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**School of Engineering and Digital Sciences
Department of Mechanical and Aerospace Engineering**

MSc Thesis

**DECISION ANALYSIS MODEL FOR RAILROAD
TRANSPORTATION OF FREIGHT CONTAINERS FROM
LIANYUNGANG TO DUISBURG**

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**This MSc thesis submitted to Nazarbayev University in partial fulfillment of the
requirements for the degree of Master of Science in Mechanical and Aerospace Engineering**

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DECLARATION

I hereby, declare that this manuscript, entitled “*Decision Analysis Model for Railroad Transportation of Freight Containers from Lianyungang to Duisburg*”, is the result of my own work except for quotations and citations, which have been duly acknowledged.

I also declare that, to the best of my knowledge and belief, it has not been previously or concurrently submitted, in whole or in part, for any other degree or diploma at Nazarbayev University or any other national or international institution.



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Date: 11.04.2026

Abstract

In recent years, the Belt and Road Initiative's (BRI) transportation service, proposed in 2013, has become a viable option for the transportation of goods to Europe. This study aims to find the best alternative railway route between Lianyungang (China) and Duisburg (Germany). A structured decision analysis model was developed to rank the alternatives under two key attributes, such as time and cost savings of transportation, which are critical factors in logistics. This was done by constructing single attribute utility functions and integrating them into a Multiattribute Utility Function (MUF) based on the expert input. The significance of research lies in the use of MUF in decision-making problems on railroad transportation systems because this method has not been studied for these kinds of problems. The study considered three alternative routes. The results indicate that Alternative 1 (route through Kazakhstan and Russia) has provided the highest utility value of 0.566, followed by Alternative 2 (via the Caspian sea) with 0.451 and Alternative 3 (via Trans-Siberian railway systems) with 0.195. These results highlight that Alternative 1 provides the optimal trade-off between cost and time savings. The analysis also shows that time saving has much higher significance than cost saving. One-way and global sensitivity analyses were conducted to assess the robustness of the results. The findings show that Alternative 1 is a dominant alternative across a range of variations of preference parameters, which proves the reliability and stability of the model. This research contributes into the development of the logistics decision analysis, showing an effective application of the Multi-Attribute Utility Theory (MAUT) to practical cases related to transportation problems.

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List of Abbreviations & Symbols

BRI	Belt and Road Initiative
MUF	Multiattribute Utility Function
MAUT	Multiattribute Utility Theory
AHP	Analytic Hierarchy Process
MCDM	Multi-Criteria Decision Making
PUI	Partial Utility Independence
UI	Utility Independence
BET	Basic Expansion Theorem
UDM	Utility Dependence Matrix

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Chapter 1 - Introduction

1.1. Background

Chinese President Xi Jinping proposed the Belt and Road Initiative (BRI) in 2013. In recent years, the transportation service has become a viable option for the delivery of goods to Europe. The EU's large deficit in goods trade with China indicates heavy imports [1]. The most traded goods are machinery and vehicles. Moreover, Eurasian rail freight is approximately 80% less expensive than air freight and twice as fast as sea freight [2]. There are several alternative rail routes between China and Europe, differing in relevant criteria shown in Figure 1. Choosing the best rail route is complex and uncertain, which requires a structured evaluation.

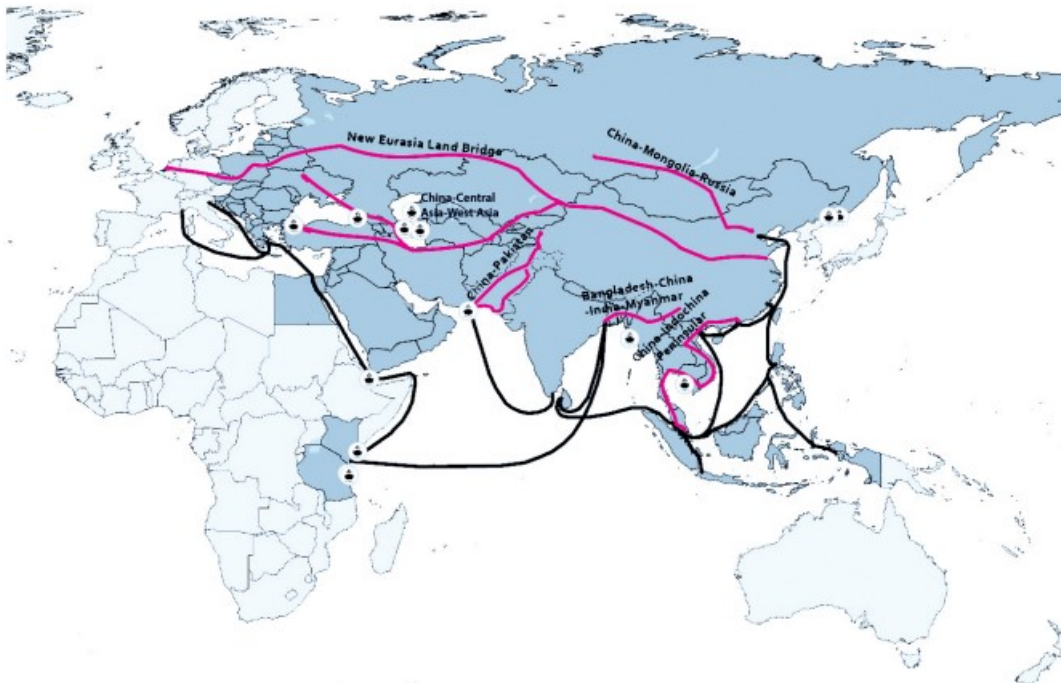


Figure 1: China-Europe rail and sea routes [3].

1.2. Motivation

Selection of an efficient decision model can potentially reduce costs and delivery times. This supports sustainable growth in trade between China and Europe. The project will use the concept of Multiattribute Utility Functions (MUF), which is part of Multiattribute Utility Theory (MAUT) [4]. This theory serves as a decision support tool for a number of areas by assisting in

evaluation alternatives based on a number of factors [5]. Being able to factor preferences and trade-offs ensures more rational and thorough decision making. This is beneficial in handling the uncertainties, and the model requires input from the decision maker. In this research, the decision maker will be a logistics expert representing companies that manage freight transport. Limited research has applied advanced decision analysis methods (e.g. MUF) to logistics and railroad transportation problems. Furthermore, the presence of multiple alternatives and differences in their characteristics complicates the decision-making process. Consequently, this study addresses these gaps by developing a decision analysis model using MUF to evaluate and rank alternative railway routes from Lianyungang (China) to Duisburg (Germany).

The research studies railway routes from Lianyungang and Duisburg. These destinations were selected for two reasons. First, the foundation of China-Kazakhstan Logistics Cooperation Base under BRI in Lianyungang opens a new Eurasian bridge to Europe to facilitate the trade between China and Central Asian countries [6]. Second, Duisburg is one of the main and largest European hub for rail freight between China and Europe that processes up to 850 000 containers annually [7]. Therefore, these two hubs represent strategically significant destination points in China-Europe railway corridor and provide a basis for evaluation of freight transportation for the study.

1.3. Research Aim and Objectives

The project aims to develop a decision analysis model using MUF to rank the most to least efficient rail route between Lianyungang, China and Duisburg, Germany. The objectives are:

- Analyze existing China–Germany railway routes and their key characteristics;
- Identify critical evaluation attributes;
- Assess decision maker's true preferences;
- Develop a mathematical model using MUF to rank-order alternative routes.

The project contributes to railway freight optimization between Lianyungang and Duisburg and along the China-Europe corridor, in general, by developing a decision analysis framework that helps to examine trade-offs between attributes for each alternative. In doing so, it provides one of the first applications of MUF approach within logistics and transportation systems, addressing the

critical methodological gap between theoretical MAUT approaches and practical freight management.

1.4. Structure of the Thesis

This study consists of six chapters that are constructed in a logical way. It aims to develop a decision analysis tool for freight transportation and shows practical implementation of MAUT in logistics. The first chapter is an introduction that covers the background, motivation, and research objectives. This is followed by literature review which gives an overview on existing transport modes between China and Germany, MAUT and its applications, and related studies in logistics. The third chapter describes the main methodology of the study, namely the construction of Multiattribute Utility Functions. The fourth chapter presents results that include outcomes of the study on ranking of alternatives and sensitivity of the decision model. The fifth chapter discusses these results, interpreting the outcomes, assessing model robustness, and outlining limitations. The last chapter concludes the thesis by summarizing the main findings of the study.

Chapter 2 - Literature Review

2.1. Overview of the Eurasian Rail Freight

The trade volumes are rapidly growing between China and Europe. According to Eurasian rail-container report, around 745900 TEU (standard container unit) were transported through the Eurasian rail route [8]. It is a 10.6% increase compared to 2023, which implies a strong continuing demand for rail freight. Moreover, trade amounted to \$748.9 billion, which is a small share of overall trade. But it is still a large share considering its advantages over other transport modes [8], [9]. Rail freight is faster than sea mode. For instance, the transit time of sea route from Shanghai to Hamburg takes 37 days, sea route from Lianyungang to Istanbul takes around 30-40 days, sea route from Shanghai to Gdynia takes around 37-42 days. On the contrary, the transit time of rail route from Shanghai to Gdynia takes 19 days, rail route from Lianyungang to European countries takes around 13-21 days, even though sea routes are much cheaper than rail [9], [10]. Rail and sea freight cost much less compared to air transport mode. Previously mentioned route from Shanghai to Gdynia costs \$37000 by air freight, while rail and sea cost \$4500 and \$2600, respectively, but the transit time is greatly reduced to 5-9 days [10]. Rail freight has some advantages over sea and air transport modes.

The Eurasian rail system offers several corridors, each using different border crossing and hubs. They are northern, middle, and southern corridors as illustrated in Figure 2.

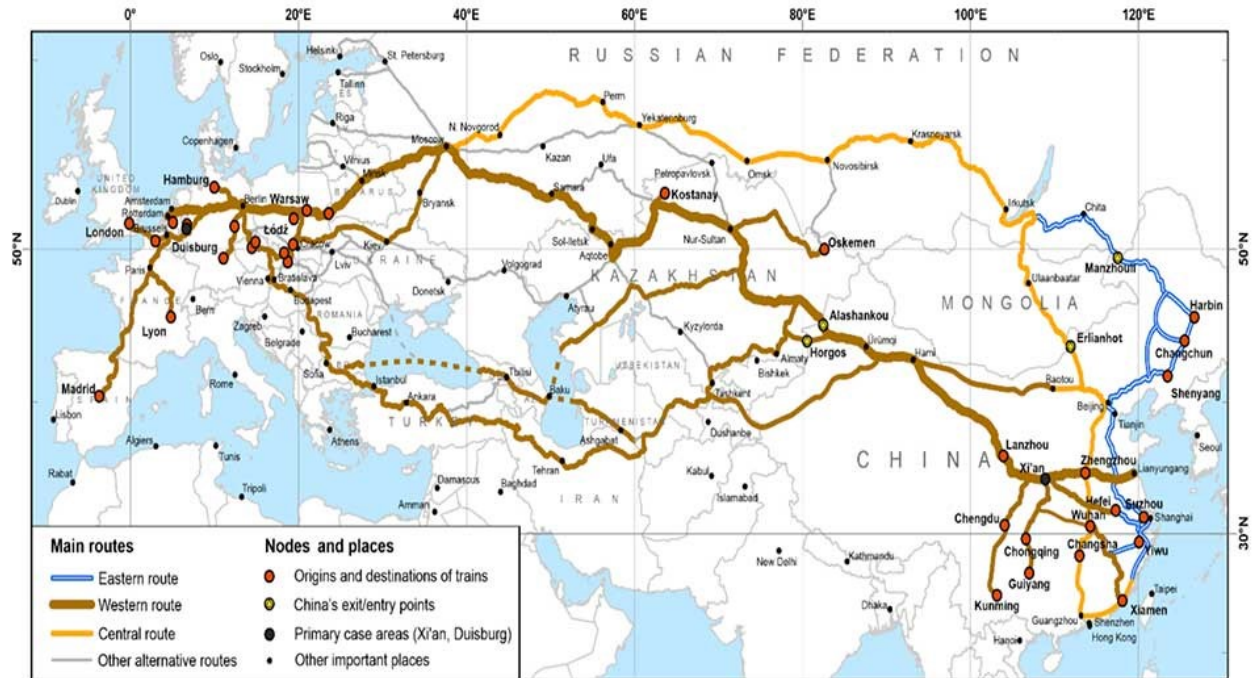


Figure 2: Rail freight main routes in Eurasian railway system [11].

2.2. Review of Related Studies

This passage discusses some studies related to logistics problems. The paper about application of MAUT in evaluation of rural roads is one of the practical examples of decision analysis [12]. This evaluation was done to increase the accessibility of rural areas in Nigeria. AHP (Analytic Hierarchy Process) techniques were used to prioritize rural roads based on the following attributes: social, economic, demographic, political, financial. 59 roads were selected as alternatives. As a result, 10 roads were identified as high priority roads for accessibility because they had far more facilities described in attributes. It can be stated that the study focused on quantitative attributes.

Compared to the previous paper, a study on optimization of coal transportation systems in Thailand, specifically from Chonburi Province to Saraburi Province, used similar methodology [13]. However, they integrated weights of criteria from AHP into function of zero-one goal programming (ZOGP). This was done to create the optimal route. Its aim was to improve logistics performance by minimizing cost, time, and risks. The study was based on quantitative (transportation cost and time) and qualitative data (transportation risk). Freight damage, security,

environment, law, financial risks are described as transportation risks. 8 transportation routes were determined including different transport modes (rail, sea, and road). The result shows that combination of ship and truck is the optimal option.

Although based on a similar problem and methodology, the study on multimodal transportation route selection in Thailand for coal industry uses slightly modified AHP method with fuzzy logic (Fuzzy Analytic Hierarchy Process (FAHP)) [14]. The paper proposes a decision-making approach to handle uncertainty in expert judgment and conflicting criteria in logistics to choose the best transport route. The model considers 9 key criteria, including transportation cost, time, and different risk factors such as environmental, law and financial risks. Data collected from expert interviews was used to evaluate and rank 8 alternatives. The result identified the most optimal route based on aggregated weighted scores, which shows the effectiveness of FAHP in handling uncertainty in multi-criteria decision making (MCDM). However, the method is based on a single case study and relies heavily on expert judgment.

The next study addresses the selection of optimal freight transportation route in large-scale multimodal systems [15]. They present a methodological framework, which can be applied to international corridors. It aims to develop a systematic decision analysis model. The method combines a five-level hierarchy of transport needs (economic, political, geographical, infrastructure, and technology) with Petri Net to simulate and compare alternatives. Each alternative is assessed across multiple attributes such as distance, cost, reliability, regulations, infrastructure quality, and technological readiness.

These studies utilize regular decision-making tools. This addresses an important gap in the literature about usage of advanced and complex decision models for railway transportation problems. Limited studies applied MUF approach to logistics problems. Table 1 shows summary of studies presented previously.

Table 1: Summary of related studies [12], [13], [14], [15].

#	Author	Title	Methodology	Attributes
1	Akpan & Morimoto (2022)	An application of Multi-Attribute Utility Theory (MAUT) to the prioritization of rural roads to improve rural accessibility in Nigeria	AHP	Social, economic, demographic, political, financial.
2	Kaewfak, Ammarapala & Huynh (2021)	Multi-objective Optimization of Freight Route Choices in Multimodal Transportation	AHP & ZOGP	Cost and time, freight damage, infrastructure and equipment, operational, security, environment, law, financial risks.
3	Ammarapala et al. (2021)	Fuzzy AHP Approach for Route Selection in Multimodal Transportation: The Case of Coal Industry in Thailand	FAHP	Transportation cost and time, freight damage, infrastructure and equipment, operational, security, environment, law/community, financial risks
4	Kabashkin (2023)	Model of Multi Criteria Decision-Making for Selection of Transportation Alternatives on the Base of Transport Needs Hierarchy Framework and Application of Petri Net	Petri Net	Distance, cost, reliability, regulations, infrastructure quality, and technological readiness.

2.3. MAUT in Decision Analysis

MAUT is a decision support tool assisting in the evaluation of the alternatives based on a number of attributes [5]. Being able to factor preferences and trade-offs ensures more rational and thorough decision-making. This is beneficial in handling the uncertainties, and the model requires input from the decision maker. The theory is versatile across diverse decision-making scenarios, particularly those characterized by risk and uncertainty, because it measures the strength of preference for riskier options [16]. Moreover, it offers a structured decision tool for decision makers to assess results by analyzing multiple criteria, enhancing informed decision-making [17]. MAUT is a structured framework that helps factor preferences of decision maker across diverse scenarios.

The practical application of MAUT has a wide range of uses, ranging from agricultural planning to urban development, pharmacy, and logistics. Theory shows potential in agriculture, for instance, it enhances decision making with numerous attributes, such as crop selection and resource distribution, to increase the reliability of decisions [4]. Khaddour et al. [18] mentioned that MAUT

can be used in urban development to rank sustainable strategies. They used attributes such as service accessibility and effective land use to evaluate and select the best option for urban planning problems. Moreover, the theory can be applied in completely different fields, such as pharmacy. According to Chung et al. [19], MAUT is used in the assessment of drugs. They evaluated cardiovascular drugs based on attributes such as efficacy, safety, patient convenience, and cost, and decided which drugs could be used in the formulary. Logistics problems are practically solved using this methodology. For example, the study selected an optimal freight route for coal transportation in Thailand by considering cost, time and risks [13]. MAUT is applicable in various fields, starting from agriculture to logistics.

There are some challenges associated with the implementation of MAUT in practical problems. These challenges lead to the biased preferences of the decision maker. The first challenge is incomplete identification of goals in applying this theory. Incomplete decision models are caused by decision maker's failure to identify all necessary goals. This leads to poor performance of the decision model, where the decision maker identifies only a part of the goals [20]. Unidentified goals might play a crucial role in MAUT problems. The next challenge is the cognitive constraints faced by decision makers during the evaluation process. It is psychologically exhausting for decision makers to decide on preferences and trade-offs in complex problems [21]. This happens because of the difficulty in representing their preferences mathematically. Moreover, different decision models under MAUT show different ranking outcomes. This occurs because of the different evaluation techniques of models [22], [23]. Selection of appropriate decision model by understanding the features of a particular problem is necessary to reach desired outcome. MAUT has some challenges, such as unidentified goals, cognitive constraints, and selection of appropriate decision models.

2.4. Research Gap Analysis

The application of MAUT has a wide range of uses, ranging from employee selection and agricultural planning to urban development and pharmacy. This methodology can be applied to ranking the best alternative railroad route between China and Germany. Moreover, MAUT faces some challenges such as unidentified goals, cognitive constraints, complexity of models, and selection of appropriate decision models. It is suitable for addressing the railroad transportation

problem because of the quantitative attributes. However, this theory relies heavily on expert judgment according to relevant studies.

An extensive literature exists on transportation route selection using MAUT approaches. However, limited studies applied Multiattribute Utility Function approach to logistics problems, particularly, to China-Europe rail route selection. Existing research primarily focuses on local or regional transportation problems, while limited research has been given to intercontinental freight networks. The study utilizes the MUF approach to the China-Europe freight transportation context.

Chapter 3 - Methodology

This study utilizes the framework of the Multiattribute Utility Theory (MAUT) to evaluate and determine the optimal railway transportation route between China and Europe. MAUT is considered to be a widely used and recognized methodology for solving complex problems that involve multiple criteria and which may have conflicting outcomes. MAUT is suitable because it can handle quantitative and qualitative information as well as the preferences of the decision maker.

MAUT is considered to be appropriate because the problem involves complex trade-offs between different criteria such as cost savings and time savings. Traditional approaches that involve single criteria cannot handle such trade-offs. The methodological principles of this research have been derived from those of Abbas and Keeney & Raiffa [24], [25].

3.1. Attribute Selection

Identifying evaluation criteria or attributes that affect railroad transportation efficiency between China and Germany is necessary to meet the study's objectives. In the previous chapter, review of related studies revealed that cost and time are the most common attributes in route selection problems [12], [13], [14], [15]. These papers studied other attributes such as infrastructure and equipment quality, freight damage, political, environmental risks. This study will focus on the two attributes due to data availability. This research will focus on the following parameters:

- Cost Savings (X_1): This will be measured in terms of US Dollars and will range between 0 to 5000 US Dollars;
- Time Savings (X_2): This will be measured in days and will range between 0 to 31 days.

Transit time is an attribute accounting for the days needed to transport goods. This attribute significantly varies among the alternative routes. The next attribute is a cost, implying fees for transit between different countries. These attributes are affected by several factors, which make a difference among the alternative routes. The main factors are border crossing charges and transshipment costs associated with transfer of goods from one vehicle (train, ship, etc.). One of the main problems of railway transportation is the different railway gauges. Most of the CIS (The Commonwealth of Independent States) countries, including Kazakhstan, Russia, Uzbekistan,

Kyrgyzstan, Turkmenistan, and Tajikistan, have a gauge of 1520 mm, in comparison to China and Europe, which have a gauge of 1435 mm [26]. Therefore, there is a need for the unloading and loading of goods. This adds additional expenses and time for the transportation of goods by railroad. Other factors are track access, storage, security, and inspection fees, etc. These attributes are significantly affected by the border crossing charges and transshipment. To summarize, the study will focus on cost and time attributes.

3.2. Identification of Alternative Rail Routes

The study focuses on the three alternatives or railroad routes between Lianyungang (China) and Duisburg (Germany). These alternatives differ in their parameters. All routes are shown below with railroad crossing countries:

- 1) Alternative 1: Lianyungang – Dostyk – Elets – Smolensk – Brest – Małaszewicze – Duisburg;
- 2) Alternative 2: Lianyungang – Altyntol – Aktau – Baku – Ganja – Istanbul – Budapest – Duisburg;
- 3) Alternative 3: Lianyungang – Manzhouli – Irkutsk – Smolensk – Brest – Małaszewicze – Duisburg.

These are major existing and available railroads, shown in Figure 3. The first and last alternatives mostly use the Trans-Siberian railway network via Russian Federation. The most common route is through Kazakhstan and then utilizes the Trans-Siberian railway network in Russian Federation. Next route is one of the Middle Corridor routes through the Caspian Sea. There is a need for transshipment of goods from train to ship, which adds additional expenses. Last alternative is uncommon route for transportation of goods from China to Europe compared to alternatives previously mentioned. All alternative routes differ in distance and transit time. Routes significantly range in transit time from 14 to 45 days. The routes via Kazakhstan are faster compared to the last alternative route. The study identified 3 alternative railroad routes between Lianyungang and Duisburg.



Figure 3: Alternative railroads of the study (1 - Lianyungang, 2 - Duisburg, Yellow line - Alternative 1, Green line - Alternative 2, Red line - Alternative 3).

3.3. Data Collection and Decision Maker's Input

The study strongly depends on the transportation data. Transit time of goods and cost associated with transportation are major data. This information will be gathered from the logistics reports, industry publications, transportation statistics, and transportation experts. Then, further refinement of this data is necessary to implement it into the decision model.

Decision maker's input will be collected using a structured questionnaire. Before the data collection, permission to administer the questionnaire to the decision maker was obtained from the Nazarbayev University Institutional Research Ethics Committee. The structured questionnaire will be developed following utility elicitation techniques, where the decision maker will be asked to evaluate the trade-offs between the attributes of the decision problem using hypothetical scenarios (see Appendix A). The quantification of the preferences will be done using indifference probabilities, which will be elicited following the standard lottery questions. The decision maker

will be an expert from the field of railroad transportation. The outcomes will determine critical trade-offs between attributes using Mathematica software and help to determine the best route.

3.4. Multiattribute Utility Framework

A major feature of this study is that possible dependency exists among attributes, particularly between cost savings (X_1) and time savings (X_2). This has made this study use Partial Utility Independence (PUI) and Utility Independence (UI).

Under this approach, some attributes are allowed to be dependent, while for others, independence exists. This approach offers a more realistic modeling of decision maker preferences than when all attributes are independent of each other. The Basic Expansion Theorem (BET) allows for the decomposition of the utility function into conditional utility components, particularly when there is partial dependency among attributes [24].

3.4.1. Utility Dependence Matrix (UDM) Construction

The first step involved in the development of the MUF is the establishment of the dependency relationship of the attributes, which can be achieved with the help of the Utility Dependence Matrix (UDM). It assists in the establishment of the relationship of the attributes and reveals if the attributes are independent or dependent from utility perspective, thereby aiding the establishment of the appropriate form of the multiattribute utility function.

The matrix development process is based on the answers obtained from the decision maker's answers on structured preference questions, which are based on lottery-type questions. These questions are used to determine if the preference of one attribute depends on the other attribute. The matrix will be used to determine the appropriate utility model, as well as the validity of the PUI or UI assumptions.

The study on the filter restoration problem of deep borehole showed practical advantage of the UDM in structuring the decision maker's preference across multiple attributes [5]. As a result, it showed PUI under experts' preferences, which implies the importance of UDM in validating the framework. An example of lottery-based question for construction of UDM can be seen below,

while other examples are represented in Figure 4 and Appendix B based on previous research of filter restoration problem [5]:

Question 1. Dependence: cost saving ($X1$) of time saving ($X2$)

a) *Given two projects A and B with 50-50 chance of receiving different saved time, which project would you prefer in Scenario 1?*

Scenario 1: in both projects $X2^* = 31$ days of time savings|

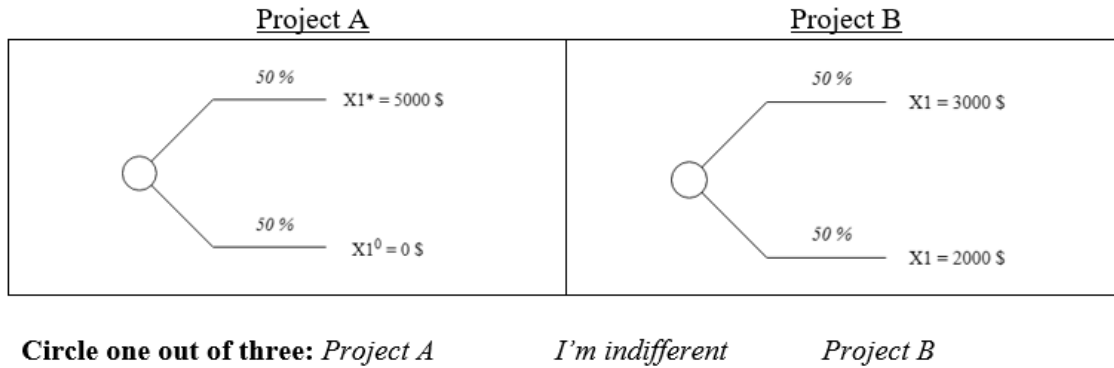


Figure 4: Lottery-based question for cost and time saving dependence

This helps to determine the interdependence structure of UDM. There are four possible outcomes that are utility dependence, one-switch independence, boundary independence, and utility independence [27]. When two attributes are dependent on each other, it leads to the utility dependence case, while the opposite is utility independence. In one-switch independence, dependence is monotonic and preferences depend on another attribute. The boundary independence case shows dependence of the first attribute, while the second attribute is independent of first attribute. This will help in the construction of the MUF.

3.4.2. Construction of the Multiattribute Utility Function

Under the PUI approach, the MUF is built up from the Basic Expansion Theorem (BET), by which the utility function may be broken down into marginal and conditional terms [24].

Under the UI assumption, where all attributes are treated as independent, the MUF takes the multilinear form [25]:

$$U(X_i, X_j) = \sum k_i U_i(X_i) + \sum k_{ij} U_i(X_i) U_j(X_j) \quad (1)$$

where: $U_i(X_i), U_j(X_j)$ are single attribute utility functions; k_i, k_{ij} are scaling constants.

The sequence of steps in the construction of MUF using PUI and UI methods is as follows:

1. UDM Construction and Dependency Identification: Construction of UDM based on answers provided in the questionnaire for identification of dependency among attributes.
2. Utility Elicitation: Single and conditional utilities are elicited from the decision-maker by asking lottery-based questions and including risk preferences of the DM, seen in Appendix A.
3. Model Specification:
 - a. Conditional utility functions for dependent attributes are obtained using BET methods.
 - b. Independent utility functions are integrated in the form of a multilinear function.
4. Estimation of Scaling Constants: Trade-offs between attributes are estimated in terms of relative importance using scaling coefficients.
5. Construction of MUF: The utility function is constructed by integrating all components of utility elicited in the above steps.
6. Consistency Check: The constructed utility function is checked for rationality in this step.

The resultant MUF will be used in evaluating each alternative by determining utility values. An alternative with the highest expected utility will be chosen as the optimal alternative, ensuring that quantitative performance and decision-maker preferences are included in the analysis.

3.5. Sensitivity Analysis

Sensitivity analysis was performed with respect to weight coefficients of the MUF to assess the robustness of the decision model. Two approaches were used that are one-way sensitivity analysis and global sensitivity analysis.

One-way sensitivity analysis was conducted to evaluate the effect of single weight variation on the expected utility of alternatives. One weight parameter was varied at a time, while the others were fixed or adjusted to satisfy the normalization constraint. Each weight parameter (k_1 , k_2 , k_{12}) was varied in a range of $\pm 20\%$ relative to its original value retrieved from expert preferences. For each variation step, the expected utility of all alternatives was recalculated using MUF. This approach helps to identify the sensitivity of each isolated parameter to the ranking and expected utilities for local changes.

Global sensitivity analysis was performed using Monte Carlo simulation approach. This was done to assess the effect of changes in the weight coefficients on the ranking of alternatives. Multiple sets of weight coefficients were randomly generated under the normalization constraint. For each generated weight combination, the expected utilities of all alternatives were computed using MUF. Consequently, the robustness of the decision model was checked by observing the possible changes to the ranking of alternatives and Pearson correlation coefficient was performed to analyze the influence of each parameter on the expected utilities. It measures the linear relationship between two variables by dividing covariance by the standard deviation of those variables. Pearson correlation coefficient shows the influence of the variation in weight parameters on the expected utility and indicates whether this influence is positive or negative. A positive correlation indicates that increase in weight parameter leads to increase in expected utility, while negative correlation shows inverse relationship. This approach allows the evaluation of the stability of the decision model. The methodology of the study is shown in Figure 5, including all the steps previously mentioned.

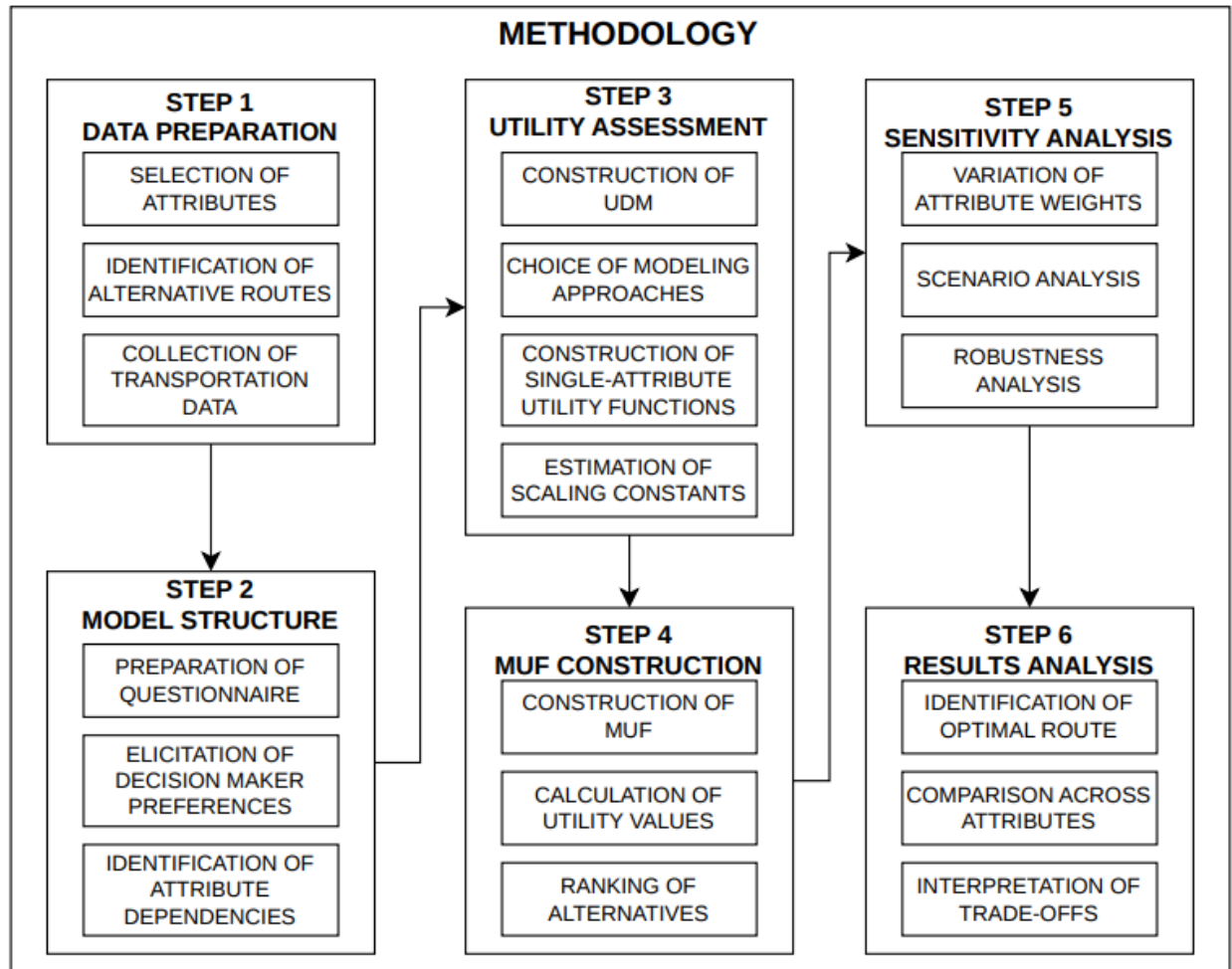


Figure 5: Methodology of the study.

Chapter 4 - Results

In this chapter, all data are provided by logistics company, and experts input. The chapter is divided into five sub-chapters that are data analysis, least squares method, UDM, ranking of the alternatives, and sensitivity analysis. The main objectives are maximization of the cost and time savings on transportation of goods across Lianyungang and Duisburg.

We identified important parameters for transportation of freight that are cost and time of transportation. Table 2 shows the ranges of cost and transit time attributes for all alternatives in the study, from best to worst possible values, including border crossing fees and transshipment costs in terms of both cost and time. Cost includes fees to transit between countries. Transit time implies time needed to transport one unit of container from China to Germany.

Table 2: Ranges of attributes for alternatives.

Attribute	Unit	Range	
		Best	Worst
Cost	US Dollars	11000	16000
Transit time	Days	14	45

4.1. Data Analysis

The identified two attributes were rearranged to show the saving parameters for logistic expert to transport freight. The first attribute is cost saving. This attribute was calculated using the difference between the highest cost and lowest cost, which shows how much money logistic companies can save using one of the identified alternative rail routes. The next attribute is time saving. Similarly to cost savings attribute, this attribute is the difference between highest and lowest transit times between Lianyungang and Duisburg. And shows how much time in days logistic companies can save using all three alternatives. As a result, Table 3 represents fractals of attributes for all alternatives of cost and time savings. The values were derived by subtracting from the maximum cost and transit time for all minimum, low, base, high, and maximum fractals (0%, 10%, 50%, 90%, and 100%, respectively).

Table 3: Fractals of attributes for all alternatives.

Alternative	Attribute	Unit	Values				
			Min.	Low	Base	High	Max.
1	X ₁ – Cost savings	USD	3000	3200	4000	4800	5000
	X ₂ – Time savings	days	15	15.8	19	22.2	23
2	X ₁ – Cost savings	USD	2000	2300	3500	4700	5000
	X ₂ – Time savings	days	0	3.1	15.5	27.9	31
3	X ₁ – Cost savings	USD	0	400	2000	3600	4000
	X ₂ – Time savings	days	0	1.5	7.5	13.5	15

Ranking of the alternatives on single attributes is not viable option to find the optimal route because some alternatives have advantage over other alternatives in terms of cost and time savings. For instance, alternative 2 has great time savings compared to other alternatives, while alternative 1 is efficient in cost saving parameter. Therefore, there is no clear best solution for the case and this methodology will help to find it mathematically using MAUT.

Then, the decision tree was constructed to show all the possible scenarios. Figure 6 shows the decision tree for construction of the MUF. There are 27 outcomes, where low, base, high fractals of each attribute need to be assessed by decision maker for every alternative to find the best railway route. X₁ refers to the cost savings and X₂ is transit time savings of each alternative. U refers to the expected utility to rank the alternatives.

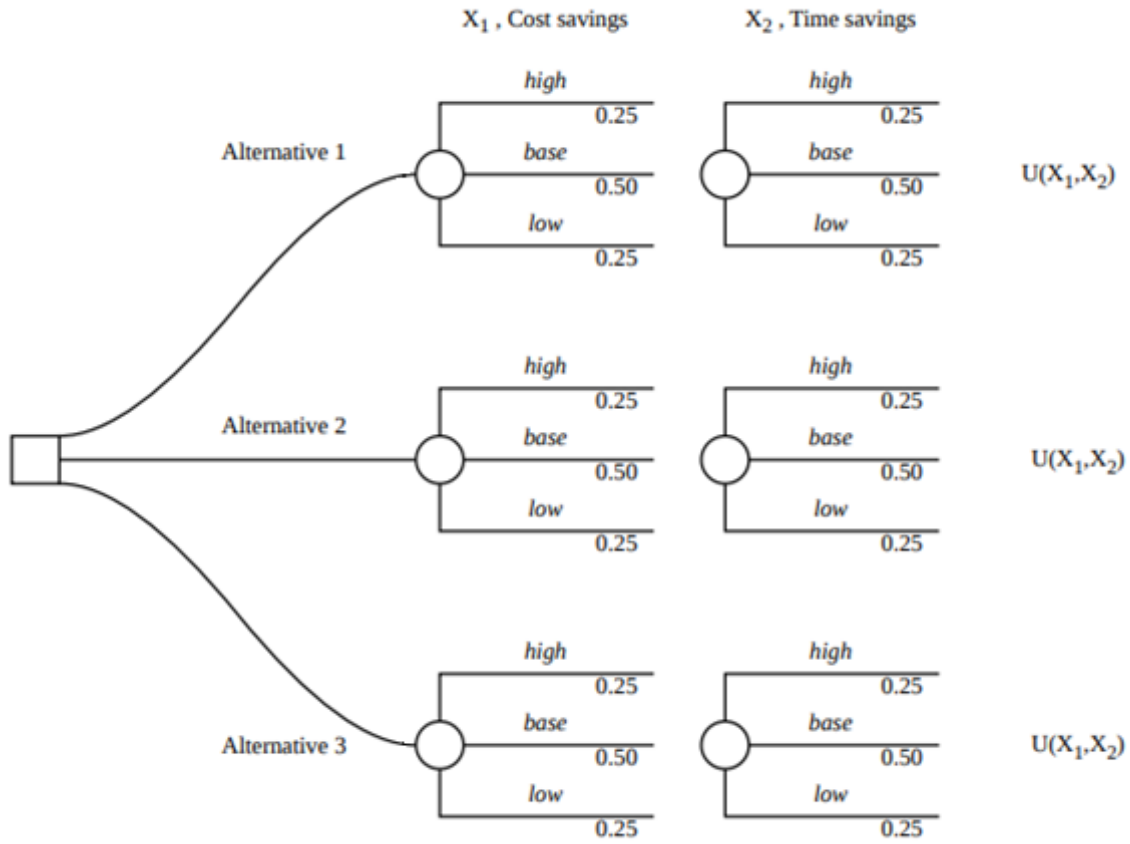


Figure 6: The decision tree for alternative routes from Lianyungang to Duisburg.

4.2. Least Squares Method

The single-attribute utility function was determined using the least squares method based on the decision maker's responses to the questionnaire adapted from [5] (see Appendix A). The utility function for attributes was identified by decision maker's indifference on choices between saving some amount of attribute value and playing a lottery, where the outcome is uncertain. The questionnaire resulted in data points to construct graphs in Figure 7 and 8. The single-attribute utility form of power function was applied to elicit preferences for a quantitative attribute in Mathematica (see Appendix C, Appendix D). The utility function and its data points table for cost saving attribute ($U(X_1)$) is shown below in Figure 7. This was derived through interviews with the decision maker.

X Axis	Y Axis
0	0
2000	0.25
3000	0.50
4000	0.75
5000	1.00

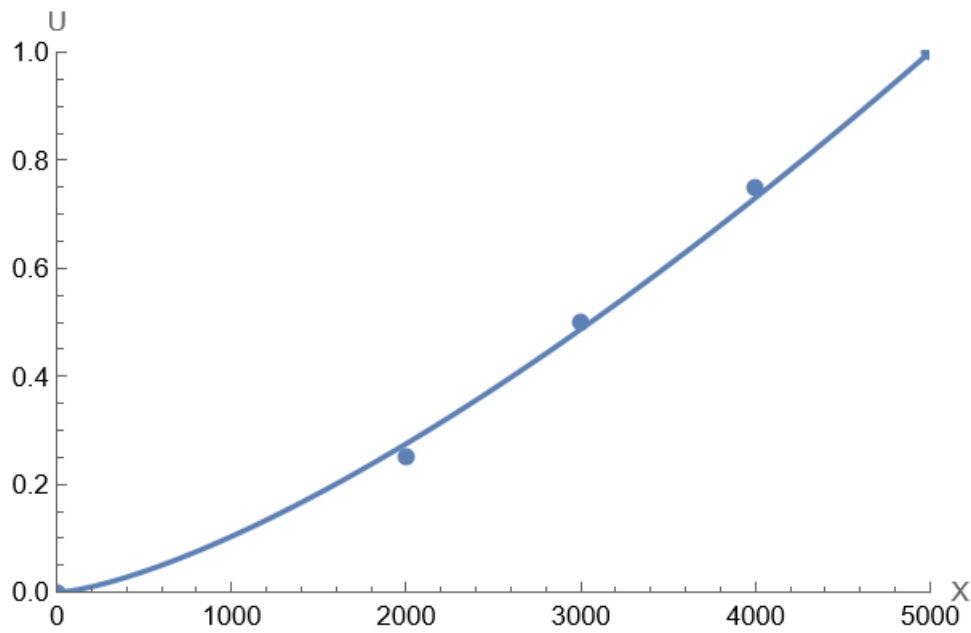


Figure 7: Curve fitting for function U(X₁).

Measure	Value
R	0.9986
R ²	0.9962

$$U(X_1) = \frac{x_1^{1.4078}}{5000} \quad (2)$$

Similarly, the utility function of time saving attribute (U(X₂)) was found using decision maker's input by questionnaire.

X Axis	Y Axis
0	0
10	0.25
15	0.50
23	0.75
31	1.00

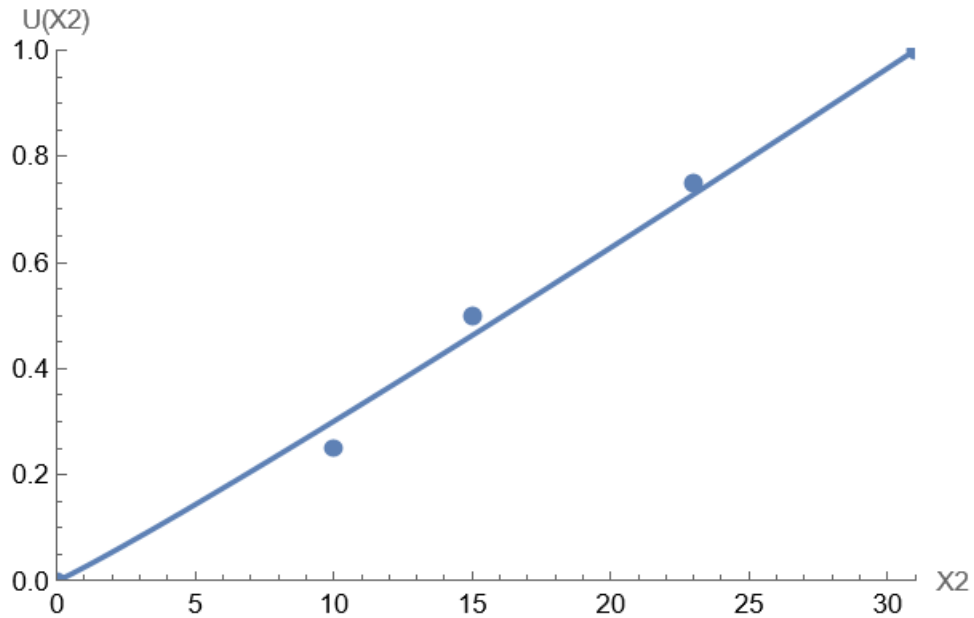


Figure 8: Curve fitting for function $U(X_2)$.

Measure	Value
R	0.9938
R^2	0.9858

$$U(X_2) = \frac{x_2^{1.0619}}{31} \quad (3)$$

4.3. UDM & MUF

At this point, one decision maker has responded to the questionnaire and the results are based only on the responses. The questionnaire answers from decision maker resulted in boundary independence case. The utility independence case is shown in matrix on the left in Figure 9. The matrix on the right shows the dependence among attributes. Specifically, we can observe X_2 is

boundary independent of X_1 and X_1 is utility independent of X_2 , where the top-right corner cell of the matrix is filled with circle. If X_1 is boundary independent of X_2 and X_2 is utility independent on X_1 , then the bottom-left corner cell of the matrix would be circled instead of top-right corner cell.

	X_1	X_2
X_1	○	
X_2		○

	X_1	X_2
X_1	○	○
X_2		○

Figure 9: The utility dependence matrix for utility independence case (left) and boundary independence case (right).

The construction of MUF will assist in best alternative determination. The function was constructed for both cases. In the independence case (Figure 8, left matrix), attributes are independent of each other and takes the following multilinear form [28]:

$$U(X_1, X_2) = k_1 U(X_1) + k_2 U(X_2) + k_{12} U(X_1) U(X_2) \quad (1)$$

where

$$U(X_1) = \left(\frac{X_1}{5000}\right)^{1.4078}; \quad U(X_2) = \left(\frac{X_2}{31}\right)^{1.0619}; \quad k_1 = 0.01; \quad k_2 = 0.80; \quad k_{12} = 0.19.$$

The coefficients were found directly from the decision maker's preferences by assigning probabilistic values for different cases of attributes in the lottery.

In the boundary independence case (Figure 8, right matrix), one of the attributes is dependent on the other and takes this form based on the BET [24]:

$$\begin{aligned}
U(X_1, X_2) = & U(X_1^*, X_2^*) U(X_2 | X_1^0) U(X_1 | X_2^*) + U(X_1^0, X_2^*) U(X_2 | X_1^0) [1 - U(X_1 | X_2^*)] \\
& + U(X_1^*, X_2^0) [1 - U(X_2 | X_1^0)] U(X_1 | X_2^0) + U(X_1^0, X_2^0) [1 - U(X_2 | X_1^0)] [1 \\
& - U(X_1 | X_2^0)]
\end{aligned} \quad (4)$$

Simplification of the multilinear form for boundary independence case into the utility independence case.

$$1) \ U(X_1, X_2) = U(X_1^*, X_2^*)U(X_2|X_1^0)U(X_1|X_2^*) + U(X_1^0, X_2^*)U(X_2|X_1^0)[1 - U(X_1|X_2^*)] + U(X_1^*, X_2^0)[1 - U(X_2|X_1^0)]U(X_1|X_2^0) + U(X_1^0, X_2^0)[1 - U(X_2|X_1^0)][1 - U(X_1|X_2^0)] \quad (4)$$

$$U(X_1^*, X_2^*) = 1; \ U(X_1^0, X_2^0) = 0;$$

$$U(X_1^*, X_2^0) = k_1 = 0.01; \ U(X_1^0, X_2^*) = k_2 = 0.8.$$

$$2) \ U(X_1, X_2) = 1 \cdot U(X_2|X_1^0)U(X_1|X_2^*) + k_2 \cdot U(X_2|X_1^0)[1 - U(X_1|X_2^*)] + k_1 \cdot [1 - U(X_2|X_1^0)]U(X_1|X_2^0) \quad (5)$$

$$U(X_1|X_2^*) = U(X_1|X_2^0) = U(X_1);$$

$$U(X_2|X_1^0) = U(X_2).$$

$$3) \ U(X_1, X_2) = 1 \cdot U(X_2)U(X_1) + k_2 \cdot U(X_2)[1 - U(X_1)] + k_1 \cdot [1 - U(X_2)]U(X_1) \quad (6)$$

$$4) \ U(X_1, X_2) = k_1 U(X_1) + k_2 U(X_2) + k_{12} U(X_1)U(X_2) \quad (1)$$

Where

$$k_{12} = 1 - k_1 - k_2.$$

X^* refers to the highest value of the attribute. On the contrary, X^0 refers to the lowest value. During the assessment of expert preferences, it was found that X_2 is utility independent of X_1 , X_1 is boundary independent of X_2 . However, it was shown that in this special case of X_2 is utility independent of X_1 and X_1 is boundary independent of X_2 leads to X_1 utility independence of X_2 , and so UDM becomes an identity matrix. The MUF takes the multilinear form, as in Formula 1. This implies X_1 is utility independent of X_2 [29]. Consequently, the multilinear form for boundary independence case is simplified to the utility independence case.

4.4. Ranking of the Alternatives

The calculation of expected utility for alternatives shows that the Alternative 1 (route through Kazakhstan and Russian Federation) is best railway route between Lianyungang and Duisburg considering two attributes (cost and time savings). The ranking of alternatives with expected utilities are shown in Table 4.

Table 4: Rank order of the alternatives.

Rank	Alternative	Expected utility
#1	1 (via Kazakhstan and Russia)	0.566
#2	2 (via Caspian sea)	0.451
#3	3 (via Trans-Siberian rail system)	0.195

4.5. Sensitivity Analysis

One-way local and global sensitivity analyses were performed for all coefficients. This was done by varying each scaling coefficient by $\pm 20\%$ while maintaining the normalization constraint in Formula 7. The results, shown in Tables 5, 6, 7, indicate that Alternative 1 remains optimal option for transportation of goods between the two presented destinations.

Table 5: One-way local sensitivity on k_1 .

k_1	Alternatives		
	1	2	3
-0.2	0.566	0.450	0.195
-0.15	0.566	0.450	0.195
-0.10	0.566	0.450	0.195
-0.05	0.566	0.450	0.195
0.00	0.566	0.451	0.195
0.05	0.566	0.451	0.195
0.10	0.567	0.451	0.196
0.15	0.567	0.451	0.196
0.2	0.567	0.451	0.196

Table 6: One-way local sensitivity on k_2 .

k_2	Alternatives		
	1	2	3
-0.2	0.541	0.421	0.170
-0.15	0.547	0.428	0.177
-0.10	0.554	0.436	0.183
-0.05	0.560	0.443	0.189
0.00	0.566	0.451	0.195
0.05	0.573	0.458	0.202
0.10	0.579	0.466	0.208
0.15	0.585	0.473	0.214
0.2	0.592	0.480	0.220

Table 7: One-way local sensitivity on k_{12} .

k_{12}	Alternatives		
	1	2	3
-0.2	0.572	0.458	0.201
-0.15	0.571	0.456	0.200
-0.10	0.569	0.454	0.198
-0.05	0.568	0.452	0.197
0.00	0.566	0.451	0.195
0.05	0.565	0.449	0.194
0.10	0.563	0.447	0.192
0.15	0.562	0.445	0.191
0.2	0.560	0.444	0.189

$$k_1 + k_2 + k_{12} = 1, \quad k_i \geq 0.$$

(7)

Global sensitivity analysis was conducted using a Monte Carlo simulation approach to evaluate the robustness of the decision outcome with respect to uncertainty in preference weights. A total of 100 000 random weight coefficients for k_1 , k_2 , and k_{12} were generated from a uniform

distribution over all possible weight combinations using normalized random draws to explore the entire weight simplex.

Table 8: Pearson correlation.

Alternative	K_1	K_2	K_{12}
1	0.846	0.042	-0.883
2	0.810	0.107	-0.912
3	0.753	0.197	-0.945

Pearson correlation coefficients were computed between each weight parameter and the expected utility values in Table 8. The table shows a strong positive correlation between k_1 and expected utility, a strong negative correlation between k_{12} and expected utility, while k_2 has weak influence on the alternatives.

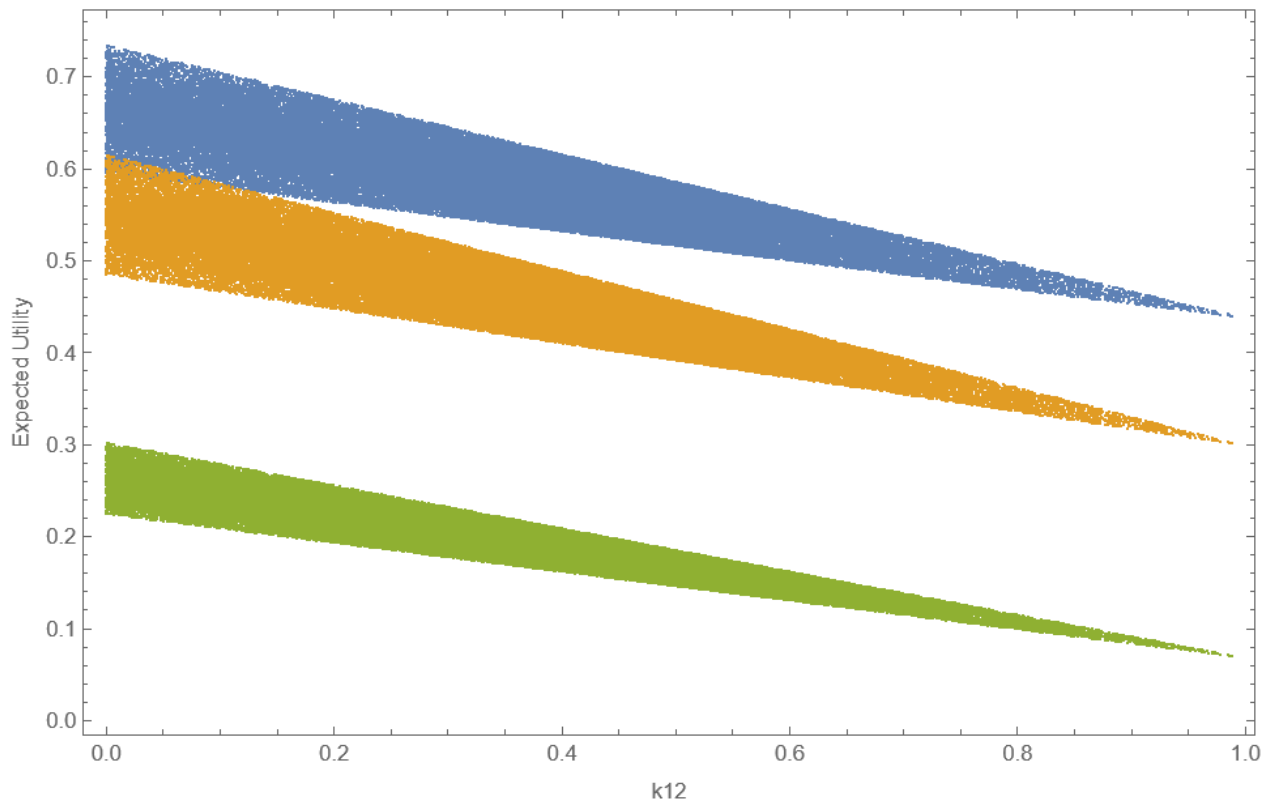


Figure 10: Monte Carlo simulation (Alternative 1 – blue dots, Alternative 2 – orange dots, Alternative 3 – green dots).

Alternative 1 is optimal in the majority of feasible weight combinations with only a few weight values, where its expected utility is comparable to Alternative 2 in Figure 10. Changing the x-axis to k_1 and k_2 does not significantly change the graph outcome because these coefficients are under the same normalization constraint in Formula 7. The graphs for k_1 and k_2 will be increasing functions because of the positive correlation.

Chapter 5 - Discussion

In this chapter, the outcomes of UDM, ranking outcomes, and sensitivity analysis are described. The outcomes of lottery-based questions corresponded to boundary independence case. Here, it is clear that X_2 is independent of X_1 . At the same time, X_1 is dependent on X_2 . It can be concluded that the value of time is higher for the decision maker compared to cost. Utility dependence matrix is constructed on the basis of this response in matrix on the right in Figure 9 [24]. After construction of MUF and calculation of expected utilities of each alternative, Alternative 1 showed the highest utility score, then Alternative 2 and Alternative 3, respectively. The third alternative had highest cost and transit time compared to other alternatives and it showed poor performance under decision maker's preferences. On the contrary, first and second alternatives were close in attributes, which increased the complexity of the decision-making. However, the implementation of MAUT into the logistic problem helped find the trade-offs between attributes and optimal route.

One-way local and global sensitivity analysis was carried out for all coefficients. The local sensitivity analysis was performed by varying all weight coefficients by $\pm 20\%$ and satisfying the normalization condition in Formula 7. Here, one of the coefficients is varied, while keeping the other weight constant and varying the third one to satisfy the condition. The decision maker's preference is known and certain. This approach allows local stability to be checked. From the outcomes in Tables 5, 6, and 7, it is clear that Alternative 1 is the best option for the transport of goods between the given two destinations and demonstrates strong robustness locally.

Global sensitivity analysis was also performed through a Monte Carlo simulation method to assess the robustness of the decision result in relation to uncertainty in preference weights (see Appendix E). In total, 100,000 random values of weight coefficients for k_1 , k_2 , and k_{12} were randomly generated through a uniform distribution on a simplex. This implied that the expected utility of each alternative was determined, and consequently, the alternative with the highest utility score was established. It was noted that Alternative 1 was optimal in all simulations for all feasible combinations of coefficients, illustrated in Figure 10. This implied that the ranking of alternatives was globally robust in relation to uncertainty in weight parameters. The structure of results was stable and no fluctuations were observed. The model can provide reliable support tool for decision-making under uncertainty.

The Pearson correlation coefficients were computed between each of the weight parameters and the expected utility values in Table 8. From the table, there was a strong positive correlation between k_1 and expected utility, a strong negative correlation between k_{12} and expected utility, while k_2 had weak influence on the alternatives. This showed that the first attribute and interaction term contributed most significantly to expected utility across the alternatives. Moreover, this implies that increasing the interaction term leads to a decrease in expected utility and it should be carefully calibrated. The different rankings across the entire weight simplex showed dominance of Alternative 1. This did not depend on any calibration of the weights, which showed strong robustness against uncertainty.

The study has limitations such as generalization problem, choice of specific method, bias, number of decision makers and attributes, and usage of static conditions. The methodology is based on a single study of freight transportation between Lianyungang and Duisburg, showing the limiting generalization and focus on a single MAUT approach. Other methods may give different results. The study relies on single expert judgment, which can introduce bias. But the approach of the study developed questionnaire to better understand the preference of the expert. The number of attributes can be increased to create a more complex and robust decision model. Lastly, the decision model considers static conditions, not real-time or dynamic changes in transportation data. These are the main limitations of the study. Consequently, the future work will focus on these limitations.

Chapter 6 - Conclusion and Future Research

This study developed a decision analysis model using MUF to evaluate and rank alternative railway routes between Lianyungang and Duisburg. The research considered a logistics challenge that involves trade-offs between minimizing costs and saving time, which cannot be effectively modeled using single-criterion approaches.

6.1. Key Findings

According to the obtained results, Alternative 1, the route between China and Germany through Kazakhstan and Russia, represents the best solution in terms of railroad transportation of freight containers with a utility value of 0.566. Next come Alternative 2 (via the Caspian sea) and 3 (via Trans-Siberian rail systems) with respective utilities of 0.451 and 0.195. This indicates that Alternative 1 represents the optimal choice for the decision maker because of its utility value. However, Alternative 2 showed better time-saving characteristic, while Alternative 1 demonstrates a more favorable trade-offs under both attributes.

The findings suggest that time saving plays more important role compared to cost saving attribute from the perspective of the decision maker as proven by the UDM and corresponding questionnaire. This highlights the importance of delivery speed in railway freight transportation decisions, especially in the context of China-Europe logistics. In addition, sensitivity analysis supports the robustness of the model. One-way and global sensitivity analysis both indicate that Alternative 1 remains the optimal choice regardless of the range of weight coefficient variations sampled. Even when the importance of cost and time varies, Alternative 1 still provides the greatest expected utility in all cases, showing stability and reliability of the decision model.

6.2. Limitations and Future Work

However, there are some weaknesses that should be noted. First, the research is based on the input of a single decision maker, which can lead to prejudice or bias. In addition, cost and time savings are the only two attributes used in the model, whereas other attributes should be taken into account in real-world logistics decisions as risk parameters. The model does not consider dynamic

changes in transportation systems and assumes static conditions. Future research can improve this study by including multiple decision makers to reduce bias and increase the generalizability. Other attributes should be included in the model, along with the use of real data.

6.3. Final Remarks

In summary, the current research shows that the use of MAUT is an efficient and practical tool for solving complicated decision-making problems in logistics. The application of MUF method in this study offers structured decision analysis model that shows the trade-offs between attributes in the context of the Lianyungang-Duisburg corridor. The developed model effectively finds the best route and provides a structured decision model for assessing alternatives.

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Appendix A

Identification of utility function for X_1 attribute (cost savings). The blank spaces need to be filled by decision maker, where decision maker is indifferent between saving some amount of money and playing a lottery on the right side.

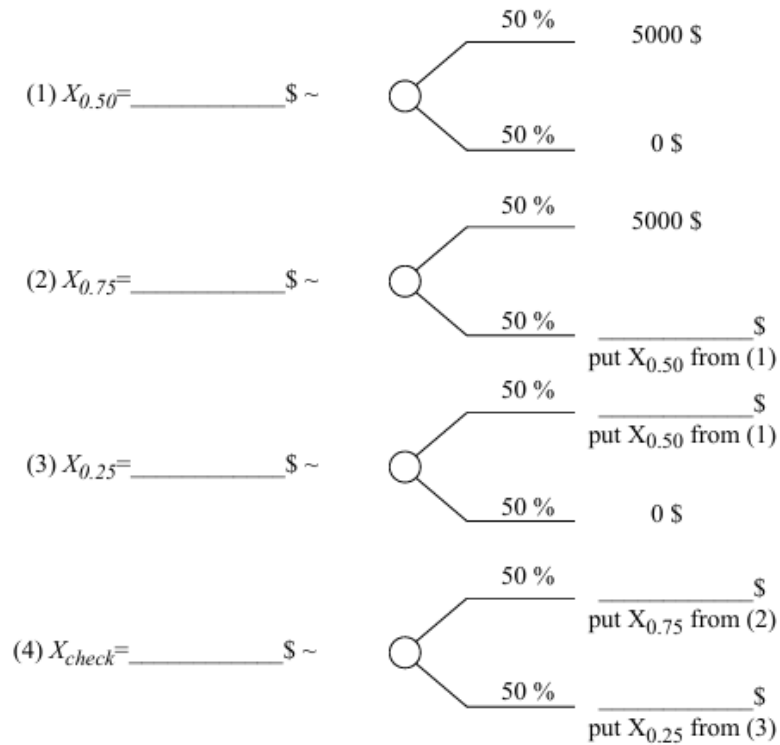


Figure 11: Identification of utility function for X_1 .

Appendix B

Table 1. Attributes

Attribute	Measure	Range	
		best	worst
$X1$ - cost saving	\$ dollars	$X1^* = 5000$	$X1^0 = 0$
$X2$ - time saving	days	$X2^* = 31$	$X2^0 = 0$

Question 1. Dependence: cost saving ($X1$) of time saving ($X2$)

a) Given two projects A and B with 50-50 chance of receiving different saved time, which project would you prefer in Scenario 1?

Scenario 1: in both projects $X2^* = 31$ days of time savings



Circle one out of three: *Project A* *I'm indifferent* *Project B*

b) Which project would you prefer in Scenario 2?

Scenario 2: in both projects $X2^0 = 0$ days of time savings



Circle one out of three: *Project A* *I'm indifferent* *Project B*

c) If the value of attribute $X1$ in Scenario 1 (or in Scenario 2) were held fixed at some other value between $X2^*$ and $X2^0$, would your answer change?

Circle your answer: *Yes* *No*

Figure 12: Identification of interdependencies between attributes $X1$ and $X2$.

Appendix C

```

data1 = {{0., 0.}, {2000., 0.25}, {3000., 0.50}, {4000., 0.75}, {5000., 1.00}};

fitData = Rest[data1];

fit1 = FindFit[fitData, (x1/5000.)^r1, {{r1, 1.0}}, x1];

rHat1 = r1 /. fit1;
u[x_] := (x/5000.)^rHat1;

(*R and R^2 computed on U scale*)
xVals = fitData[[All, 1]];
yObs = fitData[[All, 2]];
yFit = u /@ xVals;

rSquared = 1 - Total[(yObs - yFit)^2] / Total[(yObs - Mean[yObs])^2];
rPearson = Correlation[yObs, yFit];

Print["Estimated exponent   r = ", NumberForm[rHat1, {6, 4}]];
Print["Utility curve       U(x1) = (x1/5000)^", NumberForm[rHat1, {6, 4}]];
Print["R (Pearson)         = ", NumberForm[rPearson, {6, 4}]];
Print["R^2                 = ", NumberForm[rSquared, {6, 4}]];

(*Plot*)
Show[ListPlot[data1, PlotStyle -> PointSize[0.018], AxesLabel -> {"X1", "U1"}, PlotRange -> {{0, 5000}, {0, 1}},
Plot[u[x], {x, 0, 5000}, PlotRange -> {0, 1}, PlotLegends -> {"Power-law fit"}]]

```

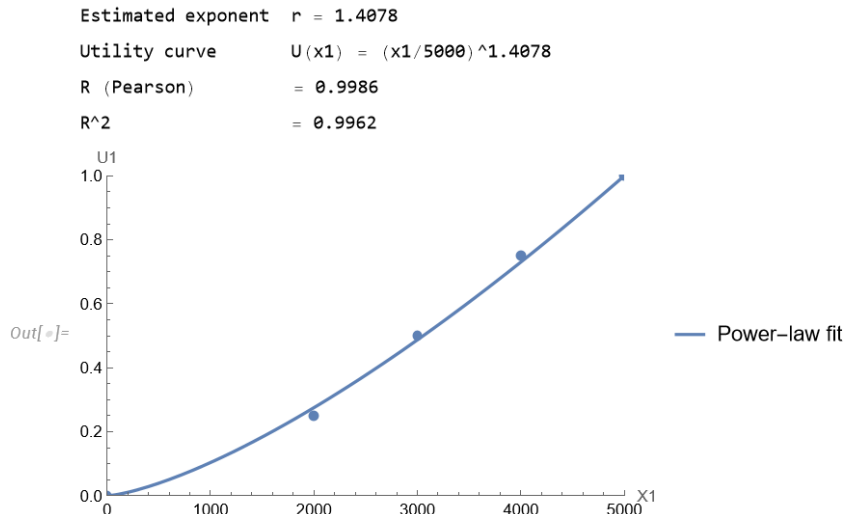


Figure 13: Mathematica code of least squares method for X_1 attribute.

Appendix D

```

data2 = {{0., 0.}, {10., 0.25}, {15., 0.5}, {23., 0.75}, {31., 1.0}};

fitData2 = Rest[data2];

fit2 = FindFit[fitData2, (x/31.)^r, {{r, 1.0}}, x];
rHat2 = r /. fit2;
u2[x_] := (x/31.)^rHat2;

xVals2 = fitData2[[All, 1]];
yObs2 = fitData2[[All, 2]];
yFit2 = u2 /@ xVals2;

rSq2 = 1 - Total[(yObs2 - yFit2)^2] / Total[(yObs2 - Mean[yObs2])^2];
rPearson2 = Correlation[yObs2, yFit2];

Print["Estimated exponent   r = ", NumberForm[rHat2, {6, 4}]];
Print["Utility curve       U(X2) = (X2/31)^", NumberForm[rHat2, {6, 4}]];
Print["R (Pearson)         = ", NumberForm[rPearson2, {6, 4}]];
Print["R^2                 = ", NumberForm[rSq2, {6, 4}]];

Show[ListPlot[data2, PlotStyle -> PointSize[0.02], AxesLabel -> {"X2", "U(X2)"}, PlotRange -> {{0, 31}, {0, 1}},
Plot[u2[x], {x, 0, 31}, PlotLegends -> {"Power-law fit"}]]

```

```

Estimated exponent   r = 1.0619
Utility curve       U(X2) = (X2/31)^1.0619
R (Pearson)         = 0.9938
R^2                 = 0.9858

```

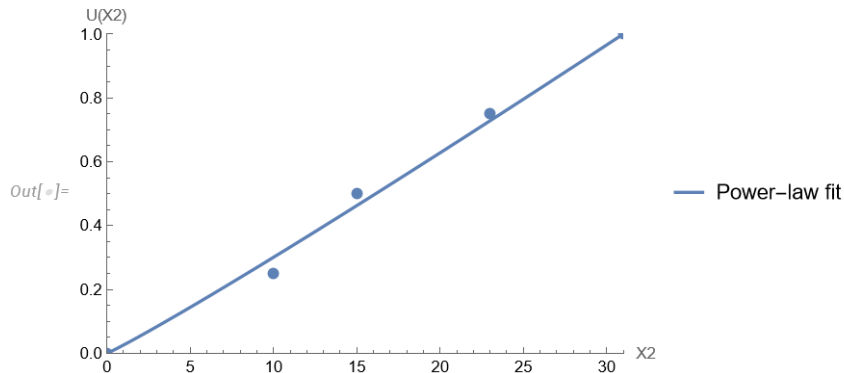


Figure 14: Mathematica code of least squares method for X_2 attribute.

Appendix E

```

ClearAll[ExpectedUtilityWithWeights, numSamples, weightSamples,
  euSamples, gsaData, k1List, k2List, k12List, corrRows, corrTable];

ExpectedUtilityWithWeights[{k1_, k2_, k12_}] :=
Module[{UIndMUFLocal, utilityMatrix, weightedMatrix},
  (*Multi-attribute utility function*)
  UIndMUFLocal[x1_, x2_] := k1 * U1Ind[x1] + k2 * U2Ind[x2] + k12 * U1Ind[x1] * U2Ind[x2];
  (*Compute utility values for each alternative*)
  utilityMatrix = Table[Table[UIndMUFLocal[costLBHInd[[i, r]], timeLBHInd[[i, c]]],
    {r, 1, 3}, {c, 1, 3}], {i, Length[rowLabels]}}];
  (*Apply probabilities*) weightedMatrix = Map[#, probMatrixInd &, utilityMatrix];
  (*Compute expected utility for each alternative*)
  Total[Flatten[#]] & /@ weightedMatrix];

numSamples = 100000;

weightSamples = Table[Module[{a1, a2, a3, sum}, {a1, a2, a3} = RandomReal[{0, 1}, 3];
  sum = a1 + a2 + a3;
  {a1, a2, a3} / sum], {numSamples}];

(*Compute expected utilities for all samples*)
euSamples = ExpectedUtilityWithWeights /@ weightSamples;

(*Combine weights and results*)
gsaData = MapThread[Join, {weightSamples, euSamples}];

(*Correlations*)

k1List = weightSamples[[All, 1]];
k2List = weightSamples[[All, 2]];
k12List = weightSamples[[All, 3]];

corrRows = Table[{rowLabels[[i]],
  Correlation[k1List, euSamples[[All, i]]], Correlation[k2List, euSamples[[All, i]]],
  Correlation[k12List, euSamples[[All, i]]]}, {i, Length[rowLabels]}}];

corrTable = Grid[Prepend[corrRows, {"Alternative", "corr(EU, k1)",
  "corr(EU, k2)", "corr(EU, k12)"}], Frame -> All, ItemSize -> All];

corrTable

(*Visualization*)

ListPlot[Table[Transpose[{k12List, euSamples[[All, i]]}], {i, Length[rowLabels]}],
  PlotLegends -> rowLabels, Frame -> True,
  FrameLabel -> {"k12", "Expected Utility"}, PlotRange -> All, ImageSize -> Large]

```

Figure 15: Mathematica code of sensitivity analysis.