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RESEARCH ARTICLE

Direction Of Arrival Estimation in the Presence of Imperfect Waveforms for Multiple Targets in MIMO Radar

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ABSTRACT Direction Of Arrival (DOA) estimation of multiple targets is a renowned challenging problem that has extensive applications in Multiple Input Multiple Output (MIMO) radar system. The introduction of sub-space based techniques escalates the accurate estimation of DOA but at the cost of increased computational complexity as they require multiple snapshots. The current work deals with the presentation of a novel approach based on Wild Horse Optimization (WHO) for the DOA estimation of multiple targets with Co-located MIMO radar in the presence of imperfect waveforms. The theory of extended array manifold vectors is incorporated in Mean Square Error (MSE) sense to develop an objective function that requires a single snapshot to gain the desired results. The deviation in MSE from the desired value is controlled through a penalty function which is the difference between the desired and actual responses of the system. A rigorous statistical analysis based on Monte Carlo simulations is carried out to validate the effectiveness of the proposed algorithm through histogram plots, box plots, Cumulative Distribution Function (CDF) plots, RMSE, robustness against noise, and estimation accuracy. Result comparison with state-of-the-art algorithms further endorses the legitimacy of the proposed WHO scheme for DOA estimation.

INDEX TERMS Antenna arrays, direction of arrival estimation, evolutionary computing techniques, monostatic MIMO radar, wild horse optimization.

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) systems attracted the research community due to their vast applications in various fields. MIMO systems offer improved data rate, better reliability and increased spectral efficacy that make them a central component of today's modern radar systems [1], [2]. Monostatic or Co-located MIMO radars got several advantages over the traditional radars such as better resolution and enhanced detection capabilities [3].

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In monostatic MIMO radars, the Direction Of Departure (DOD) and Direction Of Arrival (DOA) are the same. Several researchers have investigated the issue of DOA in MIMO radar system. One of the most frequently used methods is sub-space based approaches that work on the covariance matrices of the spatially received data. Examples of these methods are Multiple Signal Classification (MUSIC) and Estimation of Signal Parameter Through Rotational Invariance Techniques (ESPRIT) [4], [5], [6], [7]. These techniques divide the covariance matrices into the subspaces of signal and noise and hence require multiple snapshots to attain reasonable DOA's. In [8] and [9], methods based on Compressive Sensing (CS) are proposed that utilize the target sparsity in the spatial domain with fewer snapshots, where the sparse reconstruction problem is achieved by transforming the sparse targets and thus can be used for estimating the DOA's of fast moving targets [10], [11]. Typically, CS based methods are divided into two categories, i.e., greedy and norm based methods. In greedy based methods, orthogonal matching pursuits [12] and stage wise orthogonal matching pursuits [13], [14] are adopted to reconstruct the sparse signals. While, in norm based methods, Convex optimization is used to solve the L1 norm minimization problem which is achieved by transforming L0 minimization problem [15], [16]. A super resolution based method is proposed in [17]to estimate the DOA of fast-moving targets with a single snapshot by exploiting the virtual aperture obtained through orthogonal waveforms. On the other hand, researchers have also used Machine Learning (ML) based algorithms to estimate the DOAs in the field of radar signal processing [18], [19].

In today's fast growing era, the role of heuristic or evolutionary computing which is the sub field of artificial intelligence cannot be ignored. Heuristic computing has got extensive applications in every field of science and technologies [20], [21], [22]. Examples of well-known heuristic computing are genetic algorithm, differential evolution, particle swarm optimization etc. The current work deals with the presentation of a novel approach based on Wild Horse Optimization (WHO) for the DOA estimation of multiple targets with Co-located MIMO radar in the presence of imperfect waveforms. The theory of extended array manifold vectors is incorporated in Mean Square Error (MSE) sense to develop an objective function that requires a single snapshot to gain the desired results. The deviation in MSE from the desired value is controlled through a penalty function which is the difference between the desired and actual responses of the system. A rigorous statistical analysis based on Monte Carlo simulations is carried out to validate the effectiveness of the proposed algorithm in terms of computational complexity, RMSE, robustness against noise, and estimation accuracy. Result comparison with state-of-the-art algorithms available in literature further endorses the legitimacy of the proposed WHO scheme for the DOA estimation.

The salient futures of the current work are

i. A new algorithm based on WHO is proposed for estimating DOA.

ii. A mathematical model for monostatic or Co-located MIMO radar system is developed in the Presence of Imperfect Waveforms.

iii. A fitness function based on RMSE is designed and optimized.

iv. A penalty function is introduced that controls the deviation of the fitness function from the mean value.

v. An extensive Monte Carlo simulations are carried out to validate the legitimacy of the proposed algorithm.

The rest of the paper is organized as follows, a mathematical model for DOA estimation using monostaic MIMO radar

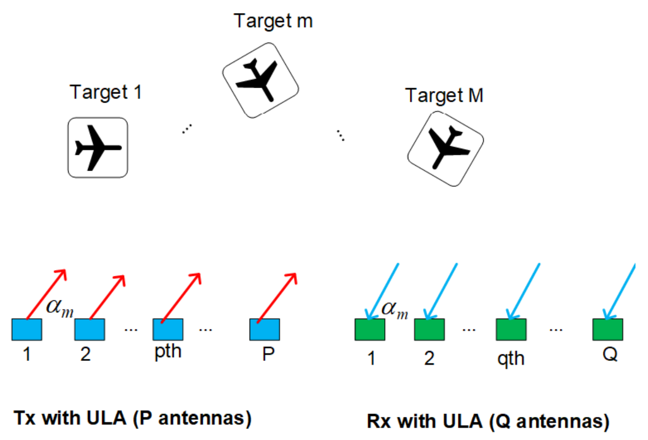

FIGURE 1. Co-Located MIMO Radar(DOD=DOA).

is presented in section II while the proposed methodology based on WHO is given in section III. In the same way, the validity and effectiveness of the WHO is discussed in section IV through several simulations by incorporating different scenarios. The conclusion of the work and future recommendations are provided in section V.

Throughout the paper, matrices are denoted by capital bold letters while vectors are represented with small bold letters.

II. DATA MODEL

The field of MIMO Radar can be divided into two types based on the distance of Transmitter (T_x) and Receiver (R_x) antenna array from each other. The system is called bi-static MIMO radar if the (T_x) array is kept far from the (R_x) array where the Direction Of Departure (DOD) is not equal to the Direction Of Arrival (DOA) for the same target in space. Similarly, if the (T_x) is placed close to the (R_x), it is called monostatic or Co-located MIMO radar system where the DOD is equal to the DOA. In general, the separation between Rx and Tx arrays in a bistatic MIMO radar should be sufficiently large to ensure that the radar waves can be considered planar for targets located in far field [23], [24]. A common practice is that the separation should be at least several times the largest dimension of the arrays. In this section, we developed data model for the Co-located MIMO radar that receives signals from M targets in the presence of imperfection signals [25]. The DOA for the *mth* target is denoted by α_m. We assumed Uniform Linear Arrays (ULAs) both at the Tx and at the Rx side that are having P and Q antenna elements respectively as shown in Figure 1.

Co-Located MIMO Radar(DOD=DOA). In this work, correlated signals are assumed whose correlation matrix for P antenna's, can be given as,

$$\mathbf{R}= [\mathrm{r}_{1,\mathrm{p}},\mathrm{r}_{2,\mathrm{p}},\mathrm{r}_{3,\mathrm{p}},\ldots,\mathrm{r}_{\mathrm{P},\mathrm{p}}]^T \tag{1}$$

where $p = 1, 2, 3, \ldots, P$. In (1),

$$\mathbf{r}_{\mathrm{p},\mathrm{p}}=[\mathrm{r}_{\mathrm{p},1},\mathrm{r}_{\mathrm{p},2},\ldots,\mathrm{r}_{\mathrm{p},\mathrm{P}}] \tag{2}$$

where $r_{1,1} = r_{2,2} = \ldots, = r_{P,P} = 1$. Clearly in (1), $\mathbf{R}$ is a conjugate symmetric matrix, where $r_{f,g} = \int_{T_p} x_f(t)x_g^*(t)dt$ such that $f, g \in \{1, 2, \ldots P\}$ and T_p represents the pulse duration. Let the waveform transmitted by the transmitter is:

$$\mathbf{x(t)} = [x_1(t), x_2(t), , x_3(t), \ldots .x_p(t)] \tag{3}$$

The received waveform reflected from the mth target can be given as:

$$\gamma_m(t, \tau) = \beta_m(\tau) {b_t}^T(\alpha_m)x(t) \tag{4}$$

where $\beta_m(\tau)$ represents the radar cross section (RCS) during the $\tau - th$ pulse for the mth target. In the same way, $b_t(\alpha_m)$ is the $P \times 1$ complex Array Manifold Vector (AMV) associated with the transmitter and is given as:

$$\mathbf{b_t(\alpha_m)} = [1, e^{-j\psi}, \ldots, e^{-j(P-1)\psi}]^T \in \mathbb{C}^{P\times 1} \tag{5}$$

where $\psi = \pi \sin \alpha_m$. The waveform at the receiver side can be represented as:

$$y(t, \tau) = \sum_{m=1}^{M} \beta_m(\tau) b_r(\alpha_m) {b_t}^T(\alpha_m)x(t) + \eta(t, \tau) \tag{6}$$

where $b_r(\alpha_m)$ is the AMV associated with the receiver array which is given as:

$$\mathbf{b}_r\mathbf{(\alpha_m)} = [1, e^{-j\psi}, \ldots, e^{-j(Q-1)\psi}]^T \in \mathbb{C}^{Q\times 1} \tag{7}$$

In the same way $\eta(t, \tau)$ is the Gaussian White Noise vector i.e.,

$$\mathbf{E}[\eta(\mathbf{t_1}, \tau)\eta^H(\mathbf{t_2}, \tau)] = \sigma^{\mathbf{2}}\mathbf{I}_Q * \delta(\mathbf{t_1} - \mathbf{t_2}) \tag{8}$$

δ is the Kronecker delta which is zero for $t_1 = t_2$. The received signal is passed through the matched filter, whose output is modeled as follows:

$$\begin{aligned} z_p(\tau) &= \int_{T_p} \mathbf{y}(t, \tau)\mathbf{x^*}_p(t)dt \\ &= \sum_{m=1}^{M} \beta_m(\tau) {b_t}^T(\alpha_m) r_p b_r(\alpha_m) + d_p(\tau) \end{aligned} \tag{9}$$

where r_p is defined in (1) and $d_p(\tau)$ is the noise added at the matched filter that can be defined as,

$$d_p(\tau) = \int_{T_p} \eta(t, \tau)x_m^*(t)dt \tag{10}$$

In the same way, the output from the entire matched filters is given as:

$$\mathbf{z}(\tau) = [\mathbf{z_1}^{\mathbf{T}}(\tau), \mathbf{z_2}^{\mathbf{T}}(\tau), \ldots, \mathbf{z_P}^{\mathbf{T}}(\tau)] \in \mathbb{C}^{PQ\times 1} \tag{11}$$

$$\mathbf{z}(\tau) = \sum_{m=1}^{M} [(R^T b_t(\alpha_m)) \otimes b_r(\alpha_m)]\beta_m(\tau) + d(\tau) \tag{12}$$

where $\otimes$ is Kronecker product and in vector-matrix form (12), can be given as,

$$\mathbf{z}(\tau) = \mathbf{Be}(\tau) + d(\tau) \tag{13}$$

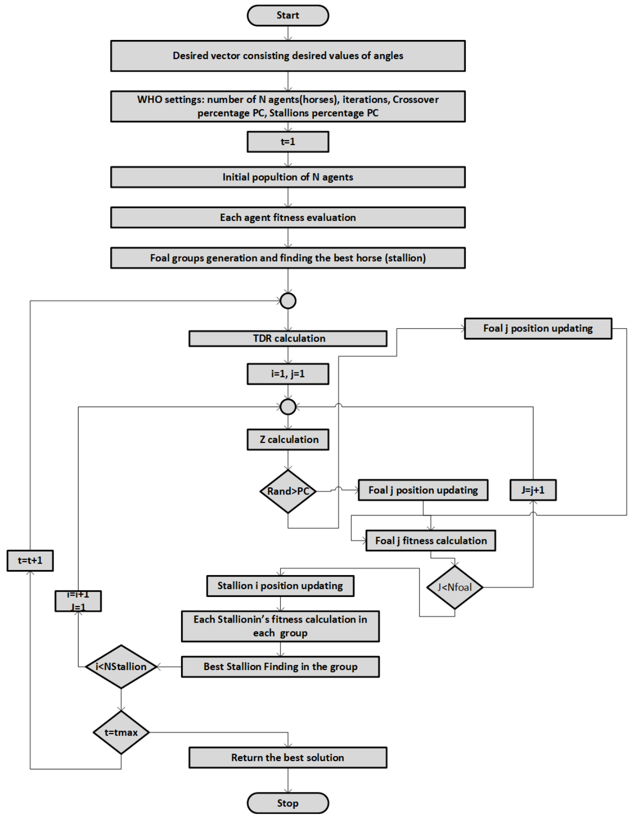

FIGURE 2. WHO flow chart.

In (13), $\mathbf{d}(\tau) = \left[d_1^T(\tau), d_2^T(\tau), \ldots, d_Q^T(\tau)\right]^T$ is the overall array noise. Similarly,

$\mathbf{e}(\tau) = [\beta_1(\tau), \beta_2(\tau), \ldots, \beta_m(\tau)]^T$ is the Radar Cross Section Vector (RCS) vector.

$\mathbf{B} = [\mathbf{b}(\alpha_1), \mathbf{b}(\alpha_2), \ldots, \mathbf{b}(\alpha_M)] \in \mathbb{C}^{PQ\times M}$ is the extended virtual Array Manifold matrix, i.e., with $\mathbf{b}(\alpha_m) = (\mathbf{R}^T\mathbf{b}_t(\alpha_m)) \otimes \mathbf{b_r}(\alpha_m)$. So the goal is to estimate α_m for $m = 1, 2, \ldots, M$ in the presence of imperfections ($\mathbf{R}$).

III. PROPOSED METHODOLOGY

The WHO algorithm, developed by Naruei and Keynia in 2021, imitates how wild horses behave [26].WHO has gained popularity and applicability in the scientific and technical fields due to its ability to resolve challenging optimization problems [27], [28], [29]. The structure of WHO is based on the social behaviors of stallions and foals. It achieves optimization that balances exploration and exploitation in a novel way to produce robust and flexible results. In this work, we have developed and implemented the WHO algorithm for the Co-located MIMO radar system for Direction of Arrival (DOA) estimation. Particularly, a new fitness function (to be discussed below) is designed and optimized through WHO. Figure 2 shows the flow diagram of the WHO algorithm, while its step-by-step pseudo code is given below.

A. STEP 1. INITIALIZATION

Like other heuristic algorithms, the first step is to randomly generate the population. In this step, we have randomly generated $\mathbf{H}$ number of horses, and the dimension of each horse is $1 \times M$ where M is the total number of unknowns (DOAs) in our problem. The upper and lower bounds of each horse are given mathematically as $|\alpha_i| < \pi$.

$$\mathbf{H} = \begin{bmatrix} \hat{\alpha}_{1,1} & \hat{\alpha}_{1,2} & \cdots & \hat{\alpha}_{1,M} \\ \hat{\alpha}_{2,1} & \hat{\alpha}_{2,2} & \cdots & \hat{\alpha}_{2,M} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\alpha}_{K,1} & \hat{\alpha}_{K,2} & \cdots & \hat{\alpha}_{K,M} \end{bmatrix} \tag{14}$$

Groups are formed on the basis of G=HxPS where PS is the stallions percentage and remaining members are equally divided into these groups. Initially the leaders(stallions) of each group are randomly selected and then later in the algorithm, they are selected on the basis of their fitness.

B. STEP 2. FITNESS EVALUATION

In this step, the fitness of each horse is calculated by using the concept of mean squared error. This fitness is based on difference of the desired and estimated responses of the system as given:

$$MSE = \frac{1}{(P \times Q)} E[|\mathbf{z} - \hat{\mathbf{z}}|^2] + PF(\mathbf{z}) \tag{15}$$

where $PF(\mathbf{z})$ is the penalty function used for controlling the deviation in the MSE from its mean value and can be defined as:

$$PF(\mathbf{z}) = 100 * \sum_{m=1}^{M} \sum_{k=1}^{K} (z_m - z_k)^2 \tag{16}$$

The multiplying factor 100 is a scaling factor for normalization. In (17), $\mathbf{z}$ is the desired solution vector as defined in (13) while $\hat{\mathbf{z}}$ is the estimated result vector that can be given as:

$$\hat{\mathbf{z}}(\tau) = \hat{\mathbf{B}}\mathbf{e}(\tau) \tag{17}$$

where $\hat{\mathbf{B}} = [\mathbf{b}(\hat{\alpha}_1), \mathbf{b}(\hat{\alpha}_2), \ldots, \mathbf{b}(\hat{\alpha}_M)]$ with $\mathbf{b}(\hat{\alpha}_m) = (\mathbf{R}^T \mathbf{b}_t(\hat{\alpha}_m)) \otimes \mathbf{b}_\mathbf{r}(\hat{\alpha}_m)$.

C. STEP 3. GRAZING BEHAVIOUR

The position of each horse in the population is updated according to the following rule:

$$x_{j,G}^k = A(Sta^k - x_{j,G}^k) + Sta^k \tag{18}$$

$A = 2adpt\cos(2\pi\rho * adpt)$, where ρ is a uniform random number in [-2,2].$x_{j,G}^k$ is the current position of the group member, Sta^k is the stallion or leader of the group position and $adpt$ is an adaptive mechanism given as:

$$\begin{aligned} P &= \rho_1 < TDR,\ IDX = (P' == 0), \\ adpt &= \rho_2 * IDX + \rho_3 * (\sim IDX) \end{aligned} \tag{19}$$

where ρ_1 and ρ_3 are random vectors and ρ_2 is a random number with uniform distribution in the range [0,1]. P' is a vector of 0 and 1. TDR represents the stopping criterion based on the maximum number of iterations achieved. The fitness of the new position is evaluated. The position update during grazing behavior introduces random perturbations to simulate the random search.

TABLE 1. System specifications.

Sr.No.	Name	Specifications
1	Operating System	Windowds 10 Pro, 64 bit
2	Software	MATLAB 2023b
3	RAM	16GB
4	Processor	Intel Core i7-8565U @ 1.80-1.99 GHz

TABLE 2. WHO parameter settings.

Sr.No.	Name	Specifications
1	Population Size	30 horses
2	Number of iterations	500
3	Lower Bound	-$\pi/2$
4	Upper Bound	$\pi/2$

TABLE 3. Parameters for the proposed WHO algorithm.

Sr.No.	Name	Values
1	Stallions Percentage	0.2
2	Crossover Percentage	0.13
3	Number of Stallions	NStallion=ceil(PS*N)
4	Number of foals	N-NStallions

TABLE 4. Estimation accuracy of 4 sources.

Name	θ_1	θ_2	θ_3	θ_4	fitness
Desired	-0.9774	0.9774	-1.1519	1.1519	-
15dB	-0.9774	0.9774	-1.1519	1.1519	1.18E-01
10dB	-0.9773	0.9774	-1.1519	1.1520	1.33E-01
5dB	-0.9772	0.9773	-1.1519	1.1520	1.84E-01
0dB	-0.9768	0.9770	-1.1514	1.1515	3.42E-01

D. STEP 4. HORSE MATING BEHAVIOR

For each pair of horses, the crossover operation is performed to produce the offspring according to the following rule:

$$\begin{aligned} X_{G,k}^{PP} &= crossover(X_{G,i}^{qq}, X_{G,j}^{zz}),\ i \neq j \neq k \\ pp &= qq = end;\ \ crossover = mean \end{aligned} \tag{20}$$

where $X_{G,j}^{zz}$ is the position of the horse zz from group j and leaves the group. Its position is replaced by a horse whose parents have left group k and i. Evaluate the fitness of the offspring. The crossover operation combines traits from two horses to produce new solutions, promoting exploration.

E. STEP 5. GROUP LEADERSHIP

The group leaders compete for leading their group to a suitable area (water hole). The dominant group captures that area not allowing other groups but is replaced when other group becomes dominant. The position of leader(Stallion) is

TABLE 5. Estimation accuracy of 3 sources.

Name	θ_1	θ_2	θ_3	fitness
Desired	-0.7156	0.7156	0.8901	-
15dB	-0.7156	0.7156	0.8901	1.08E-01
10dB	-0.7156	0.7156	0.8901	1.23E-01
5dB	-0.7156	0.7157	0.8902	1.74E-01
0dB	-0.7153	0.7158	0.8900	3.32E-01

TABLE 6. Estimation accuracy of 2 sources.

Name	θ_1	θ_2	fitness
Desired	-0.5760	0.5760	-
15dB	-0.5760	0.5760	7.50E-03
10dB	-0.5760	0.5760	2.32E-02
5dB	-0.5760	0.5760	7.43E-02
0dB	-0.5758	0.5759	2.32E-01

TABLE 7. Proximity effect for 3 sources.

Name	θ_1	θ_2	θ_3	fitness
Desired	-0.7156	0.7156	0.8901	-
Estimated	-0.7156	0.7156	0.8901	1.23E-01
Desired	-0.7156	-0.5411	0.8901	-
Estimated	-0.7156	-0.5410	0.8901	1.24E-01
Desired	-0.7156	-0.6283	0.8901	-
Estimated	-0.7154	-0.6284	0.8901	4.24E-01
Desired	-0.7156	-0.6283	-0.5411	-
Estimated	-0.7158	-0.6280	-0.5408	9.89E-01

updated using the following rule:

$$Stag_{G,j} = \begin{cases} A(WH - Stag_{G,j}) + WH \ if \ \rho_3 > 0.5 \\ A(WH - Stag_{G,j}) - WH \ if \ \rho_3 \leq 0.5 \end{cases} \tag{21}$$

Here $Stag_{G,j}$ is the next position of the leader of group j, WH is the position of the water hole, $Stag_{G,j}$ is the current position of the jth group leader. Leaders' positions are updated based on a cosine function, which ensures diverse exploration patterns.

F. STEP 6. EXCHANGE AND SELECTION OF LEADERS

For each group, compare the fitness of the leader with other group members. If a group member has better fitness than the leader, exchange their positions accordingly:

$$Stag_{G,i} = \begin{cases} X_{G,i} & if \ \cos t(X_{G,i}) < \cos t(Stag_{G,i}) \\ Stag_{G,i} & if \ \cos t(X_{G,i}) \geq \cos t(Stag_{G,i}) \end{cases} \tag{22}$$

This step ensures that the best-performing horses become leaders, maintaining a focus on promising areas in the search space.

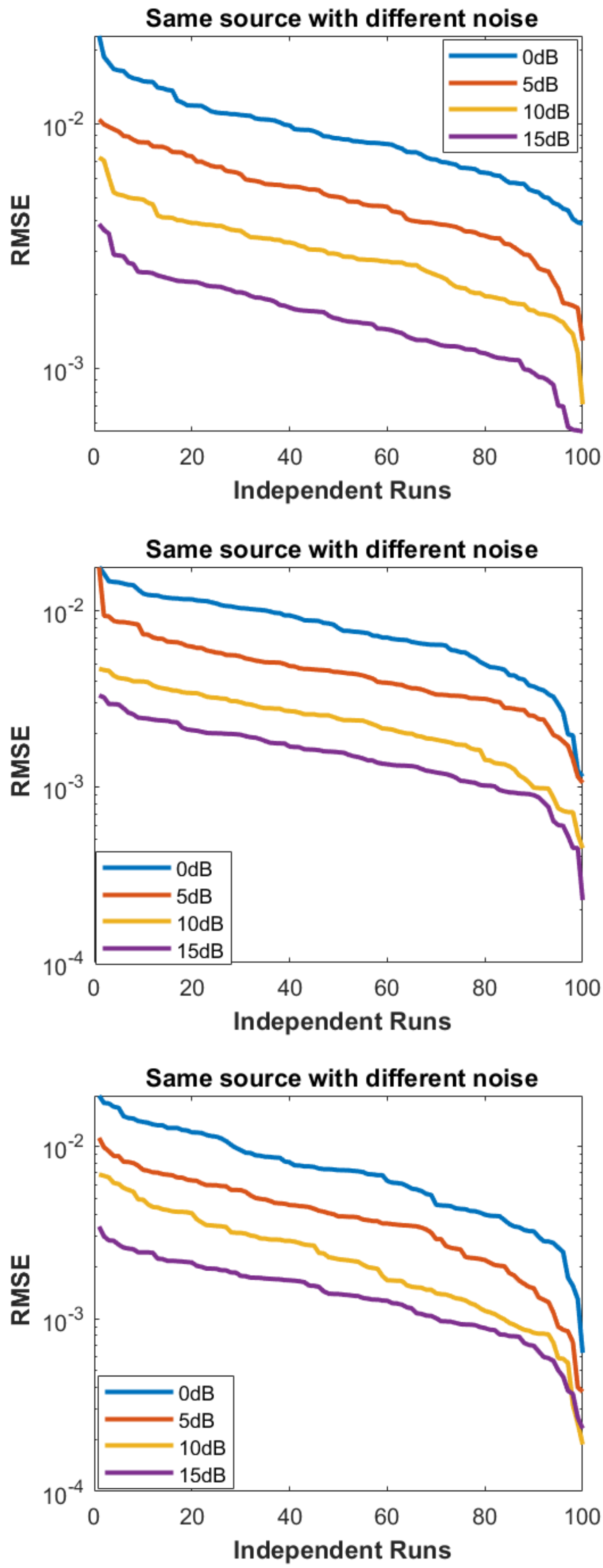

FIGURE 3. RMSE of 4, 3, and 2 sources.

G. STEP 7. UPDATE POSITIONS

Update positions of all horses in the population using the current leaders' positions. Step 8. Termination Check: If the

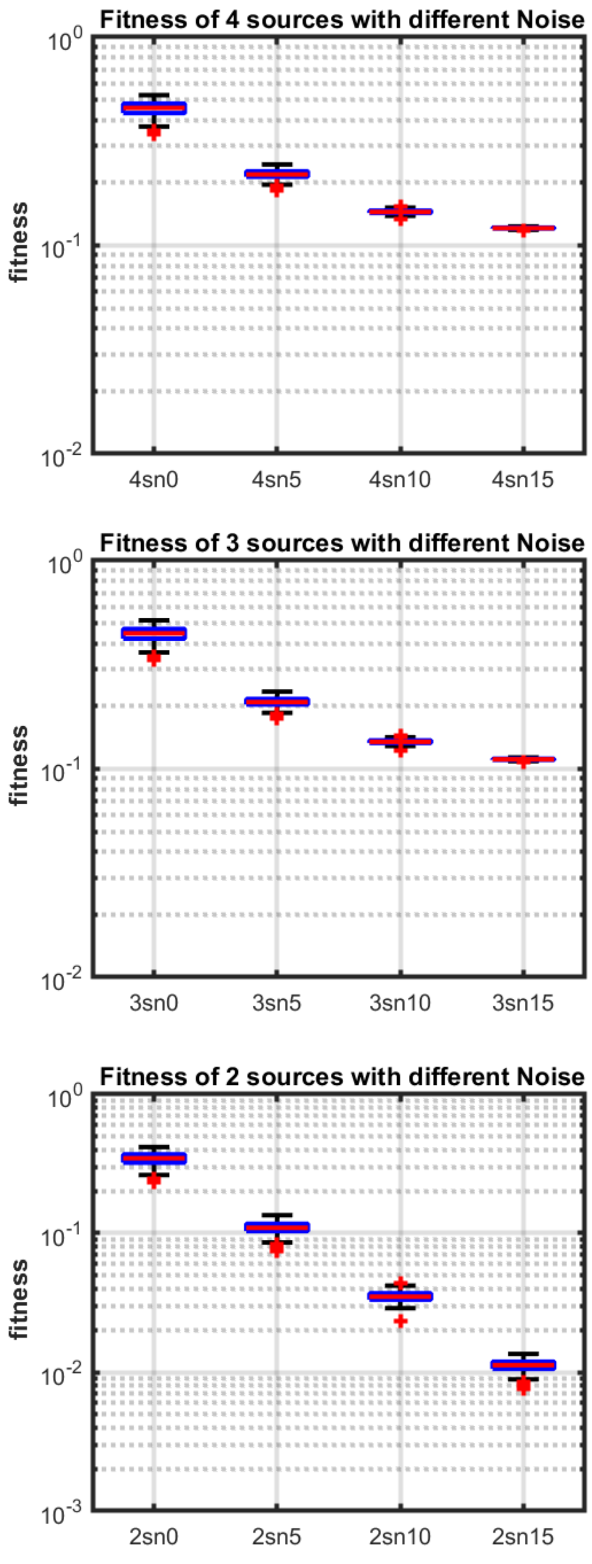

FIGURE 4. Box Plots of 4, 3, and 2 sources.

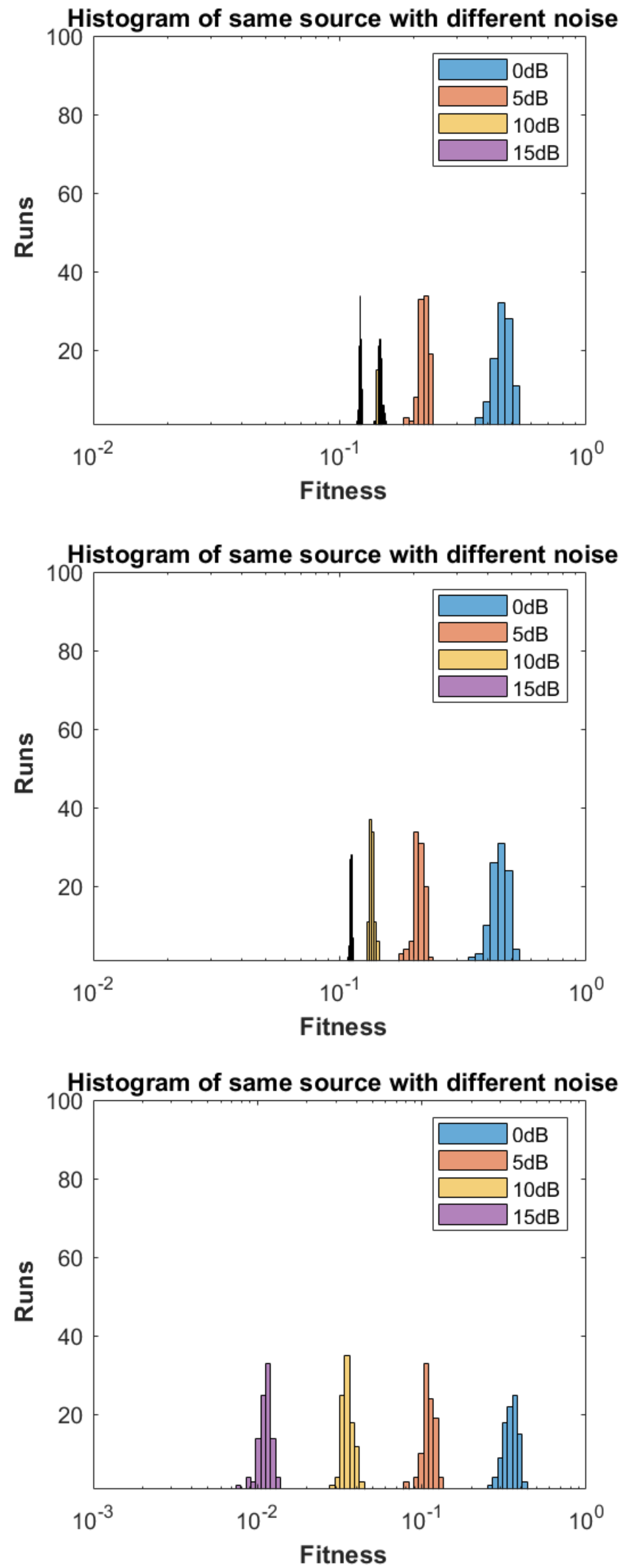

FIGURE 5. Histogram of 4, 3, and 2 sources.

maximum number of iterations T_{max} is reached, stop the algorithm. Otherwise, repeat steps 3 to 7.

Step 9. Output Return the best horse (i.e., solution) found and its fitness.

IV. RESULTS AND DISCUSSION

In this part, statistical analysis are carried out to verify the effectiveness of the proposed WHO algorithm.

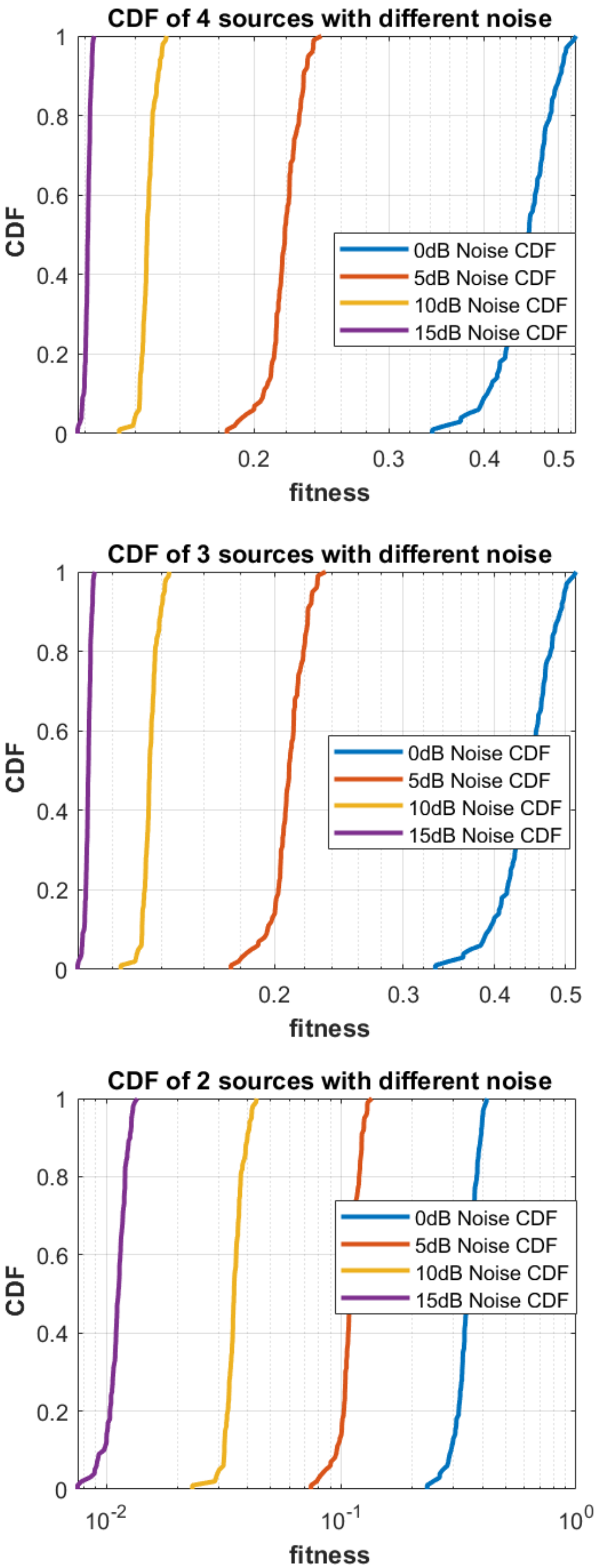

FIGURE 6. CDF Plots of 4, 3, and 2 sources.

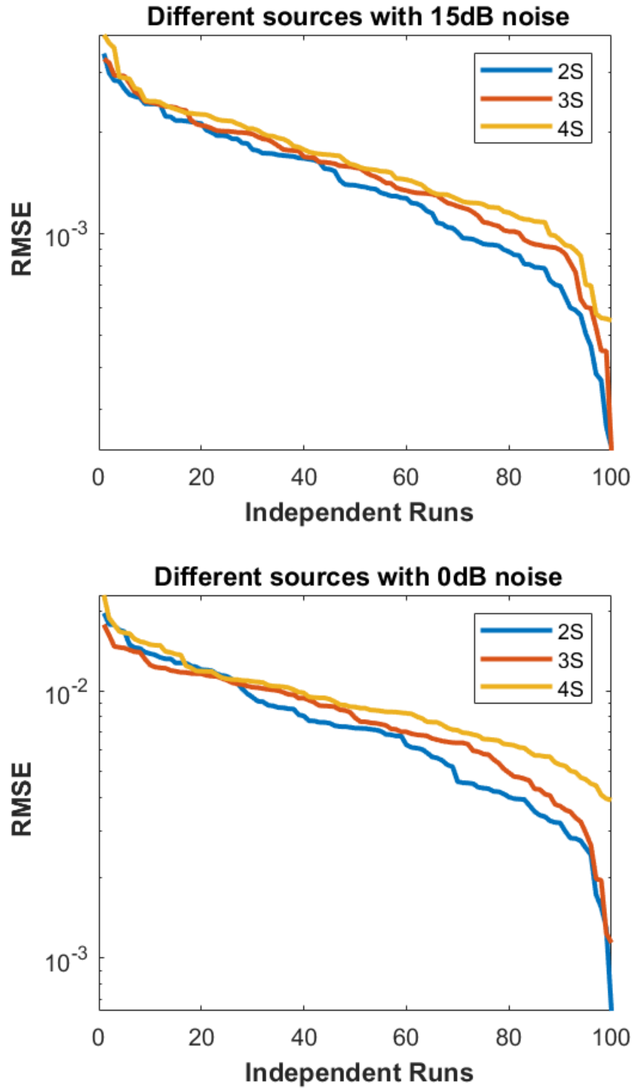

FIGURE 7. RMSE Plots of different sources.

Different scenarios based on the number of far field targets are discussed, i.e., for two, three, four and six sources respectively. The entire simulations are carried out through a machine whose specifications are given in Table 1. In the first part of this section, estimation accuracy, proximity effect, and robustness against noise are discussed, while in the second part, the proposed algorithm is compared with the state-of-the art algorithms. The $(f, g)th$ entry in the R is $r(f, g)$ where, $f, g \in \{1, 2, \ldots P\}$. Here $r(f, g) = \lambda exp(-|f - g|\mu)$, where μ is called the colour parameter. Throughout the simulations, we have used $\mu = 0.1$ and $\lambda = 0.9$. The parameter settings of WHO is given in Table 2. Throughout in this section, the desired and estimated values of the angles are taken in radians. Each case is evaluated over 100 independent trails.

A. ESTIMATION ACCURACY

This case deals with the estimation accuracy of the WHO algorithm for 4, 3, and 2 sources, respectively, in the presence of different noise levels as given in Tables 4-6. The ULAs at the transmitter and receiver of the Co-located MIMO are

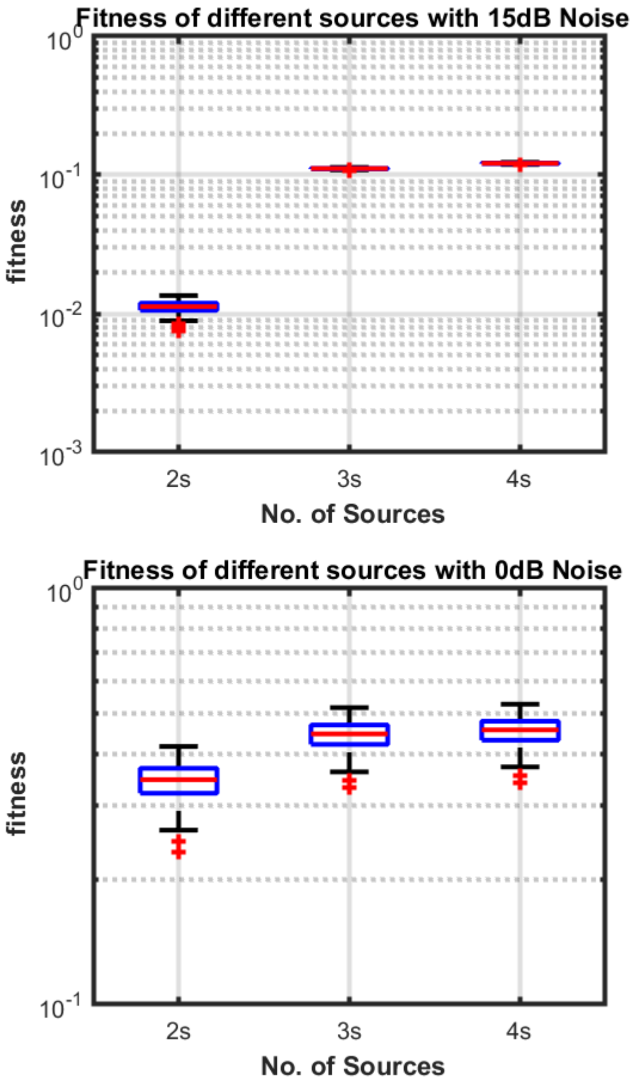

FIGURE 8. Box Plots of different sources.

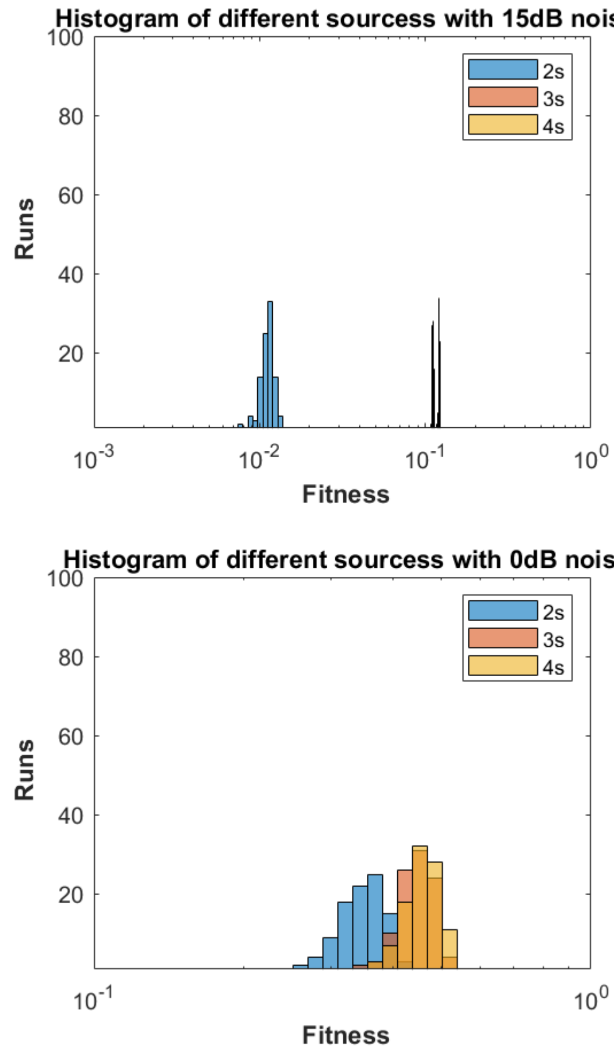

FIGURE 9. Histogram Plots of different sources.

having 6 antennas each. Although, the estimation accuracy of the WHO algorithm is degraded for low values of noise and increased number of sources, but still it has got acceptable results.

B. PROXIMITY EFFECT

In this subsection, the proximity effect of the proposed algorithm is discussed in the presence of 20dB noise for 3 sources. As provided in Table 7, the proposed WHO algorithm has obtained better results even when the sources are closely spaced from each other.

C. WHO'S PERFORMANCE IN A RANGE OF SCENARIOS: ASSESSING THE SAME SOURCES WITH DIFFERENT NOISE, DIFFERENT SOURCES WITH SAME NOISE AND WORST-CASE SCENARIOS

In this subsection, the legitimacy of WHO algorithm is verified through several RMSE, box plots, histograms, and Cumulative Distribution Function (CDF) plots as shown in Figures 3-6 for 4, 3, and 2 sources with different noise, respectively. The notations 4sn0, 4sn5, 4sn10 and 4sn15 denote 4 sources with 0dB, 5dB, 10dB and 15dB noise respectively. Similar notations have been used for 3 and 2 sources as well. The RMSE, box plots, histogram, and CDF plots are shown in Figures 7-10 for different sources with the same noise. Only the plots for 15dB and 0dB are shown but other plots can be provided if demanded. Likewise, the performance of the WHO algorithm is tested for worst-case scenarios of SNR i.e., from -15dB to 15dB as shown in Figure 11. One can see that the proposed algorithm still attains acceptable results even in this worse scenario of SNR. The proposed algorithm is also tested for an even worse scenario of different SNR levels for same number of sources as shown in Figure 12. Again, the results are fairly good. The notations 2s, 3s, 4s and 6s denote 2 sources, 3 sources, 4 sources, and 6 sources respectively.

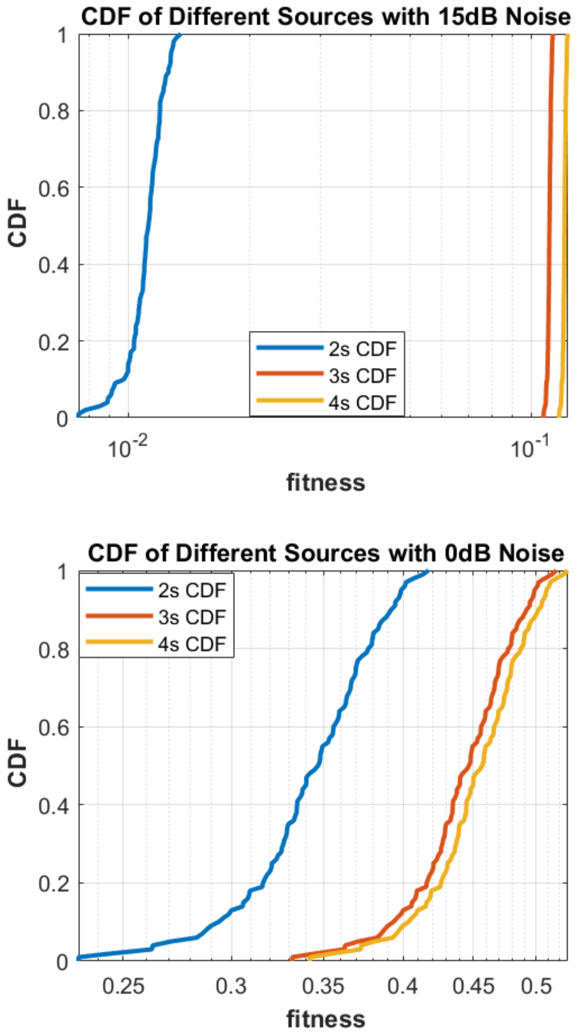

FIGURE 10. CDF Plots of different sources.

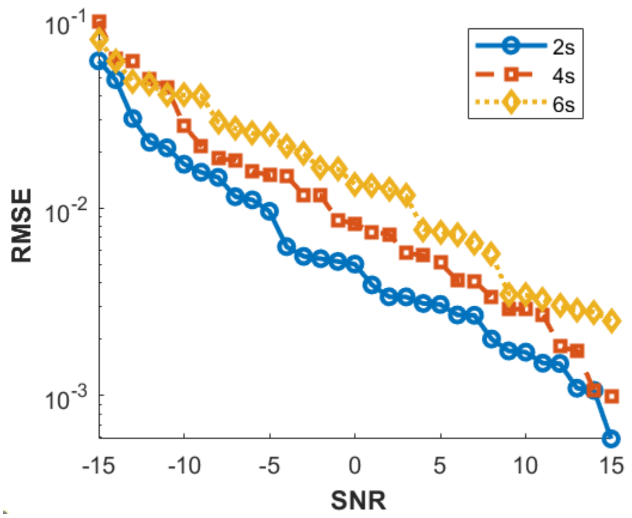

FIGURE 11. 6, 4 and 2 sources with varying noise.

D. COMPARISON

In this subsection, the validity of the proposed WHO algorithm is endorsed by comparing its results with the state-of-the art algorithms [14], [16], [17]. For this purpose, 3 sources were considered. This subsection is divided into two cases.

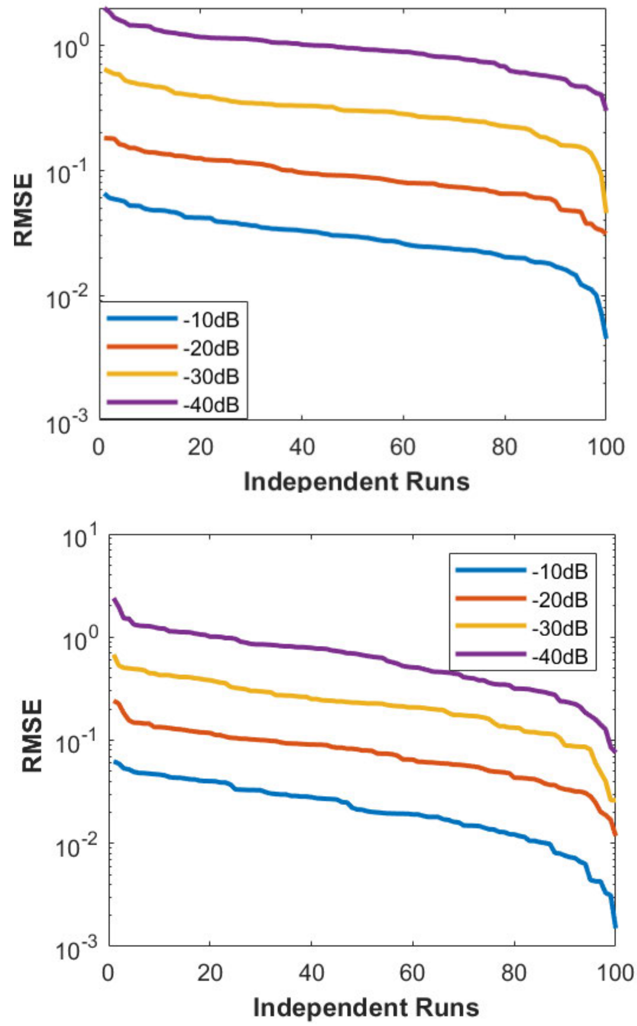

FIGURE 12. 4 and 2 sources with different noise.

TABLE 8. Estimation accuracy with P=5, Q=10.

Scheme	θ_1	θ_2	θ_3	RMSE
Desired angles	-0.8029	0.8029	0.9076	-
Proposed Sch.	-0.8029	0.8029	0.9076	1.1573E-04
OMP Sch. [14]	-0.8033	0.8019	0.9199	4.08E-01
CDSR Sch. [17]	-0.8045	0.8010	0.9106	1.30E-01
SBL Sch. [16]	-0.8033	0.7996	0.9192	3.99E-01

E. ESTIMATION ACCURACY

In this case, the ULA of Tx is kept fixed having P=5 and initially the ULA of Rx was having 10 antennas and then reduced to 4 antennas, respectively. Noise was kept at a fixed level of 20dB as given in Tables 8-9. As is clear from the results, the proposed algorithm WHO has better estimation accuracy as compared to other algorithms.

TABLE 9. Estimation accuracy with P=5, Q=4.

Scheme	θ_1	θ_2	θ_3	RMSE
Desired angles	-0.8029	0.8029	0.9076	-
Proposed Sch.	-0.8028	0.8028	0.9076	1.35E-04
OMP Sch. [14]	-0.8270	0.8220	0.8989	1.0591
CDSR Sch. [17]	-0.8169	0.8178	0.9022	7.01E-01
SBL Sch. [16]	-0.8208	0.8171	0.9104	7.62E-01

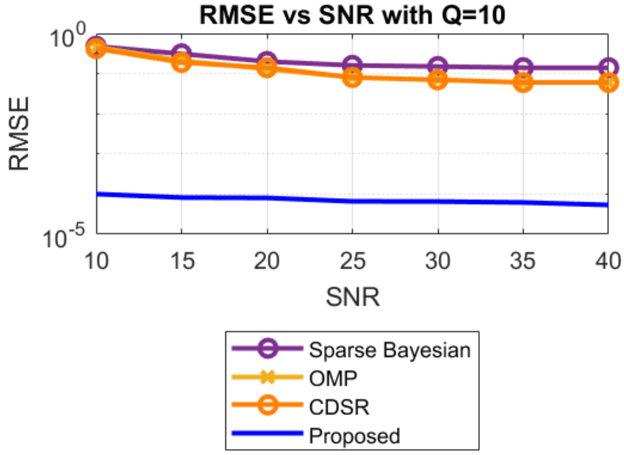

FIGURE 13. Comparison Plots with Q=10.

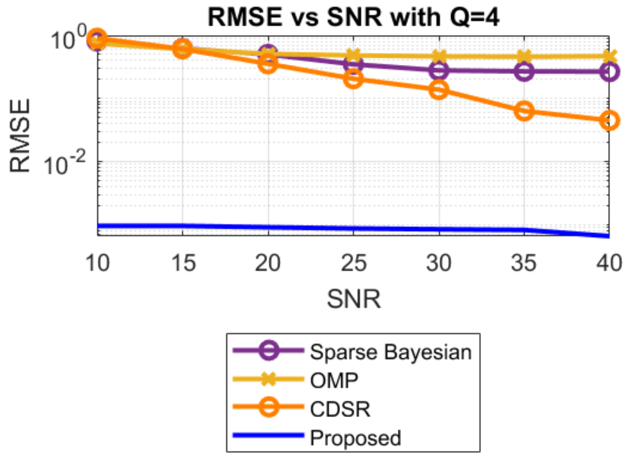

FIGURE 14. Comparison Plots with Q=4.

F. ROBUSTNESS AGAINST NOISE

In this subsection, again the ULA of Tx was kept fixed having P=5 and initially the Q of Rx was taken to be 10 and then reduced to 4. In both of these cases, the noise level was kept variable ranging from 10dB to 40dB. As is clear from Figures 14-15, the proposed algorithm has won the match.

G. COMPUTATIONAL COMPLEXITY

Table 10 shows the computational complexity. As is clear, The best computational complexity is of the OMP algorithm. The proposed algorithm has more computational time than

TABLE 10. Computational complexity.

Sr.No.	Schemes	Computational Times (s)
1	Proposed	0.9851
2	CDSR [17]	0.6037
3	OMP [14]	0.1051
4	SBL [16]	1.0702

OMP [14] and CDSR [17] however, its computational complexity is less than SBL [16].

V. CONCLUSION AND FUTURE DIRECTIONS

In this work, a new algorithm based on Wild Horse Optimization (WHO) was designed to estimate the DOA's of multiples targets in the presence of imperfect waveforms. A Co-located MIMO radar system was exploited that was having ULAs both at transmitter and receiver sides. The theory of extended AMV was incorporated to design the fitness function that was the combination of MSE and penalty function. The penalty function was used to control any deviation in the MSE. It was shown through several simulation results that the proposed WHO got good results as compared to other algorithms in terms of robustness and estimation accuracy. However, the proposed algorithm is computationally expensive than OMP and CDSR.

In the future, the same work can be compared with the latest state-of-the-art algorithms as given in [19]. Further, the same algorithm can be designed and implemented for near field sources in a bi-static radar system.

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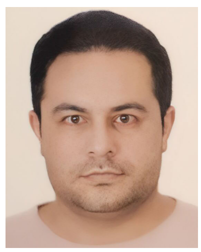

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