar{S}_t^H}{Z_t} \cdot Q_t^H + \frac{\alpha_t^L \bar{S}_t^L}{Z_t} \cdot Q_t^L \tag{49}$$

This premium-setting decision introduces a novel trade-off not typically seen in standard monopoly pricing problems. In addition to the traditional balance between price and quantity, the fund must also consider how its premium affects the composition of its portfolio through Stage 3 decisions. A higher premium incentivizes the fund to sell more shares, but it may also distort the fund's portfolio allocation toward less valuable assets, potentially reducing the fundamental value of each share. This creates an interesting interdependence between premium-setting and portfolio composition.

3.4.3 Mutual Fund's Optimization Problem: Stage 3

In the final stage, taking the information production level X_t and premium ω_t as given from previous stages, the fund makes its portfolio allocation decisions. These decisions involve determining how many stocks to purchase of each type ($\bar{S}_t^H$ and $\bar{S}_t^L$) and what fraction of each type to sell as fund shares (α_t^H and α_t^L). This stage captures the fund's core function as both a direct investor and financial intermediary.

The fund's optimization problem in this stage introduces important economic tensions between proprietary trading and intermediation. Each stock in the fund's portfolio generates expected returns through capital gains and dividends, with the precision of these returns depending on the information production from Stage 1. Alternatively, each stock can be sold as part of fund shares, generating immediate premium revenue but sacrificing future proprietary returns.

The fund maximizes:

$$\max_{\bar{S}_t^H, \bar{S}_t^L, \alpha_t^H, \alpha_t^L} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Pi_t^F \tag{50}$$

Where the per-period profit is:

$$\begin{aligned}
\Pi_t^F = & \underbrace{(1-\alpha_t^H)[p(X_t)(Q_t^H - Q_{t-1}^H) + (1-p(X_t))(Q_t^L - Q_{t-1}^L)]\bar{S}_{t-1}^H}_{\text{Capital Gains from Investment Holdings}} \\
& + \underbrace{(1-\alpha_t^L)[p(X_t)(Q_t^L - Q_{t-1}^L) + (1-p(X_t))(Q_t^H - Q_{t-1}^H)]\bar{S}_{t-1}^L}_{\text{Capital Gains from Investment Holdings}} \\
& + \underbrace{(1-\alpha_t^H)[p(X_t)D_t^H + (1-p(X_t))D_t^L]\bar{S}_{t-1}^H}_{\text{Dividends from Investment Holdings}} \\
& + \underbrace{(1-\alpha_t^L)[p(X_t)D_t^L + (1-p(X_t))D_t^H]\bar{S}_{t-1}^L}_{\text{Dividends from Investment Holdings}} \\
& + \underbrace{\omega_t \cdot V_t^F \cdot (\alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L)}_{\text{Premium Revenue from Fund Shares}} \\
& - \underbrace{(Q_t^H \bar{S}_t^H + Q_t^L \bar{S}_t^L)}_{\text{Cost of Stock Purchases}}
\end{aligned} \tag{51}$$

Subject to:

1. Budget Constraint: $$\begin{aligned} & Q_t^H \bar{S}_t^H + Q_t^L \bar{S}_t^L \\ & \leq (1-\alpha_t^H)[p(X_t)(Q_t^H - Q_{t-1}^H) + (1-p(X_t))(Q_t^L - Q_{t-1}^L)]\bar{S}_{t-1}^H \\ & + (1-\alpha_t^L)[p(X_t)(Q_t^L - Q_{t-1}^L) + (1-p(X_t))(Q_t^H - Q_{t-1}^H)]\bar{S}_{t-1}^L \\ & + (1-\alpha_t^H)[p(X_t)D_t^H + (1-p(X_t))D_t^L]\bar{S}_{t-1}^H \\ & + (1-\alpha_t^L)[p(X_t)D_t^L + (1-p(X_t))D_t^H]\bar{S}_{t-1}^L \\ & + \omega_t \cdot V_t^F \cdot (\alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L) \end{aligned} \tag{52}$$
2. Portfolio Constraints: $$\bar{S}_t^H \leq \varsigma \tag{53}$$ $$\bar{S}_t^L \leq 1 - \varsigma \tag{54}$$

The fund distributes all profits as dividends to fund shareholders:

$$D_t^F = \Pi_t^F \tag{55}$$

A distinctive feature of this stage is how the fund balances its current portfolio allocation against expected future returns. Capital gains and dividends depend on previous-period holdings, while purchasing decisions affect future periods. This intertemporal dimension creates a dynamic tension between building proprietary positions for future returns and maximizing current premium revenue through fund shares.

The fund's decision has another layer of complexity due to an economic tension between profitability and diversification. While the fund has incentives to invest more heavily in the higher-return stock type identified through its information production, diversification concerns may push it to maintain a balanced portfolio, particularly for the portion sold as fund shares. Such tension between return maximization and risk management introduces rich portfolio dynamics that interact with the fund's dual role in this economy.

3.5 Government

This model includes a government that issues one-period risk-free bonds to provide a risk-free asset in the economy. The government's flow budget constraint in nominal terms is:

$$B_t^G = (1 + r_t)B_{t-1}^G \tag{56}$$

where B_t^G is the nominal value of government bonds issued in period t and r_t is the nominal interest rate on government bonds.

3.5.1 Bond Market Structure and Non-Arbitrage Conditions

Government bonds are one-period risk-free assets that provide a nominal return r_t. For households to hold both bonds and stocks in positive quantities, the following non-arbitrage conditions must hold:

For stocks identified as H-type:

$$1 + r_t = \mathbb{E}_t \left[M_{t,t+1} \left(\frac{p_0(Q_{t+1}^H + D_{t+1}^H) + (1 - p_0)(Q_{t+1}^L + D_{t+1}^L)}{p_0 Q_t^H + (1 - p_0)Q_t^L} \right) \right] \tag{57}$$

For stocks identified as L-type:

$$1 + r_t = \mathbb{E}_t \left[M_{t,t+1} \left(\frac{p_0(Q_{t+1}^L + D_{t+1}^L) + (1 - p_0)(Q_{t+1}^H + D_{t+1}^H)}{p_0 Q_t^L + (1 - p_0)Q_t^H} \right) \right] \tag{58}$$

For mutual fund shares:

$$1 + r_t = \mathbb{E}_t \left[M_{t,t+1} \left(\frac{Q^F_{t+1} + D^F_{t+1}}{Q^F_t} \right) \right] \tag{59}$$

where $M_{t,t+1}$ is the stochastic discount factor:

$$M_{t,t+1} = \beta \left(\frac{C^J_{t+1}}{C^J_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{60}$$

Expected returns on stocks account for identification uncertainty through p_0, while fund share returns reflect the fund's improved identification probability $p(X_t)$ through Q^F_t and D^F_t.

The real return on government bonds, r^r_t, is governed by the standard Fisher equation:

$$1 + r^r_t = (1 + r_t)\mathbb{E}_t \left[\frac{P_t}{P_{t+1}} \right] \tag{61}$$

3.6 Market Clearing Conditions

For the equilibrium to be well-defined, all markets in the model must clear. The complete set of market clearing conditions is specified below.

3.6.1 Asset Markets

Stock Market Clearing

The total supply of each stock type must equal the combined demand from the mutual fund and wealthy households:

For H-type stocks:

$$\varsigma = \bar{S}^H_t + \tilde{S}^{H,W}_t \tag{62}$$

For L-type stocks:

$$1 - \varsigma = \bar{S}^L_t + \tilde{S}^{L,W}_t \tag{63}$$

Fund Share Market Clearing

The supply of fund shares must equal the demand from both household types:

$$\alpha^H_t \bar{S}^H_t + \alpha^L_t \bar{S}^L_t = \lambda S^{F,W}_t + (1 - \lambda) S^{F,P}_t \tag{64}$$

The left side represents the total amount of fund shares supplied, which consists of portions of both H-type and L-type stocks that the fund sells as shares. The right side represents the total demand from both wealthy households and poor households.

Bond Market Clearing

The government's bond issuance must equal the total household bond holdings:

$$\lambda B_t^W + (1-\lambda)B_t^P + B_t^G = 0 \tag{65}$$

3.6.2 Factor Markets

Labor Market Clearing

For H-type labor:

$$\lambda L_t^{W,H} + (1-\lambda)L_t^{P,H} = \int_0^{\varsigma} L_t^H(f)df = \int_0^{\varsigma} \frac{Y_t}{A_t^H(f)} \left(\frac{P_t^H(f)}{P_t}\right)^{-\epsilon} df \tag{66}$$

For L-type labor:

$$\lambda L_t^{W,L} + (1-\lambda)L_t^{P,L} = \int_{\varsigma}^1 L_t^L(f)df = \int_{\varsigma}^1 \frac{Y_t}{A_t^L(f)} \left(\frac{P_t^L(f)}{P_t}\right)^{-\epsilon} df \tag{67}$$

These conditions ensure that the total supply of each type of labor from households equals the total demand for that labor type from intermediate firms.

3.6.3 Goods Market

Intermediate Goods Market Clearing

For each intermediate good, production must equal demand:

$$Y_t^H(f) = A_t^H(f)L_t^H(f) = Y_t \left(\frac{P_t^H(f)}{P_t}\right)^{-\epsilon} \quad \text{for all} \quad f \in [0, \varsigma] \tag{68}$$

$$Y_t^L(f) = A_t^L(f)L_t^L(f) = Y_t \left(\frac{P_t^L(f)}{P_t}\right)^{-\epsilon} \quad \text{for all} \quad f \in (\varsigma, 1] \tag{69}$$

Final Goods Market Clearing

Total production must equal total expenditure in the economy:

$$Y_t = \lambda C_t^W + (1-\lambda)C_t^P + \gamma X_t^{\xi} \tag{70}$$

where γX_t^{ξ} is the fund's information production cost, and $\lambda C_t^W + (1-\lambda)C_t^P$ is aggregate household consumption.

These market clearing conditions, combined with the optimality conditions for all agents, fully characterize the equilibrium. The heterogeneous labor market structure ensures both firm types remain viable despite productivity differentials, while noisy signals create a meaningful role for information production.

4. Model Solution

This section outlines the solutions to the optimization problems presented in Section 3 and details the optimality conditions resulting from these optimization problems. Detailed mathematical derivations for all solutions presented herein are detailed within a supplementary appendix, available from the author upon request; their inclusion within the main body of this document was deemed impractical as it would significantly increase the manuscript's length beyond reasonable limits for a thesis of this nature. The analysis proceeds by first presenting the solutions for the production sector, covering both final and intermediate good firms, followed by the derivation of household optimality conditions, and finally outlining the solution to the mutual fund's multi-stage optimization problem.

4.1 Final good firm

The goal of solving the final good firm's optimization problem is the derivation of the optimal demand for each intermediate input. The firm chooses input quantities $Y_t^H(f)$ for $f \in [0, \varsigma]$ and $Y_t^L(f)$ for $f \in (\varsigma, 1]$ to maximize:

$$\max_{Y_t^H(f), Y_t^L(f)} P_t \left(\int_0^{\varsigma} Y_t^H(f)^{\frac{\epsilon-1}{\epsilon}} df + \int_{\varsigma}^1 Y_t^L(f)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^{\varsigma} P_t^H(f) Y_t^H(f) df - \int_{\varsigma}^1 P_t^L(f) Y_t^L(f) df \tag{71}$$

Taking first-order conditions and solving yields the standard demand functions for each intermediate input:

$$Y_t^I(f) = Y_t \left(\frac{P_t^I(f)}{P_t} \right)^{-\epsilon} \tag{72}$$

This generates the labor demand for each intermediate firm:

$$L_t^I(f) = \frac{Y_t^I(f)}{A_t^I(f)} = \frac{Y_t}{A_t^I(f)} \left(\frac{P_t^I(f)}{P_t} \right)^{-\epsilon} \tag{73}$$

It is possible to derive the associated price index:

$$P_t = \left(\int_0^{\varsigma} (P_t^H(f))^{1-\epsilon} df + \int_{\varsigma}^1 (P_t^L(f))^{1-\epsilon} df \right)^{\frac{1}{1-\epsilon}} \tag{74}$$

These demand functions play a pivotal role in the intermediate firms' decision-making, since they determine how much of each firm's output will be purchased at any given price. The demand elasticity ϵ measures the response of the quantity demanded to price changes, with higher values indicating greater substitutability between the varieties of intermediate goods.

The price index represents the minimum cost of producing one unit of the final good, taking into account the prices of all intermediate inputs. It aggregates all individual prices into a single measure, with each price weighted by its relative importance in the production process.

The total labor demand for each type is the integral over the respective firm ranges:

$$L_t^{H,D} = \int_0^{\varsigma} L_t^H(f)df = \int_0^{\varsigma} \frac{Y_t}{A_t^H(f)} \left(\frac{P_t^H(f)}{P_t} \right)^{-\epsilon} df \tag{75}$$

$$L_t^{L,D} = \int_{\varsigma}^1 L_t^L(f)df = \int_{\varsigma}^1 \frac{Y_t}{A_t^L(f)} \left(\frac{P_t^L(f)}{P_t} \right)^{-\epsilon} df \tag{76}$$

These equations were used in presenting the labor market clearing conditions and they show how labor demand depends on aggregate output, firm-specific productivity (including noise), and price dispersion. The information production activities of the mutual fund, by reducing the variance of productivity noise $\sigma_\eta^2(X_t) = \frac{\bar{\sigma}^2}{1+\phi X_t}$, affect labor allocation efficiency by making productivity more predictable.

4.2 Intermediary good firms

Intermediary firms, indexed by f and belonging to type $I \in \{H, L\}$, operate under monopolistic competition and face the production technology and demand structure outlined in Section 3. As was detailed, these firms confront uncertainty regarding their contemporaneous productivity $A_t^I(f) = A^I(1 + \eta_t(f))$, knowing only their permanent type A^I and observing the noisy signal $A_t^I(f)$. Furthermore, their price-setting opportunities are subject to Calvo-style frictions, occurring with probability $1 - \theta_I$. Within each period t, these firms undertake two key decisions. First, all firms determine their optimal production level (labor input) based on their currently observed productivity and prevailing price. Second, the subset of firms granted the opportunity engages in forward-looking price-setting. For clarity, the solutions derived from these decisions are presented sequentially below, beginning with the production choice pertinent to all firms each period.

4.2.1 Production Decision

In the production stage, which occurs every period, each firm chooses labor input $L_t^I(f)$ to maximize current-period profits, given its current price $P_t^I(f)$:

$$\max_{L_t^I(f)} P_t^I(f) Y_t^I(f) - W_t^I L_t^I(f) \tag{77}$$

Subject to production and demand constraints:

$$Y_t^I(f) = A_t^I(f) L_t^I(f) \tag{78}$$

$$Y_t^I(f) = Y_t \left(\frac{P_t^I(f)}{P_t} \right)^{-\epsilon} \tag{79}$$

Without explicitly solving the optimization problem at this stage, it is possible to obtain the expression for the marginal cost. In order to achieve this, equations (73) and (79) need to be substituted in the objective function in (77). Hence, the firm's profit can be expressed as:

$$\begin{aligned} \Pi_t^I(f) &= P_t^I(f) Y_t^I(f) - W_t^I L_t^I(f) \\ &= Y_t \left(\frac{P_t^I(f)}{P_t} \right)^{-\epsilon} \left[P_t^I(f) - \frac{W_t^I}{A_t^I(f)} \right] \end{aligned} \tag{80}$$

This shows that the firm's marginal cost is inversely proportional to its productivity, as in standard New Keynesian framework:

$$MC_t^I(f) = \frac{W_t^I}{A_t^I(f)} = \frac{W_t^I}{A^I(1 + \eta_t(f))} \tag{81}$$

4.2.2 Price-Setting Decision

In the price-setting stage, with probability $(1 - \theta_I)$, a firm of type I can reset its price. When allowed to reset, the firm chooses optimal reset price $\hat{P}_t^I(f)$ to maximize the expected present value of profits:

$$\max_{\hat{P}_t^I(f)} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_I)^k \left[\hat{P}_t^I(f) Y_{t+k}^I(f) - W_{t+k}^I L_{t+k}^I(f) \right] \tag{82}$$

Subject to:

$$Y_{t+k}^I(f) = A_{t+k}^I(f) L_{t+k}^I(f) \tag{83}$$

$$Y_{t+k}^I(f) = Y_{t+k} \left(\frac{\hat{P}_t^I(f)}{P_{t+k}} \right)^{-\epsilon} \tag{84}$$

Since the noise component $\eta_t(f)$ is assumed to be i.i.d. with mean zero, the current realization provides no information about future realizations:

$$\mathbb{E}_t[\eta_{t+k}(f) | \eta_t(f)] = \mathbb{E}[\eta_{t+k}(f)] = 0 \tag{85}$$

This leads to:

$$\begin{aligned}\mathbb{E}_t[A^I_{t+k}(f)|A^I_t(f)] &= \mathbb{E}_t[A^I(1+\eta_{t+k}(f))|A^I(1+\eta_t(f))] \\ &= A^I \cdot \mathbb{E}_t[(1+\eta_{t+k}(f))|(1+\eta_t(f))] \\ &= A^I \cdot 1 = A^I \end{aligned} \tag{86}$$

This means the firm's best estimate of its future productivity is simply the base productivity level A^I for its type, not its current noisy productivity $A^I_t(f)$.

The calculation of the expected future marginal cost, crucial for this forward-looking price decision, also simplifies due to the assumed transitory nature of the productivity noise:

$$\mathbb{E}_t\left[\frac{W^I_{t+k}}{A^I_{t+k}(f)}\middle|A^I_t(f)\right] = \mathbb{E}_t[W^I_{t+k}] \cdot \frac{1}{\mathbb{E}_t[A^I_{t+k}(f)|A^I_t(f)]} = \mathbb{E}_t[W^I_{t+k}] \cdot \frac{1}{A^I} \tag{87}$$

For better tractability, it is possible to define:

$$\mathbb{E}_t[W^I_{t+k}] = W^I_t \cdot (1+g_w)^k \tag{88}$$

where g_w is the expected growth rate of wages.

Then, taking the first-order condition and solving the optimization problem yields the optimal reset price:

$$\hat{P}^I_t(f) = \frac{\epsilon}{\epsilon-1} \cdot \frac{W^I_t}{A^I} \cdot \Omega^I_t \tag{89}$$

where Ω^I_t is an adjustment factor accounting for expected future conditions:

$$\Omega^I_t = \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\beta\theta_I)^k Y_{t+k}\left(\frac{1}{P_{t+k}}\right)^{-\epsilon}(1+g_w)^k}{\mathbb{E}_t \sum_{k=0}^{\infty}(\beta\theta_I)^k Y_{t+k}\left(\frac{1}{P_{t+k}}\right)^{-\epsilon}} \tag{90}$$

This optimal reset price equation reveals an important mechanism of the model. Namely, firms set prices as a markup $\frac{\epsilon}{\epsilon-1}$ over their expected marginal cost, adjusted by a factor Ω^I_t that accounts for expected future conditions, which is a modification of a standard markup formula in the New Keynesian model.

Notably, the results reveal that there is a fundamental disconnect between current production decisions and forward-looking price setting. Current production is based on realized noisy productivity $A^I_t(f)$, while price setting uses expected future productivity A^I. This creates potential inefficiencies when prices don't fully reflect current costs.

Another important observation is that improved information quality (higher X_t) does not directly affect forward-looking price-setting. However, as it decreases the variance of productivity noise $\sigma^2_\eta(X_t)$, current production decisions turn out to be more efficient due to reduced variability of marginal costs, which creates an interesting asymmetry in the model.

Finally, the model captures a curious interaction between informational frictions - via noisy productivity - and nominal rigidities associated with Calvo pricing, where the former affects current efficiency of production while the latter influences the response of prices to aggregate shocks. This mechanism introduces novel dynamics for the transmission of monetary policy.

4.3 Households

This subsection outlines the solutions derived from the utility maximization problems faced by the two types of households as presented in Section 3. The primary focus lies on the resulting optimality conditions that govern labor supply, consumption patterns and, critically for this study, their portfolio allocation decisions across available financial assets.

4.3.1 Wealthy Households

Wealthy households maximize expected lifetime utility subject to their budget constraint. Taking first-order conditions with respect to consumption, labor supply, and financial assets yields the optimality conditions below.

For consumption:

$$\lambda_t^W = \frac{(C_t^W)^{-\sigma}}{P_t} \tag{91}$$

where λ_t^W is the Lagrange multiplier on the budget constraint, representing the marginal utility of wealth.

For labor supply of both types:

$$\frac{W_t^H}{P_t} = (C_t^W)^{\sigma} \cdot (L_t^{W,H})^{\eta} \tag{92}$$

$$\frac{W_t^L}{P_t} = (C_t^W)^{\sigma} \cdot (L_t^{W,L})^{\eta} \tag{93}$$

These equations represent the standard intratemporal optimality conditions, where the real wage equals the marginal rate of substitution between leisure and consumption.

For bond holdings, the Euler equation is:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^W}{C_t^W} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} \cdot (1 + r_{t+1}) \right] \tag{94}$$

This expression represents the standard intertemporal optimality condition for a risk-free asset, equating the marginal utility cost of saving one unit today to the discounted expected marginal utility benefit received tomorrow.

For stock holdings (H-type and L-type) and fund shares, the solution yields:

$$Q_t^H = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^W}{C_t^W} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} [p_0(Q_{t+1}^H + D_{t+1}^H) + (1-p_0)(Q_{t+1}^L + D_{t+1}^L)] \right] \tag{95}$$

$$Q_t^L = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^W}{C_t^W} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} [p_0(Q_{t+1}^L + D_{t+1}^L) + (1-p_0)(Q_{t+1}^H + D_{t+1}^H)] \right] \tag{96}$$

$$Q_t^F = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^W}{C_t^W} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} (Q_{t+1}^F + D_{t+1}^F) \right] \tag{97}$$

These equations equate the current price of each risky asset (Q_t^H, Q_t^L, Q_t^F) to the expected discounted value of its future payoff (resale price plus dividend), adjusted by the household's stochastic discount factor. Notably, the equations for direct stock holdings incorporate the probability p_0 of correctly identifying the stock type, reflecting the imperfect information faced by wealthy households when investing directly.

To derive tractable asset demand functions from these optimality conditions, several standard assumptions common in macro-finance are employed, namely the conditional log-normality of asset returns and the application of a second-order Taylor approximation to the Euler equations. This approximation, exact under log-normality and CRRA utility, allows the household's portfolio choice problem to be represented within a tractable mean-variance framework, balancing expected return against risk. Such approach allows for a more tractable derivation of explicit asset demand functions without altering the fundamental simultaneous nature of the household's optimization problem.

With these assumptions, the portfolio optimization problem can be expressed in a mean-variance framework, where wealthy households maximize:

$$\begin{aligned} \max_{a_t^{H,W}, a_t^{L,W}, a_t^{F,W}, a_t^{B,W}} \; & \log(1+r_{t+1}) + a_t^{H,W}\pi_t^{H,W} + a_t^{L,W}\pi_t^{L,W} + a_t^{F,W}\pi_t^{F,W} \\ & - \frac{\gamma^W}{2} \begin{pmatrix} a_t^{H,W} & a_t^{L,W} & a_t^{F,W} \end{pmatrix} \Sigma_t \begin{pmatrix} a_t^{H,W} \\ a_t^{L,W} \\ a_t^{F,W} \end{pmatrix} \end{aligned} \tag{98}$$

$$\text{subject to } a_t^{H,W} + a_t^{L,W} + a_t^{F,W} + a_t^{B,W} = 1 \tag{99}$$

where $\pi_t^{I,W}$ is the risk premium for asset I, defined as:

$$\pi_t^{I,W} = \mathbb{E}_t \left[\log \left(\frac{R_{t+1}^{I,W}}{Q_t^I} \right) \right] - \log(1+r_{t+1}) \tag{100}$$

Solving this optimization problem yields the optimal portfolio allocation shares:

$$\begin{pmatrix} a_t^{H,W*} \\ a_t^{L,W*} \\ a_t^{F,W*} \end{pmatrix} = \frac{1}{\gamma^W} \Sigma_t^{-1} \begin{pmatrix} \pi_t^{H,W} \\ \pi_t^{L,W} \\ \pi_t^{F,W} \end{pmatrix} \tag{101}$$

where Σ_t is the variance-covariance matrix of log excess returns:

$$\Sigma_t = \begin{pmatrix} \text{Var}_t\left[\log\left(\frac{R_{t+1}^{H,W}}{Q_t^H}\right)\right] & \text{Cov}_t\left[\log\left(\frac{R_{t+1}^{H,W}}{Q_t^H}\right), \log\left(\frac{R_{t+1}^{L,W}}{Q_t^L}\right)\right] & \text{Cov}_t\left[\log\left(\frac{R_{t+1}^{H,W}}{Q_t^H}\right), \log\left(\frac{R_{t+1}^{F}}{Q_t^F}\right)\right] \\ \text{Cov}_t\left[\log\left(\frac{R_{t+1}^{L,W}}{Q_t^L}\right), \log\left(\frac{R_{t+1}^{H,W}}{Q_t^H}\right)\right] & \text{Var}_t\left[\log\left(\frac{R_{t+1}^{L,W}}{Q_t^L}\right)\right] & \text{Cov}_t\left[\log\left(\frac{R_{t+1}^{L,W}}{Q_t^L}\right), \log\left(\frac{R_{t+1}^{F}}{Q_t^F}\right)\right] \\ \text{Cov}_t\left[\log\left(\frac{R_{t+1}^{F}}{Q_t^F}\right), \log\left(\frac{R_{t+1}^{H,W}}{Q_t^H}\right)\right] & \text{Cov}_t\left[\log\left(\frac{R_{t+1}^{F}}{Q_t^F}\right), \log\left(\frac{R_{t+1}^{L,W}}{Q_t^L}\right)\right] & \text{Var}_t\left[\log\left(\frac{R_{t+1}^{F}}{Q_t^F}\right)\right] \end{pmatrix} \tag{102}$$

or, more compactly:

$$\Sigma_t = \begin{pmatrix} \sigma_H^2 & \sigma_{HL} & \sigma_{HF} \\ \sigma_{HL} & \sigma_L^2 & \sigma_{LF} \\ \sigma_{HF} & \sigma_{LF} & \sigma_F^2 \end{pmatrix} \tag{103}$$

where each element represents a variance or covariance of log excess returns.

Note that the share allocated to bonds can be determined from the constraint:

$$a_t^{B,W*} = 1 - a_t^{H,W*} - a_t^{L,W*} - a_t^{F,W*} \tag{104}$$

These derived optimal portfolio shares ($a_t^{H,W*}, a_t^{L,W*}, a_t^{F,W*}, a_t^{B,W*}$) determine the household's desired nominal holdings of each asset type. Specifically, the nominal amount allocated to each asset is obtained by multiplying the optimal share by the total financial wealth available for investment, $W_t^{F,W}$. For stocks and fund shares, this nominal amount is then divided by the respective asset price (Q_t^H, Q_t^L, Q_t^F) to yield the quantity demanded:

$$\tilde{S}_t^{H,W*} = \frac{a_t^{H,W*} \cdot W_t^{F,W}}{Q_t^H} \tag{105}$$

$$\tilde{S}_t^{L,W*} = \frac{a_t^{L,W*} \cdot W_t^{F,W}}{Q_t^L} \tag{106}$$

$$S_t^{F,W*} = \frac{a_t^{F,W*} \cdot W_t^{F,W}}{Q_t^F} \tag{107}$$

$$B_t^{W*} = a_t^{B,W*} \cdot W_t^{F,W} \tag{108}$$

A key observation from this solution is that the demand for each asset of wealthy households' depends not only on its own risk premium but also on how it co-varies with other available assets, which allows them to construct diversified portfolios that balance risk and return. The inverse of the variance-covariance matrix Σ_t^{-1} reflects these diversification advantages, with off-diagonal elements reflecting hedging demands.

4.3.2 Poor Households

Poor households face a similar but more restricted optimization problem, as they can only invest in mutual fund shares and bonds. Their first-order conditions for consumption and labor supply are identical to those of wealthy households:

$$\lambda_t^P = \frac{(C_t^P)^{-\sigma}}{P_t} \tag{109}$$

$$\frac{W_t^H}{P_t} = (C_t^P)^{\sigma} \cdot (L_t^{P,H})^{\eta} \tag{110}$$

$$\frac{W_t^L}{P_t} = (C_t^P)^{\sigma} \cdot (L_t^{P,L})^{\eta} \tag{111}$$

For bonds and fund shares, the Euler equations are:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^P}{C_t^P} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} \cdot (1 + r_{t+1}) \right] \tag{112}$$

$$Q_t^F = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^P}{C_t^P} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} (Q_{t+1}^F + D_{t+1}^F) \right] \tag{113}$$

Under the same assumptions as for wealthy households, their portfolio optimization problem is simpler:

$$\max_{a_t^{F,P}, a_t^{B,P}} \log(1 + r_{t+1}) + a_t^{F,P} \pi_t^{F,P} - \frac{\gamma^P}{2} (a_t^{F,P})^2 \cdot \sigma_{F,t}^2 \tag{114}$$

$$\text{subject to } a_t^{F,P} + a_t^{B,P} = 1 \tag{115}$$

where $\pi_t^{F,P}$ is the risk premium for fund shares as perceived by poor households and $\sigma_{F,t}^2 = \text{Var}_t \left[\log \left(\frac{R_{t+1}^F}{Q_t^F} \right) \right]$ is the variance of log excess returns on fund shares.

Solving this optimization problem yields the optimal portfolio allocation:

$$a_t^{F,P*} = \frac{\pi_t^{F,P}}{\gamma^P \cdot \sigma_{F,t}^2} \tag{116}$$

$$a_t^{B,P*} = 1 - a_t^{F,P*} = 1 - \frac{\pi_t^{F,P}}{\gamma^P \cdot \sigma_{F,t}^2} \tag{117}$$

And the resulting asset demands are:

$$S_t^{F,P*} = \frac{a_t^{F,P*} \cdot W_t^{F,P}}{Q_t^F} = \frac{\pi_t^{F,P}}{\gamma^P \cdot \sigma_{F,t}^2} \cdot \frac{W_t^{F,P}}{Q_t^F} \tag{118}$$

$$B_t^{P*} = a_t^{B,P*} \cdot W_t^{F,P} = \left(1 - \frac{\pi_t^{F,P}}{\gamma^P \cdot \sigma_{F,t}^2} \right) \cdot W_t^{F,P} \tag{119}$$

where $W_t^{F,P}$ is the financial wealth of poor households.

A key distinction from wealthy households is that poor households cannot diversify across different stock types, which forces them to accept a less efficient risk-return tradeoff. Their demand for fund shares depends only on the fund's risk premium and variance, without the benefit of covariance terms with other assets.

4.3.3 Aggregate Demand and Elasticity

It is important to derive the aggregate demand for fund shares as it plays an important role in the fund's optimization problem. It is defined as the sum of demands from both household types:

$$\begin{aligned} S_t^{F,D} &= \lambda S_t^{F,W} + (1-\lambda) S_t^{F,P} \\ &= \lambda \frac{a_t^{F,W} \cdot W_t^{F,W}}{Q_t^F} + (1-\lambda) \frac{a_t^{F,P} \cdot W_t^{F,P}}{Q_t^F} \end{aligned} \tag{120}$$

Similarly, the elasticity of aggregate demand with respect to the premium ω_t constitutes a crucial input for the mutual fund's premium-setting decision. Conventionally defined to yield a positive value for downward-sloping demand, this elasticity measures the percentage change in quantity demanded resulting from a percentage change in the premium:

$$\epsilon(\omega_t) = -\frac{\partial S_t^{F,D}}{\partial \omega_t} \cdot \frac{\omega_t}{S_t^{F,D}} \tag{121}$$

The negative sign incorporated into this standard definition ensures that $\epsilon(\omega_t)$ is positive when demand decreases as the premium ω_t (and thus the price Q_t^F) increases, which is the expected relationship.

To derive this elasticity, it is important to determine how each household type's demand responds to changes in the premium. Recall that the fund share price includes the premium:

$$Q_t^F = (1+\omega_t) V_t^F \tag{122}$$

where V_t^F is the fundamental value per share.

Taking the derivative of aggregate demand with respect to the premium and performing extensive algebraic manipulation, the elasticity can be expressed as:

$$\epsilon(\omega_t) = \frac{\omega_t}{1+\omega_t} + \frac{\omega_t}{(1+\omega_t)^2 V_t^F \cdot S_t^{F,D}} \cdot \left[\frac{\lambda}{\gamma^W} \cdot \Phi_W \cdot W_t^{F,W} + \frac{1-\lambda}{\gamma^P} \cdot \Phi_P \cdot W_t^{F,P} \right] \tag{123}$$

where:

$$\Phi_W = \frac{\sigma_H^2 \sigma_L^2 - \sigma_{HL}^2}{\det(\Sigma_t)} \tag{124}$$

$$\Phi_P = \frac{1}{\sigma_F^2} \tag{125}$$

This expression reveals that the elasticity consists of two components: (1) The direct price effect $\frac{\omega_t}{1+\omega_t}$, which approaches 1 as the premium increases, and (2) The portfolio substitution effect captured by the second term, which depends on the relative importance of wealthy against poor households.

The comparative analysis of Φ_W and Φ_P provides important insights into household behavior. Typically, Φ_W exceeds Φ_P because Φ_W accounts for the covariance structure across all three risky assets, reflecting wealthy households' ability to substitute between them, while Φ_P only depends on the variance of fund share returns, reflecting poor households' limited options. Moreover, the numerator of Φ_W is positive when H-type and L-type stock returns are imperfectly correlated. This provides wealthy households with diversification opportunities that are unavailable to poor households. In addition, the denominator of Φ_W is typically smaller than σ_F^2 due to the negative terms involving covariances, which further magnifies the difference between the two terms. Such difference in Φ values reveals that wealthy households are more responsive to premium changes as they can reallocate their portfolios more efficiently across multiple types of assets.

A larger proportion of wealthy households (λ) or higher relative financial wealth for wealthy households ($W_t^{F,W}/W_t^{F,P}$) increases the overall elasticity of fund share demand with respect to the premium. This is because wealthy households have more substitution options (direct stock investments) compared to poor households (only bonds). Conversely, market segmentation creates a captive clientele through poor households, who must use the fund to gain stock market exposure. This gives the fund a degree of market power, which is stronger when the proportion of poor households $(1-\lambda)$ is higher.

The fund's information production activities affect elasticity of demand through their effect on the variance-covariance structure. Better information (higher X_t) reduces variance and increases correlations between assets, affecting both Φ_W and Φ_P. This creates a direct link between the fund's information investment and its pricing power.

4.4 Mutual Fund

This section presents the solution to the mutual fund's optimization problem, as defined in Section 3. The model employs backward induction approach to solve the problem, due to the potential complexities anticipated to arise from the fund's interconnected decisions regarding information, pricing, and portfolio allocation. This approach proceeds sequentially, solving Stage 3 (Portfolio Allocation) first, followed by Stage 2 (Premium Setting), and finally Stage 1 (Information Production).

The adoption of backward induction for this problem is motivated by a number of mathematical and economic considerations with the intention of enhancing tractability and conceptual clarity.

First, it aims to mitigate potential analytical difficulties stemming from inherent circular dependencies anticipated within the fund's section of model. To clarify, the optimal premium is expected to depend upon demand elasticity, which itself is influenced by information quality (affecting return variances/covariances). At the same time, the optimal information production level may depend upon expected premium revenue. Breaking the problem into sequential stages, where decisions in earlier stages are taken as given in later stages, is a reliable technique intended to help manage such circular interdependencies.

Second, the sequential approach also builds an implicit commitment mechanism where the fund makes investments in information production before setting premium and choosing portfolio allocation. This prevents time inconsistency problems where the fund might deviate from announced strategies upon collecting fees from investors. This simulates the real-world environment where research capabilities represent long-term investments that cannot be adjusted as frequently as pricing decisions.

Third, treating the outcomes of earlier stages (information production level, premium) as state variables for subsequent stages facilitates the characterization of the solution. While intended to simplify the analysis and potentially reduce the likelihood of multiple equilibria compared to a simultaneous approach, this decomposition represents an important step towards analytical tractability, even if ultimately insufficient to fully mitigate the numerical difficulties encountered later in the simulation analysis. Although this structure does not guarantee uniqueness or eliminate all computational complexities, it effectively manages dimensionality by decomposing a high-dimensional optimization into a sequence of lower-dimensional problems.

Finally, the staged optimization reflects the different decision horizons typically observed in asset management practice. Strategic decisions regarding information infrastructure often precede tactical decisions on pricing, which in turn precede operational decisions on daily portfolio adjustments.

While this staged methodology provides a structured approach to solving the fund's complex problem, it is important to note that significant interdependencies remain, particularly between the stages, which translate to the numerical solution results.

4.4.1 Stage 3: Portfolio Allocation Decision

The complete optimization problem solved in this stage is outlined in equations (50) through (54) in Section 3 and duplicated below.

Recall that, in the final stage, taking the information production level X_t and premium ω_t as given from previous stages, the fund makes its portfolio allocation decisions $(\bar{S}_t^H, \bar{S}_t^L, \alpha_t^H, \alpha_t^L)$ to maximize the expected, discounted stream of its future per-period profits (Π_t^F):

$$\max_{\bar{S}_t^H, \bar{S}_t^L, \alpha_t^H, \alpha_t^L} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \Pi_{t+k}^F$$

where the per-period profit is defined as:

$$\begin{aligned}\Pi_t^F = & (1-\alpha_t^H)[p(X_t)(Q_t^H - Q_{t-1}^H) + (1-p(X_t))(Q_t^L - Q_{t-1}^L)]\bar{S}_{t-1}^H \\ & + (1-\alpha_t^L)[p(X_t)(Q_t^L - Q_{t-1}^L) + (1-p(X_t))(Q_t^H - Q_{t-1}^H)]\bar{S}_{t-1}^L \\ & + (1-\alpha_t^H)[p(X_t)D_t^H + (1-p(X_t))D_t^L]\bar{S}_{t-1}^H \\ & + (1-\alpha_t^L)[p(X_t)D_t^L + (1-p(X_t))D_t^H]\bar{S}_{t-1}^L \\ & + \omega_t \cdot V_t^F \cdot (\alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L) \\ & - (Q_t^H \bar{S}_t^H + Q_t^L \bar{S}_t^L)\end{aligned}$$

This maximization is subject to the following constraints:

1. Budget Constraint:

$$\begin{aligned}& Q_t^H \bar{S}_t^H + Q_t^L \bar{S}_t^L \\ & \leq (1-\alpha_t^H)[p(X_t)(Q_t^H - Q_{t-1}^H) + (1-p(X_t))(Q_t^L - Q_{t-1}^L)]\bar{S}_{t-1}^H \\ & + (1-\alpha_t^L)[p(X_t)(Q_t^L - Q_{t-1}^L) + (1-p(X_t))(Q_t^H - Q_{t-1}^H)]\bar{S}_{t-1}^L \\ & + (1-\alpha_t^H)[p(X_t)D_t^H + (1-p(X_t))D_t^L]\bar{S}_{t-1}^H \\ & + (1-\alpha_t^L)[p(X_t)D_t^L + (1-p(X_t))D_t^H]\bar{S}_{t-1}^L \\ & + \omega_t \cdot V_t^F \cdot (\alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L)\end{aligned}$$

2. Portfolio Constraints:

$$\begin{aligned}\bar{S}_t^H &\leq \varsigma \\ \bar{S}_t^L &\leq 1-\varsigma\end{aligned}$$

where the fundamental value of the fund's portfolio, V_t^F, is defined according to equations (48) and (49).

By setting up the Lagrangian with ν_t as the Lagrange multiplier on the budget constraint and ϑ_t^H and ϑ_t^L on the portfolio constraints, it is possible to derive optimality conditions for both stock holdings and allocation fractions.

The first-order conditions with respect to stock holdings $\bar{S}_t^H$ and $\bar{S}_t^L$, assuming the portfolio constraints may be binding ($\vartheta_t^H \geq 0$ and $\vartheta_t^L \geq 0$), are:

$$\begin{aligned}\vartheta_t^H = & (1+\nu_t)\Big[\omega_t \cdot \frac{\alpha_t^H \alpha_t^L \bar{S}_t^L}{Z_t} \cdot (Q_t^H - Q_t^L) + \omega_t \cdot V_t^F \cdot \alpha_t^H - Q_t^H\Big] \\ & + \beta \mathbb{E}_t \left[M_{t,t+1}^{\nu}(1-\alpha_{t+1}^H) R_{t,t+1}^{H,e}\right] \qquad (126)\\ \vartheta_t^L = & (1+\nu_t)\Big[\omega_t \cdot \frac{\alpha_t^H \bar{S}_t^H \alpha_t^L}{Z_t} \cdot (Q_t^L - Q_t^H) + \omega_t \cdot V_t^F \cdot \alpha_t^L - Q_t^L\Big] \\ & + \beta \mathbb{E}_t \left[M_{t,t+1}^{\nu}(1-\alpha_{t+1}^L) R_{t,t+1}^{L,e}\right] \qquad (127)\end{aligned}$$

where $M^{\nu}_{t,t+1} = \beta \frac{1+\nu_{t+1}}{1+\nu_t}$ represents the fund's stochastic discount factor adjusted for budget constraint multipliers, and $R^{I,e}_{t,t+1}$ denotes the expected gross return per unit from holding stocks identified by the fund as type I, accounting for potential misidentification probability $(1 - p(X_t))$:

$$R^{H,e}_{t,t+1} = p(X_t)(Q^H_{t+1} - Q^H_t + D^H_{t+1}) + (1 - p(X_t))(Q^L_{t+1} - Q^L_t + D^L_{t+1}) \tag{128}$$

$$R^{L,e}_{t,t+1} = p(X_t)(Q^L_{t+1} - Q^L_t + D^L_{t+1}) + (1 - p(X_t))(Q^H_{t+1} - Q^H_t + D^H_{t+1}) \tag{129}$$

These optimality conditions express the shadow prices of the portfolio constraints (ϑ^H_t, ϑ^L_t) in terms of current and future benefits. The first component (multiplied by $(1 + \nu_t)$) captures the current-period net benefit of acquiring stocks, including both premium revenue effects and acquisition costs. The second component represents the expected discounted future benefit from holding stocks for proprietary trading. When the constraints are non-binding ($\vartheta^I_t = 0$), these equations determine the optimal stock holdings that balance current costs against expected future returns.

Likewise, the first-order conditions with respect to the optimal allocation fractions α^H_t and α^L_t are given by:

$$\omega_t \cdot \frac{\bar{S}^H_t \alpha^L_t \bar{S}^L_t}{Z_t} \cdot (Q^H_t - Q^L_t) + \omega_t \cdot V^F_t \cdot \bar{S}^H_t = R^H_t \bar{S}^H_{t-1} \tag{130}$$

$$\omega_t \cdot \frac{\alpha^H_t \bar{S}^H_t \bar{S}^L_t}{Z_t} \cdot (Q^L_t - Q^H_t) + \omega_t \cdot V^F_t \cdot \bar{S}^L_t = R^L_t \bar{S}^L_{t-1} \tag{131}$$

where R^I_t represents the current realized return:

$$\begin{aligned} R^H_t &= p(X_t)(Q^H_t - Q^H_{t-1}) + (1 - p(X_t))(Q^L_t - Q^L_{t-1}) \\ &\quad + p(X_t)D^H_t + (1 - p(X_t))D^L_t \end{aligned} \tag{132}$$

$$\begin{aligned} R^L_t &= p(X_t)(Q^L_t - Q^L_{t-1}) + (1 - p(X_t))(Q^H_t - Q^H_{t-1}) \\ &\quad + p(X_t)D^L_t + (1 - p(X_t))D^H_t \end{aligned} \tag{133}$$

These conditions balance the marginal opportunity cost of allocating stock to fund shares (lost proprietary trading returns $R^I_t \bar{S}^I_{t-1}$) against the marginal benefits of fund share allocation, which include both direct premium revenue and the effect on portfolio composition via the fundamental value V^F_t. The equations reveal a fundamental trade-off in the fund's optimization problem: each additional unit allocated to fund shares generates immediate premium revenue but sacrifices future proprietary returns from information-enhanced trading.

Define the net benefit of holding each stock type:

$$\Pi^{\text{net}}_{H,t} = \mathbb{E}_t\left[M^{\nu}_{t,t+1}(1-\alpha^H_{t+1})R^{H,e}_{t,t+1}\right] - Q^H_t \tag{134}$$

$$\Pi^{\text{net}}_{L,t} = \mathbb{E}_t\left[M^{\nu}_{t,t+1}(1-\alpha^L_{t+1})R^{L,e}_{t,t+1}\right] - Q^L_t \tag{135}$$

Then, after solving the complete system of first-order conditions and carefully accounting for all interdependencies between variables, the optimal stock holdings for non-binding constraints are:

$$\bar{S}^{H*}_t = \frac{Z_t\left(\Pi^{\text{net}}_{L,t} + \omega_t \cdot V^F_t \cdot \alpha^L_t\right)}{\omega_t(Q^H_t - Q^L_t)\alpha^H_t\alpha^L_t} \tag{136}$$

$$\bar{S}^{L*}_t = \frac{Z_t\left(\Pi^{\text{net}}_{H,t} + \omega_t \cdot V^F_t \cdot \alpha^H_t\right)}{\omega_t(Q^L_t - Q^H_t)\alpha^H_t\alpha^L_t} \tag{137}$$

When the portfolio constraints may be binding, these expressions become:

$$\bar{S}^{H*}_t = \min\left\{\frac{Z_t\left(\Pi^{\text{net}}_{L,t} + \omega_t \cdot V^F_t \cdot \alpha^L_t\right)}{\omega_t(Q^H_t - Q^L_t)\alpha^H_t\alpha^L_t}, \varsigma\right\} \tag{138}$$

$$\bar{S}^{L*}_t = \min\left\{\frac{Z_t\left(\Pi^{\text{net}}_{H,t} + \omega_t \cdot V^F_t \cdot \alpha^H_t\right)}{\omega_t(Q^L_t - Q^H_t)\alpha^H_t\alpha^L_t}, 1-\varsigma\right\} \tag{139}$$

And the optimal allocation fractions are:

$$\alpha^{H*}_t = \frac{Z_t(R^L_t\bar{S}^L_{t-1} - \omega_t \cdot V^F_t \cdot \bar{S}^L_t)}{R^L_t\bar{S}^L_{t-1} + R^H_t\bar{S}^H_{t-1} \cdot \frac{\bar{S}^L_t}{\bar{S}^H_t} - 2\omega_t \cdot V^F_t \cdot \bar{S}^L_t} \tag{140}$$

$$\alpha^{L*}_t = \frac{Z_t(R^H_t\bar{S}^H_{t-1} - \omega_t \cdot V^F_t \cdot \bar{S}^H_t)}{R^L_t\bar{S}^L_{t-1} \cdot \frac{\bar{S}^H_t}{\bar{S}^L_t} + R^H_t\bar{S}^H_{t-1} - 2\omega_t \cdot V^F_t \cdot \bar{S}^H_t} \tag{141}$$

These expressions reveal key economic mechanisms driving the fund's portfolio decisions.

First, the optimal holding of H-type stocks ($\bar{S}^{H*}_t$) depends positively on the net benefit of L-type stocks ($\Pi^{\text{net}}_{L,t}$) and on the premium-weighted value of L-type allocation fraction ($\omega_t \cdot V^F_t \cdot \alpha^L_t$), while being inversely related to the price differential ($Q^H_t - Q^L_t$). Similarly, the optimal holding of L-type stocks ($\bar{S}^{L*}_t$) depends positively on the net benefit of H-type stocks ($\Pi^{\text{net}}_{H,t}$) and the premium-weighted value of H-type allocation fraction. Notably, in this model, the fund increases its holdings of one stock type when the other type becomes more profitable. This might seem contradicting the traditional portfolio theory where investors would simply increase their holdings of the more profitable asset, however such counterintuitive pattern actually makes economic sense considering the fund's full optimization problem. The fund optimizes across both stock types simultaneously; when

L-type stocks become more profitable, the fund increases holdings of H-type stocks to maintain an optimal balance between proprietary trading and fund shares. Given the fund's limited capital resources, if it decides to retain more L-type stocks for proprietary trading, it needs to sell more H-type stocks as fund shares to maintain portfolio balance, increasing its H-type holdings as a result.

Second, the fund's premium ω_t directly affects both stock holdings and allocation fractions. Hence, higher premiums increase the incentive to sell more stocks as fund shares, which may potentially alter the composition of the fund's portfolio to maximize premium revenue.

Third, the optimal allocation fractions presented in equations (140) and (141) illustrate that the fund allocates a larger fraction of its stock holdings to fund shares when the premium revenue ($\omega_t \cdot V_t^F$) is high relative to the expected returns from proprietary trading ($R_t^I \bar{S}_{t-1}^I$).

The equations characterizing optimal portfolio decisions - (136), (137), (140), and (141) - form a system with circular dependencies. The optimal stock holdings depend on the allocation fractions, while the optimal allocation fractions depend on the stock holdings both directly and through V_t^F. A critical interdependency emerges through $Z_t = \alpha_t^H \bar{S}_t^H + \alpha_t^L \bar{S}_t^L$, which appears in both the stock holding and allocation fraction equations, requiring simultaneous determination in equilibrium.

Importantly, the structure of these equations can potentially lead to corner solutions, where the fund gravitates toward extreme portfolio positions, unless bounded artificially. When information production creates substantial proprietary trading advantages, the fund may minimize allocation fractions to retain most stocks for its own portfolio returns. On the other hand, when the potential for premium revenue is high relative to proprietary returns, the fund might maximize its stock holdings to amplify premium revenue.

4.4.2 Stage 2: Premium Setting Decision

In the second stage, having already chosen its information production level X_t from Stage 1, the fund sets its premium ω_t to maximize premium revenue. Following proper backward induction, the fund has solved Stage 3 completely and knows how its optimal portfolio allocation variables $\{\bar{S}_t^{H*}(\omega_t, X_t), \bar{S}_t^{L*}(\omega_t, X_t), \alpha_t^{H*}(\omega_t, X_t), \alpha_t^{L*}(\omega_t, X_t)\}$ respond to changes in the premium.

The fund's problem in Stage 2 can be expressed as:

$$\max_{\omega_t} \omega_t \cdot V_t^F(\omega_t, X_t) \cdot S_t^{F,D}(\omega_t, p(X_t)) \tag{142}$$

Where $V_t^F(\omega_t, X_t)$ is the fundamental value per share:

$$V_t^F(\omega_t, X_t) = \frac{\alpha_t^{H*}(\omega_t, X_t)\bar{S}_t^{H*}(\omega_t, X_t)}{\alpha_t^{H*}(\omega_t, X_t)\bar{S}_t^{H*}(\omega_t, X_t) + \alpha_t^{L*}(\omega_t, X_t)\bar{S}_t^{L*}(\omega_t, X_t)} \cdot Q_t^H + \frac{\alpha_t^{L*}(\omega_t, X_t)\bar{S}_t^{L*}(\omega_t, X_t)}{\alpha_t^{H*}(\omega_t, X_t)\bar{S}_t^{H*}(\omega_t, X_t) + \alpha_t^{L*}(\omega_t, X_t)\bar{S}_t^{L*}(\omega_t, X_t)} \cdot Q_t^L \tag{143}$$

The first-order condition with respect to ω_t is given by:

$$V_t^F \cdot S_t^{F,D} + \omega_t \cdot \frac{\partial V_t^F}{\partial \omega_t} \cdot S_t^{F,D} + \omega_t \cdot V_t^F \cdot \frac{\partial S_t^{F,D}}{\partial \omega_t} = 0 \tag{144}$$

Before proceeding to evaluating the above condition, it is useful to define the elasticity of fundamental value with respect to the premium:

$$\text{EV}_\omega = \frac{\partial V_t^F / \partial \omega_t}{V_t^F} \tag{145}$$

This elasticity measures how the fundamental value of the fund's portfolio responds to changes in the premium. It is typically negative since higher premiums shift the fund's optimal allocation decisions in the direction of maximizing immediate premium revenue at the expense of retaining the most valuable stocks for proprietary trading.

This elasticity can be derived utilizing two complementary approaches. The fully endogenous approach applies the chain rule and implicit function theorem to the Stage 3 first-order conditions to calculate how the optimal portfolio variables respond to changes in the premium. This approach, while theoretically complete, entails significant computational challenges due to the intricate partial derivatives and complex interdependencies between variables that must be accounted for. Each change in premium affects portfolio composition through multiple channels simultaneously, creating a dense web of feedback effects that are difficult to calculate precisely. Despite these challenges, the approach provides valuable theoretical insights into the portfolio distortion mechanism. The fully endogenous approach produces the following expression:

$$\text{EV}_\omega = \frac{1}{V_t^F} \cdot \left[\frac{\bar{S}_t^H \alpha_t^L \bar{S}_t^L}{Z_t^2} \cdot (Q_t^H - Q_t^L) \cdot \frac{\partial \alpha_t^H}{\partial \omega_t} + \frac{\alpha_t^H \bar{S}_t^H \bar{S}_t^L}{Z_t^2} \cdot (Q_t^L - Q_t^H) \cdot \frac{\partial \alpha_t^L}{\partial \omega_t} + \frac{\alpha_t^H \alpha_t^L \bar{S}_t^L}{Z_t^2} \cdot (Q_t^H - Q_t^L) \cdot \frac{\partial \bar{S}_t^H}{\partial \omega_t} + \frac{\alpha_t^H \bar{S}_t^H \alpha_t^L}{Z_t^2} \cdot (Q_t^L - Q_t^H) \cdot \frac{\partial \bar{S}_t^L}{\partial \omega_t} \right] \tag{146}$$

As mentioned previously, economic intuition suggests that $\text{EV}_\omega < 0$, meaning that higher premiums lead to lower fundamental values precisely because increasing the premium incentivizes the fund to sell more fund shares, particularly from stocks with higher fundamental value, creating a portfolio distortion effect.

A simplified approach for analytical purposes develops an approximation that leverages the structural form of the Stage 3 solution. This approach begins by analyzing how the premium directly appears in the stock holdings and allocation fraction equations (136), (137), (140), and (141) derived in Stage 3.

From these relationships, it is possible to identify key factors that would drive portfolio distortion from premium changes: the price differential between stock types, the imbalance in allocation fractions, the total stock holdings, and the interaction between current holdings and returns. Combining these factors and incorporating appropriate normalization terms, the approach constructs a parametric approximation with a scaling factor $\mu > 0$ that can be calibrated to match the fully endogenous calculation:

$$\mathrm{EV}_\omega = -\mu \cdot \frac{(Q_t^H - Q_t^L)^2 \cdot |\alpha_t^H - \alpha_t^L| \cdot (\bar{S}_t^H + \bar{S}_t^L)}{R_t^L \bar{S}_{t-1}^L \cdot \bar{S}_t^H + R_t^H \bar{S}_{t-1}^H \cdot \bar{S}_t^L} \cdot \frac{\alpha_t^H \alpha_t^L \bar{S}_t^H \bar{S}_t^L}{Z_t^2} \tag{147}$$

Notably, this elasticity is also affected by information production through R_t^H and R_t^L, which incorporate the identification probability $p(X_t)$.

This expression reveals that the magnitude of the portfolio distortion effect increases with the price differential between stock types, $(Q_t^H - Q_t^L)^2$, the allocation imbalance between H-type and L-type stocks, $|\alpha_t^H - \alpha_t^L|$, and the total stock holdings $(\bar{S}_t^H + \bar{S}_t^L)$. It is further amplified by the interaction term $\alpha_t^H \alpha_t^L \bar{S}_t^H \bar{S}_t^L$, which captures how actively the fund sells shares of both stock types. Conversely, the effect is attenuated by the returns-weighted interaction between current and previous holdings in the denominator, $R_t^L \bar{S}_{t-1}^L \cdot \bar{S}_t^H + R_t^H \bar{S}_{t-1}^H \cdot \bar{S}_t^L$, and by the square of total fund shares issued (Z_t^2).

Using the elasticity of demand with respect to the premium, $\epsilon(\omega_t)$, as defined in (123), and substituting into the first-order condition (144):

$$\begin{aligned} V_t^F \cdot S_t^{F,D} + \omega_t \cdot \mathrm{EV}_\omega \cdot V_t^F \cdot S_t^{F,D} - \omega_t \cdot V_t^F \cdot \frac{\epsilon(\omega_t) \cdot S_t^{F,D}}{\omega_t} &= 0 \\ V_t^F \cdot S_t^{F,D}(1 + \omega_t \cdot \mathrm{EV}_\omega - \epsilon(\omega_t)) &= 0 \end{aligned} \tag{148}$$

Since $V_t^F \cdot S_t^{F,D} \neq 0$ in general, we get:

$$\begin{aligned} 1 + \omega_t \cdot \mathrm{EV}_\omega - \epsilon(\omega_t) &= 0 \\ \epsilon(\omega_t) &= 1 + \omega_t \cdot \mathrm{EV}_\omega \end{aligned} \tag{149}$$

Assuming that $\mathrm{EV}_\omega < 0$, this implies $\epsilon(\omega_t) < 1$ at the optimum. Solving for the optimal premium:

$$\omega_t^* = \frac{\epsilon(\omega_t^*) - 1}{\mathrm{EV}_\omega} \tag{150}$$

This appears to differ from the standard monopoly pricing result where the elasticity exceeds 1 at the optimum. However, this difference arises from the sign convention utilized in this model. The elasticity $\epsilon(\omega_t)$ as defined in (123) is already positive.

To connect to standard monopoly theory, consider that $\mathrm{EV}_\omega < 0$ as was initially assumed, so:

$$\begin{aligned}\omega_t^* &= \frac{\epsilon(\omega_t^*) - 1}{\mathrm{EV}_\omega} \\ &= \frac{\epsilon(\omega_t^*) - 1}{-|\mathrm{EV}_\omega|} \\ &= \frac{1 - \epsilon(\omega_t^*)}{|\mathrm{EV}_\omega|}\end{aligned} \tag{151}$$

Since $\epsilon(\omega_t^*) < 1$ from equation (149), $1 - \epsilon(\omega_t^*) > 0$, making $\omega_t^* > 0$ as required for a meaningful premium. If we define the magnitude of the elasticity of demand as $|\epsilon_D| = \epsilon(\omega_t)$, where ϵ_D is the conventional negative elasticity used in standard monopoly theory, then $\epsilon(\omega_t) < 1$ implies $|\epsilon_D| < 1$.

This apparent contradiction can be resolved by recognizing that the elasticity definition presented in this paper also captures the portfolio distortion effect. In standard monopoly pricing, elasticity only reflects how demand responds to price changes. In this model, the elasticity in equation (149) combines both the direct demand response and the indirect effect through changes in the fundamental value. When these effects are separated, the result becomes consistent with standard theory, with $|\epsilon_D| > 1$ at the optimum.

The optimal premium solution reveals several key economic insights.

First, market segmentation plays a pivotal role via its effect on demand elasticity. Since poor households do not have direct access to the stock market, they constitute a captive clientele with lower demand elasticity, which potentially leads to higher premiums.

Second, the analysis highlights a significant portfolio distortion effect that also acts as a self-limiting mechanism on the premium. When considering a higher premium ω_t, the fund anticipates not only lower demand $S_t^{F,D}$ but also a strategic adjustment to its optimal portfolio $\alpha_t^*, \bar{S}_t^*$ in Stage 3. This portfolio shift, which is driven by profit maximization under the new premium, typically lowers the fundamental value per share V_t^F, captured by the negative elasticity EV_ω. As a result, this interdependence results in an optimal premium ω_t^* lower than in a case when only demand elasticity were relevant. On top of that, the complex incentives generated by maximizing both premium revenue and proprietary returns, affected by this distortion, likely contribute to the model's observed tendency towards corner solutions for portfolio allocation.

Finally, information production and premium-setting decisions are strategically linked. The optimal premium depends on the information production level X_t through both the identification probability $p(X_t)$ and the portfolio allocation decisions it influences, creating a complex feedback mechanism between information quality and pricing behavior.

4.4.3 Stage 1: Information Production Decision

In the first stage, the fund chooses its information production effort X_t to maximize expected profits net of information costs as was specified in equations (44) and (45):

$$\max_{X_t} \mathbb{E}_t \Pi_t^F(X_t) - \gamma X_t^\xi$$

Subject to the resource constraint:

$$\gamma X_t^\xi \leq \kappa Y_t$$

The fund's first-order condition for optimal information production is:

$$\frac{d\mathbb{E}_t \Pi_t^F(X_t)}{dX_t} = \gamma \xi X_t^{\xi-1} \tag{152}$$

Following the backward induction approach, note that the fund has already determined the optimal premium $\omega_t^*(X_t)$ in Stage 2 and the optimal portfolio variables $\{\bar{S}_t^{H*}, \bar{S}_t^{L*}, \alpha_t^{H*}, \alpha_t^{L*}\}$ in Stage 3. By the envelope theorem, since these variables are chosen optimally in their respective stages, small changes in these variables due to changes in X_t have no first-order effect on the profit function. This means the total derivative simplifies to just the direct effect of X_t on profit through the identification probability:

$$\frac{d\mathbb{E}_t \Pi_t^F(X_t)}{dX_t} = \frac{dp(X_t)}{dX_t} \cdot \frac{\partial \mathbb{E}_t \Pi_t^F}{\partial p(X_t)} \tag{153}$$

Defining the return differential between correctly and incorrectly identified stocks:

$$\Delta R_t^H = (Q_t^H - Q_t^L) - (Q_{t-1}^H - Q_{t-1}^L) + (D_t^H - D_t^L) \tag{154}$$

$$\Delta R_t^L = (Q_t^L - Q_t^H) - (Q_{t-1}^L - Q_{t-1}^H) + (D_t^L - D_t^H) \tag{155}$$

The direct effect can then be expressed as:

$$\frac{d\mathbb{E}_t \Pi_t^F(X_t)}{dX_t} = (1-p_0)\psi e^{-\psi X_t} \cdot [(1-\alpha_t^H)\Delta R_t^H \bar{S}_{t-1}^H + (1-\alpha_t^L)\Delta R_t^L \bar{S}_{t-1}^L] \tag{156}$$

Setting this equal to the marginal cost and solving for the optimal information production level yields:

$$X_t^* = \frac{1}{\psi} \ln \left(\frac{(1-p_0)\psi \cdot [(1-\alpha_t^H)\Delta R_t^H \bar{S}_{t-1}^H + (1-\alpha_t^L)\Delta R_t^L \bar{S}_{t-1}^L]}{\gamma \xi (X_t^*)^{\xi-1}} \right) \tag{157}$$

This equation implicitly defines the optimal information production level.

Several key economic insights emerge from this analysis.

First, the fund's information production is higher when the return differential between correctly and incorrectly identified stocks (ΔR_t^H and ΔR_t^L) is larger. In other words, when high-productivity and low-productivity firms have substantially different returns, the fund has stronger incentives to invest in information.

Second, the fund's proprietary holdings $(1-\alpha_t^H)\bar{S}_{t-1}^H$ and $(1-\alpha_t^L)\bar{S}_{t-1}^L$ directly affect the return to information production. The fund invests more in information when it holds a larger share of stocks for proprietary trading rather than selling them as fund shares. This creates a self-reinforcing feedback loop that drives the model toward corner solutions: more proprietary holdings increase the return on information production, better information improves proprietary trading profits, which in turn incentivizes the fund to minimize allocation fractions and maximize proprietary positions. Without sufficient counterbalancing forces, this feedback mechanism naturally pushes the fund toward extreme allocations obtained in the numerical simulation rather than interior solutions.

Third, as captured by the factor $e^{-\psi X_t^*}$, information production exhibits diminishing returns, while higher information costs (larger γ or ξ) reduce optimal information levels. Better baseline information quality (higher p_0) similarly reduces the marginal benefit of additional investment.

Notably, information production in this model creates an interesting dual-benefit feedback loop. When the mutual fund invests in information production, it reduces productivity noise variance ($\sigma_\eta^2(X_t) = \frac{\bar{\sigma}^2}{1+\phi X_t}$), which, in turn, simultaneously improves financial returns and real economic efficiency. This creates an important externality: the fund's information production enhances overall market efficiency, but the fund is unable to fully appropriate this social benefit, potentially leading to suboptimal information production from a welfare perspective.

5. Equilibrium Characterization and Numerical Analysis

This section presents the discussion of the equilibrium characterization and numerical analysis of the model. Note that while standard macroeconomic models typically proceed from theoretical equilibrium derivation to numerical simulations, the complex nature of this framework characterized by its multistage optimization, non-linear feedback mechanisms, and intricate portfolio decisions presents significant challenges for conventional analytical analysis.

Therefore, instead of providing a closed-form analytical equilibrium, which proves mathematically intractable in the context of this model, the current section characterizes the equilibrium concept and directly examines its properties through numerical simulations. Such approach not only allows to address computational limitations but also reveals valuable economic insights about the model's structure. The very difficulties encountered in solving this model highlight the underlying tensions in financial markets with information-producing intermediaries that would remain obscured in simpler frameworks.

The analysis proceeds by first characterizing the equilibrium concept and computational challenges, followed by the explanation of the simulation approach developed to address these challenges. After that, the examination of the resulting steady state properties and corner solutions is provided. Finally, the analysis of the model's dynamic responses to monetary policy shocks is presented.

5.1 Equilibrium Concept and Computational Challenges

This model's equilibrium consists of allocation decisions by all agents, prices for all assets, and market clearing in all markets. In the equilibrium, the optimality conditions for households, firms, and the mutual fund must be satisfied, while ensuring that all markets clear through appropriate price adjustments.

The challenge of finding an equilibrium stems from a number of sources. First, as was highlighted in Section 4, the optimal outcomes of the mutual fund's staged decision process contain interdependencies between information production, premium setting, and portfolio allocation. Second, the fund's information production affects firms' current production decisions by reducing the variance of productivity noise, thereby improving the precision of firms' observed productivity signals. This creates a direct channel through which financial market activities enhance real economic efficiency, even as firms' forward-looking price-setting remains based on permanent productivity components. Third, market segmentation between wealthy and poor households generates asymmetric elasticities of demand for fund shares, which influences the fund's market power and premium-setting behavior.

These challenges collectively produce a complex system of non-linear equations that must be solved simultaneously in equilibrium. On top of that, the interdependencies between fund decisions, production efficiency, and market segmentation also lead to three distinct sources of strategic complementarity that significantly complicate the analysis:

1. **Information-Premium Complementarity:** Higher levels of information production may result in higher premiums through improved portfolio composition, which, in turn, may fund more information production

2. **Information-Portfolio Complementarity:** Better information increases the efficiency of portfolio allocation decisions, which thereafter increases returns. These reruns may be reinvested back in information production

3. **Premium-Portfolio Complementarity:** Higher premiums may change the incentives for portfolio allocation, incentivizing the fund to allocate more assets to fund shares to maximize premium revenue. This may have an effect on the fundamental value of fund shares, which reflects back on the optimal premium. Even though the sequential structure was designed to mitigate this feedback effect by having the premium decision precede the portfolio decision, the overly realistic forward-looking nature of the fund's optimization problem means the fund still accounts for how its premium will affect subsequent allocation choices, which creates a tension between the commitment mechanism and the rational expectations of the interaction of these decisions

The above complementarities facilitate the theoretical possibility of both "high-information, high-premium" equilibria, where sophisticated active management coexists with passive investment, and "low-information, low-premium" equilibria, where minimal information production occurs. Such potential for multiple equilibria captures the real-world observation that financial markets can operate under different regimes with varying levels of informational efficiency, which contributes to realism of the model.

However, the mathematical structure of these complementarities contributes to significant challenges when obtaining the numerical solution. In particular, the model's forward-looking variables and complex expectations formation lead to violations of the Blanchard-Kahn conditions necessary for a unique stable solution using standard perturbation techniques. Importantly, this violation is most likely not a mere technical issue but rather it may to some extent reflect a fundamental property of the economic structure being modeled, as there might be the inherent instability in financial markets with information-producing intermediaries unless additional frictions or constraints are introduced.

To address these challenges while preserving the model's key economic mechanisms, two equilibrium characterization approaches were created:

1. **Tractable Equilibrium with Preserved Mechanisms:** This approach was designed to maintain all essential economic channels and mechanisms while introducing computational simplifications to enhance solvability of the model. The key features of this approach include:

   - **Parameterized Variance-Covariance Structure:** Rather than deriving the variance-covariance matrix endogenously, this approach utilizes predefined functional forms where variances decrease linearly with information quality: $\sigma_H^2(X_t) = \sigma_{H,0}^2 - \sigma_{H,1}^2 \cdot p(X_t)$ and correlation increases linearly: $\rho(X_t) = \rho_0 + \rho_1 \cdot p(X_t)$. This helps alleviate computational complexity while preserving the core feature that better information reduces return volatility.
   - **Simplified Elasticity Calculation:** The elasticity of fundamental value with respect to the premium is calculated using a simplified formula give in equa-

tion (147) and discussed in Section 4: $EV_\omega = -\mu \cdot \frac{(Q_t^H - Q_t^L)^2 \cdot |\alpha_t^H - \alpha_t^L| \cdot (S_t^H + S_t^L)}{R_t^L S_{t-1}^L + R_t^H S_{t-1}^H} \cdot \frac{\alpha_t^H \alpha_t^L S_t^H S_t^L}{Z_t^2}$. This expression captures key economic dependencies without requiring complex derivatives: it scales with price differentials, portfolio imbalances, and interaction effects between allocation fractions and stock holdings.

- **More Lenient Convergence Criteria:** This approach accepts steady states with market clearing errors up to 5%, allowing for progress even when perfect market clearing is computationally problematic.

2. **Endogenous Equilibrium:** As opposed to the tractable approach, this approach preserves more endogenous relationships and employs a more sophisticated solution algorithm. Its distinctive features include:

- **Endogenously Derived Covariance Structure:** The variance-covariance matrix is directly computed from price differentials and information quality. The key formulas are:

$$\sigma_H^2 = (1 - p_X) \cdot p_X \cdot (Q_H - Q_L)^2 + (p_X^2 + (1 - p_X)^2) \cdot \sigma_\eta^2 \tag{158}$$

$$\sigma_L^2 = (1 - p_X) \cdot p_X \cdot (Q_H - Q_L)^2 + (p_X^2 + (1 - p_X)^2) \cdot \sigma_\eta^2 \tag{159}$$

$$\sigma_{HL} = (1 - p_X) \cdot p_X \cdot (Q_H - Q_L)^2 - p_X \cdot (1 - p_X) \cdot \sigma_\eta^2 \tag{160}$$

These formulas consider both main sources of uncertainty in the model: misidentification risk (first term) and productivity noise (second term). For covariance, the productivity noise term is negative, which creates an offsetting interaction between stock types when identification is imperfect.

- **Structured Approximation for Elasticity:** For the computation of the elasticity of fundamental value with respect to the premium, a slightly modified version of the formula in (147) is utilized, which is a structured approximation rather than the full theoretical expression. This specifications avoids the computational complexity of calculating multiple partial derivatives while preserving the key economic dependencies. The formula used is:

$$EV_\omega = -\mu \cdot \frac{(Q_H - Q_L)^2 \cdot |\alpha_H - \alpha_L| \cdot (S_H + S_L) \cdot \alpha_H \alpha_L S_H S_L}{Z_t^2 \cdot \max(V_F, \text{min\_var})} \tag{161}$$

The difference between this expression and the one used in the tractable approach is the explicit regularization term in the denominator, which prevents division-by-zero issues that may potentially arise with the full derivative calculation during iterations when V_F approaches zero.

- **Nested Iterative Structure:** The solution mechanism in this approach employs a nested fixed-point algorithm with an inner loop specifically targeting portfolio allocation and premium setting. This allows for closer convergence of these highly interdependent variables.
- **Highly Adaptive Damping:** The solution algorithm also utilizes a sophisticated damping mechanism that automatically adjusts from approximately 0.5 to as low as 0.006 based on convergence progress. This enables more reliable convergence despite the highly nonlinear relationships.
- **Stricter Market Clearing Requirements:** This approach also requires market clearing errors that are below 0.1% for a valid steady state, ensuring greater economic consistency.

While the tractable approach sacrifices certain degree of theoretical completeness for computational tractability, the endogenous approach focuses on more detailed modeling of feedback channels at the cost of added complexity. Notably, notwithstanding the methodological differences, both approaches produce the qualitatively similar results in regards to the inherent tensions in the fund's dual role, albeit with opposite corner solutions for stock holdings.

The combined results of the implementation of these two approaches provide complementary perspectives on the relationship between passive investment and the transmission of monetary policy, improving confidence in the findings that emerge consistently across both methodologies.

5.2 Simulation Approach and Methodology

The model is simulated in MATLAB using a first-order perturbation approach, linearizing the equilibrium conditions around the steady state. This approach is an appropriate choice considering the model's complexity and the focus on business cycle dynamics rather than higher-order effects.

The model is calibrated using standard parameter values from the macroeconomic literature, as detailed in Table 1. Several parameters require additional explanation: portfolio constraints (min_{S_H}, min_{S_L}, min_{α}) represent lower bounds imposed to prevent degenerate solutions; and the wealthy households' wage premium ($W^{W}_{premium}$) captures labor income inequality. The full endogenous model incorporates slight adjustments to the information production parameters to enhance stability, notably in the information effectiveness (ψ) and signal precision effect (ϕ) parameters. In addition to this baseline calibration featuring a relatively high share of wealthy households ($\lambda = 0.7$), an alternative calibration using a lower, potentially more standard value ($\lambda = 0.2$) was also utilized for robustness checks.

Table 1: Baseline Calibration

Parameter	Value	Description
Household Parameters		
β	0.99	Discount factor (quarterly)
σ	2	Risk aversion coefficient
η	1	Frisch elasticity of labor supply
λ	0.7	Share of wealthy households in baseline
γ^W	2	Risk aversion for wealthy households
γ^P	4	Risk aversion for poor households
Production Parameters		
ϵ	6	Elasticity of substitution between varieties
ς	0.5	Share of H-type firms
A^H	1.5	Productivity of H-type firms
A^L	1.0	Productivity of L-type firms
θ_H	0.75	Calvo parameter for H-type firms
θ_L	0.85	Calvo parameter for L-type firms
$W^W_{premium}$	1.5	Wealthy households' wage premium
Fund Parameters		
γ	0.2	Cost scaling for information production
ξ	1.5	Cost elasticity for information production
ψ	2.0-2.5	Information effectiveness parameter
p_0	0.5	Baseline identification probability
ϕ	0.5-0.6	Information effect on signal precision
σ_{bar}	0.05	Baseline signal variance
μ	0.3	Scaling factor for premium elasticity
Portfolio Constraints		
min_{S_H}	0.01-0.02	Minimum H-type stock holdings
min_{S_L}	0.01-0.02	Minimum L-type stock holdings
min_{α}	0.01	Minimum allocation fraction
Monetary Policy Parameters		
ϕ_π	1.5	Taylor rule inflation response
ϕ_y	0.125	Taylor rule output gap response (quarterly)
ρ_i	0.8	Interest rate smoothing parameter

The solution process follows several key steps.

First, the deterministic steady state is computed using an iterative fixed-point algorithm with adaptive damping to ensure convergence. The two model variants show significant differences in convergence behavior. The tractable model fails to achieve full convergence even after 2000 iterations, with the continued occurrence of relatively large changes and minimal market clearing errors. In contrast, the endogenous model achieves proper mathematical convergence through highly adaptive damping, automatically reducing the factor from approximately 0.5 to 0.006 as iteration progresses, indicating that while both models share similar theoretical structures, the endogenous model's steady state is computationally more stable, perhaps because of the more theoretically consistent approach to calculating the variance-covariance matrix.

In practice, both model variants converge toward corner solutions, albeit of different types. The tractable model consistently drives the fund toward maximum allowable stock holdings with minimum allocation fractions. Interestingly, the endogenous model produces the opposite corner solution, with the fund holding minimum allowable stock quantities while still setting allocation fractions to minimum values. This stark contrast is most likely due to a fundamental difference in how the two model specifications treat variance-covariance relationships. However, both models share the common feature of pushing allocation fractions to their lower bounds, which suggests that this aspect of the fund's behavior is robust to modeling assumptions.

Once the steady state is obtained, the system is linearized to produce a linear rational expectations structure of the form $A\hat{y}_t = B\mathbb{E}_t[\hat{y}_{t+1}] + C\varepsilon_t$, where $\hat{y}_t$ represents deviations from steady state, ε_t is the vector of structural shocks, and A, B, and C are matrices of coefficients derived from the model's equilibrium conditions. The linearized system is solved using the Sims (2002) method implemented in the `gensys` function, which applies QZ decomposition to derive the state transition equation.

Both model variants encounter significant challenges in their dynamic solutions despite different steady state properties. In fact, both models fail at the same step and for identical structural reasons: they have 9 forward-looking variables but insufficient unstable eigenvalues to satisfy Blanchard-Kahn conditions. After trying different calibration parameters, this violation remained, suggesting that it must be an underlying property of the model structure rather than an implementation issue and should be addressed when.

The solution algorithm attempts multiple different approaches before resorting to a fallback mechanism. First, it tries a standard solution using the Sims (2002) method. When this fails, it applies matrix regularization to improve conditioning but after these attempts prove unsuccessful, both models ultimately rely on the same artificial "stabilized policy function" approach that creates a transition matrix with controlled eigenvalues. This matrix uses diagonal elements of 0.7 for moderate persistence, adds small cross-effects based on normalized system matrices, and caps eigenvalues at 0.95 to ensure stability. This artificial approach produces the qualitative patterns seen in the impulse responses, which will be discussed in the upcoming sections, but explains why the response magnitudes are negligible (around 10^{-11} to 10^{-17}).

5.3 Steady State Properties and Corner Solutions

The numerical solution and analysis reveal distinct steady state characteristics across the two model specifications, each suggesting different economic forces at play. The most striking finding is the emergence of contrasting corner solutions for the mutual fund's stock holdings, a pattern robustly observed across different calibrations, alongside a consistent tendency towards minimal fund share allocation.

Specifically, the tractable model specification consistently drives the fund toward maximum allowable stock holdings. Under the baseline calibration ($\lambda = 0.7$) and the alternative calibration ($\lambda = 0.2$), the fund seeks to hold 100% of the available supply of both

H-type and L-type stocks (corresponding to $S_H = \varsigma$ and $S_L = 1 - \varsigma$). Simultaneously, it sets the allocation fractions, representing the portion of its holdings sold as fund shares, to the minimum allowable level (e.g., $\alpha_H = \alpha_L = 0.01$). This outcome suggests that when the complex variance-covariance structure of returns is simplified using parameterized approximations, the fund perceives strong incentives to maximize its proprietary trading activities. It aims to hold all available stocks, presumably to fully leverage any informational advantage derived from X_t, while minimizing its intermediation role by offering only the smallest possible fraction of its portfolio to investors. The potential benefits from premium revenue (ω_t) and the broader market effects of its information appear insufficient in this setup to outweigh the perceived gains from internalizing the full value of its stock positions.

In stark contrast, the endogenous model, which derives the variance-covariance structure directly from underlying price differentials and identification probabilities, produces the opposite corner solution for stock holdings, yet still achieves proper mathematical convergence. Across both baseline and alternative calibrations ($\lambda = 0.7$ and $\lambda = 0.2$), the fund converges to the minimum allowable stock holdings (e.g., 2-4% of the available supply, depending on the specific bound parameters) while, similar to the tractable model, setting allocation fractions to their minimum allowable values ($\alpha_H = \alpha_L = 0.01$). This result is particularly noteworthy as it arises despite the shared core theoretical structure. It implies that the endogenous computation of risk and return relationships fundamentally alters the perceived trade-offs. Instead of maximizing participation, the fund opts to minimize its direct holdings in the stock market, effectively minimizing its exposure and potentially its role as a significant direct investor. However, it persists in minimizing its intermediation activity via low allocation fractions. This suggests an equilibrium where the fund primarily exists as a marginal participant, perhaps focusing on extracting small informational rents or simply minimizing operational scale, rather than actively engaging in either large-scale proprietary trading or broad intermediation.

The emergence of these opposing corner solutions for stock holdings, robustly linked to the specific modeling choice regarding the variance-covariance structure rather than the share of wealthy households, underscores the model's sensitivity to assumptions about financial risk characteristics. It highlights that the incentives governing an information-producing intermediary's market participation (scale of holdings) versus its intermediation activity (share allocation) are complex and highly dependent on how risk interplays with information advantages and premium-setting opportunities. Furthermore, the existence of these distinct, stable steady-state regimes suggests the possibility of multiple equilibria or regime-switching behavior in markets with such intermediaries, driven perhaps by structural changes, regulatory shifts, or evolving market conditions that might alter the perceived risk-return landscape in ways analogous to the differences between the tractable and endogenous specifications. Understanding the conditions that might favor one regime over the other presents an important avenue for future research.

5.4 Dynamic Properties and Computational Boundary

Attempts to analyze the model's dynamic behavior through impulse response functions (IRFs) to monetary policy shocks using standard first-order perturbation methods did produce numerical results. However, both model variants failed to satisfy the Blanchard-Kahn (BK) conditions necessary for a unique, stable dynamic solution. Critically, this failure proved robust, persisting across different calibrations including the alternative specification with $\lambda = 0.2$, indicating the violation is likely an inherent property of the model structure.

Despite this violation of standard stability conditions, it was deemed viable to implement a controlled fallback solution method—specifically employing an artificial stabilization matrix—to explore whether the model could still generate qualitative insights about monetary policy transmission and to identify precisely how the structural tensions in the model manifest in dynamic responses. However, these IRFs exhibited magnitudes on the order of 10^{-11} to 10^{-17} (Figure 1), effectively representing numerical noise rather than economically meaningful responses. These negligible magnitudes are a direct consequence of the artificial stabilization technique employed after the standard solution method failed due to the BK condition violation. The patterns observed in these IRFs likely reflect properties of the artificial stabilization technique rather than genuine model dynamics, making them unsuitable for quantitative analysis of monetary policy transmission.

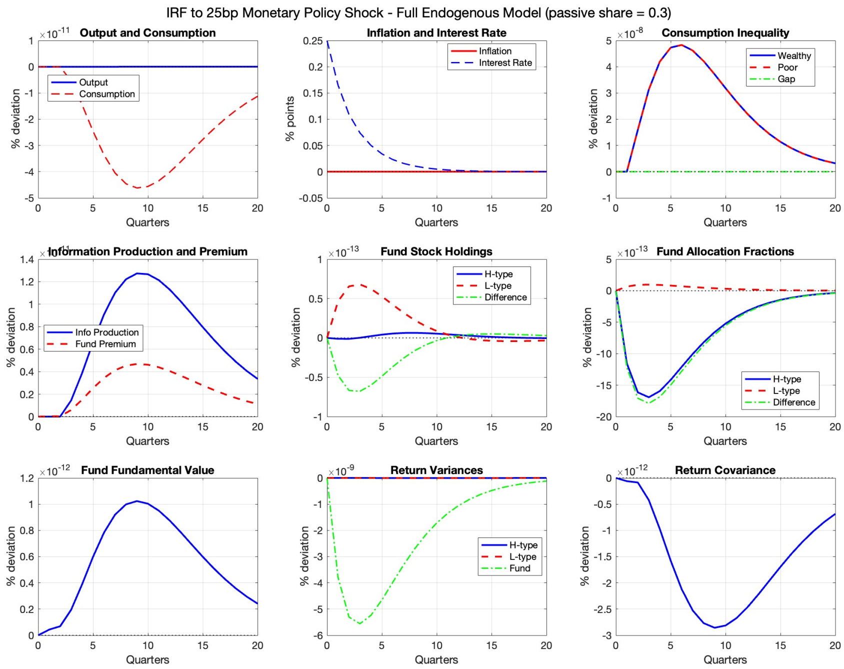

Figure 1: Impulse responses to a contractionary monetary policy shock in the endogenous model. Note: The responses have magnitudes of 10^{-11} to 10^{-17}, rendering them effectively numerical noise. These patterns reflect properties of the artificial stabilization technique rather than genuine model dynamics.

Notably, to some extent this computational boundary is not merely a technical limitation but rather a reflection of certain fundamental properties of the economic structure being modeled. When institutional investors, represented by the mutual fund in this model, already occupy extreme positions in the steady state (either maximum or minimum stock holdings with minimal allocation fractions), they have rather limited capacity to adjust portfolios in response to policy shocks within a linearized framework, which inevitably constrains traditional transmission channels. On top of that, the structural complementarities identified earlier may create potential instabilities in the transition dynamics that standard perturbation methods struggle to capture.

Rather than interpreting these quantitatively unreliable impulse responses, this analysis suggests that the computational challenges encountered may indicate limitations in standard dynamic analysis techniques when applied to this particular model structure. Interestingly, the contrasting corner solutions obtained in the steady state analysis—with funds gravitating toward either maximum or minimum stock holdings depending on modeling assumptions—do suggest the possibility that financial markets with information-producing intermediaries might exhibit regime-switching behavior rather than smooth transitions. This characteristic, while requiring further research to fully validate, would imply that monetary policy transmission might operate differently depending on which market regime prevails. Central banks might therefore benefit from considering how financial market structure influences policy effectiveness, particularly in markets with evolving active-passive investment shares.

Overall, these findings emphasize the need for alternative modeling approaches to study dynamic interactions between passive investment, information production, and monetary policy transmission.

6. Key Results and Discussion

Building on the analytical and computational results presented in the previous sections, this section focuses on interpreting the broader economic implications of the results, discussing their significance for financial market theory and policy, and, importantly, discussing limitations of the current research and proposing potential avenues for future analysis.

6.1 Economic Interpretation of Corner Solutions

The opposing corner solutions observed in both specifications of the model reveal an underlying economic tension inherent in information-producing financial intermediaries. This tension highlights what might be called the "information producer's dilemma", which occurs when an entity producing valuable information faces significant incentives to capture that value privately rather than sharing it with clients.

This result has a direct connection to the literature on conflicts of interest in financial institutions. As Bolton et al. (2007) note, integrated financial institutions that combine proprietary trading with client services face inherent tensions between these functions. Therefore, one inference that could be made from the simulation outcomes is that the extreme corner solutions in this model formalize this intuition, suggesting that without countervailing forces (regulatory constraints, reputational concerns, or organizational structures, etc.), the natural economic tendency is drawn toward either hoarding information value or minimal market participation.

In addition, the model's outcomes also provide a novel perspective on the Grossman-Stiglitz paradox (1980), which identifies a tension between market efficiency and information production incentives. This model demonstrates how such tension can take the form of discontinuous market structures rather than smooth equilibrium adjustments, particularly in cases when information producers also act as intermediaries.

6.2 Passive Investment and Financial Market Stability

This analysis also makes valuable contributions to the examination of the growing trend toward passive investment. While the conventional concern about passive investment generally focuses on declining price efficiency (as in Jaquart et al., 2023), this model provides an alternative perspective by identifying a more structural concern characterized by the potential for discontinuous shifts in market organization.

If it is indeed the case that information-producing intermediaries exhibit regime-switching behavior rather than smooth transitions as suggested by the model's corner solutions, the growth in passive investment may not result in gradual changes in market functioning but rather it could trigger sudden shifts in market structure once critical thresholds are crossed. To a certain degree, this perspective may provide a theoretical foundation for recent empirical observations of market fragility during periods of rapid passive investment growth.

Additionally, the consistent finding that funds minimize their allocation fractions regardless of total stock holdings makes important suggestion that the intermediation function itself may be systematically undervalued in current market structures. While the specific mechanism somewhat differs from phenomena observed empirically, this interpretation draws significant parallel support from evidence on 'closet indexing' (Cremers & Petajisto, 2009). Their research shows numerous funds charging fees characteristic of active management, essentially constituting compensation for intermediation, while maintaining portfolios with minimal deviation from market benchmarks, effectively limiting active risk-taking. Even though minimizing allocation fractions in this model serves to prioritize proprietary trading rather than benchmark-hugging, both findings point to a common structural issue: the market permits funds to receive compensation associated with active intermediation without fully committing to the core function (be it serving clients extensively via providing shares in the model, or taking significant active risk empirically). This observation suggests that the market structure, in both the model's logic and observed empirical practice, does not sufficiently value or incentivize genuine, unconstrained active intermediation relative to the perceived benefits of alternative strategies like proprietary trading or low-risk fee collection.

6.3 Implications for Monetary Policy Effectiveness

The findings from this model have particularly important implications for central banks operating in evolving financial markets. Traditional monetary policy models typically assume smooth transmission channels operating through gradual portfolio adjustments. The possibility of regime-switching behavior in financial intermediaries suggests that policy effectiveness might vary dramatically depending on which market regime prevails.

When intermediaries operate at corner solutions (either maximum holdings/minimum sharing or minimum holdings overall), their capacity to adjust portfolios in response to policy changes becomes constrained. This constraint could create nonlinearities in policy transmission that standard models fail to capture. For instance, interest rate changes might have dramatically different effects depending on whether markets are in a high-information/proprietary-trading regime or a low-information/minimal-participation regime.

These findings suggest that central banks may need to develop regime-specific modeling approaches that account for structural differences in transmission mechanics across different market configurations. The framework developed in this paper provides a starting point for such regime-specific analysis, identifying key mechanisms through which regime characteristics might influence policy effectiveness.

6.4 Information Production as a Public Good

Another significant insight from this analysis concerns the public good aspects of information production in financial markets. In this model, fund-produced information enhances the precision of firms' productivity signals, creating economy-wide benefit spillovers beyond just improved investment returns for the fund. However, the fund's demonstrated tendency toward corner solutions featuring minimal sharing highlights a crucial misalignment: since the fund incurs the costs of information production but cannot directly monetize the resulting economy-wide efficiency benefits, its profit-maximizing strategy leads it to fully prioritize the easily capturable private benefits derived largely from proprietary trading, rather than performing its intermediation role. This outcome heavily suggests that purely market-driven incentives may lead to an under-provision or under-utilization of the socially beneficial aspects of financial information.

This finding suggests a potential role for policy intervention to support the public good role of financial information production. Such interventions might take the form of subsidies for information-sharing activities, regulations requiring minimum allocation fractions, or the development of specialized institutions explicitly devoted to producing and disseminating market information as a public service.

Ultimately, this analysis emphasizes the critical public good nature of financial information within this framework. The identified tension, where the private fund bears production costs but cannot capture the resulting economy-wide spillovers, leads to suboptimal sharing in the model's steady state. Theoretically formalizing these positive externalities thus provides a useful lens for evaluating market structure and a motivation for policy discussions that may treat financial information as part of public infrastructure rather than purely private good.

6.5 Limitations and Directions for Future Research

The analysis indicates a number of important limitations in the current modeling approach that present promising directions toward future research avenues. These limitations fall into several main categories: clarity of information assumptions, structural design choices potentially creating unintended consequences, incentive misalignments, and significant methodological constraints revealed during the analysis.

First, a significant limitation stems from potential ambiguity in how the model specifies the information available to intermediate firms, particularly regarding their contemporaneous productivity shocks ($\eta_t(f)$). The model structure, where firms know their base productivity (A^I) and observe their actual output per worker ($A_t^I(f)$), implies they can perfectly deduce the current noise realization ($\eta_t(f)$). This deduction makes the period-by-period production decision reactive—simply hiring the labor required to meet demand determined by preset prices—rather than an active optimization choice. This structural implication contrasts somewhat with the exposition's language emphasizing uncertainty and noisy signals regarding contemporaneous productivity, potentially creating interpretative challenges. While the core economic decision lies in the forward-looking price-setting stage, where firms correctly account for the transitory nature of the noise, the precise mechanism and information set governing contemporaneous production requires refinement for consistency. Furthermore, the price-setting problem relies on simplified expectations formation, notably the assumption of a deterministic expected wage growth path, given in equation (88), abstracting from potential uncertainty in future factor costs. Future research should revisit the modeling of firms' contemporaneous information acquisition, potentially incorporating imperfect signal extraction regarding $\eta_t(f)$ to better align structure and narrative, and explore the implications of richer expectations formation processes.

Second, a fundamental challenge arises from the interaction between heterogeneous labor inputs and information asymmetry. The assumption that firms hire perfectly type-specific labor, while intended to ensure the viability of low-productivity firms, may inadvertently create a mechanism that perfectly reveals firm types to the market. If households can observe which type of labor a firm hires, the uncertainty regarding firm types

– central to the fund's information production role and the direct investment decisions of wealthy households – is potentially eliminated. This structural feature could undermine the model's core premise regarding the value of costly information acquisition and the fund's specific advantage. The current framework implicitly assumes some friction preventing this perfect inference from labor choices, or this revealing aspect represents an oversight. Addressing this critical structural limitation is vital for future research. Potential avenues include: modeling labor markets with imperfect specialization, where firms might use a mix of labor types or where labor skills are less distinct identifiers; introducing explicit frictions in observing firms' factor inputs; reconsidering the fund's value proposition beyond just type identification (e.g., focusing on its ability to better predict the magnitude of noise $\eta_t(f)$, offering superior diversification, or lower transaction costs); or exploring alternative mechanisms to support low-productivity firm viability that do not rely on type-revealing factor inputs.

Third, key structural design choices influence the model's outcomes and present limitations that interact with calibration assumptions. The three-stage sequential optimization for the mutual fund, designed to manage complex interdependencies, appears to create strong feedback loops between stages based on rational expectations of future actions. This amplification of forward-looking behavior may contribute significantly to the observed numerical instability and the tendency towards extreme steady-state allocations. Evaluating the robustness of the findings to alternative modeling of the fund's decision process (e.g., simultaneous optimization, different sequencing, or models incorporating learning/bounded rationality) is an important direction. Relatedly, the initial calibration choice of a high proportion of wealthy households ($\lambda = 0.7$) was investigated. As detailed in Sections 5.3 and 5.4, robustness checks using $\lambda = 0.2$ demonstrated that while quantitative levels adjusted, the qualitative nature of the steady-state corner solutions (maximal vs. minimal holdings depending on variance-covariance treatment) and the fundamental dynamic instability (BK failure) persisted. This reinforces the interpretation that the model's core challenges are structural, rooted in the fund's optimization and risk-modeling framework, rather than being solely calibration-dependent.

Finally, the analysis starkly revealed the model's computational fragility and limitations of standard solution methods for this class of problem. The emergence of contrasting steady-state corner solutions highlights significant sensitivity to modeling assumptions regarding risk. More critically, the robust failure across different calibrations to satisfy the Blanchard-Kahn conditions confirms that standard first-order perturbation methods are insufficient for characterizing the dynamics of this system. The negligible impulse responses obtained via artificial stabilization are computational artifacts, not reflective of economic behavior. This computational boundary strongly suggests that the underlying economic system, with its inherent feedback loops, potential for multiple equilibria (implied by the contrasting steady states), and state-dependent constraints (corner solutions), possesses complex nonlinear dynamics. Consequently, assessing the dynamic impact of passive investment share on monetary policy effectiveness requires moving beyond linearization. Future research should prioritize alternative solution methodologies such as global solution techniques, agent-based modeling, or possibly Markov-switching frameworks capable of handling potential regime shifts, discontinuities, and nonlinearities. Furthermore, incorporating explicit economic mechanisms that might provide countervailing forces against extreme allocations—such as portfolio adjustment costs, reputational concerns tied to intermediation volume, or regulatory constraints—could yield models with more stable interior solutions and potentially more tractable dynamics.

7. Conclusion

This paper has proposed a novel extension of the New Keynesian framework incorporating information-producing mutual funds to investigate how passive investment might influence monetary policy transmission. By modeling a financial sector with information production capabilities, the model presented in this paper managed to implement a direct channel linking financial market activities to real economic outcomes through enhanced precision of firms' productivity signals affected by the fund's information production activities.

The analysis reveals underlying tensions inherent in financial intermediaries with dual roles as information producers and investment vehicles. Two model specifications, differing mainly in their treatment of variance-covariance structure of assets returns, produced contrasting corner solutions—one driving funds toward maximum stock holdings in the tractable model specifications and the other toward minimum holdings in the endogenous setup—while both consistently minimized fund share allocation. These contrasting outcomes within each specification proved qualitatively robust to alternative calibrations of household heterogeneity ($\lambda = 0.7$ vs. $\lambda = 0.2$), as discussed in Section 5.3. These results formalize the well-known conflicts of interest in integrated financial institutions and provide a structural perspective on the Grossman-Stiglitz paradox regarding market efficiency and information production incentives.

The encountered computational challenges, particularly the failure to satisfy Blanchard-Kahn conditions when linearizing the model, highlight difficulties in characterizing a unique stable dynamic path using standard numerical methods. This failure was also found to be robust across calibrations (Section 5.4) and ultimately prevented a reliable quantitative assessment of the model's dynamic properties. When considered alongside the contrasting steady-state corner solutions, which represent strikingly different modes of intermediary behavior, these computational issues suggest the possibility that the underlying economic system might exhibit complex nonlinear dynamics, potentially including regime-switching behavior rather than smooth transitions. If this characteristic is indeed reflective of real-world market structures, it has important implications for monetary policy. Since transmission mechanisms may operate differently across distinct market regimes, central banks may need to develop adaptive approaches that account for potential discontinuities in financial market structure rather than assuming uniformly smooth transmission.

In addition to its insights for monetary policy, the framework developed in this paper emphasizes the public good characteristics of information production in financial markets. The model formalizes how fund-produced information generates positive externalities by enhancing the precision of firms' productivity signals, which leads to improved real economic efficiency across the whole model. However, due to the inability of the private fund to fully capture these widespread benefits despite bearing the information production costs, a potential market failure arises, suggesting that, without external interventions, information production might be suboptimal from a societal viewpoint. This result provides a strong theoretical foundation for considering certain aspects of financial information as public infrastructure and for exploring policy interventions specifically designed to facilitate information production or better align private incentives with social benefits.

While computational limitations prevented a proper quantitative assessment of how passive investment share affects policy effectiveness, the theoretical structure developed here offers valuable qualitative insights into the convoluted relationships between information production, market structure, and monetary policy transmission. Acknowledging the specific limitations detailed in Section 6.5, including the computational boundary and certain structural assumptions, identifying how the incentives inherent in information-producing intermediaries can drive their portfolio choices toward extreme allocations, as seen in the simulation results, and highlighting the potential implications for market functioning—possibly involving discontinuities rather than smooth adjustments—represents a key contribution of this paper to understanding evolving financial markets.

Several promising directions exist for building upon this framework. Methodologically, alternative solution techniques designed for potential regime-switching dynamics could help overcome the computational challenges identified in this study. Structurally, introducing balancing mechanisms—like reputation effects or adjustment costs—that better align information production and intermediation incentives could address the tendency toward corner solutions in the model's steady state. Refining specific structural elements, such as the interaction between labor market specification and information revelation, is also warranted based on the limitations discussed. From a policy perspective, incorporating explicit regulatory constraints and taxation allows for evaluating interventions that support optimal information production while maintaining market stability. As passive investment continues to reshape financial markets, such refinements will be vital for understanding the evolving interplay between market structure and monetary policy effectiveness.

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