

Ambiguity in asset pricing: evidence from Kazakhstan

BY

TUSUPOVA KLARA

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Advisor: Dr. Francesco Rocciolo

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Abstract

This research analyses a role of pricing factors in Kazakhstan. To conduct our research, we used methodology of a reference paper. According to the paper, they argue that investor fear of ambiguity incorporated as compensation in asset price (Brenner & Izhakian, 2018). Hence, for scope of our research, it is interesting to take Kazakhstan, which considered emerging market with a less developed and more vulnerable financial market. Particularly, the concept of ambiguity, risk and absolute deviation are remains unstudied in Kazakhstan. Therefore, this thesis aims to examine how KASE index reflect various factors, including ambiguity, absolute deviation and risk, which are embedded in index pricing.

Key words: ambiguity, asset pricing, emerging market, equity premium, expected return.

1. Introduction

In traditional asset pricing, risk is defined as the known probability of favourable returns and is compensated for bearing it. But, we usually cannot differentiate between ambiguity and risk. Likewise, we need distinguish them as its origin is not the same.

Emerging market countries often face different financial challenges. Inefficient political regulation, regional geopolitical tensions and dependence on raw materials contribute to market unpredictability. This unpredictability named as ambiguity. Ambiguity is characterised by uncertainty over a probability distribution. Investors often face situations where probability of the returns is uncertain and making decision more complex.

Notably, we investigate ambiguity within context of Kazakhstan emerging market., where ambiguities of returns are usually overlooked. Therefore, we will examine existence of the ambiguity and whether it incorporated in the pricing of assets or not.

2. Literature Review

The discussion begins with the definition of ambiguity, introduced by Knight in 1921(Knight, 1921). In his early work of Risk, Uncertainty, and Profit, Knight assumes that risk and ambiguity are not derived from the same origin, and subsequent theoretical models present only one condition that needs to be held, e.g. CAPM shows that higher risk measured by beta is equal to higher expected returns, whereas it still fails to include additional premium coming from investors' ambiguity (Knight, 1921). This exclusion of

potential compensation is especially thought-provoking to observe in developing and less transparent market (Knight, 1921).

Later, in 1961, Ellsberg revealed the Ellsberg Paradox, which highlight individual's preference. The study refers to sub-branch of finance to behavioural finance (Ellsberg, 1961). The Ellsberg Paradox showed how individuals tend to choose a known probability distribution over an unknown probability distribution. Commonly, the experiment included two baskets with different compositions of balls inside (Ellsberg, 1961). First, A basket presented an equal number of balls, 50 red and 50 black, with an acknowledged probability of one over two to get one over another coloured ball, whereas another basket B contained just 100 balls without any portion to be revealed (Ellsberg, 1961).

During the trial, most of the correspondents prefer basket A as it expected outcome is directly known and people deliberately choose to avoid an unfamiliar option, resulting in a version of ambiguity even if the results may be the same (Ellsberg, 1961). Ambiguity becomes a key topic, along with increase in concerns of payment equity premium to investors (Ellsberg, 1961).

Furthermore, decision-making based on worst-case scenarios by ambiguity-averse individuals was discovered by Gilboa and Schmeidler in 1989 (Gilboa & Schmeidler, 1989). Previous works of Knight and Ellsberg became the foundation for an extended version of the conventional Maximum Expected Utility model (Gilboa & Schmeidler, 1989). While people tend to maximize their utility, decision making under ambiguity illustrated in Ellsberg paradox, defined as conservative and worst case (Gilboa &

Schmeidler, 1989). Consequently, individuals make investment decisions based on focusing on the worst-case expected utility rather than the optimistic scenario. In the financial market, such actions lead investors to behave wisely, and only in the case of certain information (Gilboa & Schmeidler, 1989). A higher expected return comes from less transparent conditions, unknown probability of returns, or reliability estimates of the model.

This thesis builds upon the empirical findings of Brenner and Izhakian (2018), whose demonstrated that ambiguity is not a theoretical concept. Hence, it is a factor in the asset pricing model which is required to price in the S&P500 (Brenner & Izhakian, 2018). Brenner and Izhakian conducted research on the U.S. stock market and calculated the degree of ambiguity first. In a few words, ambiguity is diverged from investors' expectations, applying to the normal probability distribution function (Brenner & Izhakian, 2018).

What is more that results interpretation reveals again that in practice, investors require an equity premium for higher ambiguity which is describe previous studies. The model itself confirms that ambiguity is an independent risk factor, and ambiguity included vs excluded values are different (Brenner & Izhakian, 2018).

In the stock markets, authors empirically investigated ambiguity under different conditions and the findings reflect that unclear regulation and inefficient policy directly affect the price of compensation in Turkey (Cevheroglu-Acar & Karahan, 2023). Moreover, state dependence increases during recessions is characterised by ambiguity,

and the concept itself is differently priced in developed and developing financial markets (Cevheroglu-Acar & Karahan, 2023).

Zhao suggest that ambiguity extends beyond investor's behaviour and form new division of asset pricing framework in financial market (Zhao etc., 2022). During policy changes in China, macroeconomic variables are also change in expansions and recessions (Zhao etc., 2022). In China, policy uncertainty affects ambiguity premium pricing. Similarly, higher policy ambiguity leads to higher expected returns (Zhao etc., 2022). Based the review of ambiguity in developed market, this paper discusses how ambiguity influences return pricing in the emerging market.

One of the earliest studies on emerging market and equity premium is written by Bekaert & Harvey (Bekaert & Harvey, 1997). They studied return volatility in emerging market. Specifically, they conducted time-series and cross-sectional analysis. Also, they examine capital market reforms in 12 different countries (Bekaert & Harvey, 1997). The distributional summary table shows that empirically equity returns have excess kurtosis and not normally distributed. The asymmetry of return can be described as effect of idiosyncratic shocks (Bekaert & Harvey, 1997). In essence, they conclude that the volatility in developing markets is unstable and can be exposed to changes in regime.

A particularly relevant paper is "Asymmetric volatility and risk in equity market" which claim that negative returns increase return's volatility (Bekaert & Wu, 2000). The claim derived from the reaction to volatility during negative news. In this research there is an evidence that market response to positive development is rare. The paper support the

notion that returns are non-normal, skewed, leptokurtic and clustered in emerging market(Bekaert & Wu, 2000).

Another noteworthy paper about stocks returns volatility, which witness a jump in spikes (Cremers et al., 2012). In this research, the traditional variance of returns became poor proxy for risk. On the other hand, mean absolute deviation is more economically meaningful as a proxy in non-Gaussian distributions. Moreover, the study shows that returns are usually fat-tailed, skewed and subjected to periodic jump in risk(Cremers et al., 2012).Also, when there is a control for beta and volatility, they find that mean absolute deviation captures high fluctuations. Therefore, if market participant faces large absolute deviation in returns, they will require additional compensation (Cremers et al., 2012).

Finally, Barberis, Mukherjee, and Wang examine the emotional reaction of investors to ambiguity in 2016 (Barberis et al., 2014). Their research shows that during periods of uncertainty and negative news, market participants tend to overreact and set negative expectations in market. Alternatively, in case of good news, people choose safe results and show risk-averse attitude (Barberis et al., 2014). Individuals reaction to negative news transmit as fear in price, whereas in good condition prices are stable. Based on the example of 2008, ambiguity drove excess fluctuations and shift predictability of expected return. It illustrates the importance of examining the concept of economic business cycle fluctuation and behavioural finance.

2. Methodology and Data

For the scope of observation, it was chosen to take the daily returns of KASE Index from 1990 to 2025. The Kase Index is good choice as the single market indices. Moreover, in order to create business cycle dynamics, we collected GDP, unemployment, government and household consumption, investment, GDP growth and created a recession index quarterly from 2001 to 2024. We found that the recession index can be calculated on deviations in GDP and the rate of unemployment. These variables can be implemented to observe any fluctuations in the equity premium of assets during downturn and development.

The methodological calculation based on Brenner&Izhakian paper (Brenner & Izhakian, 2018). Mainly, starting from estimating the degree of ambiguity. We included separate calculations mean and standard deviation of daily KASE Index returns. The mean was calculated based on a rolling window between time(t) and t+1, simply averaging returns and corresponding returns to another pair, whereas the standard deviation was corrected by suggestions. The suggested formula defines the volatility or standard deviation square as the sum of the difference of returns and expected returns in square, and the sum of the change of returns and expected returns today and the day before (Scholes & Williams, 1977). The formula represented in the following way:

$$\sigma_t^2 = \sum_{i=1}^{N_t} (r_{t,i} - E[r_{t,i}])^2 + 2 \sum_{i=2}^{N_t} (r_{t,i} - E[r_{t,i}])(r_{t,i-1} - E[r_{t,i-1}])^{[21]}$$

The bin for the standard normal distribution has a range of -6% and +6% with 62 bins(intervals). Every day, we gathered aggregation of favourable return in terms of probability. To follow up, we created a pair of μ, σ was summed up to set in a normalized probability distribution. The formula can be pictured:

$$P(r \geq r_f) = 1 - \Phi(r_f; \mu, \sigma)$$

Adding all in one formula according to the reference paper (Brenner & Izhakian, 2018):

$$\begin{aligned} \mathcal{U}^2[r] = & \frac{1}{w(1-w)} * (E[\Phi(r_0; \mu, \sigma)]Var[\Phi(r_0; \mu, \sigma)] \\ & + \sum_{i=1}^{60} E[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)] * Var[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)] \\ & + E[1 - \Phi(r_{60}; \mu, \sigma)]Var[1 - \Phi(r_{60}; \mu, \sigma)]) \\ r_0 = & -0.06, w = r_i - r_{i-1} = 0.002 \end{aligned}$$

$\mathcal{U}_t$ = is the variance of the daily probabilities of returns over the month

v_t = is the variance of the daily probabilities of returns over the month

$\frac{1}{w(1-w)}$ = variability of probability calculated as a weighted average on bin dimensions

To define weigh of the degree of ambiguity, we used correction result, to check whether the size of the bin affects the degree of ambiguity, and with the intent to control size effect (Schneeweiss & Komlos, 2009).

Each coefficient of expected ambiguity, expected volatility, and expected absolute deviation was derived from- series ARMA (p, q) model (Brenner & Izhakian, 2018):

$$\ln\sqrt{v_t} = \psi_0 + \varepsilon_t + \sum_{i=1}^p \psi_1 \cdot \ln\sqrt{v_{t-1}} + \sum_{i=1}^q \theta_i \cdot \varepsilon_{t-i}$$

$$\ln\mathfrak{U}_t = \psi_0 + \varepsilon_t + \sum_{i=1}^p \psi_1 \cdot \ln\mathfrak{U}_{t-i} + \sum_{i=1}^q \theta_i \cdot \varepsilon_{t-i}$$

$$\ln\vartheta^E = \psi_0 + \varepsilon_t + \sum_{i=1}^p \psi_1 \cdot \ln\vartheta_{t-i}^E + \sum_{i=1}^q \theta_i \cdot \varepsilon_{t-i}$$

$\mathfrak{U}_t = \text{expected ambiguity}$

$v_t^E = \text{expected volatility}$

$\vartheta^E = \text{expected absolute deviation of returns from the expected return}$

Taking the results of the ARMA model and using them to calculate the equity premium (Brenner & Izhakian, 2018):

$$\text{Excess Return}_t(rx_m) = \alpha + \gamma \cdot v_t^E + \eta \cdot (\mathfrak{U}^2)_t^E + \varepsilon_t$$

$v_t^E = \text{expected volatility}$

$(\mathfrak{U}^2)_t^E = \text{expected ambiguity}$

η : measures whether ambiguity is rewarded ($\eta > 0$) or penalized ($\eta < 0$)

$$rx_m = \alpha + \gamma v_t^E + \eta((\mathfrak{U}^2)_t^E * \vartheta_t^E) + \theta (\mathfrak{U}^2)_t^E + \omega \vartheta_t^E + \varepsilon_t$$

$$rx_m = \alpha + \gamma v_t^E + \eta((\mathfrak{U}^2)_t^E * \vartheta_t^E) + \varepsilon_t$$

Apart from finding pricing in ambiguity, volatility and absolute deviation, there are regressions to check the business cycle fluctuation of equity premium by the following formulas:

$$rx_m = \alpha + \beta X_t + \delta D_t^{rec} + \theta (X_t * D_t^{rec}) + \varepsilon_t$$

$$X_t \in \{\vartheta_t^E, \mathbb{U}, \nu_t^E\}$$

β – pricing coefficient in expansion

D_t^{rec} – recession dummy

δ – coefficient of shift in recession

θ – pricing change in recession

4. Results

Estimating volatility and the degree of ambiguity

We choose to test based on Kazakhstan Stock Exchange(KASE) daily returns from January to January next, between 1990 and 2025, 420 months in total. KASE represents an index, aggregated shares of all listed companies on the KASE exchange market of Kazakhstan. All firms report sound financial data, and their stocks are considered moderately liquid. Each stock shows weight related to its capitalization (KASE description)

To find the current weight of different stocks, we need to multiply the price of a share by the number of outstanding shares and the proportion of available shares on the exchange (free-floating factor), divided by the total amount of all weights (KASE description). The KASE Index is the price of stocks multiplied by the number of stocks and the free-floating factor, all divided by the normalized basis point. As the risk-free rate, it was decided to choose TONIA, which represents the risk-free rate on the money market (KASE description).

In general, during calculation of ambiguity, risk free will be constant over its mean and use T-bill 3 months, bills or notes of the U.S. short-term as risk free rate can be acceptable. When it comes to the excess return e.g. defining equity premium for the Kazakhstan market with incorporated expectation of Kazakhstani participants, after discussion of ways representing risk-free, TONIA still becomes the best suitable tool. Furthermore, TONIA shows average-weighted daily (short-term) open repo transactions conducted on the market by government securities. Its compounded floating rate maturity is considered to be six months (Government press).

First of all, by proceeding to code in Stata, we adjusted returns of the KASE Index based on by correction (Scholes and William, 1977). Referring to the methodological part, we used the same logic and came up with Figure 1. Mainly, the graph represents adjusted monthly volatility according to correction formula. Normally, we can see that spikes of the graph fluctuate in the period of economic instability, any global crisis, e.g. Russian crisis between 1998 and 1999, the global financial crisis 2007-2008, Covid-19, and other

factors. The financial market responds to both domestic and external shocks, reflecting their influence through volatility and price changes.

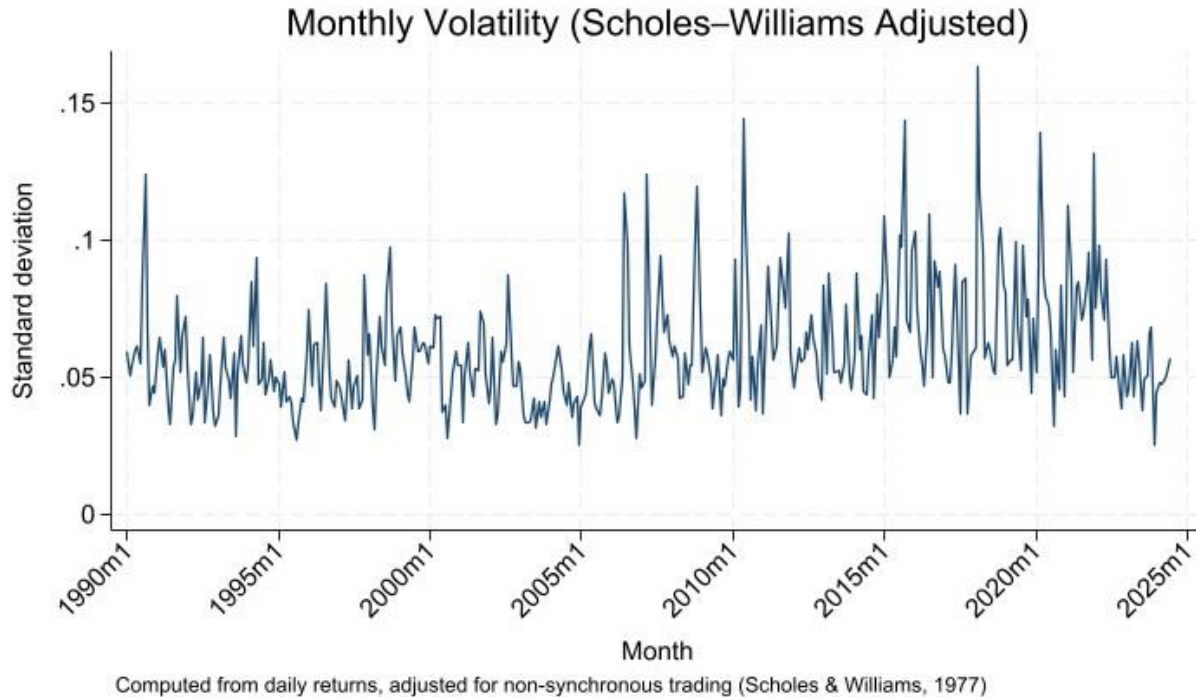


Figure 1. Adjusted volatility of the KASE Index

Furthermore, degree ambiguity is reflected in the plot and indicated by key events in every spike. Interestingly, remarkable ambiguity spikes correspond to economic events or major regional uncertainty. Historically, in 1991, after Kazakhstan became independent, along with establishing the country from scratch, people experienced insecurity related to how the country would develop. So, we argue that ambiguity rose as citizens were worried about the new country's performance.

Coming to the major spike of these years was the collapse of the USSR. Well-known event worldwide for its uncertainty. Though the independence, every government entity started to work, introduced a new currency, developed global recognition, involved

trade, and wrote laws for the country. But still resulted in expectation of uncertainty from the new flow of privatization and hyperinflation, along with no proper banking system, aggregate to a sharp increase in ambiguity at the beginning of 1994. So, an independent country on the edge of control own currency. Missed the basic monetary policy and raw trade interaction.

Turning point was an introduction new currency- tenge. In addition, the establishment of the National Bank on 14th April 1993 led to set gradual control over monetary policy and the financial market (Tadzhiyakov, 2013). When the goal of the National Bank was to replace the ruble and build a new banking system based on the prior Kazakh Soviet Socialist Republic to tackle current challenges.

Over time, the National Bank continued to work in terms of price stability and inflation by introducing financial instruments like securities for short-term source funds for second-tier banks to allow building deposit and credit relations, implemented a refinancing rate, allocated funds to intervene, and sustaining liquidity in the market of newly issued government securities(Tadzhiyakov, 2013). All such efforts indeed affected to economy by undertaking hyperinflation of 2,265 percent in 1993 to the lowest inflation rate of 1.9 percent in 1998(Tadzhiyakov, 2013).

Furthermore, after 7 years of independence, people's expectations of dependence on the previous system had changed, and they did not experience any shock from the crisis of the main trade partner of the Russian Federation. Even though close ties were created

with Russia, it did not create a spillover effect to Kazakhstan, but did result in high volatility.

Follow up, establish dependency on the source of income from the National Fund in any case of further development or upcoming major oil market shocks. It directly hit the economy and created a short-run ambiguity rise. Though Kazakhstan continues to diversify its economy, oil price volatility and its impact can be observed in 2004, 2005, 2014, and in the following years. In 2014, the Ukraine conflict created uncertainty about actions of Russia Federation towards Crimea, subsequently, the oil market perceived fear and responded with fluctuation of prices (Suleyman O. Altiparmak, etc., 2013).

The next attention-grabbing possible explanation for the increase in the degree of ambiguity between 2007 and 2009 is. For Kazakhstan, with an unstable banking system, the Global Financial Crisis brought default of the large banks of BTA, Alliance, and Temirbank (M & A, 2019). Consequently, the authority of the country had to bail out and react quickly. For citizens without strong credibility in the National Bank and currency under constant inflation, it was one more nightmare.

Government money injection took place through the National Bank, which provided low-interest-rate credit for second-tier banks. Moreover, the government started to purchase shares of banks through Samruk-Kazyna Sovereign Wealth Fund (Alexander, 2011). These measures somehow decrease pressure on the monetary system and result in stabilization of the banking sector by avoiding systematic risk of default (Alexander, 2011). However, the financial market and the economy were under ambiguity, and all

such volatility was incorporated in the price of the Kase Index from 2007 to 2010. Also, one of possible the indirectly effect of fear based on a demonstrative case of the Greek debt crisis (Mariana A.,2024).

After the collapse and a slight recovery of the crisis in the banking sector, more events contributed to a new wave of growth in the degree of ambiguity. Specifically, on the 20th of August in 2015, known in Kazakhstan as the occasion of shifting from a fixed exchange rate to a floating exchange rate.

The National Bank steer money to keep the exchange rate on fixed amount. Such action took cost to sustain at single rate until August of 2015. When based on decision between the government and the national bank, floating exchange rate was introduced on 20th August 2015 (Mckenzie, 2015). In results, in a day, tenge depreciated by 23% and in two days 27.7% according to official statistics (Mckenzie, 2015). Such a sudden acknowledge of loss of power of currency among citizens lead to concerns and destabilized at this time low individuals' confidence.

Moreover, as imports became more expensive and inflation started to be incorporated in prices, it created a huge burden of intervention from the Former President and again revealed the inefficiency of monetary policy (Jarosiewicz, 2015). In our graph, this episode of the country's failure can be clearly observed by a sudden jump and continued uncertainty surge during 2015 and 2016.

Over and above that, oil market volatility as a consequence of disagreement among OPEC members and other interested parties, and low credibility in the financial market,

pushed for monetary policy tightening further. The National Bank introduction of inflation targeting subsequently replaced the managed exchange rate, and had unclear communication strategies. Now, it was important to build a consolidated monetary framework with a lower target for the long run. Specifically, settled target 6-8% in 2017 to 5-7% in 2018, and 4-6% in 2019 (Bank, 2017).

One of the feasible causes of the increase in uncertainty can be due to the resignation of the first president of the country on the 19th of March 2019(Aljazeera, 2019). Specifically, after long years since independence governor whole country gave his seat to less less-known president, which was quite unexpected (Theguardian, 2019) . The new president, Kassym-Jomart Tokayev started to be a head of the country and won based on the results of the election on 9th June 2019 (Gleboff, 2019).

Previously, it is mentioned was unexpected political uncertainty and followed up with the next tremendous pressure on ambiguity come up from Covid-19. Mainly, lockdown, closed borders, limitations on transportation and flights, and putting the state of emergency on 15th March 2020 resulted in a slowing economy (Halpin, 2020). Shock from strict rules of quarantine and one more in July-August were preventing mechanisms, but, panic and uncertainty already shifted market expectations.

The next events that probably cause spikes in the graph are civil unrest in January of 2022 and following and Ukraine war. After uncertainty coming from what kind of direction can take place for the country, citizens expectations were still unstable. Soon, after time passes, ambiguity degree gets down.

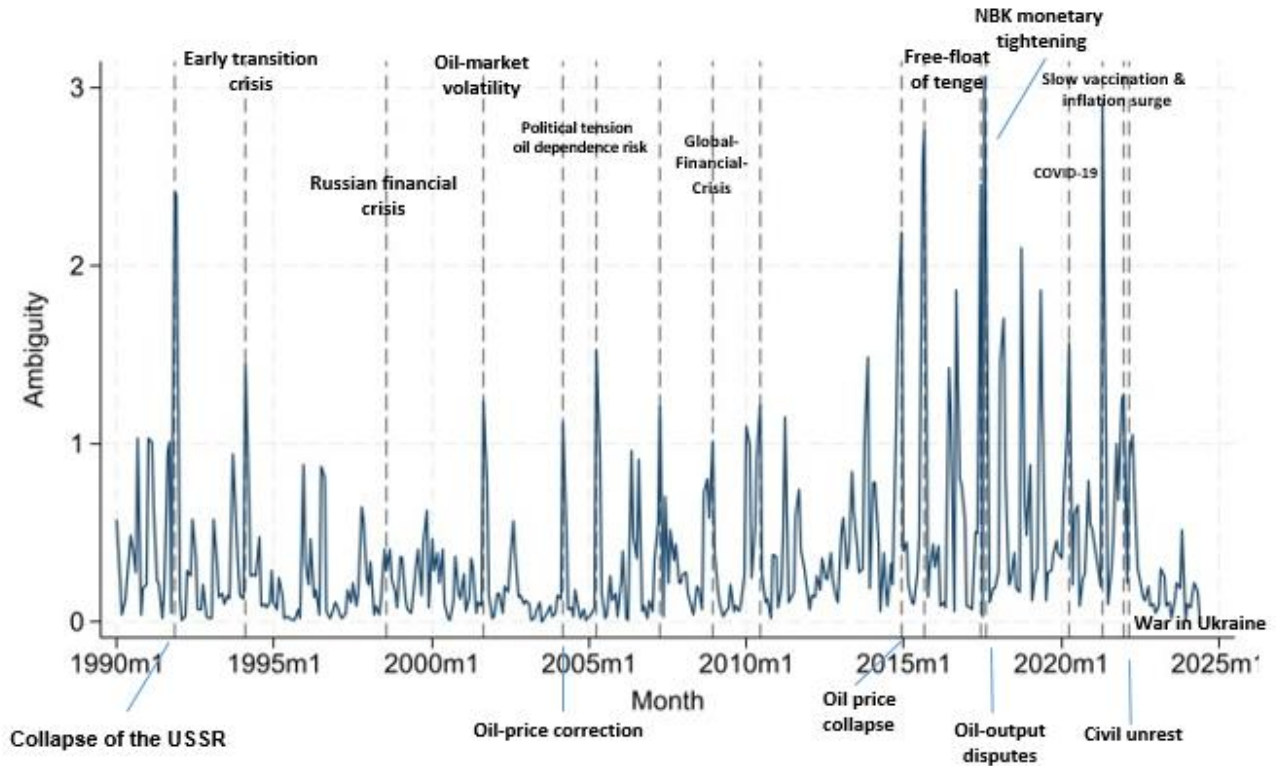


Figure 2. Calculated uncertainty indicator

Furthermore, excess returns were observed monthly in Figure 2 from 2000 to 2025. During the period, the trend witnessed frequent fluctuations, during periods of crisis, shock, and economic events. However, referring Brenner & Izhakian, ambiguity is a measure based on behavior, information dispersion and expectation of market participants, whereas excess returns or market premium is a reflection of price change relative to risk-free rate or incorporated reaction in price (Brenner & Izhakian, 2018). It can be seen based on the highlights of the same events that ambiguity and excess returns have relatively different spikes. When the expectation of the market was not settled, ambiguity measurements were high, whereas excess return indicators were low.

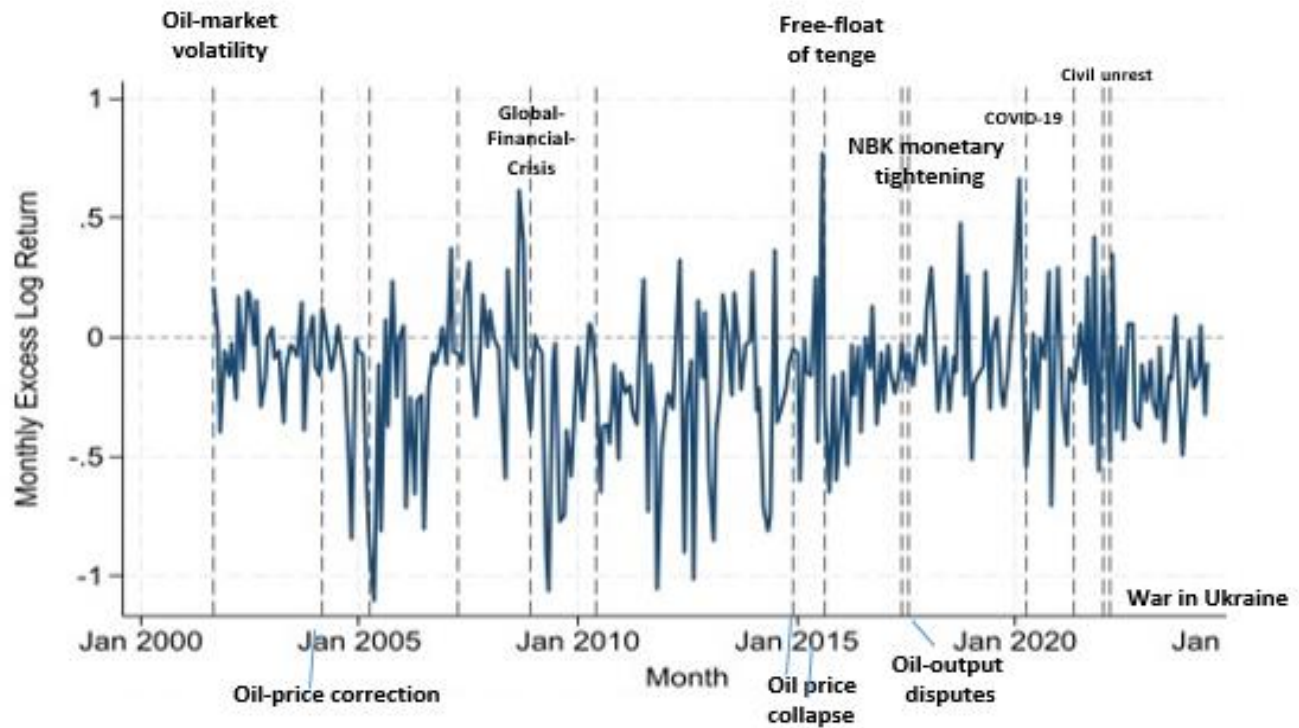


Figure 3. The time series of excess return

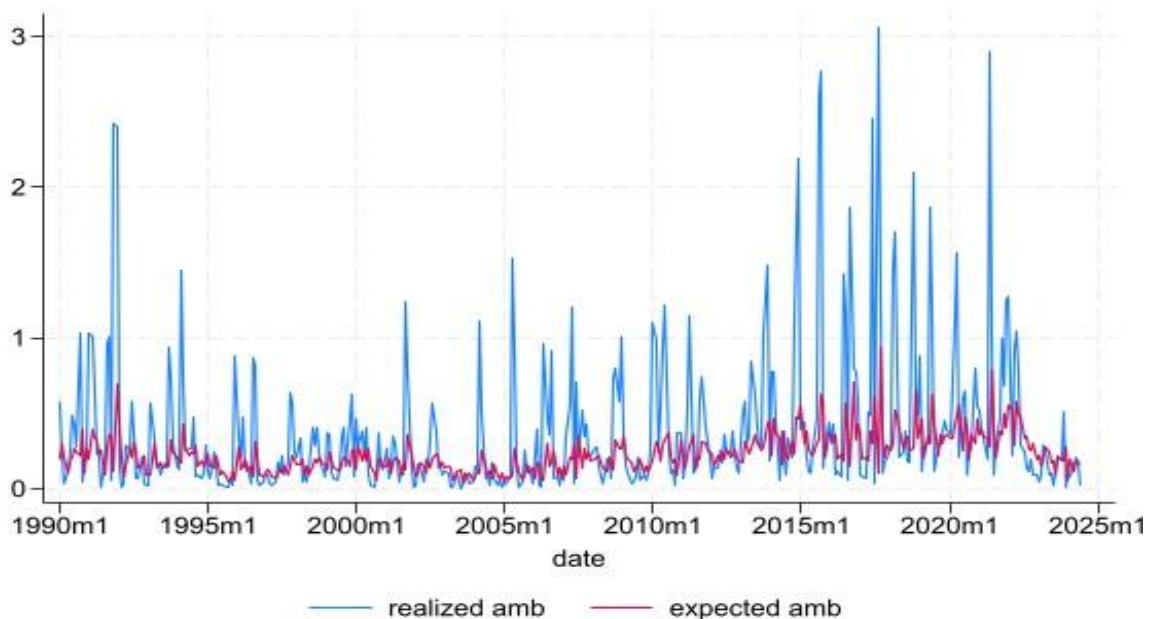
The ARMA model are visualized through three graphs, each comparing to projected and actual variables of financial indicators. To compare starting from the volatility Figure 4, from first of January 1990, to January 2025. Here, basically, a similar pattern, except that the red line (expected volatility), in contrast to the blue line (realized volatility), experiences less in scale of frequent fluctuation.

The main difference can be observed during the crisis period, with a smooth transition from the red line, which is expected to show a trend, excluding excessive volatility in rare cases of crisis. One of the feasible examples can be considered: The Global Financial Crisis and COVID-19. So, for forecasting purposes, expected volatility

is gone under the model in order to capture volatility tendency, and realized it is still shown as any variation covering short-term volatility.

The next graph can be described similarly, but it is about absolute deviation. The blue line (realized absolute deviation) represents higher peaks compared to the red line (expected absolute deviation). Similarly, the trend in the Figure can be seen as a separation of the long-term trend from the short-term movement and variability of lines differencing in forecasting power. Actual absolute deviation changes sharply downwards, whereas predicted is smoothed to capture the predictable element of the figure.

To follow up, we need to consider the ambiguity, which is illustrated actual and expected. In Figure 4, of expected pattern of ambiguity is definitely different from the realized. Alternatively, the variability and spike fluctuations react similarly to those in previous cases. The trend of expected ambiguity illustrates asymmetry in ambiguity and can be used for the following step of research.



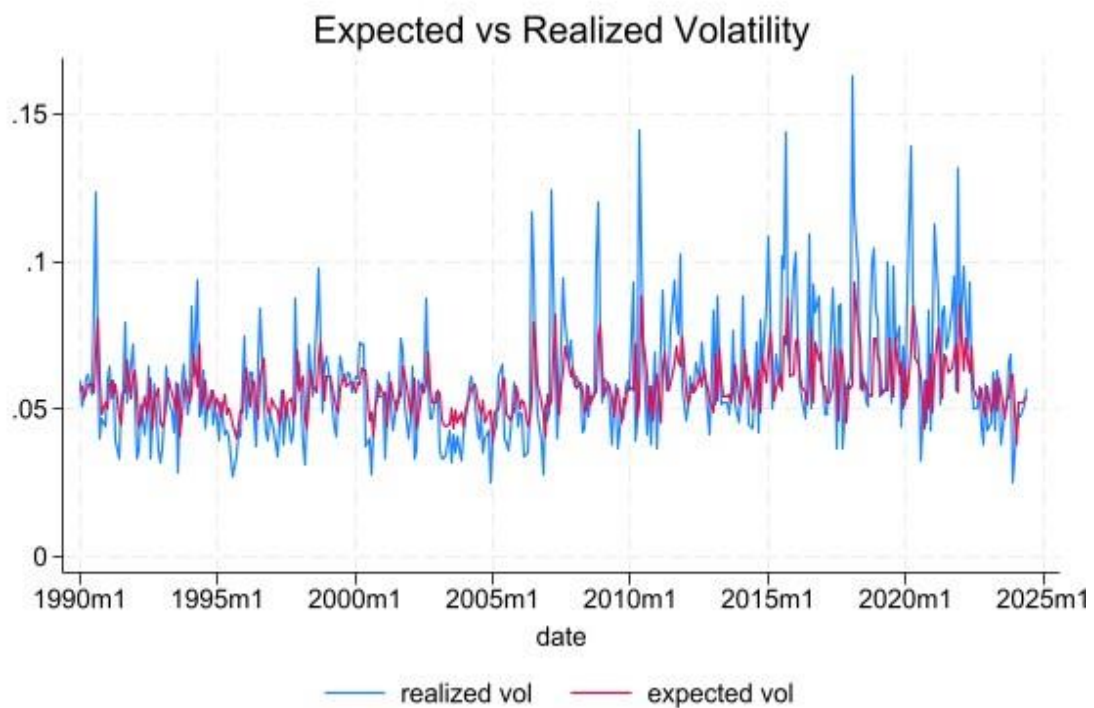
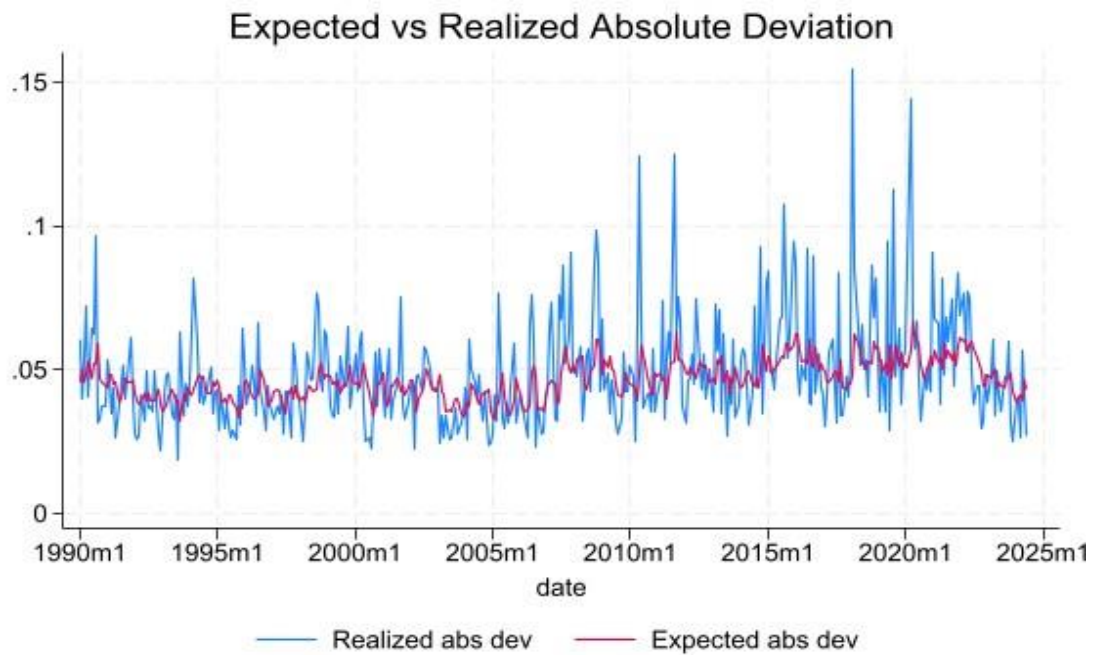


Figure 4. Time-Series Plot of Expected and Realized Measures

ARMA model p, q were chosen based on the lowest AICC in Table 1. For each model separately for log square root volatility, log uncertainty, log of absolute deviation followed the same path of looping the best fit model. Likewise, model clarity in need of calibration precise p and q as in case of insufficient of one, lead either an increase in standard error or creating omitted variables. So, based on searching till the 10th cycle, it was determined that Model Lags with the best explanatory power for the given variables.

Furthermore, regarding relationships between the results of ARMA, e.g, expected variables were tested by the augmented Dickey–Fuller test. Based on Table 1, it can be observed that all variables except returns are non-stationary, so taking the difference of the variable itself and its lag was decided. Moreover, the cointegration between these mentioned variables was not found during testing Engle-Granger.

Furthermore, for each given variables in Table 1, we calculated its descriptive statistics, based on the same sample of 274 observations. The starting point is a mean of return accounting for 2.20%, whereas negative median indicate that return shows a negative tendency more frequently than positive. The skewness and kurtosis of monthly return reflect non-asymmetry and fat tail or positive kurtosis (leptokurtic).

The absolute deviation of returns shows the positive skewness, which tends to the right and heavy-tailed distribution, similarly, volatility and ambiguity. Conversely, the ambiguity represents the largest kurtosis and right-skewed distribution, and it cannot be overlooked that its standard deviation is 0.523, which can be a reasonable explanation of uncertainty market participants face.

The risk is represented based on variance, not in terms of standard deviation. In panel B, we demonstrate that the returns deviation, excess return, and volatility are positive and not highly correlated, whereas volatility and probability of favorable return correlate along with absolute deviation. The degree of ambiguity mostly correlates with volatility, which will be handled further in regression. Then the degree of ambiguity positively correlates with absolute deviation and probability of favorable return, and the lowest excess return.

Regarding the findings of Brenner and Izhakian, it can be observed that risk and ambiguity are correlated, whereas ambiguity and excess returns are not (Brenner & Izhakian, 2018). In our Table 1, 0.060 shows an insignificant indicator between excess return and volatility, which is similar to the reference paper case. Moreover, ambiguity and excess return are 0.163, indicating a similar pattern. Also, all absolute deviation, ambiguity and favorable return are highly correlated, resulting in action to take care of feasible multicollinearity between given financial indicators.

Table 1. Summary statistics

ARMA	(2,1)	(1,2)	(2,1)
p	0.0053	0.0130	0.0040

	Variable	N	Mean	Median	Min	Max	Std.dev	Skewness	Kurtosis
Panel A: Monthly return related statistics									
r_m	Monthly Retrun	274	0.0220	-0.0162	-0.4590	1.3457	0.2377	1.5452	7.9543
rx_m	Excess Retrun	274	-0.1408	-0.1280	-1.0914	1.2601	0.3193	0.1469	5.3418
v_t	Volatility	274	0.0596	0.0559	0.025	0.1631	0.0209	1.4381	5.9735
ϑ	Abs.dev.	274	0.0489	0.0444	0.0187	0.1548	0.0187	1.6529	7.5966
P	Prob.	274	0.1562	0.1487	0.0162	0.3418	0.0690	0.2895	2.3880
$\mathcal{U}_t$	Ambiguity	274	0.4297	0.2364	0.0032	3.0551	0.5228	2.4060	9.7863
Panel B: Cross-correlations									
	$r_m - rf$	v_t	ϑ	P	$\mathcal{U}_t$				
$r_m - rf$	1	0.060	0.2905	-0.0955	0.1632				
v_t		1	0.7464	0.8788	0.5692				
ϑ			1	0.5202	0.4743				
P				1	0.4772				
$\mathcal{U}_t$					1				

Follow up, table 2 represents autocorrelation for monthly variables and their first difference of logarithmic change. In the way to describe the pattern of volatility, we may start from its first lag at the smallest (ρ_1) to the median lags (ρ_5), indicating a gradual decrease. Finally, reaching its last lag, ended up with twice less than the first lag. According to Brenner and Izhakin, the volatility of SPDR experienced a similar downward

trend, except highest lag, where there is still a downward slope (Brenner & Izhakian, 2018). The change of volatility represents a negative trend from scratch, following mean reversion.

Similarly, absolute deviation, mean probability, and ambiguity exactly witness a persistent decrease till median lag, where they change their pattern towards a slight upturn till the last lags. Also, their changes do not reflect any difference in the previously mentioned pattern, representing a distinct negative and mean-reverting trend. Mostly, all variables follow a persistence rather than a random walk. We need to filter the values by ARMA model and differentiate between expected and realized variables.

Table 2. Autocorrelations

lags	1	2	3	4	5	6	7	8	9	10
v_t Risk	0.436	0.204	0.172	0.128	0.113	0.140	0.179	0.188	0.232	0.202
Volatility change	-0.291	-0.186	0.025	-0.032	-0.048	0.001	0.036	-0.035	0.070	0.006
Absolute dev. (ϑ)	0.310	0.200	0.185	0.128	0.107	0.118	0.131	0.135	0.122	0.179
Absolute dev. change	-0.434	-0.037	-0.001	0.000	-0.034	0.00	-0.01	0.011	0.022	-0.031
Mean prob. (P)	0.448	0.259	0.245	0.187	0.152	0.229	0.264	0.248	0.282	0.232
Mean prob. change	-0.334	-0.150	0.048	-0.053	-0.069	0.058	0.008	-0.011	0.045	0.21
Ambiguity ($\mathcal{U}_t$)	0.339	0.094	0.051	0.006	0.08	0.043	0.108	0.122	0.184	0.209
Ambiguity change	-0.271	-0.165	-0.083	0.023	0.014	-0.033	0.038	-0.034	-0.027	0.085

Furthermore, Table 3 reflects previously mentioned ARMA executed data summarization. Specifically, in Table 3, forecasted values based on 414 observations and the cross-correlation between variables. The expected volatility and expected absolute deviation have means of 5.71% and 0.63%, and relatively low standard deviations of 0.88% and 0.055% respectively. The expected ambiguity has 22.67% mean and 11.73% standard deviation still relatively high than other variables. To sum up, these numbers can be justified as a phenomenon of uncertainty.

All variables are skewed to the right and are fat tailed and their values shrink after ARMA model, providing smoother values. Regarding their correlation, all of them capture a strong correlation and move in one direction, but still statistically different from each other.

Table 3. Expected values

Panel A: Main statistical measures								
Variable	N	Mean	Median	Min	Max	Std.dev.	Skewness	Kurtosis
v_t^E	414	0.0571	0.0561	0.0383	0.0932	0.0088	0.8516	4.1091
ϑ_t^E	414	0.0463	0.0456	0.0349	0.0682	0.0055	0.7659	3.9012
$\mathfrak{U}_t^E$	414	0.2267	0.2045	0.0322	0.6999	0.1173	1.0591	4.2475
Panel B: Cross-correlations of expected values of the variables								
	v_t^E	ϑ_t^E	$(\mathfrak{U}^2)_t^E$					
v_t^E	1							
ϑ_t^E	0.8453	1						
$(\mathfrak{U}^2)_t^E$	0.6667	0.5997	1					

This part of research referring to one of important objections, does ambiguity explains the expected return. Definitely, until this point, equity premium in case of ambiguity and ambiguity dispersion interaction considered priced, risk and absolute deviation not priced in the reference paper for SPDR return, but in some instances, results are differentiate (Brenner & Izhakian, 2018).

For emerging market of Kazakhstan, it was revealed only absolute deviation is priced in case of all regression in Table 4. According to panel A, all three approach of applying regression showed, it does significant all time. Investors demand compensation not exactly for taking ambiguity and volatility(risk), but for dispersion in return itself.

From prior checking of the correlation between expected volatility, expected ambiguity and expected absolute deviation that can be observed in panel C, it was decided to proceed with separate regressions and interactions for each regressor. From panel C, 0.67, 0.81 correlation for volatility & ambiguity and volatility & dispersion reveal relation of change together is pretty high. Ambiguity and dispersion correlation is 0.54, indicating that ambiguity does not move in the same direction as dispersion.

Table 4. Main regression results

$rx_m = \alpha + \gamma v_t^E + \eta((U^2)_t^E * \vartheta_t^E) + \theta (U^2)_t^E + \omega \vartheta_t^E + \varepsilon_t$								
$rx_m = \alpha + \gamma v_t^E + \eta((U^2)_t^E * \vartheta_t^E) + \varepsilon_t$								
#	α	γ	η	θ	ω	N	R ²	Adj.R ²
Panel A: Regression results								
1	-0.0012 (-0.13)	-0.1925 (-1.24)				273	0.006	0.002
2	0.00644 (0.50)		-0.2638 (-1.07)			273	0.0025	0.001
	-0.0013 (-0.13)				-0.8032* (-3.60)	273	0.0439	0.0404
3	0.0049 (0.38)	-0.1792 (-1.13)	-0.2128 (-0.85)			273	0.0075	0.0002
4	-0.0042 (-0.32)	0.2742 (0.85)	0.1005 (0.41)	-0.0032 (-0.07)	-1.0936* (-2.30)	273	0.0507	0.0365
5	-0.0013 (-0.12)	0.2662 (0.82)		-0.0007 (-0.01)	-1.0706* (-2.31)	273	0.0504	0.0398
Panel B: Business cycle								
$rx_m = \alpha + \beta X_t + \delta D_t^{rec} + \theta(X_t * D_t^{rec}) + \varepsilon_t$								
$X_t \in \{\vartheta_t^E, U, v_t^E\}$								
	α	β_{exp}	δ	Interaction (θ)		N	R ²	Adj.R ²
v	-0.1364* (-4.57)	0.0089* (3.60)	-0.1482* (-4.52)	-0.0204* (-7.70)		92 959	0.0	0.0651
ϑ	-0.1357* (-4.52)	0.01677* (3.94)	-0.1178* (-2.90)	-0.0337* (-6.64)		92 962	0.0	0.0654
U	-0.1351* (-4.42)	0.0010 (1.26)	-0.1381* (-3.72)	-0.0062* (-2.95)		92 197	0.0	0.0137
Panel C: Cross correlation								
	v_t^E	$(U^2)_t^E$	ϑ_t^E					
v_t^E	1							
$(U^2)_t^E$	0.6706	1						
ϑ_t^E	0.8082	0.5418	1					

Regression on all macro variables, including GDP, unemployment, recession index, expected volatility, expected ambiguity and expected absolute deviation illustrated in Panel B. Given the recession time, all of the forecasted values are priced, whereas in the expansion case, volatility and absolute deviation are priced.

To follow up, changes in the pricing of all expected variables during recession are significant. Market participant cares about all factors affecting price in recession. Moreover, from perspective of business cycle, we may say that most of variables pricing e.g. absolute deviation and volatility, follow counter cyclical.

When expected absolute deviation rises in market, the need to be compensated with premium increase for taking dispersion. However, both volatility and ambiguity imply that short-term expectations of risk or ambiguity do not strongly explain return fluctuations in the market.

In less transparent and less liquid emerging market of KASE, people react for every change from average information. Probably, the demand premium for absolute deviation and volatility in expansion and in recession are followed up. Regarding ambiguity, it starts to be priced only in case of recession, when the fear for decision making is exaggerated. Moreover, lack of financial literature among citizens and lack of profound credibility in institutions substantially affect to decision making of market participants too.

5. Conclusion

In the way of conclusion, I would like to say that we tried to conduct research as in reference paper. Generally, our empirical analysis showed that KASE Index is priced in

Kazakhstan. Also, it is important to note that limited access to extension part of macro analysis, own coding in stata and minor sample of date with low frequency may be limitations of our results. Nevertheless, earliest attempts to investigate in the ambiguity in emerging market have done.

At the beginning we discussed several objections of thesis, which now have been covered. The most noteworthy part is that the equity premium for absolute gain or loss incorporated in the emerging market (Barberis, Huang & Santos 2001). But, according to our results, we have not found ambiguity priced in Kazakhstan compare to developed market of the USA. Moreover, during different economic condition, ambiguity became only significant in case of recession. This finding proven by the reference paper.

To conclude, despite the limitations, we have plenty room to further investigation and continue to examine full picture of asset pricing in the emerging market.

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