A NEW GENERALIZATION OF THE NORMAL DISTRIBUTION: THE GAMMA - NORMAL DISTRIBUTION AND ITS APPLICATIONS

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Introduction

Statistical distributions are commonly applied to describe real world phenomena. Due to the usefulness of statistical distributions, their theory is widely studied and new distributions are developed. The interest in developing more flexible statistical distributions remains strong in statistics profession. Many generalized classes of distributions have been developed and applied to describe various phenomena. Recently, Alzaatreh et al. (2014) developed a new method to generate family of distributions and called it the *gamma-X* family of distributions. They used the family to propose a new generalization of the normal distribution as follows:

If *X* is a normal random variable with PDF $\phi(x)$ and CDF $\Phi(x)$, we define the gamma-normal distribution with parameters ***a, /3, fl*** , and *O* as

$$g(x) = \text{^ ^ (*) [- l 0 g (l - Φ (x)) Π 1 - Φ (.) Γ - > - c o <}_{x} \text{< oo,}$$

where $a>0,\ J3>0,\ cr>0$ and $-\text{°°}<ju<\text{°°}$.

When $a = fi = \backslash$, the gamma-normal distribution reduces to the normal distribution with parameters № and ***O.*** Thus, the gamma-normal family is a generalization of the normal distribution. In this project, general statistical properties for the gamma-normal distribution are studied in detail. The limiting behaviors, moments, mean deviations, dispersion, and Shannon entropy for the gamma-normal distribution are provided. Bounds for the non-central moments are obtained. The method of maximum likelihood estimation is proposed for estimating the parameters of the gamma-normal distribution. Two real data sets are used to illustrate the applications of the gamma-normal distribution.