Enhancing Exchange Rate Forecasting: The Impact of Outlier Removal on USD/KZT Predictions

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Abstract

Exchange rate forecasting presents significant challenges due to the influence of market shocks. This study evaluates forecasting models (ARIMA, SARIMAX, and LSTM) for predicting the USD/KZT exchange rate following Kazakhstan's 2015 transition to a floating exchange rate regime. Specifically, it investigates whether preprocessing through outlier removal via Changepoint Analysis and Isolation Forest improves forecasting accuracy. Using daily data from 2015 to 2024 and incorporating key exogenous variables, model performance is compared with and without outlier removal. The results show that outlier removal leads to narrower confidence intervals and improved model stability, particularly in ARIMA and SARIMAX forecasts. These findings contribute to understanding optimal preprocessing techniques for exchange rate forecasting in markets subject to external shocks.

1 Introduction

In August 2015, Kazakhstan transitioned from a managed exchange rate regime to a free-floating exchange rate policy, abandoning the exchange rate band as part of a shift toward inflation targeting as its new monetary policy framework.

The National Bank of Kazakhstan (NBK) declared that it would limit interventions in the foreign exchange market to prevent excessive volatility that could disrupt the financial system, rather than directly influencing the tenge price.

The exchange rate has substantial implications for Kazakhstan's economy, heavily dependent on oil exports, foreign investments, and trade with neighboring countries [1]. Accurate forecasting is crucial for monetary policymakers, businesses engaged in international trade, investors, and financial institutions operating in Kazakhstan. However, sudden market shocks and unusual periods of volatility challenge the capture of market trends for forecasting models.

The field of exchange rate forecasting has evolved significantly since the seminal work of Meese and Rogoff [2], who established that structural models based on macroeconomic fundamentals were unable to outperform a simple random walk model in out-of-sample forecasting. As de Souza Vasconcelos and Hadad Júnior [3] demonstrate in their bibliometric analysis, exchange rate forecasting methodologies have diversified considerably, incorporating both linear time series approaches such as ARIMA and SARIMAX, and nonlinear methods including neural networks such as LSTM (Long Short-Term Memory).

These time series models are important tools in economic forecasting, yet their effectiveness can be considerably compromised by the presence of outliers, especially in volatile emerging markets. The work by Qu et al. [4] concludes that implementing preprocessing techniques to manage these anomalous data points can enhance forecasting accuracy and robustness. In financial time series, Isolation Forest is one of the common techniques for detecting unusual patterns [5]. Additionally, the study by Manner [6] highlights the necessity of detecting changepoints in volatility, as these shifts can greatly influence risk and behavior in financial markets. Both techniques, along with manual intervention, are applied in this study to remove known major shocks from Kazakhstan's market.

In this paper, the identification and exclusion of anomalous periods through Changepoint Analysis and Isolation Forest is examined on whether outlier detection improves forecasting accuracy for the USD/KZT exchange rate. By comparing the forecasts from the ARIMA, SARIMAX, and LSTM models with and without outlier removal preprocessing, this research evaluates different methodological approaches to exchange rate prediction in a market characterized by periodic volatility.

The study makes two key contributions: (1) comparing multiple forecasting models for the USD/KZT exchange rate; (2) evaluating the effect of outlier removal on prediction accuracy.

2 Methodology

2.1 Data Collection and Preprocessing

This study analyzes daily USD/KZT exchange rate data from September 9, 2015, to December 31, 2024, obtained from the National Bank of Kazakhstan. Using data from TradingEconomics, several external variables potentially influencing the exchange rate are also included, as detailed in Table 1.

Variable	**Description**
usdkzt	USD/KZT Exchange Rate (target variable)
usdrub	USD/RUB Exchange Rate
kase	Kazakhstan Stock Exchange (KASE) Index
brent	Brent Crude Oil Price
fx	Kazakhstan's Foreign Exchange Reserves
usinterest	US Interest Rate
usinflation	US Inflation Rate
usgdp*	US Quarterly GDP growth
kzinterest*	Kazakhstan's Interest Rate
kzinflation*	Kazakhstan's Inflation Rate
kzgdp*	Kazakhstan's Quarterly GDP growth

* Variables excluded from the final analysis based on feature selection criteria

Table 1: Dataset Variables Description

The preprocessing workflow includes:

1. Handling missing values: FX reserves data, initially available monthly, is converted to daily frequency using forward-filling. Similarly, missing values for market closure days are forward-filled.

2. Stationarity testing using the Augmented Dickey-Fuller (ADF) test to determine appropriate differencing.

3. Feature selection based on correlation analysis to retain the most relevant variables.

Figure 1 shows the daily USD/KZT exchange rate for the study period. The time series exhibits several notable periods of volatility, particularly during global market shocks (COVID-19 pandemic), regional geopolitical events (Russia-Ukraine conflict), and oil price fluctuations.

Figure 1: Daily USD/KZT exchange rate from September 9, 2015, to December 31, 2024

The correlation analysis between the target variable and potential predictors is visualized in Figure 2, guiding feature selection for the SARIMAX and LSTM models.

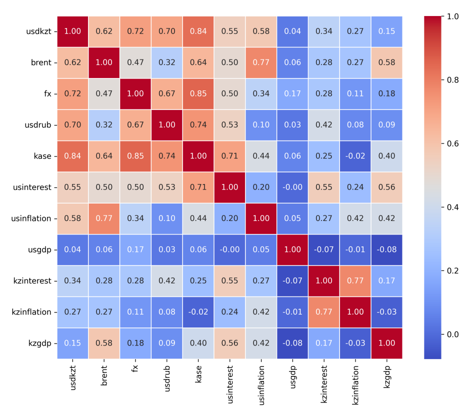


Figure 2: Feature correlation heatmap showing relationships between the USD/KZT exchange rate and external variables.

2.2 Outlier Detection Methods

Two complementary outlier detection techniques are employed:

2.2.1 Changepoint Analysis with Chebyshev's Inequality

In this study, a non-parametric approach for detecting outliers is employed, utilizing a combination of changepoint analysis and Chebyshev's inequality. The aim of changepoint analysis is to identify points where the distribution of residuals $R(x_1), \ldots, R(x_n)$ changes significantly [7]. The nonparametric E-Divisive method [8] is implemented, identifying points where the statistical properties of the time series shift without assuming any specific distribution.

Once the residuals are partitioned into homogeneous segments

$$S_j = \{R(x_{\hat{\tau}_{j-1}+1}), \ldots, R(x_{\hat{\tau}_j})\}, \tag{1}$$

Chebyshev's inequality is applied to identify outliers within each segment. This distribution-free approach states that for any random variable with finite variance, the probability of an observation deviating from its mean by more than L standard deviations is at most $1/L^2$, regardless of the underlying distribution [7].

For each segment S_j, threshold limits are constructed as:

$$\pm L\hat{s}_j \tag{2}$$

where $\hat{s}_j$ represents the sample standard deviation of residuals in segment S_j, and L is a parameter determining the sensitivity of outlier detection.

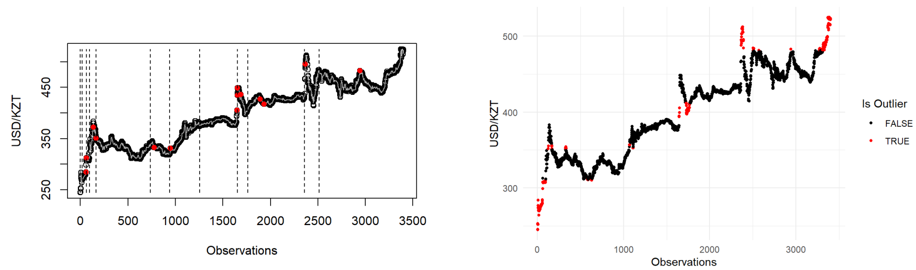
2.2.2 Isolation Forest

Isolation Forest exploits two key properties of anomalies: (1) they are few in number, and (2) they possess attribute values significantly different from normal instances. The algorithm constructs isolation trees (iTrees) that partition data recursively until all points are isolated. Due to their distinct characteristics, anomalies require fewer partitions and thus have shorter path lengths in the resulting trees [9].

The anomaly score s for an instance x is calculated as:

$$s(x, n) = 2^{-\frac{E(h(x))}{c(n)}} \tag{3}$$

where $E(h(x))$ is the average path length of x across multiple iTrees, $c(n)$ is the average path length of unsuccessful search in Binary Search Trees, and n is the subsample size. When s approaches 1, the instance is more likely to be an anomaly; values close to 0.5 suggest normal points.



(a) Outliers detected using Changepoint Analysis

(b) Outliers detected using Isolation Forest

Figure 3: Comparison of outlier detection methods applied to USD/KZT exchange rate data. Red dots indicate detected outliers. The Changepoint Analysis (a) identifies regime shifts and sustained anomalous periods, while Isolation Forest (b) detects point anomalies and short-term deviations.

Additionally, known macroeconomic shock periods (e.g., currency devaluation, COVID-19 pandemic, Russia-Ukraine conflict) are manually identified and incorporated into the outlier detection framework.

2.3 Forecasting Models

2.3.1 ARIMA

The AutoRegressive Integrated Moving Average model is specified as:

$$y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \tag{4}$$

where y_t is the time series (possibly after differencing), c is a constant, ϕ_i are the AR parameters, θ_j are the MA parameters, and ϵ_t is white noise [10]. The model is denoted as ARIMA(p, d, q) where:

- p is the order of the autoregressive terms (determined using PACF)
- d is the degree of differencing required for stationarity (determined using ADF test)
- q is the order of the moving average terms (determined using ACF)

Model parameters are estimated using Maximum Likelihood Estimation (MLE), and optimal model specification is selected based on the Akaike Information Criterion (AIC).

$$\text{AIC} = -2\ln(L) + 2k \tag{5}$$

$$\text{BIC} = -2\ln(L) + k\ln(n) \tag{6}$$

where L is the maximum value of the likelihood function, k is the number of estimated parameters, and n is the sample size. Lower values of AIC and BIC indicate better model fit, with BIC generally favoring more parsimonious models due to its stronger penalty term for additional parameters.

2.3.2 SARIMAX

The Seasonal ARIMA with eXogenous variables extends the ARIMA model by incorporating seasonal components to capture periodic patterns and exogenous variables to account for external factors.

The model is formulated as:

$$\Phi(B^s)\phi(B)(1-B^s)^D(1-B)^d y_t = \delta + \Theta(B^s)\theta(B)\epsilon_t + \sum_{i=1}^{r}\beta_i X_{i,t} \tag{7}$$

where B is the backshift operator, Φ and Θ are seasonal AR and MA polynomials, s is the seasonal period, D is the seasonal differencing order, $X_{i,t}$ are exogenous variables, and β_i are their coefficients.

For this study, the exogenous variables include the USD/RUB exchange rate, oil prices, KASE index, and FX reserves.

Additionally, a seasonal component with $s = 5$ was included to capture weekly patterns in the exchange rate, reflecting the business week cycle in financial markets.

2.3.3 LSTM

Long Short-Term Memory networks are specialized recurrent neural networks capable of learning long-term dependencies in time series data. The LSTM network architecture consists of:

$$
\begin{aligned}
f_t &= \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \\
i_t &= \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \\
\tilde{C}_t &= \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \\
C_t &= f_t * C_{t-1} + i_t * \tilde{C}_t \\
o_t &= \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \\
h_t &= o_t * \tanh(C_t)
\end{aligned} \tag{8}
$$

where f_t, i_t, and o_t represent the forget, input, and output gates respectively; C_t is the cell state; h_t is the hidden state; W and b are learnable parameters; σ is the sigmoid activation function; and $*$ denotes element-wise multiplication [11].

2.4 Evaluation Framework

To assess forecasting performance and the impact of outlier removal, the following steps are considered:

1. Data splitting: 80% for training and 20% for testing
2. Model training on both original and outlier-removed datasets
3. Forecasting horizon: 90-day ahead predictions
4. Performance metrics:

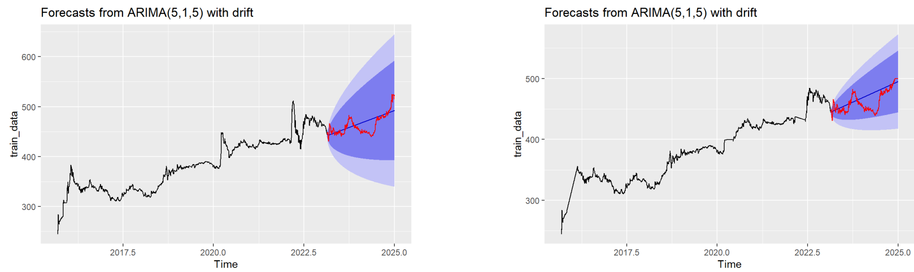
Metric	**Formula**
Root Mean Square Error (RMSE)	$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$
Mean Absolute Error (MAE)	$\frac{1}{n}\sum_{i=1}^{n}\lvert y_i - \hat{y}_i\rvert$
Mean Absolute Percentage Error (MAPE)	$\frac{100}{n}\sum_{i=1}^{n}\frac{\lvert y_i - \hat{y}_i\rvert}{\lvert y_i\rvert}$
R-squared (R^2)	$1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$

Table 2: Performance metrics and their formulas

3 Results

3.1 ARIMA

The time series was fitted with an ARIMA model using the auto.arima () function. The optimal model was chosen according to AIC values.



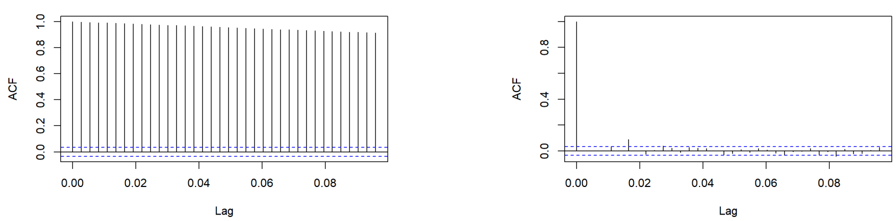
(a) ARIMA forecast (before)

(b) ARIMA forecast (after)

Figure 4: Comparison of the ARIMA forecasts before and after outlier removal.

Figure 4 presents the ARIMA(5,1,5) forecasts with drift, where the shaded area shows 80% and 95% confidence intervals. After outlier removal, the forecast displays narrower confidence intervals, suggesting improved model stability.

The ACF plots (Figure 5) illustrate the autocorrelations of the raw data and the differenced data. The ACF of the raw data reveals high autocorrelation values for all lags, suggesting that the data is non-stationary. After applying differencing, the ACF shows a significant drop, with only the first lag exhibiting a notable correlation. The remaining lags oscillate around zero and stay within the confidence bounds.



(a) ACF of raw data

(b) ACF after differencing

Figure 5: Comparison of the ACF plots.

Table 3 provides a comparative summary of the ARIMA model's performance in forecasting the USD/KZT exchange rate, both before and after outlier removal. While there is a slight increase in RMSE, MAE, and MAPE values after preprocessing, the model shows significant improvements in likelihood-based model selection criteria, such as AIC, AICc, and BIC. This suggests that, despite the minor increases in forecast error, the overall model fit has improved.

Metric	**Before**	**After**
RMSE	15.3385	15.4061
MAE	11.6815	11.8734
MAPE	2.5323%	2.6102%
R^2	0.3969	0.2061
Log Likelihood	-5953.53	-4898.71
AIC	11931.05	9643.42
AICc	11931.17	9643.53
BIC	12001.95	9714.31

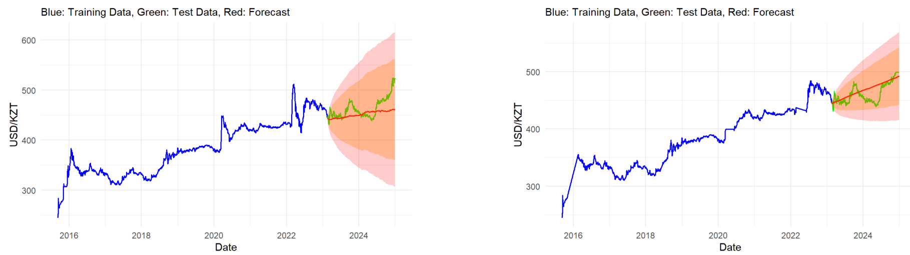
Table 3: ARIMA model performance metrics before and after outlier removal

These findings suggest that while outlier removal may not immediately improve prediction accuracy for the ARIMA model in this specific case, it enhances model stability and overall statistical fit, potentially leading to more reliable long-term forecasting performance for the USD/KZT exchange rate.

3.2 SARIMAX

The SARIMAX model was fitted to the time series with the inclusion of exogenous variables and seasonal components. The model selection process yielded a SARIMAX(0,1,2)(2,0,0)[5] specification with six exogenous variables.

Figure 6 illustrates the forecasting results of the SARIMAX model for the USD/KZT exchange rate before and after outlier removal, respectively. After outlier removal, the confidence interval is slightly narrower, suggesting improved model stability and reduced uncertainty. The post-processed model appears more aligned with recent exchange rate trends, indicating that removing anomalies enhances predictive performance.



(a) SARIMAX forecast (before) (b) SARIMAX forecast (after)

Figure 6: Comparison of the SARIMAX forecasts before and after outlier removal.

Coefficients:					
	ma1	ma2	sar1	sar2	drift
	0.1058	-0.0095	-0.0095	-0.0242	0.0754
s.e.	0.0174	0.0181	0.0173	0.0181	0.0278
	brent	fx	usdrub	kase	
	-0.0071	-1e-04	0.0131	-0.0003	
s.e.	0.0059	1e-04	0.0245	0.0015	

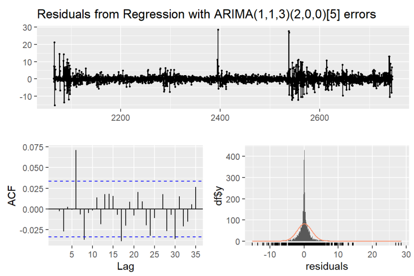
Table 4: SARIMAX coefficient estimates and standard errors

The SARIMAX model shows a substantial improvement in all performance metrics after outlier removal (Table 5). The RMSE decreased by 23.4%, while the AIC and BIC values also show considerable improvement, indicating better model fit. However, R^2 remains low in both cases (7.8% and 29.3%), suggesting that the model still captures only a limited portion of the variance in the time series data.

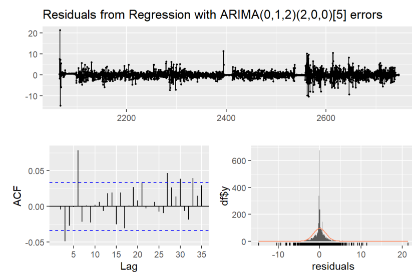
Metric	**Before**	**After**
RMSE	18.9671	14.5325
MAE	14.2152	11.1791
MAPE	2.9806%	2.4558%
R^2	0.0779	0.2936
Log Likelihood	-5964.59	-4834.58
AIC	11945.18	9687.16
AICc	11945.23	9687.22
BIC	11992.44	9740.33

Table 5: SARIMAX model performance metrics before and after outlier removal

Figure 7 compares SARIMAX residual analyses before and after outlier removal. Panel (a) shows the ARIMA(1,1,3)(2,0,0)[5] model's pre-removal residuals with several extreme spikes ($> \pm 10$), significant autocorrelation in the ACF plot, and a heavy-tailed distribution. Panel (b) displays the ARIMA(0,1,2)(2,0,0)[5] model's post-removal residuals, exhibiting fewer extreme values (within ± 5 range), reduced autocorrelation, and improved normality in the distribution. The outlier removal process resulted in a simpler model with improved diagnostic properties, indicated by the transition from ARIMA(1,1,3) to ARIMA(0,1,2) in the non-seasonal component.



(a) SARIMAX residual analysis (before)



(b) SARIMAX residual analysis (after)

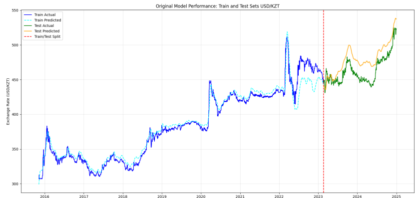
Figure 7: Comparison of the SARIMAX residual analysis before and after outlier removal.

3.3 LSTM

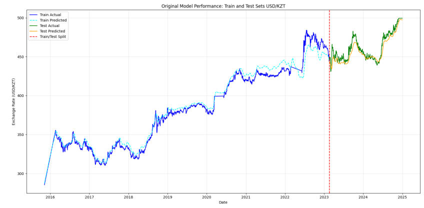
The predictive model is based on a stacked LSTM architecture, implemented using the Keras deep learning library. The architecture consists of two LSTM layers with 50 units each. The first LSTM layer is configured to return sequences to allow stacking, followed by a dropout layer to mitigate overfitting. The second LSTM layer outputs a single sequence, which is passed through another dropout layer and two dense (fully connected) layers, with the final layer producing a single output value corresponding to the predicted exchange rate.

The model is trained using the Adam optimizer and the Mean Squared Error (MSE) loss function. To enhance generalization and prevent overfitting, early stopping is employed based on the validation loss, with the best model weights restored upon termination of training.

The figure 8 shows the impact of outlier removal on LSTM model performance for exchange rate prediction. In both panels, the green line represents actual values while the orange line shows predictions, with the red vertical line indicating the train/test split point.



(a) LSTM train/test set performance (before)



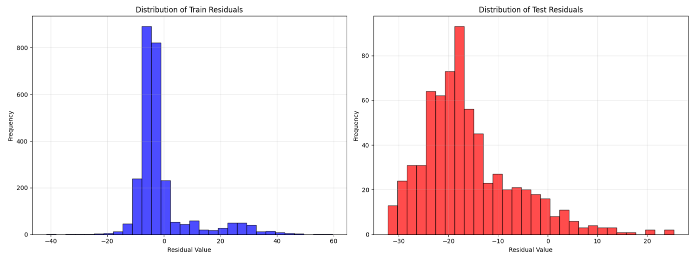
(b) LSTM train/test set performance (after)

Figure 8: Comparison of LSTM test set performances before and after outlier removal.

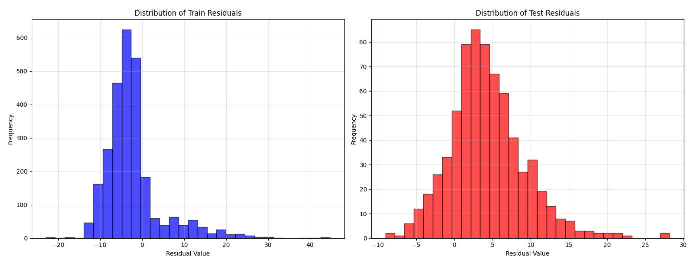
The closer alignment between predicted and actual values in panel 8b suggests that outliers were indeed introducing noise that obscured underlying patterns in the exchange rate data, which the LSTM model was subsequently able to learn more effectively after their removal.

Figure 9 illustrates the distribution of residuals from the LSTM model on both the train and test sets. Each panel contains blue histograms for train residuals and red histograms for test residuals.

In the "before" panel 9a, the test residuals are skewed left (negative), suggesting an overestimation of exchange rates. After outlier removal in panel 9b, both distributions become more compact and symmetric around zero. The train residuals concentrate more tightly between -15 and 15, while the test residuals show a more balanced distribution centered approximately at zero, with reduced extreme values.



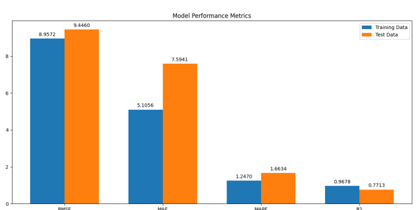
(a) LSTM test set residuals (before)



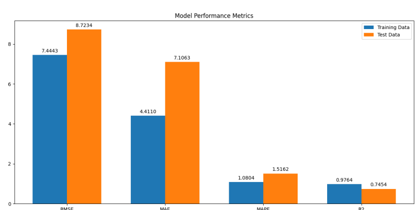
(b) LSTM test set residuals (after)

Figure 9: LSTM test set residuals

Table 6 and figure 10 summarize the performance of the LSTM model before and after outlier removal. The training and test set metrics are provided to assess the model's generalization ability.



(a) LSTM metrics (before)



(b) LSTM metrics (after)

Figure 10: Comparison of the LSTM metrics before and after outlier removal.

The results indicate that the LSTM model trained on data without outliers exhibits a significantly different trend compared to the model trained on the unprocessed dataset. In particular, the outlier-removed approach suggests more accurate and unbiased predictions as evidenced by the more normally distributed residuals and improved RMSE(-7.65%), MAE (-6.42%), MAPE (-8.85%), and R^2 (+10.90%).

Metric	Training (Before)	Test (Before)	Training (After)	Test (After)
RMSE	8.9572	9.4460	7.4443	8.7234
MAE	5.1056	7.5941	4.4110	7.1063
MAPE	1.2470	1.6634	1.0804	1.5162
R^2	0.9678	0.7713	0.9764	0.8554

Table 6: LSTM model performance metrics before and after outlier removal.

3.4 Comparative Model Performance

To assess the relative performance of the different forecasting approaches and quantify the impact of outlier removal, a comprehensive comparison of all models is presented in Table 7.

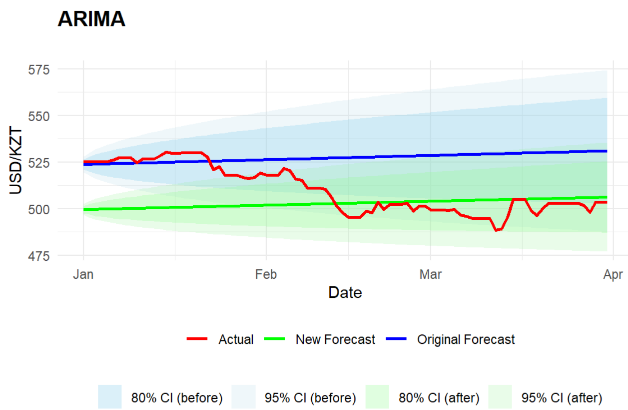
Model	Preprocessing	RMSE	MAE	MAPE	R^2
ARIMA	Original data	15.34	11.68	2.53%	0.40
	Outliers removed	15.41	11.87	2.61%	0.21
SARIMAX	Original data	18.97	14.22	2.98%	0.078
	Outliers removed	14.53	11.18	2.46%	0.29
LSTM	Original data	9.45	7.59	1.66	0.77
	Outliers removed	8.72	7.11	1.52	0.86

Table 7: Comparative performance of all models before and after outlier removal. Percentages in parentheses indicate change relative to the original data.

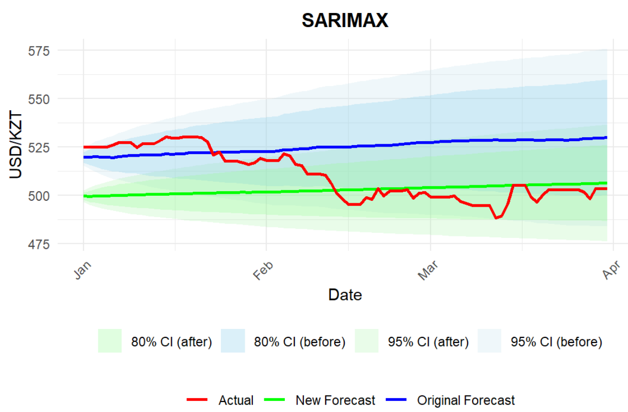
3.5 2025 Forecasting Results

For this phase, each model (ARIMA, SARIMAX, and LSTM) was re-trained using the complete dataset from September 2015 to December 2024. Following training, each model generated predictions for the first 90 days of 2025 (1 January to 31 March). These forecasts were then evaluated against the actual USD/KZT exchange rates observed during this period, providing a realistic assessment of each model's predictive capabilities in a practical application scenario.

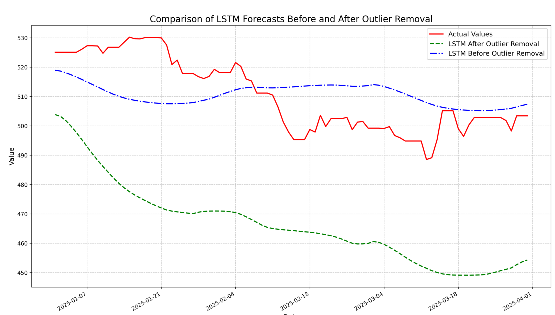
The ARIMA and SARIMAX models display similar confidence interval bands, with the actual exchange rate (red line) frequently falling outside these bands prior to outlier removal. Post-outlier removal forecasts (green line) demonstrate improved stability in later periods (from mid-February). Both models show that outlier removal produces more conservative forecasts that generally pre-



(a) ARIMA forecast for 2025



(b) SARIMAX forecast for 2025



(c) LSTM forecast for 2025

Figure 11: 2025 forecast comparison of all models before and after outlier removal. The red line represents actual values, the blue line shows forecasts without outlier removal, and the green line indicates forecasts with outlier removal.

dict lower exchange rate values compared to their standard counterparts. The LSTM model exhibits markedly different behavior. Without outlier removal, the LSTM forecast (blue line) demonstrates closer alignment to the actual values. The outlier-removed LSTM forecast (green dashed line) shows a consistent downward trend throughout the forecast period, diverging significantly from the actual values.

Across all models, removing the outliers leads to lower predicted values, suggesting that outlier removal may introduce a systematic bias toward underestimating the exchange rate in markets characterized by periodic volatility. For the Kazakhstani tenge, this indicates that preprocessing techniques must be carefully calibrated to avoid obscuring legitimate market dynamics, which might be misclassified as anomalies.

The comparative analysis demonstrates that while outlier removal enhances forecast stability, the optimal preprocessing approach depends on the specific forecasting objectives—whether prioritizing response to short-term market movements or long-term trend prediction.

4 Conclusion

In this study, the impact of outlier removal on the forecasting accuracy of the USD/KZT exchange rate was examined using ARIMA, SARIMAX, and LSTM models. By conducting a thorough analysis of the time series data, the models were applied to the original dataset containing outliers and then to a cleaned version with outliers removed. The comparison of forecasting performance revealed that removing outliers implies improved model accuracy, as evidenced by lower error metrics such as RMSE, MAE, MAPE, and AIC (for ARIMA and SARIMAX) in the cleaned dataset. The outlier removal process appears to have the greatest relative impact on the SARIMAX model.

It suggests that the SARIMAX model's performance is particularly sensitive to the presence of outliers in the dataset, and preprocessing steps to address anomalous observations are especially valuable when implementing this forecasting approach.

The analysis also indicates that Kazakhstan's currency valuation is predominantly influenced by fluctuations in global oil prices, inflation rates, interest rate parity conditions, and geopolitical developments affecting Kazakhstan's economic landscape. Statistical modeling confirms robust correlations between these variables and exchange rate movements, showing that both macroeconomic fundamentals and external market conditions are instrumental in determining the tenge's value relative to the dollar.

Future research could explore more advanced methodological approaches, such as vector autoregression (VAR) or sophisticated machine learning techniques, to enhance predictive capabilities. Additionally, examining other exchange rates that experienced periods of strengthening against the USD would be beneficial for investigating the effects of outlier removal further.

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