

**HOW DO VIX AND VVIX INFORM VOLATILITY RISK PREMIA AND TRADABLE
VOLATILITY STRATEGIES?**

BY

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THESIS

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Abstract

This thesis examines how the VIX and VVIX indices capture compensation for bearing variance and volatility-of-volatility risk, and whether these risk premia can be converted into profitable VIX futures strategies. Using daily data on the S&P 500 index, VIX, VVIX, and front-month VIX futures from 2005 to 2024, I first assess the ability of VIX and VVIX to forecast future realized volatility of the equity market and of VIX itself. The analysis uses overlapping 22-day timeframes, Newey-West standard errors, and both in-sample and out-of-sample evaluation to compare historical volatility measurements with implied volatility. Building on these forecasts, I construct variance and vol-of-vol risk premia as the difference between implied variance (from VIX and VVIX) and model-based expectations of realized variance. These premia are then used to design simple timing rules for synthetic variance and volatility-of-volatility swaps, as well as for long-short strategies in front-month VIX futures. The results show that VIX and VVIX contain substantial information about future realized volatility and volatility-of-volatility, and that the variance risk premium is on average positive while the vol-of-vol risk premium is on average negative and strongly time-varying. At the index level, these premia are economically meaningful and help explain returns on synthetic swaps. However, when the same signals are applied to tradable VIX futures, the resulting timing strategies offer weak or negative risk-adjusted performance. This divergence highlights a gap between statistically significant risk premia in variance terms and their practical exploitability in simple VIX futures strategies, suggesting that term-structure dynamics and implementation frictions materially limit tradability.

Keywords: Variance risk premium, Volatility-of-volatility, VIX, VVIX, Realized volatility, VIX futures, Economic significance.

1. Introduction

Over the past three decades, volatility has shifted from a background risk factor to an asset class in its own right. The creation of VIX futures and options gave investors the ability to trade equity market volatility directly, rather than only implicitly through options on the underlying index. At the same time, the CBOE Volatility Index (VIX), constructed from S&P 500 index options, has become a widely quoted “fear gauge” and a central input in both academic research and market practice (Whaley, 2009; Bekaert & Hoerova, 2014).

A key insight from the literature is that implied volatility embeds not only expectations of future realized volatility but also compensation for bearing volatility risk. Model-free results show that VIX can be interpreted as a synthetic variance swap rate, so that the difference between implied and realized variance can be viewed as a variance risk premium (VRP) (Jiang & Tian, 2005). Empirical work documents that this premium is, on average, positive and time-varying, and that it helps explain the behavior of equity returns and option prices (Bollerslev, Tauchen, & Zhou, 2009). From this perspective, volatility is not merely a by-product of market conditions but a distinct source of systematic risk.

More recently, attention has turned from variance to volatility-of-volatility. The introduction of the VVIX index, which is constructed from VIX options, provides a market-based measure of expected variability in VIX itself. This makes it possible to define an additional premium on volatility-of-volatility analogous to the variance risk premium. Evidence suggests that this “vol-of-vol” risk is priced and that it helps to organize the joint dynamics of VIX, option prices, and tail risk (Park, 2013, 2015; Huang et al., 2019; Albers, 2023; Guo et al., 2024). In other words, investors seem to demand compensation not only for being exposed to high volatility, but also for the risk that volatility may change abruptly.

However, much of the empirical evidence on VRP and vol-of-vol premia is based on idealized, non-tradable payoffs such as model-free variance swaps. These constructs are informative about the pricing of risk under the risk-neutral measure, but they do not necessarily translate into feasible investment strategies (Carr & Wu, 2006, 2009, 2016; Mencía & Sentana, 2013; Johnson, 2017; Simon & Campasano, 2014; Post, 2023). In practice, investors who want to trade on volatility views typically do so via VIX futures and options, which are subject to term-structure effects, roll yields, and market frictions (Alexander & Korovilas, 2013). A central question, therefore, is whether the premia inferred at the index level can be meaningfully exploited using actual traded instruments.

This thesis takes that question as its starting point. It has two main aims. The first is to examine the informational content of VIX and VVIX for forecasting future realized volatility of both the S&P 500 index and the VIX index, relative to simpler historical volatility measures. The second is to investigate whether variance and vol-of-vol risk premia, constructed from VIX and VVIX, can be used to design simple timing strategies in front-month VIX futures. By comparing index-level premia to futures-based performance, the thesis evaluates the extent to which statistically significant premia are also economically exploitable. The analysis is organized around three research questions:

1. How well do VIX and VVIX, relative to historical volatility, predict future realized volatility of the S&P 500 index and the VIX index?
2. Are the variance and volatility-of-volatility risk premia, constructed from implied and model-based expected variance, positive and time-varying, and do they generate economically meaningful payoffs in simple synthetic variance-swap strategies on the S&P 500 and on VIX?
3. Do these premia translate into profitable and robust timing strategies in front-month VIX futures, or is there a gap between index-level premia and realized futures performance?

The contribution of the thesis is threefold. First, it provides a joint evaluation of VIX and VVIX as forecasting tools for both volatility and volatility-of-volatility over a long sample of daily data,

complementing earlier work that typically focuses on one dimension at a time. Second, it constructs variance and vol-of-vol risk premia in a consistent framework and studies their behavior across different market regimes. Third, it links these premia to straightforward, implementable VIX futures strategies, thereby moving from statistical properties of risk premia to their practical relevance for investors.

2. Literature review

This chapter reviews the main strands of literature that relate to variance and volatility-of-volatility risk premia and to the information content of the VIX, VVIX and VIX futures. It provides the conceptual background for the empirical work of this thesis.

2.1. VIX as model-free implied variance.

The analysis starts by explaining VIX as an independent tool which measures upcoming market volatility. The research conducted by Christensen & Prabhala (1998) showed that option-implied volatility includes future market information which traditional historical volatility methods do not have. Jiang and Tian (2005) developed a model-free implied variance from S&P 500 options which demonstrated superior performance in predicting future realized volatility compared to Black–Scholes implied volatility and past realized volatility. The study confirms that the VIX functions as a synthetic variance swap rate for the S&P 500 index through its model-free calculation method. The research by Barndorff-Nielsen and Shephard (2002) and Andersen et al. (Bollerslev (2003) developed high-frequency realized volatility measures which showed that option-implied variance serves as an effective predictor for upcoming realized volatility. The study proves VIX indices function as vital volatility prediction instruments and variance risk premium elements because they contain implied variance data under conditions of sufficient market liquidity and operational efficiency to follow variance swap rates.

2.2. Variance risk premia from options.

The difference between implied and realized variance is typically interpreted as the variance risk premium. Bakshi and Madan (2006) and Carr and Wu (2009) show that this premium can be extracted from option prices and that it captures compensation for exposure to volatility risk. Bollerslev, Tauchen, and Zhou (2009) document that the equity variance risk premium explains a sizable portion of the time-series variation in aggregate stock returns, with periods of high premia tending to be followed by higher future returns. Drechsler and Yaron (2011) propose an equilibrium model in which time-varying volatility and variance risk premia help to account for several asset-pricing puzzles. These studies motivate treating variance risk premia not only as by-products of option pricing, but as state variables that may forecast both volatility and returns (Park, 2013, 2015; Huang et al., 2019; Drechsler & Yaron, 2011). In this thesis, the VIX-based variance risk premium serves as a central object for building volatility strategies and for evaluating the economic significance of volatility risk at the index level.

2.3. Volatility-of-volatility and VVIX.

More recent work emphasizes volatility-of-volatility as an additional dimension of risk. The introduction of VIX options made it possible to construct VVIX, a model-free index of the expected volatility of VIX itself. Park (2013) shows that VVIX is closely related to the pricing of tail-risk hedges and that high levels of VVIX are associated with larger compensation for bearing extreme downside risk. In a follow-up study, Park (2015) analyzes the performance of strategies that are long and short volatility protection and finds that volatility-of-volatility helps explain variation in their returns. Other papers interpret VVIX and related vol-of-vol measures as indicators of market uncertainty about the volatility process (Lu & Zhu, 2010; Mencía & Sentana, 2013; Johnson, 2017; Simon & Campasano, 2014; Guo et al., 2024). For example, Bekaert and Hoerova (2014) decompose the VIX into expected variance and a variance risk premium component and argue that this premium reflects both risk aversion and economic uncertainty. These studies suggest that volatility-of-volatility carries distinct information beyond the level

of volatility and may have its own risk premium, which motivates the construction of a vol-of-vol risk premium (VVRP) in this thesis.

2.4. VIX futures, term structure and tradability.

A critical aspect for this project is the tradability of volatility risk. While VIX and VVIX are non-tradable indices, VIX futures and VIX options provide instruments through which investors can take positions in volatility. Carr and Wu (2006) study the joint behavior of the VIX and VIX futures and show that the term structure of VIX futures embeds a significant volatility risk premium. Trolle and Schwartz (2009) model the stochastic volatility and convenience-yield components of the VIX futures curve and document large and time-varying deviations between spot VIX and futures prices. These findings highlight that VIX futures do not simply replicate the index; instead, they reflect additional effects such as the shape of the term structure, roll yields and the cost of carrying volatility exposure (Jiang & Tian, 2005; Carr & Lee, 2009; Bakshi & Madan, 2006). As a result, trading strategies that appear attractive when evaluated on synthetic index-based payoffs may perform very differently when implemented with actual futures. This distinction between index-level and tradable measures of volatility risk premia is a central theme of the empirical analysis in this thesis.

2.5. Gap in the literature.

The existing literature has established three key points. First, VIX-type indices are informative about future realized volatility and can be interpreted as synthetic variance swap rates. Second, the variance risk premium derived from these indices is related to expected equity returns and measures compensation for volatility risk. Third, volatility-of-volatility and VVIX contain additional information about tail risk and the pricing of volatility protection. However, there is comparatively little work that jointly analyzes variance and vol-of-vol risk premia at the index level and then evaluates whether these premia can be converted into profitable strategies using real VIX futures. Most studies either focus on variance risk alone or evaluate strategies directly on index-based or option-based payoffs without contrasting them with

tradable implementations. This thesis fills that gap by constructing both VRP and VVRP from VIX and VVIX, assessing their predictive power for realized volatility and returns, and then testing whether signals based on these premia deliver economically meaningful performance when applied to front-month VIX futures.

3. Data

This study uses daily data on the S&P 500 index, the VIX and VVIX indices, and the front-month VIX futures contract from 2005 to 2024, subject to availability. Daily closing levels of the S&P 500 are taken from Investing.com and are used to compute log returns. VIX data come from the FRED series VIXCLS and represent the market's risk-neutral expectation of 30-day S&P 500 volatility in annualized percentage points. Historical VVIX data are obtained from the CBOE and provide an analogous measure of expected volatility of VIX based on VIX options. All three series are converted to a common daily date format and merged by calendar date.

To incorporate tradable exposure to volatility, the study takes VIX futures prices from Bloomberg. The analysis focuses on the generic first VIX futures contract, UX1 Index, which rolls into the next maturity as the front contract approaches expiration. For each trading day, the last traded price of UX1 is recorded as the front-month VIX futures price. Observations with missing or zero prices in any of the series are removed at this stage.

From the merged dataset, daily log returns on the S&P 500 and on VIX are computed as first differences of the logarithm of the closing levels. Historical and realized volatility measures are constructed as annualized standard deviations of daily returns over 22-day rolling windows, corresponding to approximately one trading month. The same procedure is applied to VIX log changes to obtain historical and realized volatility-of-volatility. Implied variances are recovered by squaring the volatility indices in decimal form, and these quantities are later combined with the realized measures in forecasting regressions and in the construction of variance and vol-of-vol risk premia and trading-strategy payoffs.

4. Methodology

The empirical approach consists of three main steps. First, one-month measures of historical and realized volatility are constructed from daily data using overlapping 22-day windows for the S&P 500 and for the VIX, together with implied variances from VIX and VVIX. Second, forecasting regressions are estimated that replicate the model-free implied volatility framework of Jiang and Tian (2005) for the S&P 500 and extend it to VVIX and VIX volatility. Third, the resulting variance and vol-of-vol risk premia are translated into simple trading rules on synthetic variance-swap payoffs and on front VIX futures to assess their economic relevance.

4.1. Volatility and vol-of-vol measures

Let P_t denote the closing level of the S&P 500 on trading day t . Daily log returns are defined as

$$r_t = \ln \left(\frac{P_t}{P_{t-1}} \right).$$

A “one-month” horizon is approximated by 22 trading days. Historical volatility (HV) is the annualized standard deviation of past daily returns over the previous 22 days,

$$HV_t = \sqrt{252} \sqrt{\frac{1}{22} \sum_{j=1}^{22} (r_{t+1-j} - \bar{r}_t)^2},$$

where $\bar{r}_t$ is the mean return in that window. Realized volatility (RV) is defined symmetrically using the next 22 trading days of returns,

$$RV_t = \sqrt{252} \sqrt{\frac{1}{22} \sum_{j=1}^{22} (r_{t+j} - \bar{r}_t)^2},$$

and is therefore only known ex post. This forward-looking realized volatility over fixed 22-day horizons is in line with the realized variance construction used by Jiang and Tian (2005) for the S&P 500.

To describe volatility of the VIX index, daily VIX log-changes are defined as

$$\Delta v_t = \ln \left(\frac{VIX_t}{VIX_{t-1}} \right).$$

Applying the same rolling 22-day windows to Δv_t yields historical and realized volatility-of-volatility, denoted HV_t^{VIX} and RV_t^{VIX} . These series are used to analyze vol-of-vol risk premia and the predictive content of VVIX for future VIX volatility.

Both VIX and VVIX are quoted as annualized percentage volatilities. Implied one-month variance from the VIX is computed as

$$IVAR_t = \left(\frac{VIX_t}{100} \right)^2,$$

and implied one-month variance of VIX from the VVIX as

$$IVAR_t^{VIX} = \left(\frac{VVIX_t}{100} \right)^2.$$

Under the usual interpretation of these indices, these variances represent risk-neutral expectations of variance over the next 30 calendar days, derived from out-of-the-money options on the S&P 500 and on VIX.

4.2. Forecasting regressions for volatility and vol-of-vol

The first set of regressions examines the information content of implied volatility for future realized volatility, following Jiang and Tian (2005). For the S&P 500, the dependent variable is log realized volatility, $\ln RV_t$, and the explanatory variables are log historical volatility and log VIX, $\ln HV_t$ and $\ln VIX_t$. The following specifications are estimated on the pre-COVID subsample from 2005 to 2019:

$$\ln RV_t = \alpha_1 + \beta_1 \ln HV_t + \varepsilon_t,$$

$$\ln RV_t = \alpha_2 + \beta_2 \ln VIX_t + \varepsilon_t,$$

$$\ln RV_t = \alpha_3 + \beta_3 \ln VIX_t + \gamma_3 \ln HV_t + \varepsilon_t.$$

Because realized volatilities are computed over overlapping 22-day windows, the error terms are serially correlated by construction (Jiang & Tian, 2005; Christensen & Prabhala, 1998; Christoffersen et al., 2010). Newey–West heteroskedasticity and autocorrelation consistent standard errors are therefore used, with 21 lags to match the forecasting horizon.

To assess the incremental information in VVIX for S&P 500 volatility, the regressions are extended by including $\ln VVIX_t$ as an additional regressor. The main specification is

$$\ln RV_t = \alpha + \beta \ln VIX_t + \delta \ln VVIX_t + \gamma \ln HV_t + \varepsilon_t,$$

estimated on the full sample from 2005 onward. This formulation captures the joint contribution of VIX, VVIX and historical volatility to one-month ahead realized volatility.

For the vol-of-vol dimension, analogous regressions are estimated with log realized VIX volatility as the dependent variable, $\ln RV_t^{VIX}$, and log VVIX, log VIX and log VIX historical volatility as predictors:

$$\ln RV_t^{VIX} = \alpha + \beta \ln VVIX_t + \delta \ln VIX_t + \gamma \ln HV_t^{VIX} + \varepsilon_t.$$

These regressions evaluate whether VVIX primarily captures expectations of future VIX volatility once the current VIX level and recent VIX volatility are controlled for. Newey–West standard errors are again employed to address autocorrelation arising from overlapping windows.

4.3. Out-of-sample forecast evaluation

To compare the predictive performance of different information sets, the sample is divided into a training period (2005–2015) and an evaluation period (2016–2024). All forecasting regressions are estimated using only observations in the training period.

For each model, let $\ln \widehat{RV}_t$ denote the fitted value of log realized volatility. The corresponding forecast of realized volatility is obtained by exponentiation,

$$\widehat{RV}_t = \exp(\ln \widehat{RV}_t),$$

and the forecast of realized variance is

$$\widehat{VAR}_t = \widehat{RV}_t^2.$$

Forecast accuracy is evaluated by the mean squared error of predicted variance on the evaluation subsample,

$$MSE = \frac{1}{N_{eval}} \sum_{t \in \tau_{eval}} (RV_t^2 - \widehat{VAR}_t)^2.$$

MSE values are compared across models based on historical volatility only, VIX only, VIX and historical volatility, and the full specification including VVIX. The estimated coefficients are then used to generate out-of-sample forecasts of realized volatility for every day in the evaluation period (2016–2024). This provides a direct measure of whether VVIX improves out-of-sample forecasts of S&P 500 variance.

4.4. Construction of variance and vol-of-vol risk premia

The variance risk premium on the S&P 500 is defined as the difference between implied variance and model-based expected realized variance over the next month. In practice, two versions of the S&P 500 variance risk premium are constructed: one based on a model with VIX and historical volatility only, and one based on the full model that also includes VVIX (Carr & Lee, 2009; Carr & Wu, 2009; Bollerslev et al., 2009; Bekaert & Hoerova, 2014). The latter serves as the baseline specification in the analysis. Let

$IVAR_t$ denote the implied variance from VIX and $\widehat{VAR}_t^{SPX}$ the expected realized variance from the log-linear forecasting model that includes VIX, VVIX and historical volatility. The S&P 500 variance risk premium is

$$VRP_t^{SPX} = IVAR_t - \widehat{VAR}_t^{SPX}.$$

On the VIX side, a vol-of-vol risk premium is constructed in a similar way. The implied VIX variance $IVAR_t^{VIX}$ is derived from VVIX, and the expected realized VIX variance $\widehat{VAR}_t^{VIX}$ is obtained from the VIX volatility regression with log VVIX, log VIX and log historical VIX volatility. The vol-of-vol risk premium is defined as

$$VVRP_t = IVAR_t^{VIX} - \widehat{VAR}_t^{VIX}.$$

Both premia are expressed in variance units and measure by how much risk-neutral expectations of variance exceed model-based expectations under the physical measure.

4.5. Synthetic variance-swap payoffs and timing rules

For the synthetic variance-swap strategies on the S&P 500 and on VIX, positions are determined using a zero threshold: when the relevant premium (VRP or VVRP) is positive, the strategy takes a short position in variance, and when the premium is non-positive it takes a long position. In contrast, for the VIX futures strategies the trading signal is based on whether the premium is above or below its sample median, which treats the premia as regime indicators and avoids fine-tuning of numerical thresholds (Bakshi & Madan, 2006; Carr & Lee, 2009; Carr & Wu, 2016). To link the premia to economic quantities, synthetic one-month variance-swap payoffs are constructed. For the S&P 500, realized variance over the next month is given by RV_t^2 , while the fixed leg is $IVAR_t$. The payoff to a long variance swap initiated at date t is

$$\Pi_t^{SPX} = RV_t^2 - IVAR_t.$$

The payoff to an always-short strategy is $-\Pi_t^{SPX}$. Three synthetic SPX strategies are analyzed: an always-short variance strategy; a VRP-based timing strategy that is short when $VRP_t^{SPX} > 0$ and long otherwise; and a variant where the expected variance is taken from the model that includes VVIX.

For VIX, realized variance of the index over the next month is $(RV_t^{VIX})^2$, and the fixed leg is given by $IVAR_t^{VIX}$. The corresponding payoff to a long VIX variance swap is

$$\Pi_t^{VIX} = (RV_t^{VIX})^2 - IVAR_t^{VIX}.$$

The strategies considered are an always-short VIX variance strategy and a VVRP-based timing strategy that is short when $VVRP_t > 0$ and long otherwise. Daily payoffs are aggregated to monthly returns, and performance is summarized by average monthly returns, volatility and annualized Sharpe ratios. This approach follows the idea in the option-pricing literature that statistical pricing errors should be evaluated jointly with the out-of-sample performance of trading strategies based on those errors.

4.6. VIX futures strategies and tradability

The final step examines whether the variance and vol-of-vol premia measured from non-tradable indices can be exploited in tradable VIX futures. Let F_t denote the price of the generic front VIX future (UX1). Daily log returns on the front future are

$$r_t^{FUT} = \ln\left(\frac{F_t}{F_{t-1}}\right).$$

An always-short VIX futures strategy earns $-r_t^{FUT}$ each day.

Premium-based timing rules are then applied to the VIX future. For the VRP-based futures strategy, the position is defined as

$$pos_t^{VRP} = \begin{cases} -1, & \text{if } VRP_t^{SPX} > \text{median}(VRP^{SPX}), \\ +1, & \text{otherwise,} \end{cases}$$

and the daily return on the strategy is $pos_t^{VRP} r_t^{FUT}$. An analogous rule is used for the VVRP-based futures strategy, with the position determined by whether $VVRP_t$ is above or below its sample median. Using median thresholds keeps the rules simple and avoids overfitting the data.

For comparison with the index-level analysis, synthetic “VIX futures” returns based on log changes in spot VIX can be constructed and subjected to the same timing rules (Simon & Campasano, 2014; Lu & Zhu, 2010; Mencía & Sentana, 2013; Johnson, 2017; Post, 2023). Comparing the performance of timing rules on synthetic variance-swap payoffs, on synthetic VIX returns, and on real VIX futures allows a direct assessment of how far variance and vol-of-vol premia measured from indices carry over to actual tradable instruments.

5. Results

5.1 Descriptive statistics and volatility measures

Table 5.1 reports descriptive statistics for daily S&P 500 log returns, VIX and VVIX levels, and the constructed historical and realized volatility measures over 2005–2024. Given the well-known non-normality of equity returns, it is not surprising that S&P 500 returns have a mean near zero with negative skewness and excess kurtosis. As indices of market stress, VIX and VVIX both have substantial volatility and right-skewed, leptokurtic distributions.

Figures 5.1 and 5.2 summarize the behaviour of volatility at a monthly frequency. Figure 5.1 shows that realized volatility and VIX move closely together, with sharp co-movement during the global financial crisis, the euro-area debt episode and the COVID-19 shock. VIX tends to lie above realized volatility, suggesting that option markets price in more variance than is subsequently realized. Figure 5.2 compares realized volatility with historical volatility computed from past returns; the relationship is visibly weaker, and realized volatility often jumps beyond what recent return history alone would predict.

5.2 VIX and historical volatility as predictors of S&P 500 volatility

Table 5.2 contains the Jiang and Tian (2005) regressions for the pre-COVID sample of 2005–2019. When log realized volatility is regressed on log historical volatility only, the coefficient has the correct sign and is very highly significant; but the model accounts for less than half of the variability in future volatility. When log realized volatility is regressed on log VIX, the slope becomes larger and the R^2 increases to about 0.58, which suggests that implied volatility has more predictive power than basic historical measurement.

In the simultaneous regression with both log VIX and log historical volatility, the β_1 test is large in absolute value and highly significant, whereas the one on σ_s becomes small and insignificant. The R^2 is essentially unchanged relative to the VIX-only specification. Overall, the replication confirms that VIX behaves like a model-free variance swap rate that largely subsumes the information contained in historical volatility.

5.3 VVIX and volatility-of-volatility

Table 5.3 examines the role of VVIX for S&P 500 volatility. In a univariate regression, log VVIX significantly predicts log realized volatility: high VVIX is associated with more volatile future index returns. However, once log VIX is included, VVIX loses explanatory power and often enters with a negative coefficient, while VIX remains the dominant predictor. Adding historical volatility does not materially change this conclusion. VVIX therefore adds, at most, limited incremental information for forecasting S&P 500 volatility beyond what is already contained in VIX.

By contrast, Table 5.4 shows that VVIX is very informative for volatility-of-volatility. In regressions of log realized VIX volatility on log VVIX, log VIX and log historical VIX volatility, VVIX is strongly significant in all specifications. The coefficient on VVIX remains large and positive when controlling for the level of VIX and recent VIX volatility, while the additional regressors are small or only weakly significant. This pattern supports the interpretation of VVIX as an index of expected volatility-of-volatility, rather than as an additional predictor of equity volatility.

5.4 Out-of-sample forecast performance

Coefficients are estimated on the training sample from 2005 until 2015 and one-month-ahead nowcasts of realized variance are produced over the period 2016–2024. The mean squared error (MSE) of the predicted variance for each model is shown in Table 5.5.

The HV-only model has the highest MSE, indicating that past returns provide the weakest forecasts. The VIX-only model results in a substantial reduction of MSE relative to historical volatility, consistent with the in-sample findings. Adding historical volatility to VIX gives only a slight additional improvement, and the inclusion of VVIX does not further reduce MSE meaningfully. The full model with VIX, VVIX and historical volatility performs similarly to, or slightly worse than, the simpler VIX + HV specification. Overall, the out-of-sample evidence indicates that VIX is the key predictor of S&P 500 variance, while VVIX does not materially improve forecast accuracy at the one-month horizon.

5.5 Variance and vol-of-vol risk premia and synthetic strategies

Variance and vol-of-vol risk premia are defined as the difference between implied variance (from VIX or VVIX) and model-based expected realized variance from the log-linear forecasting regressions. Table 5.6 summarizes the corresponding daily premia. The S&P 500 variance risk premium is positive on average and right-skewed, confirming that implied index variance typically exceeds expected realized variance. The VIX vol-of-vol risk premium is, by contrast, negative on average and displays large dispersion and high kurtosis, indicating frequent episodes in which model-based expected VIX variance exceeds VVIX-implied variance.

Figure 5.3 plots the monthly averages of the two premia. The S&P 500 VRP is relatively smooth and mostly positive, whereas the VVRP for VIX is more volatile and exhibits pronounced spikes around major stress episodes. This visual evidence highlights that vol-of-vol risk is concentrated in a smaller number of extreme periods.

Table 5.7 reports the performance of synthetic variance-swap-style strategies. For the S&P 500 (Panel A), the always-short variance strategy earns a positive average payoff with a moderate Sharpe ratio. Timing strategies that condition the position on the sign of the variance risk premium, with and without VVIX in the expected-variance model, generate very similar risk–return profiles. Figure 5.4 shows that their cumulative P&L tracks the always-short strategy closely, with only modest differences around major volatility episodes.

Panel B of Table 5.7 shows a much stronger effect for VIX variance swaps. An always-short VIX variance position produces large negative average payoffs and a strongly negative Sharpe ratio, indicating that unconditionally selling VIX variance is highly unattractive. In contrast, a timing strategy that goes short VIX variance only when the vol-of-vol risk premium is positive and long otherwise delivers clearly positive average payoffs with a much higher Sharpe ratio. Figure 5.5 illustrates this contrast: the cumulative P&L of the always-short strategy trends downwards, while the VVRP-timed strategy exhibits a stable upward drift, avoiding the worst losses around volatility spikes. These results suggest that vol-of-vol premia are economically meaningful for VIX variance-swap-style payoffs.

Monthly regressions of S&P 500 returns on average VRP and VVRP (not tabulated) provide some complementary evidence. The variance risk premium itself does not display robust predictive power for next-month equity returns once controls are included, whereas the vol-of-vol premium enters with a positive coefficient that is marginally significant. This pattern is consistent with a modestly priced vol-of-vol factor in equity returns.

5.6 VIX futures and tradable strategies

Finally, the analysis turns to tradability via front VIX futures (UX1). The correlation between spot VIX and UX1 is around 0.98, and a simple regression of the futures price on VIX yields an R^2 of about 0.96, confirming that the future closely tracks the index while embedding term-structure effects.

Table 5.8, Panel A, reports the performance of an always-short UX1 strategy and two timing rules based on the S&P 500 variance risk premium and the VIX vol-of-vol premium. The always-short futures strategy yields average returns close to zero with a very low Sharpe ratio, indicating little unconditional risk premium in front VIX futures over the sample. Both premium-based timing strategies perform substantially worse: switching between long and short futures when the premia are above or below their medians produces negative average returns and clearly negative Sharpe ratios. Figure 5.6 shows that the cumulative P&L of the timing rules drifts down relative to the flat or slightly positive path of the always-short benchmark.

Panel B of Table 5.8 applies the same timing rules to synthetic “VIX futures” constructed from spot VIX log changes. The qualitative pattern is similar: unconditional short exposure earns little reward, and VRP- and VVRP-based timing fails to generate robust gains. Taken together, the index-level and futures results indicate a gap between the clear premia observed in synthetic variance-swap space—especially for VIX—and the difficulty of monetizing these premia in simple, tradable strategies on front VIX futures.

6. Discussion

First, the replication and extension of the Jiang and Tian (2005) framework confirms that the VIX index remains a powerful model-free measure of expected variance. VIX strongly dominates historical volatility in forecasting S&P 500 realized volatility, both in-sample and out-of-sample, while VVIX adds little incremental information for equity volatility once VIX is included. This suggests that, at the index level, the bulk of forward-looking information is already incorporated into the level of implied volatility.

Second, VVIX is shown to be a genuine vol-of-vol index. It is the key predictor of future VIX volatility and remains highly significant even after controlling for the level of VIX and recent VIX volatility. The associated vol-of-vol risk premium is sizable, volatile and economically important for synthetic VIX variance-swap payoffs: timing positions based on VVRP flips a strongly losing always-short

strategy into one with positive average performance and a much higher Sharpe ratio. This indicates that investors require compensation for bearing vol-of-vol risk, and that VVIX captures this risk dimension effectively.

Third, the link between index-level premia and tradable strategies is weak. The same information that matters for synthetic variance swaps does not translate into simple profitable timing rules on front VIX futures. Premium-based long–short strategies on UX1 underperform even a near-zero-return always-short benchmark, likely due to term-structure effects, roll yields and market frictions that are absent in the synthetic setups. This gap between statistical mispricing in index or variance-swap space and economic value in tradable contracts is important for interpreting results in the volatility literature: evidence of a risk premium does not automatically imply an exploitable trading strategy.

From an asset-pricing standpoint, the limited information content of the vol-of-vol premium for equity returns implies that vol-of-vol risk is weakly priced in the cross-section of equities, though this effect is small and not robust to specification. Taken together, the results are consistent with a perspective in which VIX efficiently summarizes variance information, VVIX is mostly vol-of-vol, and economically meaningful premia exist but are partly absorbed by the pricing structure of the VIX derivatives market.

7. Conclusion

This thesis has investigated the information content and economic relevance of VIX and VVIX for S&P 500 volatility, VIX volatility and volatility-related risk premia over the period 2005–2024. The empirical strategy combined a replication of the Jiang and Tian (2005) model-free implied volatility framework with extensions to VVIX, volatility-of-volatility and tradable VIX futures strategies.

The main findings are as follows. First, VIX is a strong and robust predictor of future S&P 500 volatility, clearly outperforming historical volatility, while VVIX adds little incremental information for equity volatility once VIX is accounted for. Second, VVIX is highly informative about volatility-of-

volatility: it is the dominant predictor of future VIX volatility, even after controlling for the level and recent volatility of VIX. Third, the constructed variance and vol-of-vol premia are statistically and economically meaningful for synthetic variance-swap-style payoffs. The S&P 500 variance risk premium is positive on average, supporting the view that investors pay to hedge equity variance, while the VIX vol-of-vol premium is negative on average but can be exploited through timing strategies that condition positions on its sign.

At the same time, the analysis shows that these premia are difficult to monetize in simple strategies on front VIX futures. Premium-based timing rules on UX1 and on synthetic VIX futures underperform a trivial benchmark and do not deliver robust risk-adjusted profits. Finally, the vol-of-vol premium exhibits some, but limited, predictive power for future equity returns once standard controls are included.

These results imply that VIX and VVIX provide rich information about the conditional distribution of future volatility and its higher-order dynamics, and that investors are compensated for bearing volatility and vol-of-vol risk at the index level. However, the mapping from these premia to tradeable opportunities is weak, underlining the need to consider market structure, contract design and transaction costs when interpreting empirical evidence on volatility risk premia.

Future research could extend the analysis to longer-dated VIX futures, option-based replicating strategies or other asset classes, and could explore alternative modelling approaches for expected variance that relax the log-linear framework. It would also be interesting to study how the behaviour of VRP and VVRP changes across different monetary policy regimes and crisis periods, and whether more sophisticated dynamic trading strategies can bridge the gap between index-level premia and realistic trading opportunities.

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8. Appendix

Table 5.1 – Descriptive statistics (2005–2024)

	count	mean	sd	skewness	kurtosis	min	max
SPX log return	4475	0.0003	0.0125	-0.531	15.67	-0.1277	0.1096
VIX level	4475	19.6759	8.9661	2.464	11.97	9.1400	82.6900
VVIX level	4475	92.8190	16.6076	1.099	6.02	15.7100	207.5900
HV(SPX)	4475	0.1610	0.1181	3.358	19.28	0.0302	1.0648
RV(SPX)	4475	0.1608	0.1183	3.354	19.27	0.0309	1.0877
HV(VIX)	5054	1.0922	0.5102	1.690	7.04	0.1680	4.0986
RV(VIX)	5054	1.0919	0.5163	1.740	7.28	0.2697	4.1099

Table 5.2 – Jiang & Tian (2005) Replication: ln RV on ln HV and ln VIX

	(1) HV only	(2) VIX only	(3) VIX + HV
ln_hv	0.668*** (0.049)		0.034 (0.064)
ln_vix		1.161*** (0.054)	1.116*** (0.087)
Constant	-0.683*** (0.107)	-5.369*** (0.158)	-5.171*** (0.365)
Observations	3297	3297	3297

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5.3 – VVIX Extension: ln RV(SPX) on ln VIX, ln VVIX, ln HV

	(1) VVIX only	(2) VIX + VVIX	(3) VIX + VVIX + HV
ln_vvix	0.792*** (0.193)	-0.241** (0.115)	-0.219* (0.112)
ln_vix		1.177*** (0.057)	1.073*** (0.083)
ln_hv			0.079 (0.053)
Constant	-5.578*** (0.868)	-4.334*** (0.451)	-3.972*** (0.502)
Observations	4475	4475	4475

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5.4 – Vol-of-Vol Regressions: $\ln RV(VIX)$ on $\ln VVIX$, $\ln VIX$, $\ln HV_VIX$

	(1) VVIX only	(2) VVIX + VIX	(3) VVIX + VIX + HV VIX
$\ln_vvix$	0.668*** (0.093)	0.686*** (0.103)	0.575*** (0.117)
$\ln_vix$		-0.020 (0.063)	-0.037 (0.060)
$\ln_hv_vix$			0.106** (0.050)
Constant	-3.007*** (0.422)	-3.028*** (0.419)	-2.482*** (0.494)
Observations	4475	4475	4475

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5.5 – Out-of-sample mean squared error of variance forecasts (2016–2024)

	MSE
HV only	0.006768
VIX only	0.006195
VIX + HV	0.006181
VIX + VVIX + HV	0.006220

Table 5.6 – Descriptive statistics of SPX variance and VIX vol-of-vol risk premia

	N	Mean	SD	Skew.	Kurt.	Min	Max
VRP(SPX): VIX + HV	4475	0.018084	0.014403	3.052	16.89	0.004396	0.155967
VRP(SPX): VIX + VVIX + HV	4475	0.017979	0.015276	4.348	36.65	-0.000541	0.243747
VVRP(VIX)	4475	-0.161469	0.123681	2.145	15.31	-0.500910	1.300479

Table 5.7 – Panel A: SPX variance-swap strategies (monthly performance)

	Mean (m)	SD (m)	N	Ann. mean	Ann. SD	Sharpe
Always short	.0068344	.0503129	225	.082013	.174288	.470558

VRP					9	
VRP timing (VIX + HV)	.0068344	.0503129	225	.082013	.174288	.470558
VRP timing (VIX + VVIX + HV)	.0068222	.0503129	225	.0818659	.174289	.4697136

Table 5.7 – Panel B: VIX variance-swap strategies (monthly performance)

	Mean (m)	SD (m)	N	Ann. mean	Ann. SD	Sharpe
Always short VIX variance	-.5888827	1.192619	225	-7.066593	4.131352	-1.710479
VVRP timing	.5167706	1.158229	225	6.201247	4.012224	1.545588

Table 5.8 – Panel A: VIX futures (UX1) strategies (monthly performance)

	Mean (m)	SD (m)	N	Ann. mean	Ann. SD	Sharpe
Always short UX1	-.0000309	.009595	225	-.0003708	.0332408	-.0111562
VRP timing (UX1)	-.006069	.011467	225	-.0728282	.0397242	-1.833344
VVRP timing (UX1)	-.0077573	.013008	225	-.0930871	.045061	-2.065803

Table 5.8 – Panel B: Synthetic VIX futures strategies (monthly performance)

	Mean (m)	SD (m)	N	Ann. mean	Ann. SD	Sharpe
Always short synthetic VIX	.0000869	.0129304	225	.0010429	.0447922	.0232822
VRP timing (synthetic)	-.0087977	.0150262	225	-.105572	.0520522	-2.028194
VVRP timing (synthetic)	-.0112723	.0171425	225	-.135268	.0593833	-2.27788

Figure 5.1 – VIX vs realized volatility (monthly)

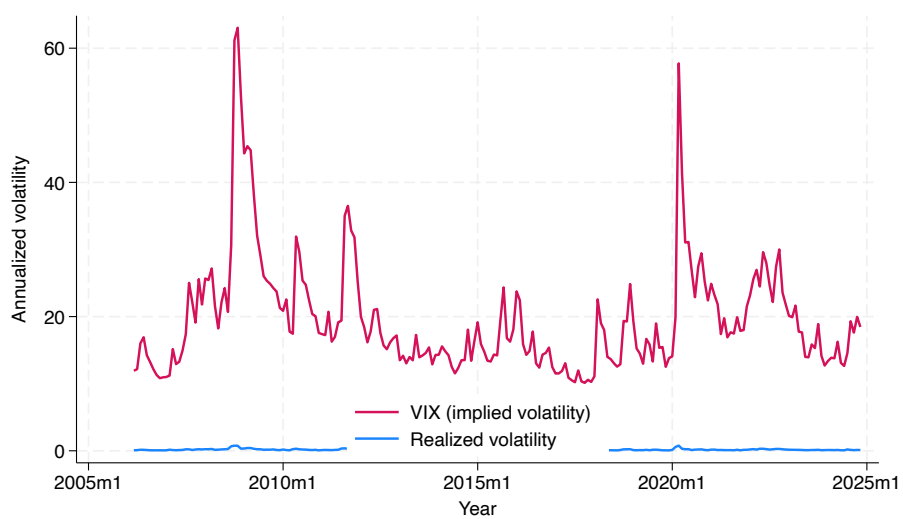


Figure 5.2 – Historical vs realized volatility (monthly)

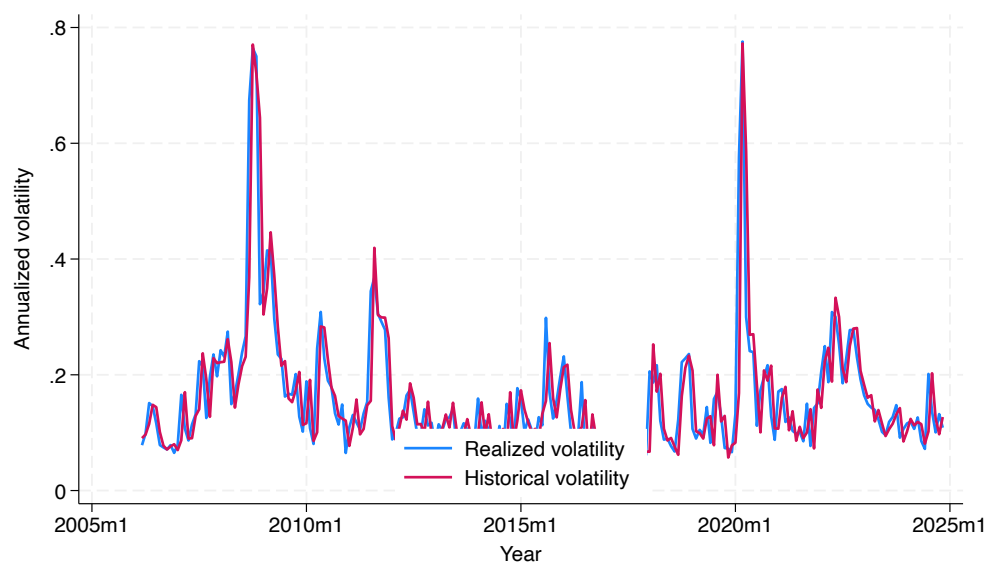


Figure 5.3 – VRP(SPX) and VVRP(VIX) over time (monthly premia)

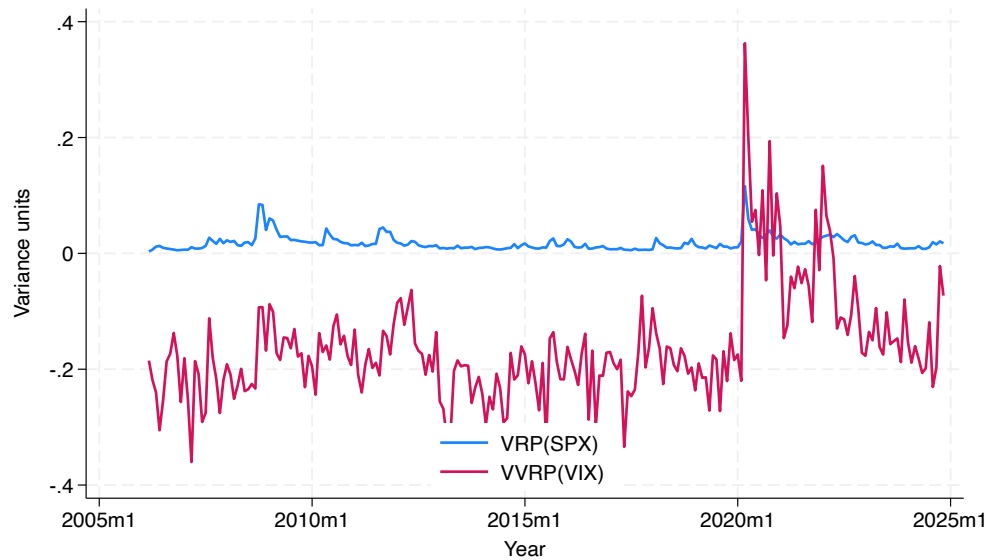


Figure 5.4 – Cumulative P&L of SPX variance-swap strategies

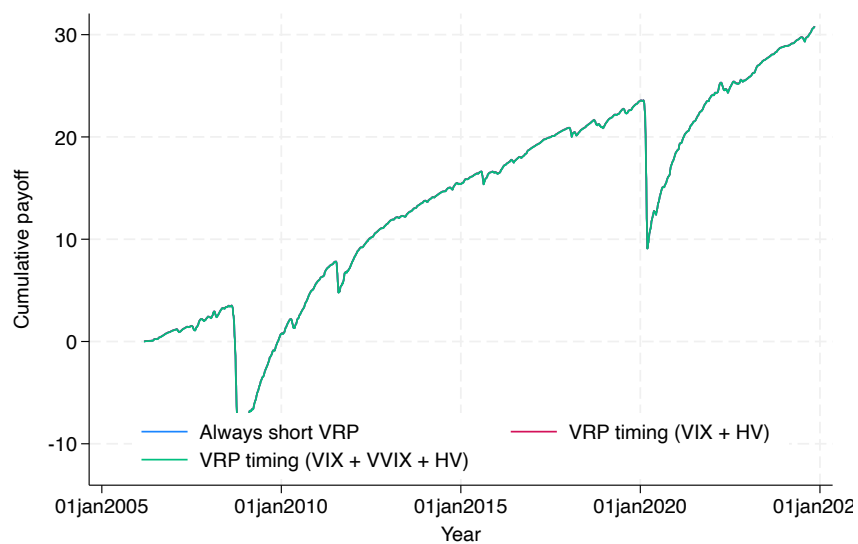


Figure 5.5 – Cumulative P&L of VIX variance-swap strategies

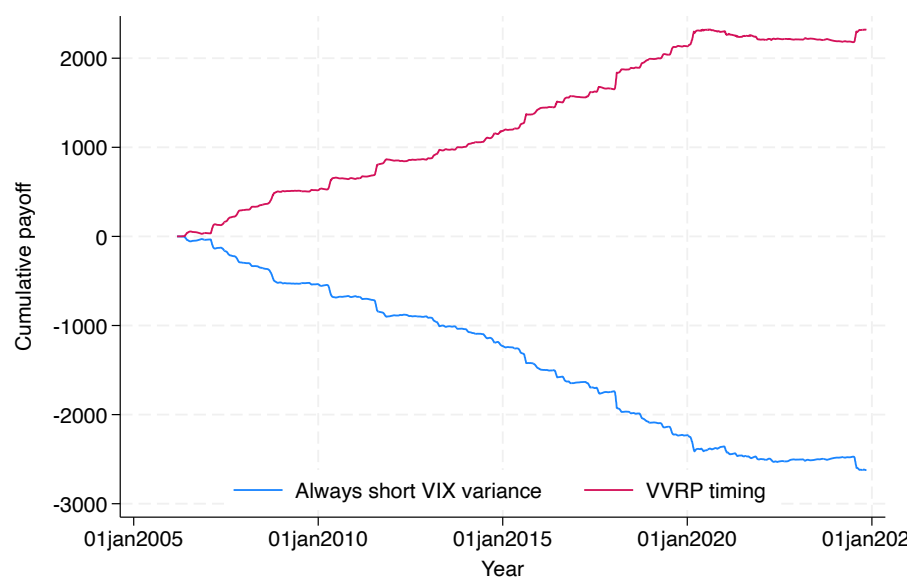


Figure 5.6 – Cumulative P&L of *real* VIX futures strategies

