

**Cost Drivers Affecting the Ultimate Pit Limit
Determination in Open-Pit Mining:
A Case of Kazakhstan**

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ORIGINALITY STATEMENT

I, Dilara Aitmagambetova, declare by this statement that the thesis title “*Cost Drivers Affecting the Ultimate Pit Limit Determination in Open-Pit Mining: A Case of Kazakhstan*” is entirely the result of my own work. To the extent of my understanding, it does not include content previously written or published by others, nor work that has been used for the awarding of another degree or diploma at Nazarbayev University or elsewhere, unless appropriate credit and citations have been provided.

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ABSTRACT

Ultimate pit limit (UPL) identification is a foundational step of open-pit mine planning, since it determines the economically recoverable limit of a deposit within a specified technical and economic conditions. While current optimization methods are computationally precise, their outputs are strongly contingent on the economic assumptions applied in block assessment. In numerous real-world uses, operating costs are generalized, although their composition and territorial variability may substantially influence the final pit layout. This research examines the impact of cost structure and price fluctuations on the definition of UPL in open-pit mining, with particular emphasis on the economic context of Kazakhstan.

This thesis is based on the actual porphyry copper deposit presented in Avalos and Ortiz (2020), which is adopted as a static technical reference. Geological, geotechnical, and recovery variables remained constant. In contrast, the economic data were modified using Kazakhstan-specific price and cost data from KazMinerals, including the period from 2016 to 2020. Economic block values were recomputed for each case and optimized using a pseudoflow-based implementation of the Lerchs–Grossmann pit optimization. Revenue factor analysis was then used to generate nested pit shells and to assess the sensitivity of the optimized pit to changes in economic conditions.

The results suggest that region-based economic variables critically affect the spatial distribution of economic block values, the ultimate pit size, and the total project value. Compared to the reference case, Kazakhstan scenarios generate more economically feasible shells with higher project values. At the same time, the average grade remains relatively constant. This indicates that pit expansion is mainly constrained by economic factors rather than by variations in mineralization quality. The study demonstrates that deterministic UPL optimization is very sensitive to variations in cost and price assumptions. These results highlight the importance of integrating region-specific economic parameters into mine planning operations. Furthermore, the analysis reveals the limitations of top-down costing and emphasizes the adoption of bottom-up uncertainty-sensitive and time-dependent optimization approaches in the future.

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Table of Contents

LIST OF FIGURES	vii
LIST OF TABLES	ix
1. INTRODUCTION.....	1
1.1 Background/Problem Definition.....	1
1.2 Objectives of the Thesis.....	4
1.2.1 Main Objectives.....	4
1.2.2 Specific Objectives.....	4
1.3 Justification of the R&D.....	5
1.4 Scope of Work.....	5
2. LITERATURE REVIEW	6
2.1 Introduction	6
2.2 Contemporary Approaches to UPL Optimization and Pit Design	7
2.3 Economic Block Valuation and Cut-Off Grade Theory	9
2.4 Cost Formation and Distribution in Open-Pit Mining	10
2.4.1 Mining Cost Structure	10
2.4.2 Processing Costs and Comminution	12
2.4.3 Cost Representation in Economic Models.....	12
2.5 Cost Variability and Economic Uncertainty	12
2.6 Benchmarking Evidence on Cost Distribution	13
2.7 Regional Context: Kazakhstan	14
2.8 Synthesis and Research Positioning.....	15
3. Methodology.....	17
3.1 Economic Block Value Formulation.....	17
3.2 Cost and Price Data Derivation	19
3.2.1 KazMinerals: Price and Cost Inputs (2016–2020).....	19
3.3 Pseudoflow Representation of the UPL Problem	21
3.4 Graph Construction and Slope Constraints	23
3.5 Implementation and Nested Pit Generation	24
4. RESULTS.....	26
4.1 Case Study	26
4.2 Generation of Optimization Results.....	27
4.3 Results of Kazakhstan Scenarios (2016 - 2020) based on KazMinerals data	28
4.3.1 Analysis of Results for 2016	28
4.3.2 Analysis of Results for 2017	33
4.3.3 Analysis of Results for 2018	36

4.3.4	Analysis of Results for 2019	40
4.3.5	Analysis of Results for 2020	43
4.3.6	Analysis of Results of Avalos and Ortiz (2020)	47
4.4	Comparison of reference case and KazMinerals Results (RF = 1.0)	51
5.	<i>Discussion</i>	55
6.	<i>Conclusions and Recommendations</i>	57
6.1	Conclusion	57
6.2	Recommendations	59
7.	<i>References</i>	60

LIST OF FIGURES

Figure 2.1. Methodology for UPL determination in open-pit mine design (A.D. Akbari et al., 2009).....	8
Figure 3.1. Example of arc creation in a 2D block model. Economic block value (left), external arcs source-node (center-bottom), external arcs node-sink (center-top), and internal arcs with 3 top block precedence (right).	23
Figure 3.2. Workflow of UPL determination.	26
Figure 4.1 Copper grade block model: a) whole block model, b) block model in slices, c) block model legend.....	29
Figure 4.2. 2016 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	30
Figure 4.3. 2016 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	31
Figure 4.4. 2016 Revenue Factor Analysis.....	32
Figure 4.5. 2017 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	33
Figure 4.6. 2017 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	35
Figure 4.7. 2017 Revenue Factor Analysis.....	35
Figure 4.8. 2018 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	37
Figure 4.9. 2018 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	38
Figure 4.10. 2018 Revenue Factor Analysis.....	39
Figure 4.11. 2019 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	40
Figure 4.12. 2019 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	42
Figure 4.13. 2019 Revenue Factor Analysis.....	43
Figure 4.14. 2020 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	44

Figure 4.15. 2020 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3.....	46
Figure 4.16. 2020 Revenue Factor Analysis.....	47
Figure 4.17. Avalos and Ortiz (2020), Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3	48
Figure 4.18. Avalos and Ortiz (2020) UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3	50
Figure 4.19. Avalos and Ortiz (2020) Revenue Factor Analysis.....	51
Figure 4.20. Comparison of Total Tonnage at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case.....	53
Figure 4.21. Comparison of Ore Tonnage at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case	53
Figure 4.22. Comparison of Average Grade at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case.....	54
Figure 4.23. Comparison of Total Economic Value at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case	54

LIST OF TABLES

Table 3.1 Literature-based evidence supporting the simplified cost allocation	20
Table 3.2 Node mapping	24
Table 4.1 Parameters for block economic assessment based on Avalos and Ortiz (2020)	27
Table 4.2 Economic Parameters Used for Kazakhstan Scenarios	28
Table 4.3 Total Tonnage (RF = 1.0).....	51
Table 4.4 Ore Tonnage (RF = 1.0)	52
Table 4.5 Average Grade (RF = 1.0)	52
Table 4.6 Total Value (RF = 1.0)	52

INTRODUCTION

1.1 Background/Problem Definition

Open-pit mining is believed to be the dominant method employed for extracting shallow ore bodies. This approach is typically used for substantial ore reserves because it enables the extraction of significant material.

One of the most critical considerations in the initial mine planning is establishing the Ultimate Pit Limit (UPL). The UPL defines the pit's maximum possible expansion, balancing size with profitability. It indicates the maximum extent to which mining remains economically viable, beyond which further excavation would not be profitable.

The UPL geometry is set independently from production planning and is essential for the project's economic success. It affects the volume of waste to extract, the amount of valuable ore recovered, the operation timespan of a mine, and the essential financial expenditure. Professionals planning the UPL seek to obtain a balance, where the pit needs to maximize economic profits while complying with safety standards, such as slope constraints. Therefore, determining the UPL is a critical phase in maintaining risk prevention and the economic feasibility of an open-pit mine. Alternatively, the choice of the UPL has a straightforward influence on major planning indicators, such as the stripping ratio, the amount of recoverable reserves, the overall life of mine, and the distribution of capital assets (Hustrulid & Kuchta, 2006). Such varied obstacles led to the conclusion that determining an appropriate UPL is one of the most important steps in open-pit mine planning.

Based on the Lerchs-Grossmann methodology, the pit that maximizes undiscounted profit while complying with slope constraints is the theoretical foundation for modern pit limit optimization (Lerchs & Grossmann, 1965). It has been developed into commonly employed parametric optimization methods, particularly Whittle-based algorithms, enabling planning departments to construct nested pit shells (Whittle, 1999). These algorithms are theoretically strong and exhibit strong practical performance. However, the outcomes are wholly contingent upon economic input variables.

The essential element of UPL optimization is assessing the block's economic value. The value of each block in a geological model is determined by the revenue generated from extraction

and processing, minus the relevant expenses. Upon alteration of the pricing parameters, the block's classification transitions from ore to waste. This may profoundly affect the UPL. Research demonstrates that variations in costs and pricing can profoundly affect mining methods based on conventional economic assumptions. This may affect the project's profitability and the related risks (Dimitrakopoulos, 2011). Subsequent research indicates that failing to account for uncertainty in mine planning may result in financially adverse decisions (Goodfellow & Dimitrakopoulos, 2017).

In optimization processes, capital and operating costs are frequently regarded as fixed inputs. Drilling, blasting, loading, transportation, fuel, labor, maintenance, and overhead represent various costs associated with operating expenses (OPEX). The overall cost is greatly affected by the depth of the hauling. As the pit deepens, the time required to transport materials increases, fuel consumption increases, and the equipment may operate less efficiently. As the pit deepens, the expenses associated with mining at the periphery increase, reducing the likelihood that the project will be profitable (Lane, 1988). Neglecting depth-sensitive elements may cause the pit size to exceed the economically feasible boundary.

The primary investment required to secure equipment, develop infrastructure, build waste dumps, handle tailings, and set up processing plants shapes the project's discount rate and its overall profitability. Initiation of a large open-pit operation requires a financial investment. Fluctuations in equipment costs and currency exchange rates can significantly impact the world market. To estimate the cost of a mining project, both direct and indirect factors, including regional economic features, need to be studied (O'Hara and Suboleski, 1992).

Nonetheless, although capital expenditure is economically vital, it is not explicitly integrated into the ultimate pit limit model used in this thesis. The Lerchs–Grossmann / pseudoflow approaches employed here are based on undiscounted EBV, which analyzes whether each block is economically extractable according to revenue and per-tonne operating costs. By comparison, CAPEX mostly influences project timing, financing discounting, and broader capital analysis, and thus becomes more applicable in later phases, such as production scheduling, discounted cash-flow evaluation, and NPV-based project analysis. In the current research, CAPEX is hence regarded as a component of the broader economic context rather than as a direct optimization variable in EBV-based pit limit determination.

These factors are crucial as mining constitutes a vital component of Kazakhstan's economy. The mining of copper, uranium, gold, chromium, and iron ore is a substantial industry for the country and a key source of its external commerce. The World Bank (2023) claims that Kazakhstan's economy remains relevant to extractive segments for development. The predominant technique for extracting solid minerals in the country is open-pit mining. The substantial resources and the land's suitability for mining account for this situation.

Kazakhstan's mining conditions exhibit unique cost components that distinguish it from other countries. Transportation of assets and developing new facilities are challenging due to the country's long distances to the deposits. Cold winter temperatures and other severe climate conditions can influence the efficiency of both the workforce and equipment, fuel quality, and maintenance requirements. Global supply chain and fluctuations in currency exchange rates are likely to impact processes, including heavy equipment from foreign markets. Latest studies on Kazakhstan's mining sector suggest that enterprise expenses are increasing due to rising energy prices, employees' salaries, and legislative changes (MINEX Kazakhstan, 2024).

The energy-demanding characteristic of mining notably affects energy prices in Kazakhstan. The costs of transportation and processing vary instantly with changes in diesel fuel and energy prices. Kazakhstan has many large open-pit mines that have been in operation for a long time. Minor fluctuations in business operating costs can substantially alter the economic pit boundaries and discounted cash flow estimates. The majority of studies on UPL optimization use generalized or global cost estimates rather than data from specific areas. These factors that influence costs are essential for long-term planning. Approaches to pit optimization that ignore Kazakhstan's specific capital and operational expenses could either overstate the economically recoverable ore reserves or underestimate financial risks.

Modeling deterministic costs does not accurately reflect the natural variability observed in real-world mining projects. In practice, both operating and capital costs can vary over time. These changes directly impact the cost boundaries of a pit. Relying solely on a fixed-cost calculation can result in designs that seem feasible in theory but may fail in real-world situations as economic conditions fluctuate. Recent findings in mine planning show that optimization models must account for uncertainty by using probabilistic or scenario-driven approaches needed to evaluate various cost scenarios (Dimitrakopoulos, 2011). Planners may carry out sensitivity or probabilistic analysis of key cost factors to analyze how shifts in OPEX and CAPEX influence the final pit

limit. Mining makes a key contribution to Kazakhstan's economy, and it has specific factors that shape the cost structure. The need for in-depth analysis of the impact of capital and operating cost factors on the UPL is highlighted. A thorough analysis of cost fluctuations and risk-based optimization ensures more logical, practical, and strategically solid decisions.

A deterministic structure was considered adequate for the current research, since the goal was not to evaluate the full uncertainty range of future pit outcomes, but to separate the directional impact of Kazakhstan price and cost inputs on UPL under constrained technical conditions. By keeping geology, recovery, and slope constraints fixed and varying only the economic inputs, the evaluation provides a transparent first-order sensitivity assessment of how EBV assumptions affect pit shape. In this background, deterministic optimization is suitable as a comparative reference because it determines whether regional economic assumptions significantly modify the pit boundary before more computationally intensive stochastic developments are applied.

This research seeks to evaluate the effect of capital and operational cost drivers. Specifically, it is tailored to the mining context in Kazakhstan. Moreover, this study aims to determine the optimal, risk-balanced UPL. Also, it aims to incorporate regional cost components into the optimization process to analyze their influence on pit design, stripping ratio, and overall project valuation. This will reduce the likelihood of failure in Kazakhstan's mining operations in the event of an economic downturn.

1.2 Objectives of the Thesis

1.2.1 Main Objectives

To analyze the impact of the structure, composition, and variation of operating costs in determining the UPL in open-pit mining. This research focuses specifically on Kazakhstan's economic and regional context. This study seeks to evaluate the effects of cost distributions and cost-variation scenarios on the block economic value (EBV), ore cut-off grade, pit design, and total project value.

1.2.2 Specific Objectives

- To analyze the connection between block economic value, cost composition, and pit boundary calculation under deterministic optimization methods

- To assess the structure, distribution, and instability of capital and operating costs in open-pit mining and their effect on cut-off grade and pit design
- To analyze the responsiveness of ultimate pit boundaries to different cost allocations and cost volatility scenarios
- To examine the effects of Kazakhstan's economic and operational environment on cost-based pit optimization results

1.3 Justification of the R&D

The research is significant as it analyzes how economic assumptions shape UPL determination in open-pit mining. Although approaches such as the Lerchs-Grossmann algorithm and pseudoflow are mathematically rigorous, their outputs rely on the economic parameters attributed to blocks. In practice, these estimates are derived from simplified cost assumptions. However, changes in cost structure and metal prices may impact ore-waste classification and the final pit shell. This study illustrates the impact of costs on pit boundaries.

The thesis is relevant to the Kazakhstan environment because local conditions influence mining costs, including energy prices, workforce, facilities, and broader economic factors. The analysis is more realistic because it uses regional price and cost inputs rather than generalized worldwide assumptions. Subsequently, the research facilitates more applied, long-term mine planning and shows how region-specific economic factors can alter pit size and total economic value. This is beneficial for reducing the risk of economically unjustified pit expansion and for advancing mine-planning strategies under varying economic conditions.

1.4 Scope of Work

- **Literature Review:** review earlier research papers on UPL optimization, economic block value composition, cut-off grade concept, cost structure in open-pit mining fluctuation, and local determinants shaping operations in Kazakhstan.
- **Reference Deposit Preference:** apply the porphyry copper deposit model of Avalos and Ortiz (2020) as the technical reference scenario, maintaining the primary geological model, block dimensions, recovery, and geotechnical constraints for regulated evaluation.

- **Economic Data Acquisition:** collect yearly copper price and operating cost data from mining companies operating in Kazakhstan to form Kazakhstan-relevant economic scenarios.
- **Cost Computation and Distribution:** adjust recorded corporate costs to model-aligned segments and allocate them to mining, crushing, and processing cost elements for use in economic block value estimates.
- **Economic Block Valuation:** re-estimate economic block values for the reference case and for every Kazakhstan scenario, applying the adjusted annual economic variables
- **Pit Optimization:** implement the pseudoflow procedure in Python to produce UPL under deterministic assumptions.
- **Revenue Factor Assessment:** conduct sensitivity analysis applying a revenue factor interval to produce nested pit shells, analyze the impact of evolving economic conditions on the scale of the pit, ore tonnage, grade, and revenue.
- **Relative Evaluation:** contrast the optimized findings for the Kazakhstan scenarios with the Avalos and Ortiz (2020) reference case to assess how local cost and price context affects pit shape and economic efficiency.
- **Interpretation:** evaluate the limitations of the outcomes for mine planning in Kazakhstan, determine methodological shortcomings, and recommend approaches for subsequent advancement in economic modeling and optimization

In the current application, the scenario analysis is explicitly implemented for operating cost components and metal price inputs within the EBV structure, whereas capital cost is considered part of the economic context in further stages of scheduling rather than a direct optimization parameter in the UPL model.

2. LITERATURE REVIEW

2.1 Introduction

Identifying the UPL is the most important step in planning the open-pit mine. The UPL is the process of establishing the final mining boundaries of a mineral deposit within specific

parameters so that the mining operation would be as profitable as possible (Hartman, 1992). This is mostly done by making the disparity among the revenue made from mining the desired mineral and the earnings spent on mining the waste material that comes with it as large as possible.

This part goes into detail about the ideas behind UPL optimization, economic block value, cut-off grade, cost formation, and its variability. Furthermore, the concept of benchmarking evidence and how Kazakhstan's economy fits into it will be discussed.

2.2 Contemporary Approaches to UPL Optimization and Pit Design

In 1965, Lerchs and Grossman developed the principle of UPL determination. It established the optimum open-pit design for a block model with primary constraints to denote slope conditions. In such a structure, the optimal pit aligns with the possible set of blocks, which enhances total economic value.

Developing from this basis of pit design during the research base, the orebody is converted into a discrete grid of blocks, and the value of each block is calculated by contrasting market prices with the mining and processing costs. The UPL is then determined by computing the pit design inputs with the right balance of both cumulative economic value and final pit boundary (Newman et al., 2010). A series of pits is produced by the conventional method of changing theoretical price hypotheses, leading to the formation of nested pit arrangements. The ultimate pit is chosen, and transitional shells are organized into pushbacks from these developed nested pit configurations (Newman et al., 2010). This method positions UPL evaluation within a wider mine planning background, which contains scheduling, constraints, and numerical formulas.

Although the Lerchs–Grossmann (LG) approach delivers a precise result for UPL identification, its applied implementation shows several limitations. In accordance with Newman et. Al. (2010), the conventional LG structure employs a partial approach, where pit boundaries are identified independently from production scheduling, and the time-sensitive correlation of mining choices is not evaluated. Moreover, nested pits are typically produced by applying fixed cut-off grades and theoretical rising price assumptions, which may diminish the economic accuracy of the terminal design. A further limitation is that LG-based models are usually less adaptable in depicting more complex geotechnical and hydrological parameters.

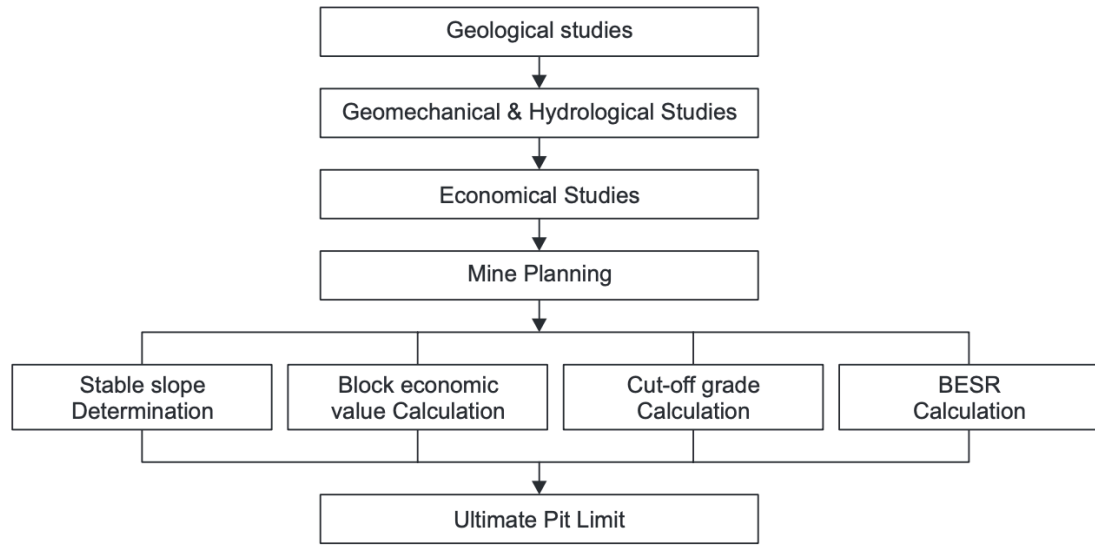


Figure 2.1. Methodology for UPL determination in open-pit mine design (A.D. Akbari et al., 2009).

In addition to the grounded in graph theory methods, approximation-based approaches, such as the floating cone algorithm, were created to estimate optimal pit limits. The floating cone method is used to assess inverted cones located over the ore blocks. The blocks in which the cones with the positive cumulative value are then incorporated accordingly (Newman et al., 2010).

Despite its computational simplicity, the floating cone approach has a few essential limitations that stop it from repeatedly reaching an actual UPL. A key shortcoming is its limitation to analyze every potential adjacent block set. Specifically, the procedure regularly fails to detect that many overlapping cones may together be even when they seem impractical when assessed separately (Ares et al., 2022). Additionally, the heuristic is strongly sequence-dependent, as the final pit design differs based on the sequence in which base blocks are handled (Newman et al., 2010). Such restrictions can cause an unintentional incorporation of non-profitable blocks and a decreased total value relative to exact approaches like LG or Pseudoflow algorithms.

Contemporary approaches are then being expanded by introducing a systematic methodology to compute UPL as well as identify pushbacks by structured economic analysis. Using established operating costs, the multiple cases are estimated by fluctuating the selling price in fixed inputs throughout a specific range. High-grade zones, including profitable stripping ratios, are economically realistic when inputting low prices. Supplemental ore zones transition to cost-

efficient, and the pit extends after the increase in price. Different selling price limits align with the apparent mined tonnage increase. These thresholds are applied to describe discrete mining pushbacks. Economic shifts are directly regulating the expansion of the pit instead of only geometric factors (Díaz et al., 2021).

To handle the computational and operational limitations of traditional LG applications, enhanced numerical approaches have been established, among which the pseudoflow algorithm denotes a notable advancement. Pseudoflow algorithms compute a similar maximum feasible subset (minimum cut) formulation as the preliminary Lerchs-Grossmann approach. However, it has refined performance for large block models. In contrast to conventional applications, pseudoflow shows refined computational performance and scalability, which makes it applicable for large block models and complex open-pit cases (Avalos and Ortiz, 2020).

Notably, economic block value is defined directly as an expression of price, recovery rate, tonnage, mining costs, processing cost, and crushing cost. This expression allows a clearer and more versatile incorporation of economic variables into the optimization procedure. Such a conception emphasizes that, despite the algorithmic application, UPL results are essentially defined by the economic inputs integrated into block value (Avalos and Ortiz, 2020).

2.3 Economic Block Valuation and Cut-Off Grade Theory

The economic reasoning regulating the UPL is integrated into block value determination, even though optimization methods establish pit geometry. Each block inside the discrete orebody block model is given a value obtained from projected revenue, connected extraction, and processing costs. Optimization runs straightforward on these figures (Newman et al., 2010). Cut-off grade theory controls the categorization of blocks as ore or waste. Ore is economically defined by obtaining value from value maximization over the whole mining system and is not considered as a fixed geological boundary (Lane, 1988). The break-even cut-off grade is based on metal price, mining cost, processing cost, and recovery in this system. Altering one of these variables changes the economic boundary between ore and waste, affecting the degree of economically recoverable metal (Lane, 1988). Due to the computation of block value as the subtraction of mining and processing costs from revenue, deviations in cost parameters immediately adjust the total value allocated to each block (Lane, 1988; Newman et al., 2010). A rise in operating costs increases the

cut-off grade and decreases the number of blocks classified as ore. On the other hand, minimizing costs extends the economically extractable boundary. Therefore, the UPL formed using deterministic optimization is profitability dependent upon the built-in cost structures in block value (Lerchs & Grossmann, 1965; Lane, 1988).

In real-world practice, cost inputs are usually oversimplified. Pre-feasibility analysis generally depends on combined cost predictions instead of comprehensive distributions. This might be suitable for the initial steps of mine development, but generalizations present uncertainty in block classification. Especially for break-even blocks situated near the economic limit (Camm, 1992). Minor modifications in supposed mining or processing costs might transfer a significant amount of material between ore and waste categories. Focus on price and geology uncertainty is broadly researched among many pit optimization literature (Dimitrakopoulos et al., 2002; Akbari et al., 2008). Yet, the cost input formation in evaluating block value is given less emphasis. Nonetheless, deterministic ultimate pit optimization functions exactly on cost-driven block valuations (Newman et al., 2010). Hence, UPL outcomes are consistent due to the correct estimation and distribution of mining and processing costs inside the block model. Consequently, EBV delivers a vital connection between operating cost composition and pit geometry. Cut-off grade theory helps to understand the influence of economic parameters definition on the ore and waste boundary (Lane, 1988). Later deterministic optimization converts this boundary into a pit arrangement (Lerchs & Grossmann, 1965).

2.4 Cost Formation and Distribution in Open-Pit Mining

Earlier chapters demonstrated the influence EBV has on UPLs. At the same time, the inner structure and allocation of operating costs demand additional analysis. Literature on mining economics highlights that operating costs emerge from several parameters rather than fixed technical parameters (Runge, 2009). Unit cost response is mutually formed by deposit geometry, throughput, equipment setups, haulage, and production capacity. Hence, the cost inputs in the optimization process indicate complicated system correlations (Runge, 2009).

2.4.1 Mining Cost Structure

In open-pit mining, overall mine cost usually comprises capital expenditure (CAPEX) and operating expenditure (OPEX) (O'Hara & Suboleski, 1992). Capital costs consist of initial mine preparation equipment procurement, facilities development, and other preliminary allocations necessary to begin the operation. These expenditures are affected by components such as scheduled production rate, equipment size, haulage network, and supporting site amenities. However, capital cost is not explicitly integrated in UPL determination, since the UPL question is time-sensitive and is expressed on the principle of undiscounted block economic value. At this phase, pit optimization analyzes whether a block is economically mineable based on its revenue and unit operating cost without evaluating the timing of investment or cash flow allocation over the mine lifecycle. CAPEX becomes more applicable only in subsequent periods of mine planning, especially in production planning, discounted cash flow analysis, and net present value (NPV) review. By comparison, operating costs are used in UPL optimization since they constitute the daily cost of mining each block. In open-pit mining, operating cost is generally formulated per tonne of material moved and consists of drilling, blasting, loading, haulage, pumping, road maintenance, lighting, and electric power use. Together, they compose the mining costs used in EBV estimations.

Runge (2009) supports this system by highlighting that unit mining cost is governed by overburden removal intensity, haulage arrangement, equipment performance, as well as production capacity. With the increase of pit depth, there is an increase of haulage distances and cycle time, leading to greater fuel consumption and equipment wear, therefore inflating the operating cost per ton (Runge, 2009). Hence, mining cost is fundamentally related to pit geometry with throughput rate. The mining cost concept is additionally improved by differentiating between waste and ore mining within OPEX (Díaz et al., 2021). Although both contain drilling, blasting, loading, hauling, and ore mining combines extra grade regulation and haulage to the processing plant. Such a distinction means that mining cost factors applied in economic models may fluctuate based on material type and operational environment (Díaz et al., 2021). Research findings validate the prevalence of specific cost drivers. Within the open pit OPEX system, hauling demonstrated the foremost cost share in mining procedure, followed by loading and blasting (Losaladjome Mboyo et al., 2025). These outcomes support the significance of tonnage and haulage as the main components of mining cost composition.

2.4.2 Processing Costs and Comminution

Crushing, grinding, concentration, operational staff, repair operations, additives, and power usage comprise processing costs (Díaz et al., 2021). Processing expenditure is highly dominated by ore properties, processing plant layout, as well as processing rate approach, leading it to be technically adjustable (Runge, 2009). Comminution is constantly recognized as the energy-demanding phase of mineral processing. Grinding processes consider a significant share of overall plant power consumption (Alvarado, 1998). Further elaborated breakdown at the process level shows more significant expenditure occurring in the comminution stage (Losaladjome Mboyo et al., 2025).

2.4.3 Cost Representation in Economic Models

Operational research reveals substantial internal variation in the cost breakdown. However, optimization structures usually include mining and processing costs as unified scalar values. Block economic value is determined by metal price, recovery, mining costs, crushing-grinding costs, and processing costs, showing how individual cost components are applied in deterministic optimization models. Simultaneously, the literature reveals that open-pit operating costs are formed by depth, scale, power consumption, ore characteristics, and process setup (Runge, 2009; O'Hara & Suboleski, 1992; Díaz et al., 2021). Deterministic pit optimization implements these costs as numerical inputs, whereas empirical validation displays that their internal allocation is highly inconsistent, especially because of the prevailing role of comminution and haulage (Curry et al., 2014; Losaladjome Mboyo et al., 2025). Therefore, cost segment fluctuations like energy for grinding or transportation fuel may have an unequal impact on block economic value and UPL. Consequently, it is essential to comprehend the structure and allocation of operating costs for analyzing how cost assumptions affect the stability of UPL by deterministic optimization.

2.5 Cost Variability and Economic Uncertainty

Although deterministic pit optimization utilizes fixed economic inputs, there is a pattern of literature review showing that both price and operating costs are ambiguous during the life of mine (Runge, 2009). Preliminary cost estimations are naturally uncertain, indicating that UPLs determined from deterministic variables are contingent based on economic input assumptions

(Runge, 2009). During the time that a great attention has been prioritized on price uncertainty, operating cost variability is of equal importance. Treating operating costs as fixed inputs may falsify project valuation because cost variability is powered by market fluctuations, policy adjustments, technological advancements, and strategic decisions, which consequently impact Net Present Value (Dehghani and Ataee-pour, 2012). The binomial modeling method presented that integrating operating cost uncertainty establishes distinct and commonly realistic economic outcomes (Dehghani & Ataee-pour, 2012).

From an operating viewpoint, cost instability occurs from variations between budgeted and real outcomes. Three primary cost distortions were identified, which comprise resource price per unit, degree of utilization, and operational efficiency (Botín and Vergara, 2015). Moreover, cost uncertainty is related to cost uncertainty fluctuations. It proves that insufficient or excessive production compared to desired goals can have a great impact on profitability, especially in operations with low throughput (Inthavongsa et al., 2015).

2.6 Benchmarking Evidence on Cost Distribution

In large copper open pit mining, only milling covers 59.1% of total operating expenditure, crushing accounted for 11%, and comminution contributes almost 70% of OPEX (Losaladjome Mboyo et al., 2025). These results match with comprehensive processing research, which emphasizes the energy consumption in grinding systems. Comminution can contribute up to 80% of total plant energy usage. Its dominant structure within processing operations is being highlighted (Foggiatto et al., 2014). Empirical evidence from a SAG mill circuit demonstrates that grinding circuits might account for up to 70% of total processing costs (Mahmoud, 2013). Inside this, grinding material, energy, as well as protective liners contribute to the main cost drivers. Together, they cover a substantial distribution of processing plant operating costs. Operating cost data from 63 mining procedures were evaluated. The distribution of total operating expenditure is about 43:43:14 to 45:45:10 for Mine:Mill:General and Administrative (G&A) costs (Curry et al., 2014). These results show that mining and milling often add around the same shares to total operating costs.

Cost analysis further shows that grinding OPEX per tonne can surpass mining, flotation, or crushing costs separately. Cost aggregation stays noticeable at the overall project level. Research analysis suggests that loading and hauling account for around 31.2% of total project costs (Mahmoud, 2013). At the same time, fine grinding contributes around 24.5%, and flotation and similar operations account for 29.4%. On the other hand, drilling and blasting cover only 0.6-1% of the total project expenditure. Although their insignificant portion of shares, changes in blast power can impact loading expenses, transportation, and crushing costs. This illustrates the mutual reliance between cost elements. Maintenance performance evaluation delivers further confirmation of structural diversity. Maintenance contributes to roughly 44% of total mine production costs in Chilean open-pit operations (Knights and Oyanader, 2005). Individual mines span from 40% to 50%. As a part of maintenance costs, contractors show 50%, spares and repairable 32%, labor 14%, and overhead 4%. These outcomes express that the dependence of contractors and maintenance approach has a great impact on the cost structure.

2.7 Regional Context: Kazakhstan

Structure and variability of mining costs are influenced by local economic conditions and impact the economic inputs applied in deterministic UPL determination. In the environment of Kazakhstan, efficiency patterns, energy regulation, exploration activities, governance quality, and economic fluctuations altogether form the cost settings in the estimated block.

Decreasing productivity growth is identified as a main structural difficulty in Kazakhstan's economy (World Bank, 2019). Productivity was resilient in the early 2000s, providing about six percentage points annually. Later, it fell to below two percent during the 2010s and had an adverse impact during the 2014-2016 commodity decline. Decline indicates diminishing industry-specific efficiency instead of structural change. In the mining processes context, decreased efficiency means greater unit operating costs. It implies reduced asset usage, extended downtime, and lower capital productivity. Deterministic pit optimization supposes stable cost inputs. Such structural efficiency limitations enhance the probability of modeled mining costs diverging from actual operating conditions.

Industry-specific findings uncover variation in cost trends among mining sectors. Between 2011 and 2019, labor efficiency in non-ferrous metals expanded by 6% annually. A decline in productivity in ore mining of -1.0 percent was observed annually. Coal mining underwent an increase in labor by 9.1 percent per year, leading to a growth of employment's share of overall cost from 16.71 percent to 25.37 percent. At the same time, the capital-to-labor ratio decreased from 85.39 to 45.24 (Atakhanova and Azhibay, 2023). The mining cost system in Kazakhstan is implied to be resource-specific rather than homogeneous. Together with high-labor demand sectors displaying diverse marginal cost behavior when compared to high-investment copper operations. Cost formation is also influenced by depletion and resources exploration level. A decrease in productivity of iron extraction can be explained by the lack of exploration activity in relation to extraction rates. This leads to observable declines in ore reserves and natural resources (Atakhanova and Azhibay, 2023). Diminished resource replacement may decrease grades of ore and increase stripping ratios. Therefore, it might lead to a boost in marginal mining cost and narrow the economic pit limit.

Cost dynamics consist of another crucial element, which is the energy structure. Research emphasizes the country's dependence on coal-based power. Energy-intensive sectors, such as mining and mineral processing, are suspected to possible cost increases linked to tightening carbon pricing systems. Comminution prevails in processing expenditure in large-scale copper mining. Considering that, a rise in electricity price would explicitly increase processing cost per tonne and raise cut-off grade limits.

Other impactful operating cost stability constituents are institutional performance and governance. Constraints on productivity and investment in Kazakhstan's mining economy are identified as regulatory inflexibility and low market competition (OECD, 2018). Another report showed governance metrics lower than global averages. Presented company-specific data indicates that corruption can diminish company productivity by 30-36 percent (The World Bank, 2019).

2.8 Synthesis and Research Positioning

The literature demonstrates that UPL determination is mathematically determined by optimization algorithms, but economically driven by the assumptions applied in block valuation.

Approaches such as Lerchs–Grossmann, floating cone, nested pits, and pseudoflow all depend on EBV as the basis for categorizing material as ore or waste, and thus for determining the final pit boundary (Lerchs & Grossmann, 1965; Newman et al., 2010; Avalos & Ortiz, 2020; Ares et al., 2022). Due to the direct dependence of EBV on metal price, recovery, and operating costs, any fluctuation in these parameters alters the economic limit of the pit. Although uncertainty in price and geology has been broadly analyzed in prior research (Dimitrakopoulos et al., 2002; Akbari et al., 2008), the intrinsic composition and variation of operating costs have obtained relatively less focus, regardless of their direct impact on cut-off grade and pit geometry.

Current research also indicates that mining and processing costs are not consistent parameters. However, they are affected by depth, energy, demand, equipment setup, ore properties, and production scale (O’Hara & Suboleski, 1992; Runge, 2009; Díaz et al., 2021). Reference literature show that cost composition varies across processes, with comminution, haulage, and maintenance often being principal cost drivers (Mahmoud, 2013; Knights & Oyanader, 2005; Losaladjome Mboyo et al., 2025). Simultaneously, deterministic pit optimization uses simplified and fixed cost variables. However, operating costs are defined by approximation inaccuracies, inefficiencies, market fluctuations, and production risk (Dehghani & Ataee-pour, 2012; Botín & Vergara, 2015; Inthavongsa et al., 2015). This produces a gap between computationally precise pit optimization techniques and the economic validity of the cost assumptions they employ. The present thesis is placed within this gap by analyzing how Kazakhstan-specific cost compositions affect UPL results under a deterministic optimization system

The regional environment of Kazakhstan strengthens this sensitivity. Cost behavior is shaped together with the variation in productivity, increased labor costs in coal mining, decreased reserves, high energy consumption, exposure to carbon pricing, and regulatory limitations (World Bank, 2019; OECD, 2018; Atakhanova & Azhibay, 2023; Howie & Atakhanova, 2022; Kabdygaliyeva, 2022). They indicate that global cost principles might not represent regional economic conditions.

3. Methodology

3.1 Economic Block Value Formulation

The estimation of the UPL in the thesis is grounded in the Lerchs-Grossmann (1965) approach, which is expressed as a maximum closure problem and addressed using a minimum s-t cut method. The block model and graph structure are based on the pseudoflow formulation (Avalos and Ortiz, 2020).

The procedure starts with the computation of economic block values (EBV), that indicate the net economic impact of each unit inside the reserve.

Block tonnage

$$T = B^3 \rho \quad (1)$$

Where:

B = block size

ρ = density

Block tonnage reflects the amount of content stored within every segment and is needed for the estimation of economic value.

The economic block value (EBV) for each block i is determined using the expression (Avalos and Ortiz, 2020):

$$EBV_i = (P - C_p) * R * T * \alpha * Z - T * C_m - T * C_c \quad (2)$$

Within this formula, P represents the metal price, C_p indicates the payable cost factor subtracted from revenue, R signifies metallurgical recovery, T corresponds to block tonnage, α represents the unit conversion factor used in the base case, Z denotes the ore grade. C_m and C_c correspond to mining and crushing per tonne, respectively.

This approach assesses every block by contrasting projected revenue with related mining and processing costs. Blocks with favorable EBV add to the project's worth, whereas unprofitable blocks indicate financial loss.

$$EBV_i(RF) = (RF * P - C_p) * R * T * \alpha * Z - T * C_m - T * C_c \quad (3)$$

The revenue factor (RF) adjusts the revenue part of the block's value while maintaining cost variables unchanged. This enables the creation of nested pit shells under different economic scenarios.

Ore – Waste Classification

$$Ore_i = \begin{cases} 1, & Z_i \geq g_c \\ 0, & Z_i < g_c \end{cases} \quad (4)$$

Blocks are categorized as ore or waste material based on whether their grade surpasses the Break-Even cut-off grade. This categorization strongly affects the allocation of economic value within the block model.

Break-Even Cut-Off Grade

$$g_c = \frac{C_m + C_p}{P * R * \alpha} \quad (5)$$

The cut-off grade determines the economic limit between ore and waste. Fluctuations in cost and price variables alter this limit and thus impact the final pit shape.

Geological and geotechnical characteristics of the orebody, comprising grade distribution patterns, block tonnage, recovery, and unit conversion coefficient were kept unchanged as documented. The sole adjusted parameters in this thesis are commodity price and operating cost elements. This approach allows clear evaluation between the reference UPL derived using the original Avalos and Ortiz (2020) economic data and scenario-based UPLs calculated from Kazakhstan market-based prices and cost inputs covering the years 2016 to 2020.

Total Pit Economic Value

$$V_{pit} = \sum_{i \in V} EBV_i \quad (6)$$

The overall pit value indicates the total economic gain of mined blocks, while the stripping ratio shows the relation between waste extraction and ore recovery.

Stripping Ratio

$$SR = \frac{T_{waste}}{T_{ore}} \quad (7)$$

3.2 Cost and Price Data Derivation

Economic inputs for the Kazakhstan cases were obtained from available annual reports for the period 2016 – 2020. Copper price and cost data were retrieved from KazMinerals’ annual reports and converted into input parameters essential for the EBV formulation. Data acquisition and preparation were carried out in Excel to preserve the original documentation and to provide a clear transition between the published information and the unit cost variables applied in the block model.

3.2.1 KazMinerals: Price and Cost Inputs (2016–2020)

For each year from 2016 to 2020, the published copper price was employed as the metal price input P in the EBV calculation. Cost data documented in reports were standardized prior to application in the optimization model. In cases where values were disclosed in US cents per pound, they were converted into USD per tonne, applying a consistent unit conversion factor such that all variables aligned with the dimensional specifications of the block model and the EBV formulation. The produced annual inputs were thereafter structured into mining, crushing-grinding, and processing cost components for scenario-based analysis.

Since the annual reports do not explicitly reveal all operating cost elements in the specific required structure for the pseudoflow block valuation model, a distribution process was required. In this research, total operating cost was allocated using a 30:50:20 division for mining, crushing-grinding, and processing, respectively. This distribution was applied as a simplifying formulation to convert aggregated cost documentation into the cost classifications necessary for the EBV model. The determined allocation is compatible with the overall result in the literature that mining and comminution prevail over open-pit operating costs, whereas other process-phase costs constitute a smaller proportion. Therefore, the distribution was applied not as a site-specific

evaluated cost composition but as a scenario-based estimate that maintains the comparative dominance of material movement and size-reduction costs in copper operations.

The 30:50:20 distribution was applied as a simplified, literature-based model-building assumption rather than a site-specific operating-cost reassembly. It was chosen to maintain three consistent trends identified in the literature. First, mining and mill-related costs usually constitute the two main components of total open-pit operating costs. Second, crushing-grinding is often the greatest single element within plant cost. Because KazMinerals annual reports do not publish costs in the exact EBV structure needed by the pseudoflow approach, this distribution was employed to adapt cost data into analytically uniform mining, crushing-grinding, and processing for relative scenario evaluation.

To support the simplified cost distribution applied in this study, Table 3.1 outlines representative literature findings on open-pit cost structure together with the role of the Avalos and Ortiz (2020) reference case used for scenario comparison.

Table 3.1 Literature-based evidence supporting the simplified cost allocation

Source	Reported evidence / reference information	Implication for this study	Relevance to the present modelling framework
Curry et al. (2014)	An approximate Mine: Mill: G&A distribution of 43:43:14 to 45:45:10 is shown by operating cost data from several mining operations.	Mining and mill-related costs usually constitute the two main operating-cost segments.	Supports the application of a cost structure, where mining and plant-related costs are both major units.
Mahmoud (2013)	Up to 70% of processing expenses may be attributed to grinding circuits and project-level data also emphasises the significance of loading, trucking, and fine grinding.	Processing costs are internally uneven, with grinding/comminution dominating plant costs.	Supports allocating the largest single share to crushing-grinding in the simplified proportion.
Losaladjome Mboyo et al. (2025)	In open-pit copper operations, comminution continues to be a major cost center, with milling accounting for 59.1% of total operating expense and crushing for 11%.	Mill-related costs can surpass mining costs, especially in copper operations with energy-intensive comminution.	Supports the assumption that crushing-grinding should obtain a larger portion than the rest of processing.
Foggiatto et al. (2014)	Up to 80% of the energy used by plants can come from combustion.	The energy burden of comminution supports its significance within plant costs.	Gives additional support for a high crushing-grinding share in the simplified allocation.
Knights and Oyanader (2005)	In open-pit copper operations, maintenance accounts for about 44% of the overall cost of mine output.	Mining cost is internally complex and greatly shaped by maintenance and equipment performance.	Supports keeping a substantial mining-cost share even when plant costs are highlighted.

Avalos and Ortiz (2020)	Processing costs are 0.315 USD/lb, recovery is 80%, mining costs are 4.0 USD/t, and crushing-grinding costs are 10.0 USD/t, according to Avalos and Ortiz. This suggests a about 21.6% mining: 54.0% crushing-grinding: 24.4% processing-equivalent cost structure.	The Avalos and Ortiz reference case is not issued as a fixed 30:50:20 split, but its implied reference-grade cost structure is generally consistent with a model in which crushing-grinding is the dominant cost component.	Supports the use of Avalos and Ortiz (2020) as the technical and economic reference for evaluation, while revealing that the present 30:50:20 assumption is broadly consistent with the prevalence of crushing-grinding.
This study (assumption)	Mining : Crushing-grinding : Processing = 30 : 50 : 20	Literature-based top-down allocation adapted to the EBV categories required by the model.	Used as a scenario-building assumption to maintain the comparative dominance of mining and comminution while allowing direct comparison with the Avalos and Ortiz reference case and the Kazakhstan annual scenarios.

Therefore, the implemented cost allocation is not aimed at reproducing a universal proportion but to stay consistent with the broader cost structure patterns recognized in prior research. Specifically, the literature supports the prevalence of mining- and mill-related costs, particularly the large contribution of crushing within plant expenditure and haulage and maintenance within mining costs. On this foundation, the 30:50:20 allocation was applied as a justifiable top-down approximation for comparative pit optimization across the Kazakhstan cases.

The goal of this distribution was to produce internally aligned Kazakhstan-based input scenarios instead of reproducing a precise site-specific operating budget. Therefore, the produced cost structure should be evaluated as a top-down projection appropriate for comparative pit optimization, not as a replacement for bottom-up costing. This assumption represents one of the primary limitations of the research and is reviewed later in the Discussion section.

3.3 Pseudoflow Representation of the UPL Problem

The Lerchs–Grossmann formulation can be expressed as a maximum closure challenge on a directed graph and similarly addressed as a minimum s - t cut problem (Lerchs & Grossmann, 1965; Avalos & Ortiz, 2020). In the present depiction, the block model is converted into a directed graph $G=(V,A)$, such that every unit maps to one node. Two supplementary nodes, a source s and a sink t are also included.

Blocks containing positive economic value are linked to the source with a capacity equivalent to each block's EBV, whereas blocks with negative economic value are linked to the sink with the capacity corresponding to the absolute amount of EBV. Slope constraints are represented using priority arcs among blocks. In case of block j has to be extracted prior to block i , a directed arc from i to j with unlimited capacity is implemented. This setup guarantees that all valid solutions satisfy geotechnical constraints.

Maximum Closure Formulation:

$$\max \sum_{i \in V} EBV_i x_i \quad (8)$$

subject to:

$$\begin{aligned} x_i &\leq x_j \quad \forall (i, j) \in A \\ x_i &\in \{0, 1\} \end{aligned} \quad (9)$$

where:

- x_i = decision variable (1 if block is mined, 0 otherwise)
- EBV_i = economic block value
- A = set of precedence arcs (slope constraints)

This expression indicates the maximum closure challenge addressed by the Lerchs-Grossmann and pseudoflow approach. The UPL relates to the block group that maximizes overall economic value subject to slope stability constraints.

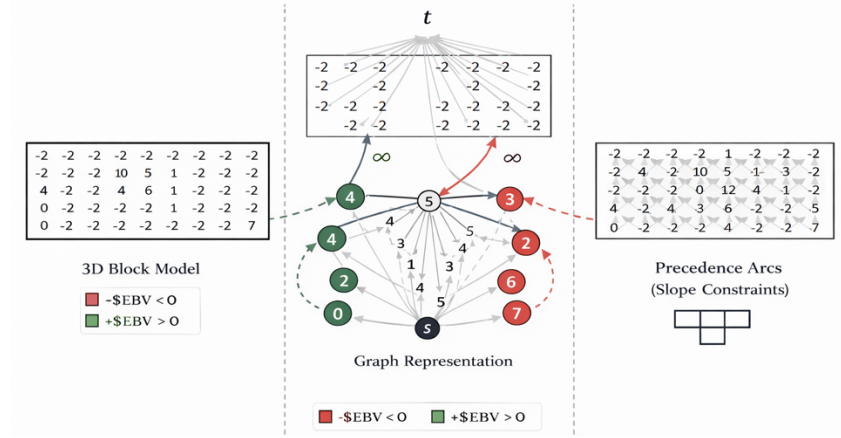


Figure 3.1. Example of arc creation in a 2D block model. Economic block value (left), external arcs source-node (center-bottom), external arcs node-sink (center-top), and internal arcs with 3 top block precedence (right).

Following the economic evaluation of the block model, the ore deposit is represented as a closure graph.

3.4 Graph Construction and Slope Constraints

Every unit in the 3D block grid is assigned to a node within the graph. Node indexing is based on the structured conversion between (x,y,z) and a unique graph position p_i .

$$p_i = 1 + x + nx * y + nx * ny * z \quad (10)$$

This transforms block positions into graph nodes.

External arcs are formed by connecting profitable blocks to the source node and economically negative blocks to the sink node. The limit of every external arc matches the total size of the block's value. Thus, the limited capacities of source-linked and sink-linked arcs embed the block's undiscounted economic value within the graph.

Table 3.2 Node mapping

<i>Node number p_i</i>	<i>x</i>	<i>y</i>	<i>z</i>
1	0	0	0
2	1	0	0
3	2	0	0
...	...	...	...
$(n_x n_y n_z)$	$(n_x - 1)$	$(n_y - 1)$	$(n_z - 1)$

The sequencing proceeds in the x-direction, continuing along y and z, maintaining uniform mapping of the entire block set.

Internal arcs denote spatial priority restrictions. Each block has specific overlying blocks that must be extracted before complying with slope conditions. According to Avalos and Ortiz (2020). These precedence rules are enforced by connecting directed arcs from a block to required prior blocks with infinite throughput. Two precedence patterns (1×5 or 1×9) may be set based on slope arrangement. This research retained the original deposit precedence pattern for consistent comparison. This arrangement guarantees that all accurate minimum cut solutions satisfy slope constraints.

3.5 Implementation and Nested Pit Generation

The optimization process was executed in Python to ensure a coherent and repeatable system for examining the reference case and the Kazakhstan-based economic cases. The block model was managed as a numerical matrix employing NumPy, while the graph arrangement needed for pit optimization was generated utilizing NetworkX. The pseudoflow package was then implemented to compute the minimum-cut challenge and determine the economically extractable set of blocks, which establishes UPL. Furthermore, Pandas was applied to arrange the resulting metrics and separate a three-dimensional visualization of the resulting pit shell based on the Mayavi library. For refined spatial interpretation, the final pit boundary visualization was performed in Isatis.neo using the exported block model results.

The numerical process adhered to the methodological order defined in the previous sections. Initially, the block model was imported, and the economic block value of every block

was recomputed for the designated economic parameters. This phase was carried out via the *EconomicValue_RF()* function, which modified block values as a relation of copper price, recovery, mining cost, crushing-grinding cost, processing cost, density, block size, and cut-off grade. In the execution, the revenue factor adjusted only the revenue term, while the cost components were fixed. This allowed the production of nested pits under more beneficial revenue assumptions.

Subsequent to economic block valuation, the optimized pit boundary was identified by the *Pseudoflow_UPL()* approach. A directed graph was constructed, with external arcs assigned based on whether each block exhibited a positive or negative EBV, while internal precedence arcs were included to preserve slope constraints. The graph was then computed using the pseudoflow-based minimum-cut solver, and the determined nodes were recorded back into the block model. The implementation maintained the 1×5 precedence pattern executed in the code structure.

To evaluate the sensitivity of the optimization outcomes across economic scenarios, the procedure was iteratively run over a revenue factor interval from 0.7 to 1.3 with a step of 0.1. This iterative procedure was performed through *GenerateNestedPits()* function. It recomputed the economic block values, formulated and resolved the pit optimization problem for each scenario, and documented the outputs in a standardized format. For each generated pit shell, the *ComputePitStatistics()* function calculated main metrics, including the number of blocks, total tonnage, ore tonnage, average ore grade, and total undiscounted value. These measures were subsequently used for comparison between the reference case and the Kazakhstan scenarios for 2016-2020.

The RF range of 0.7 to 1.3 was selected to cover a sufficiently broad conservative-to-optimistic economic span, while staying focused on the base case of RF = 1.0. In real-world terms, values under 1.0 represent less beneficial revenue factors, and this more limiting pit shells, while values above 1.0 model greater conditions and enable analysis of shell enlargement. This range was also regarded as suitable for detecting the economic threshold at which marginal blocks turn mineable, which is one of the main objectives of the research. A step size of 0.1 was chosen to offer an evident sequence of nested pits without redundant repetition, allowing uniform visual and quantitative evaluation across all scenarios.

All cases were computed using the same geological model, block dimensions, recovery, and slope constraints. Therefore, the observed variations in optimization results represent only changes in the economic input variables.

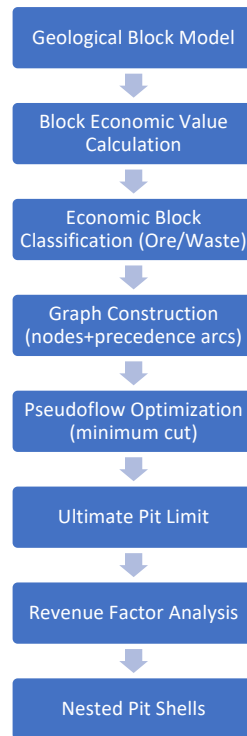


Figure 3.2. Workflow of UPL determination.

Figure 3.2 illustrates the overall workflow of the executed methodology for UPL determination. The process initiates with the geological block model and EBV computation, then advances through economic classification, graph formation, and pseudoflow optimization to produce the final pit limit and nested pit shells.

4. RESULTS

4.1 Case Study

The provided study in this research is grounded on a simulated porphyry copper deposit model proposed by Avalos and Ortiz (2020), which is the reference case for pit optimization analysis. The deposit is modeled by a three-dimensional block model consisting of $44 \times 62 \times 26$

blocks, every block with dimensions of $16 \times 16 \times 16$ meters. These measurements produce in a total of 70,928 blocks. Each block includes spatial points and linked properties, such as copper grade, economic block value (EBV), and UPL classification

Since the research was structured as a controlled comparative analysis, the deterministic structure is explained here as a reference sensitivity tool instead of a full uncertainty model.

The copper grade allocation inside the model is defined through a core high-grade area enclosed by a lower-grade mass, which represents common porphyry copper deposits. The spatial variation serves a central function in defining the allocation of economic block values and explicitly affects the shape of the optimized pit. Table 4.1 presents the estimated values of each variable.

Table 4.1 Parameters for block economic assessment based on Avalos and Ortiz (2020)

Price (P)	1.8	usd/lb
Processing cost (Cp)	0.315	usd/lb
Recovery (R)	80	%
Tonnage (T)	$2.7 \cdot 10^3$	Ton
Conversion factor (α)	2204	lb/ton
Mining cost (Cm)	4.0	usd/ton
Crushing-grinding cost (Cc)	10.0	usd/ton

After the economic assessment, the block model is converted into a directed graph model. In this graph, each block matches a node, and two supplementary nodes are added to illustrate the source and sink. Blocks with beneficial economic value are linked to the source, while blocks containing unfavorable economic value are attached to the sink. Geometrical slope constraints are applied via internal arcs with unbounded capacity, guaranteeing that all possible outcomes meet extraction constraints. This depiction allows the implementation of the pseudoflow algorithm to establish the optimal pit layout.

4.2 Generation of Optimization Results

The optimization findings were achieved through the implementation of the pseudoflow algorithm to both the reference ore deposit dataset and the Kazakhstan-based economic cases. For

every scenario, economic block values were initially computed employing the related price and cost variables. The analyzed block model was later adapted to a directed graph system, following the minimum cut method being applied to identify the optimal group of block which determine the UPL.

To assess the sensitivity of pit layout to economic factors, the revenue factor was adjusted within the range of 0.7 to 1.3. Concerning every revenue factor value, an updated set of economic block values was computed, and the optimization process was performed again. This process produced a sequence of nested pit boundaries showing various economic cases.

Table 4.2 Economic Parameters Used for Kazakhstan Scenarios

Year	Copper Price (\$/lb)	Processing (20%) (\$/lb)	Mining (\$/t)	Crushing (\$/t)
2016	2.30	0.27	4.85	8.08
2017	2.75	0.29	5.06	8.43
2018	2.52	0.319	5.65	9.41
2019	2.42	0.315	5.58	9.30
2020	3.04	0.33	5.79	9.65

For every scenario and revenue factor, the results contain the spatial distribution of economic block values and the resulting UPL. This process was executed uniformly for both the reference case and the Kazakhstan scenarios for the period from 2016 to 2020. It allows direct evaluation of the pit response considering diverse economic conditions.

4.3 Results of Kazakhstan Scenarios (2016 - 2020) based on KazMinerals data

4.3.1 Analysis of Results for 2016

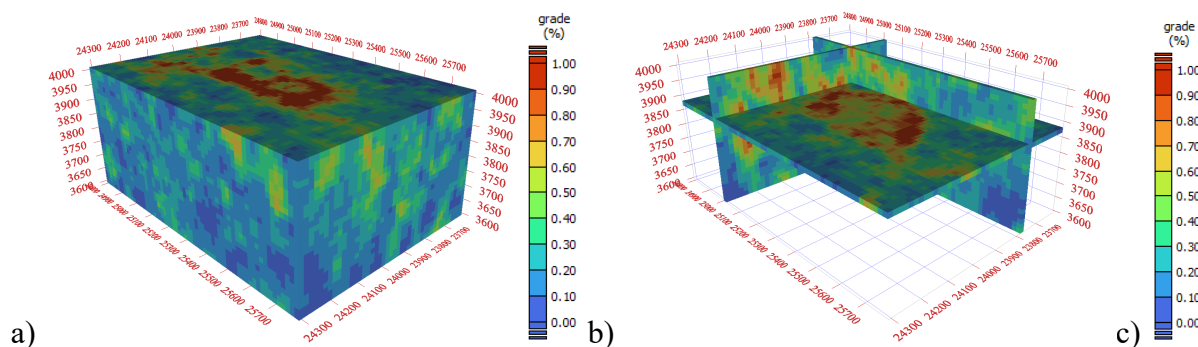
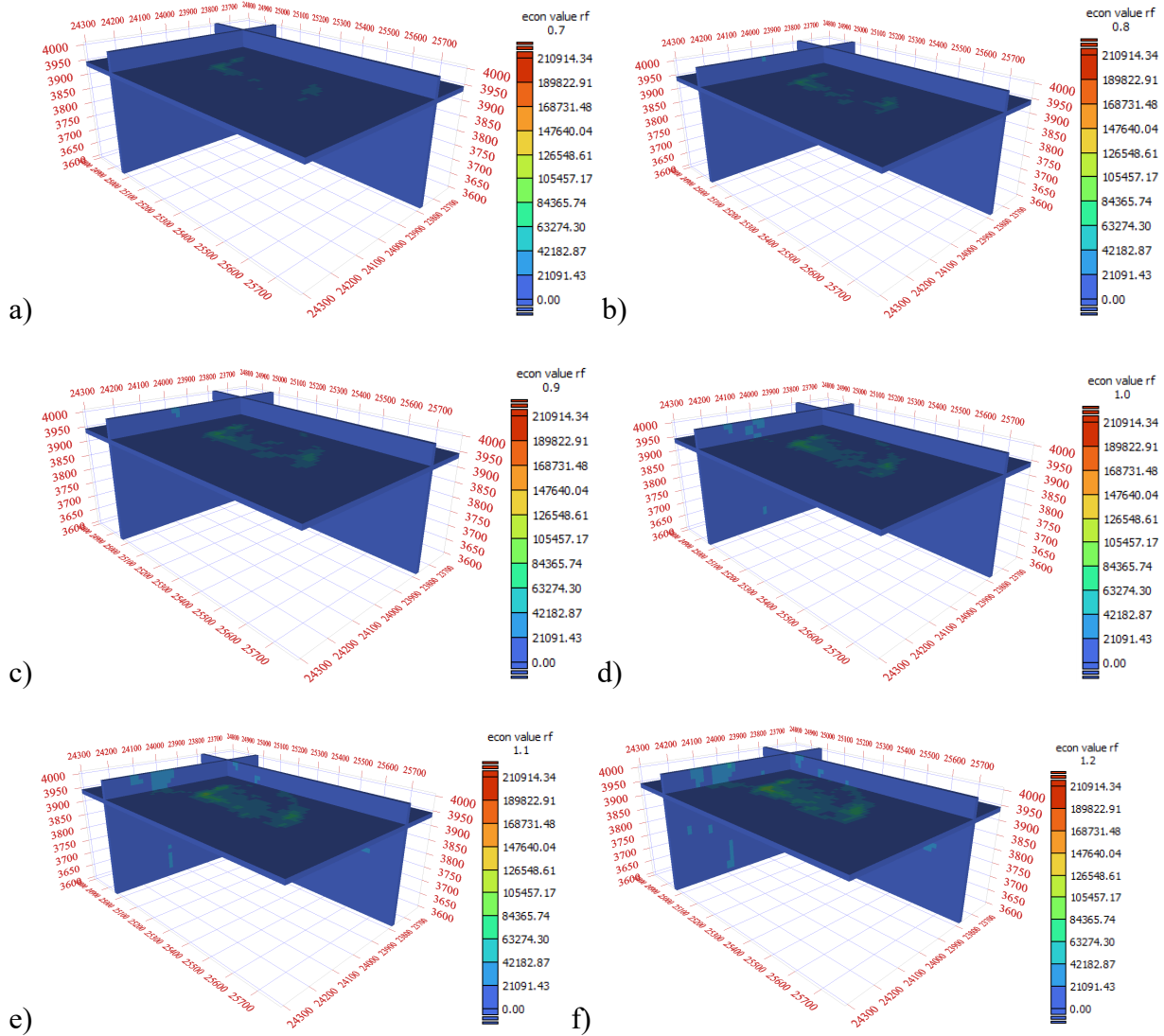


Figure 4.1 Copper grade block model: a) whole block model, b) block model in slices, c) block model legend

The copper grade block model illustrated in Figure 4.1 shows a heterogeneous spread of ore concentration with richer zones clustered in the central area of the deposit and lower grades near the edges. The grade values span roughly from near 0 up to 1.0% Cu, having majority of the deposit displaying medium grades between 0.3 and 0.9%. The cross-section verifies vertical consistency of the higher-grade core. It suggests that economically positive zones are spatially concentrated. This pattern persists unchanged across every scenario and directly affects both EBV and pit formation.



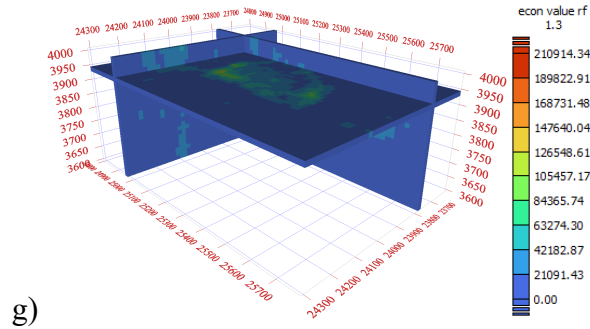
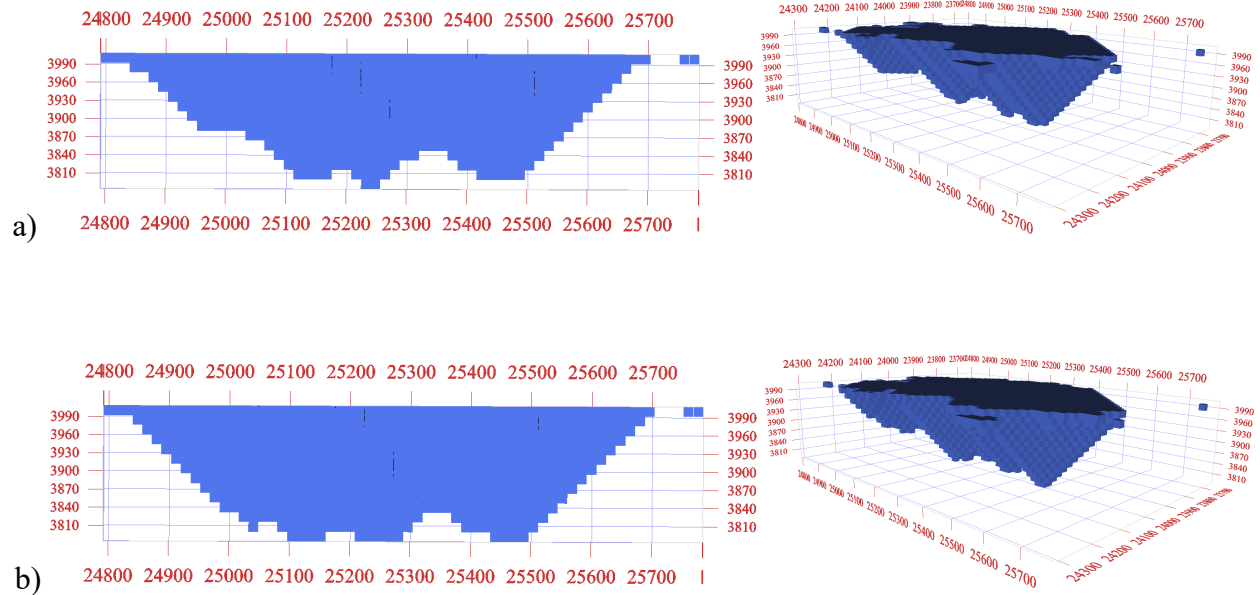


Figure 4.2. 2016 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The economic block value (EBV) arrangement for 2016 spans from 0 to approximately 210,000 USD. Yet, the majority of blocks remain under 120,000 USD. At reduced revenue factors (RF = 0.7 – 0.8), only a minor core area shows positive quantities, generally around 20,000–80,000 USD. A distinct change appears at RF = 0.9, in which the economic area turns increasingly continuous, and values rise to roughly 80,000–120,000 USD. At RF = 1.0, a steady economic core is created, having EBV values achieving about 80,000–140,000 USD. Additional growth in RF (1.1-1.3) leads to the extension of the economic zone. However, this increase is primarily caused by the addition of marginal blocks with values typically under 100,000 USD.



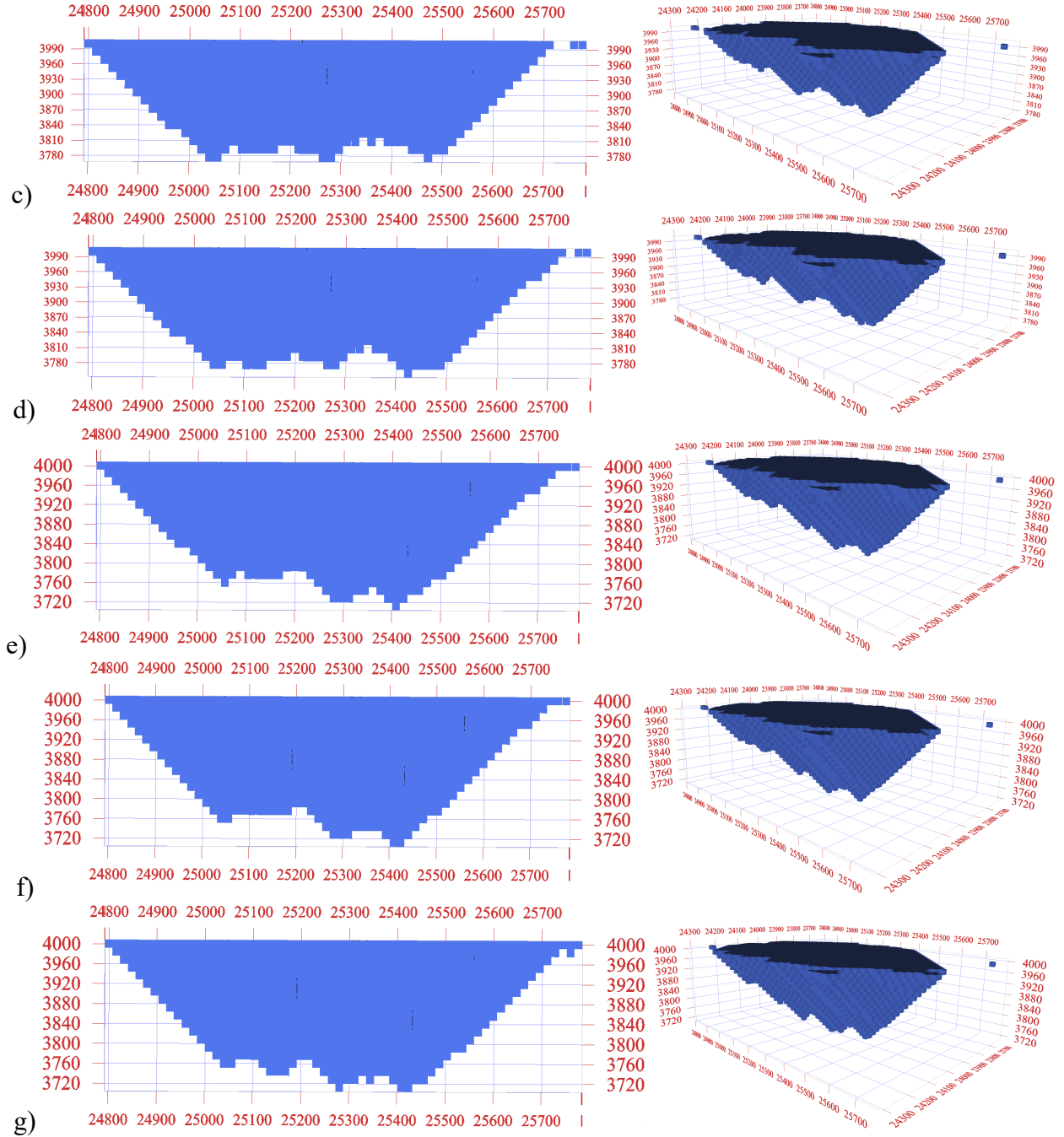


Figure 4.3. 2016 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The UPL outcomes for 2016 in Figure 4.3 display gradual pit expansion with rising Revenue Factors. At RF = 0.7, the pit remains comparatively small and confined to the central zone. At RF = 1.0, the pit attains its fundamental layout and is well-structured. Meanwhile,

subsequent rises to $RF = 1.3$ lead to further lateral and vertical enlargement. The overall pit shape persists stable as a result of slope constraints, with the highest depths reaching about 3700-3800 m elevation. Associated quantitative findings derived from the model computations show that at $RF = 1.0$, the overall tonnage is roughly 15.3 million tonnes, containing about 8.88 million tonnes of ore.

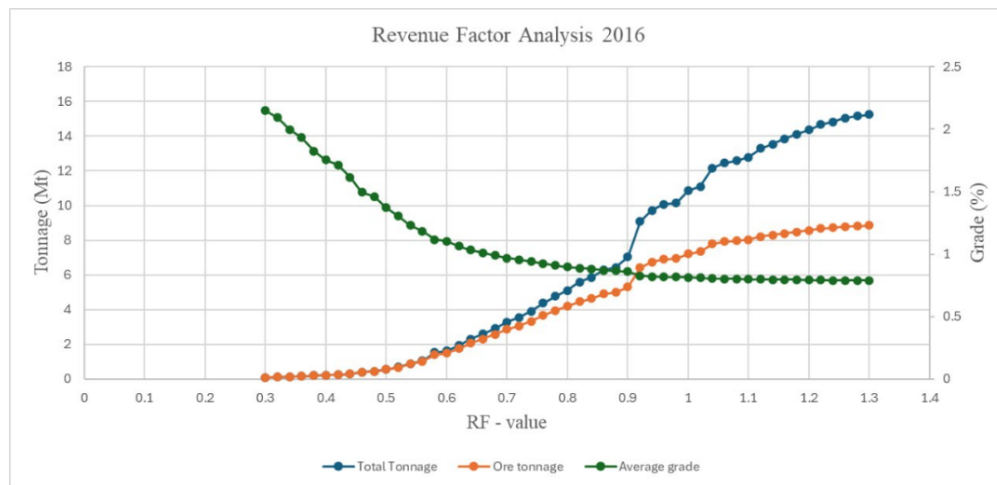


Figure 4.4. 2016 Revenue Factor Analysis

The Revenue Factor evaluation for 2016 in Figure 4.4 emphasizes the correlation between pit scale and grade. Total tonnage escalates gradually from about 6-7 Mt at lower RF values to over 15 Mt at $RF = 1.3$. Ore tonnage rises from close to 0 at low RF to nearly 8.8 Mt at $RF = 1.0$ and keeps increasing progressively. In comparison, the average grade declines from approximately 1.6% at low RF values to about 0.78% at $RF = 1.0$ and further marginally decreases with higher RF. This evidently shows the balance between volume and grade. Where higher RF values result in larger pits but lower average grades. The shift around $RF = 1.0$ indicates the optimal equilibrium between economic feasibility and grade in the deposit.

4.3.2 Analysis of Results for 2017

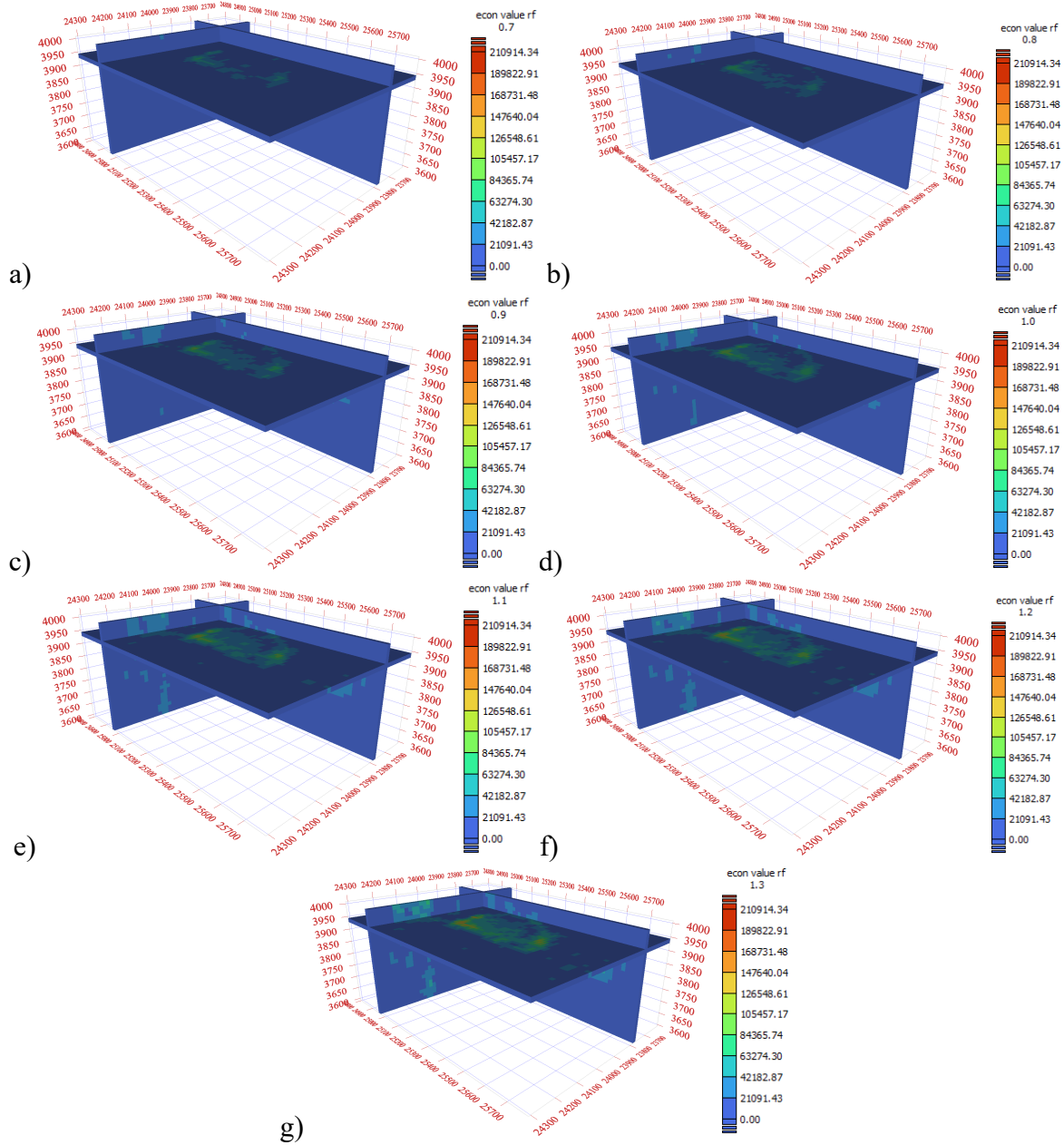
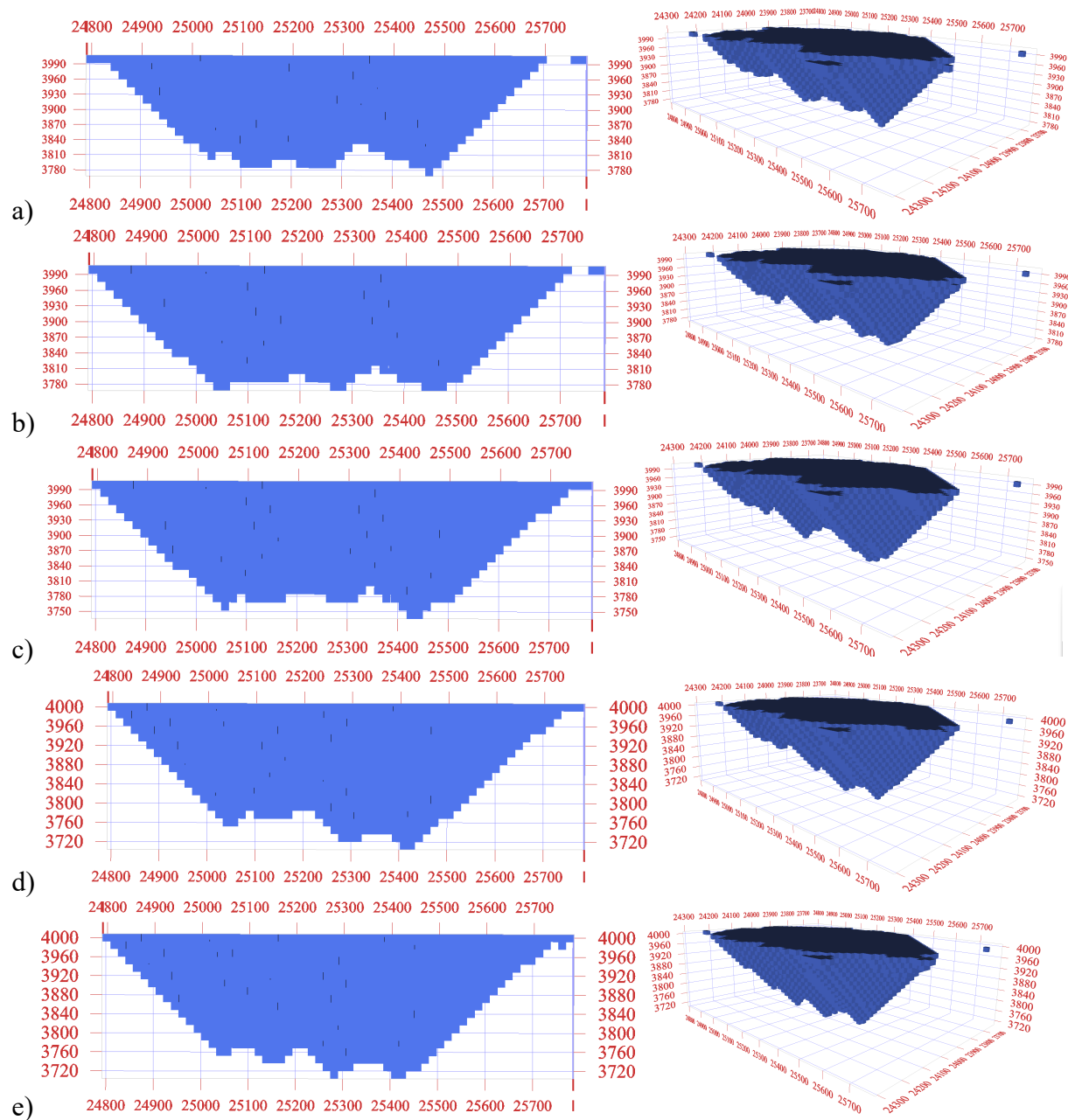


Figure 4.5. 2017 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The EBV allocation for 2017 illustrates a distinct enhancement relative to 2016, featuring a greater share of blocks in higher value ranges. At RF = 0.7 – 0.8, positive blocks are more

widespread with values typically in the span of 40,000–100,000 USD, showing prior economic feasibility. At $RF = 0.9$, a distinct and continuous economic area is established with EBV values rising to about 80,000–140,000 USD. At $RF = 1.0$, the middle core attains its maximum formation with many blocks in the range of 100,000–160,000 USD, and several reaching greater levels in the range. Additional rises in RF (1.1 – 1.3) cause substantial spatial extension, including further blocks generally measured at 60,000–120,000 USD. High-value core stays consistent at the same time.



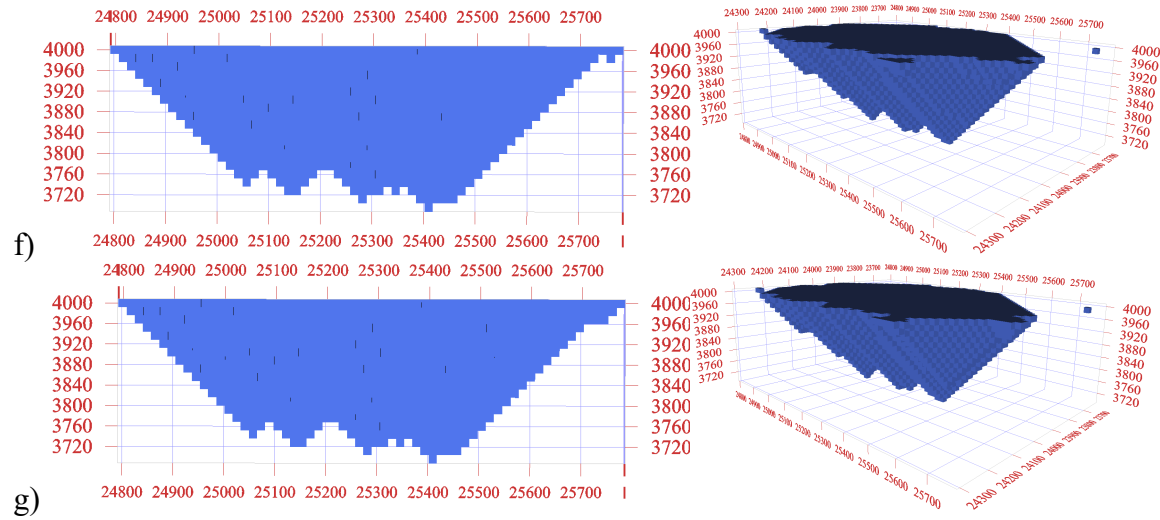


Figure 4.6. 2017 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The UPL outcomes for 2017 indicate a distinct and prior growth of the pit relative to 2016, indicating enhanced economic scenarios. At RF = 0.7, the pit is comparatively formed, spreading beyond the central region and extending to intermediate depths. As RF grows to 0.8 – 0.9, the pit enlarges both horizontally and vertically with a more continuous and broader bottom. It shows that further blocks are economically feasible. At RF = 1.0, the pit achieves a consistent and clearly delineated structure with peak depths reaching nearly 3700-3750 m elevation. Further increases to RF = 1.1 – 1.3 lead to extra expansion, especially across the pit edges and at depth. While the total shape stays uniform because of the slope constraints.

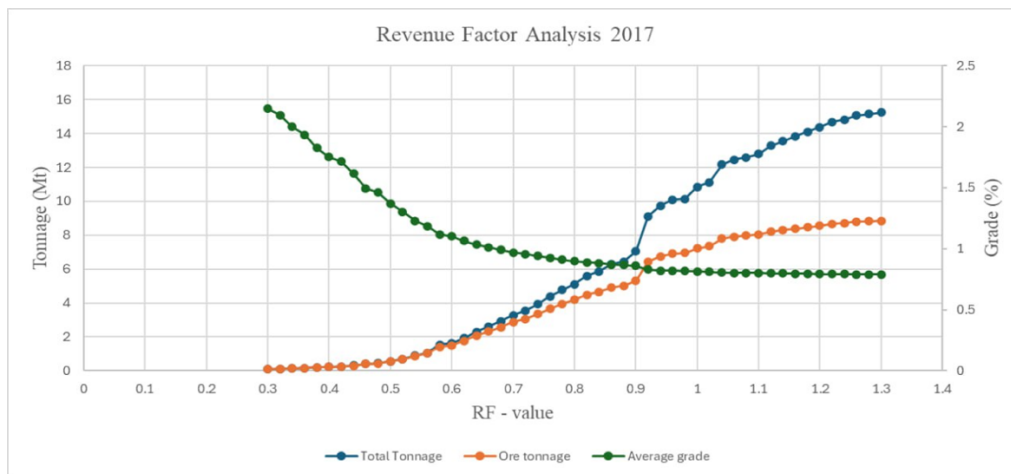
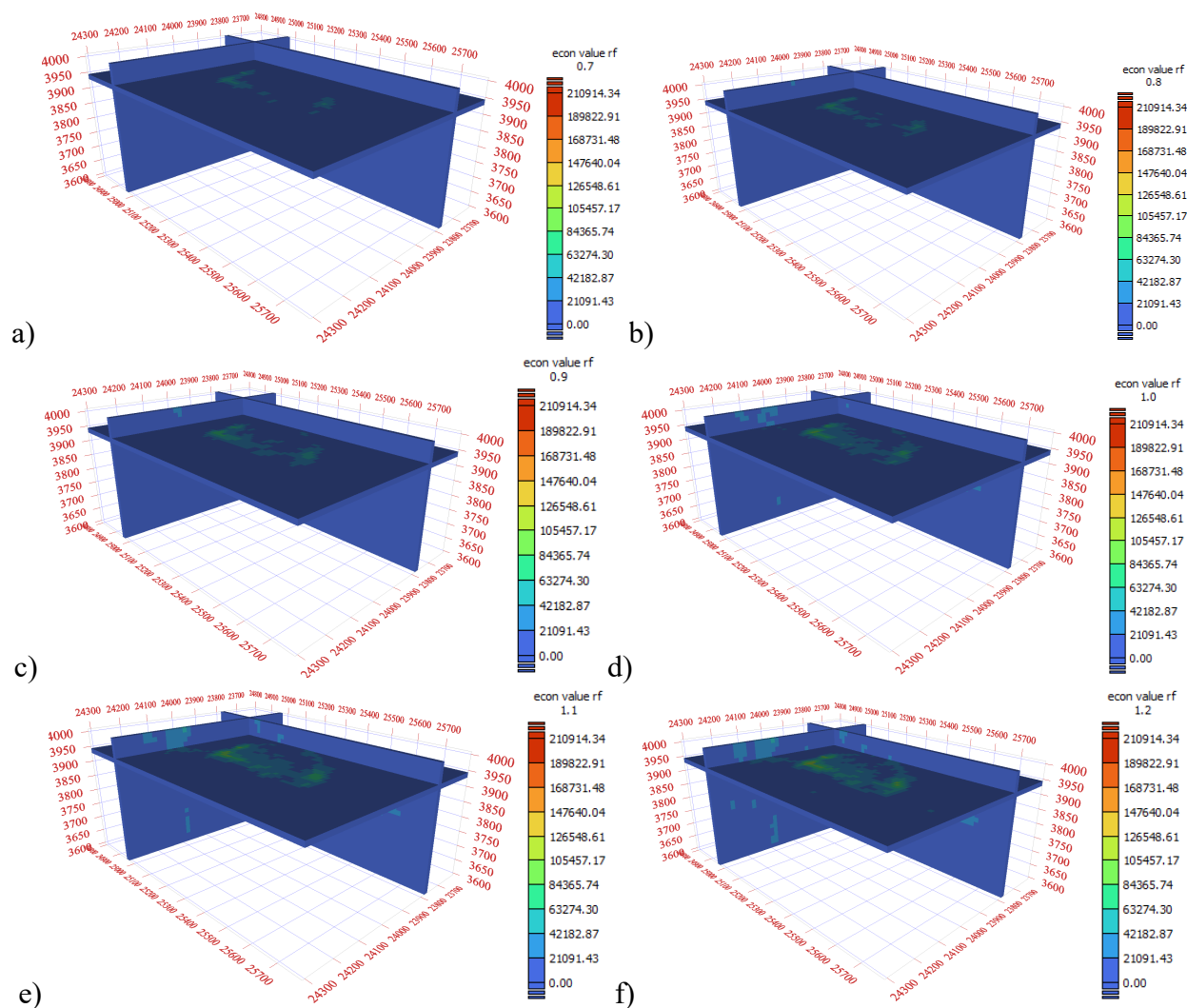
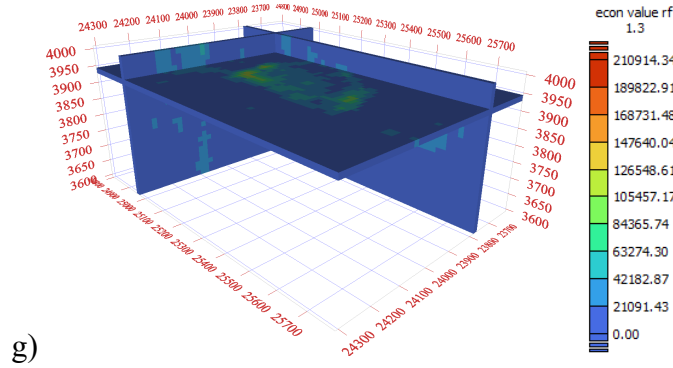


Figure 4.7. 2017 Revenue Factor Analysis

The RF evaluation for 2017 in Figure 4.7 demonstrates that total tonnage rises from about 5.8 Mt at RF = 0.8 to over 15 Mt at RF = 1.3, whereas ore tonnage increases from around 3 Mt at low RF = 0.7 to about 9 Mt at greater RF values. In contrast, the average grade declines from nearly 1.5-1.6% at low RF values to around 0.78% at RF = 1.0, and continues to decrease slightly as RF rises. A visible change happens near RF = 0.9 – 1.0, where pit enlargement intensifies.

4.3.3 Analysis of Results for 2018

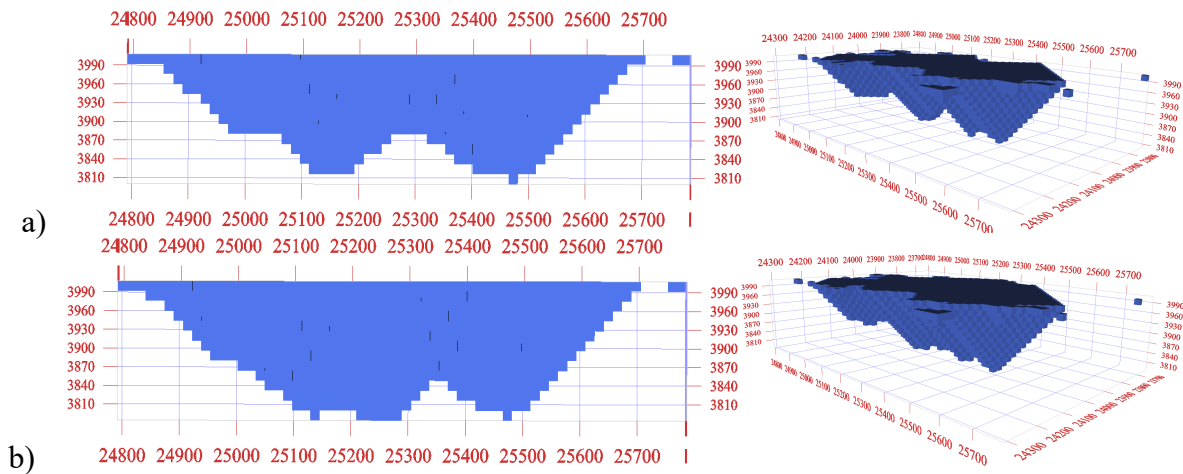




g)

Figure 4.8. 2018 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The Economic Block Value distribution for 2018 demonstrates an intermediate economic outcome, with a smaller share of high-value blocks relative to 2017. At RF = 0.7 – 0.8, only a restricted core region is economically favorable, having values primarily in the range of 20,000–80,000 USD, suggesting constrained economic viability. At RF = 0.9, an increasingly continuous economic zone emerges with EBV values rising to roughly 80,000–120,000 USD. However, the spatial extent stays more confined than in 2017. At RF = 1.0, the core transforms into a distinctly defined form with characteristic values approximately 80,000–140,000 USD. However, a smaller number of blocks attain higher value ranges. As RF rises to 1.1 – 1.3, growth takes place progressively, primarily integrating marginal blocks with values in the 40,000–100,000 USD range, whereas the high-value core persists comparatively limited.



a)

b)

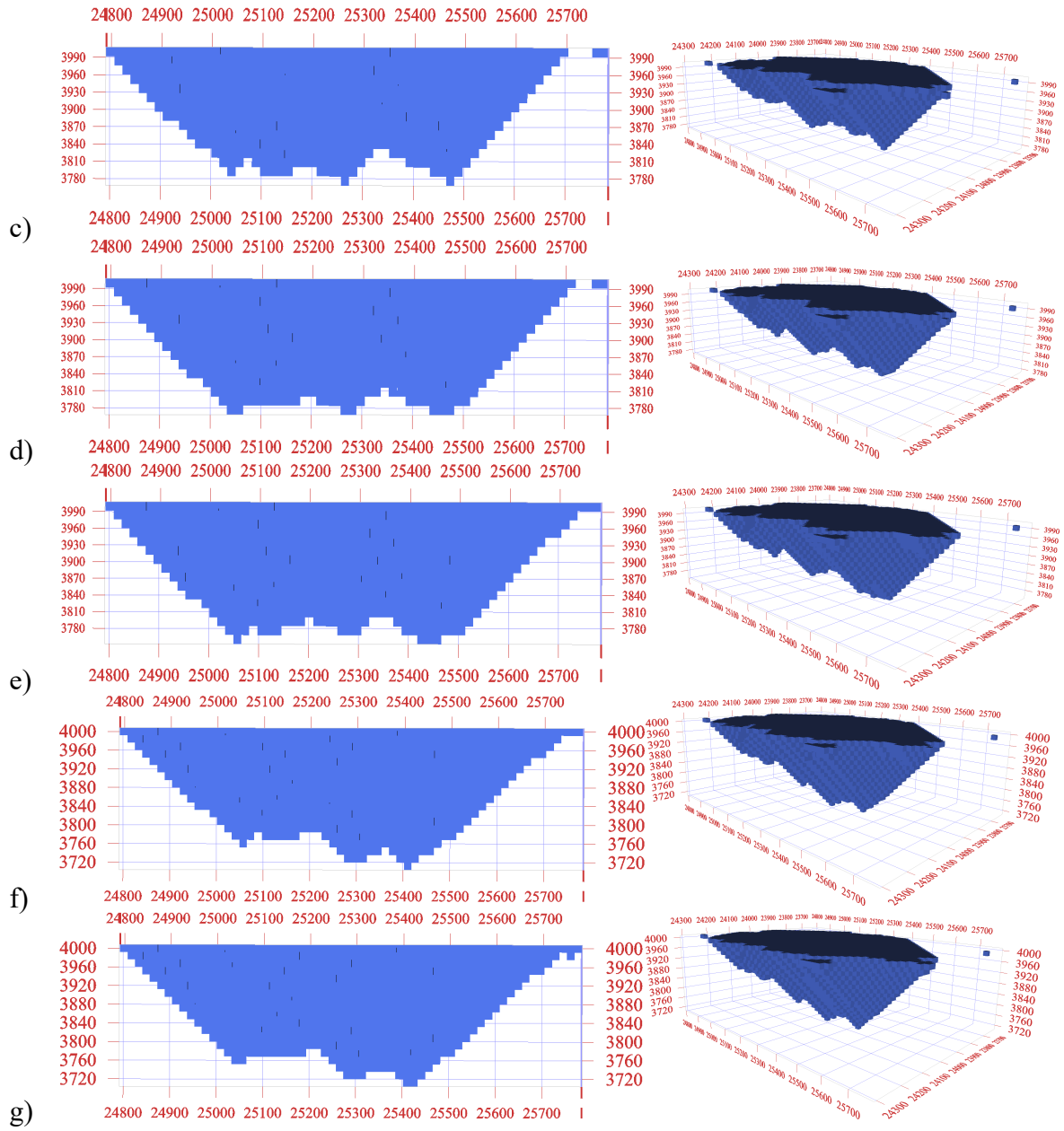


Figure 4.9. 2018 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The UPL findings for 2018 suggest a moderate pit expansion in comparison to 2017, signifying marginally reduced economic conditions. At RF = 0.7, the pit is comparatively constrained and shallow, focused primarily in the core area with a more defined foundation. As RF increases to 0.8 – 0.9, the pit extends progressively, with enhanced horizontal consistency and greater vertical

extent, although the spatial extent persists moderately limited. At $RF = 1.0$, a consistent and explicitly delineated excavation geometry is established, with peak depths reaching around 3780 m elevation. However, it has a slightly smaller footprint than recorded in 2017. Additional increases to $RF = 1.1 - 1.3$ lead to further expansion, especially at the excavation boundaries and deeper elevations, although the general geometry stays uniform as a result of slope constraints. In comparison with the year 2017, the expansion is less significant, and more material is incorporated gradually instead of rapidly.

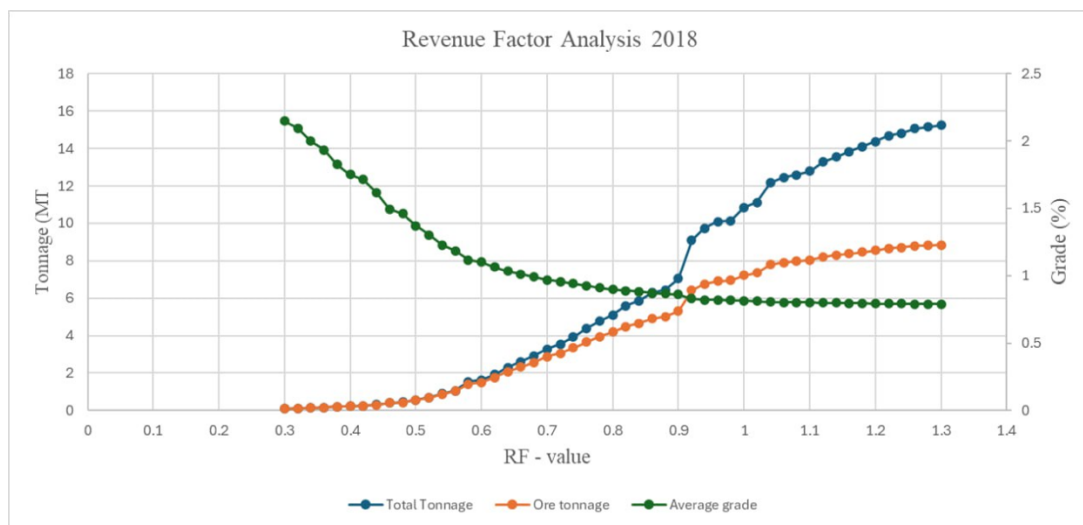


Figure 4.10. 2018 Revenue Factor Analysis

The RF evaluation for 2018 in Figure 4.10 demonstrates a comparable pattern to earlier years, with total tonnage escalating progressively as RF increases. Total tonnage increases from around 3.5 Mt at $RF = 0.7$ to about 15 Mt at $RF = 1.3$, while ore tonnage grows from near zero to roughly 9 Mt at higher RF values. By contrast, the average grade declines from approximately 2.2% at low RF values to about 0.75% at $RF = 0.9$ and remains the same with further increases in RF up to $RF = 1.3$. A transitional point is identified at $RF = 0.9-1.0$, where pit growth intensifies.

4.3.4 Analysis of Results for 2019

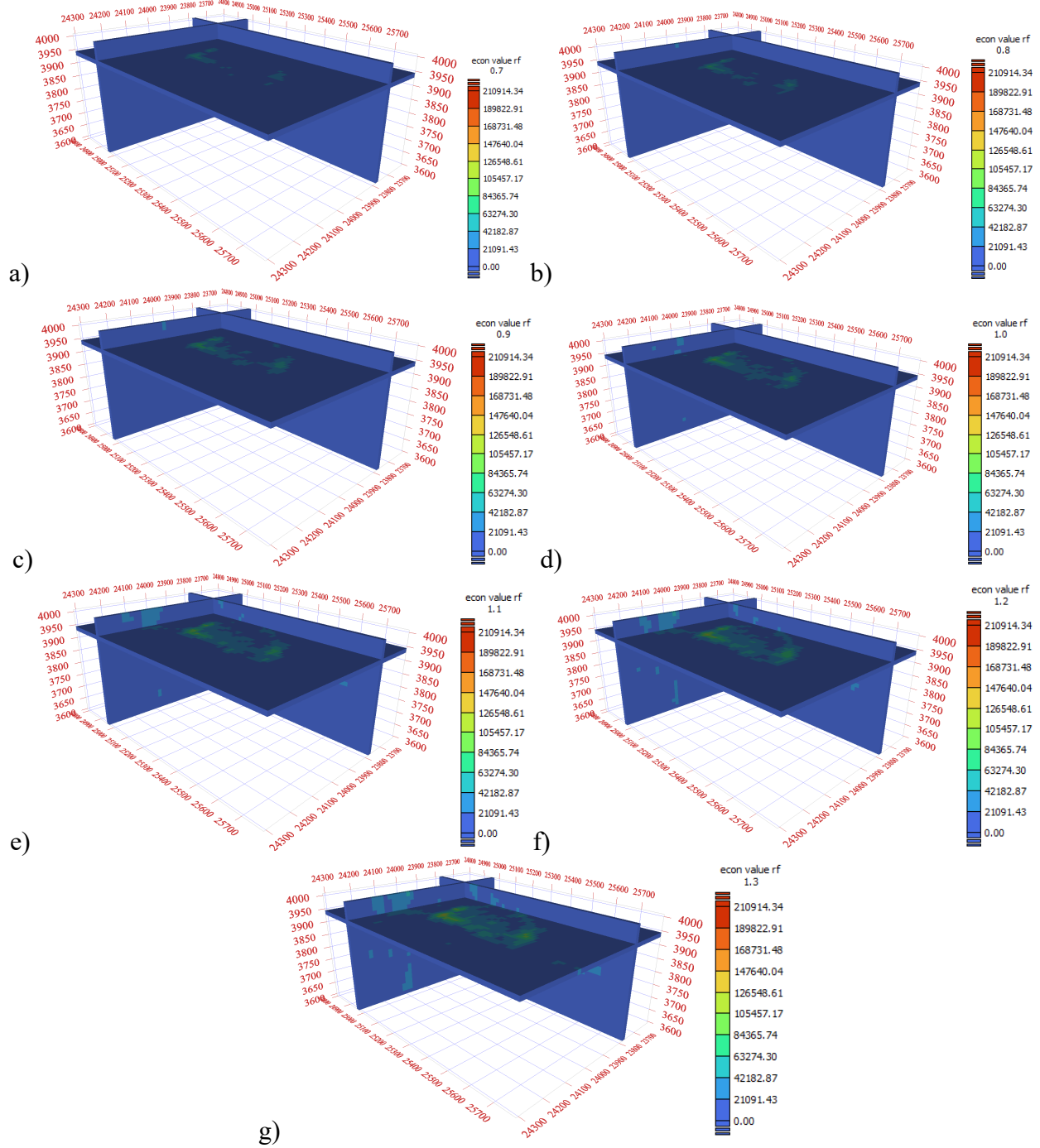
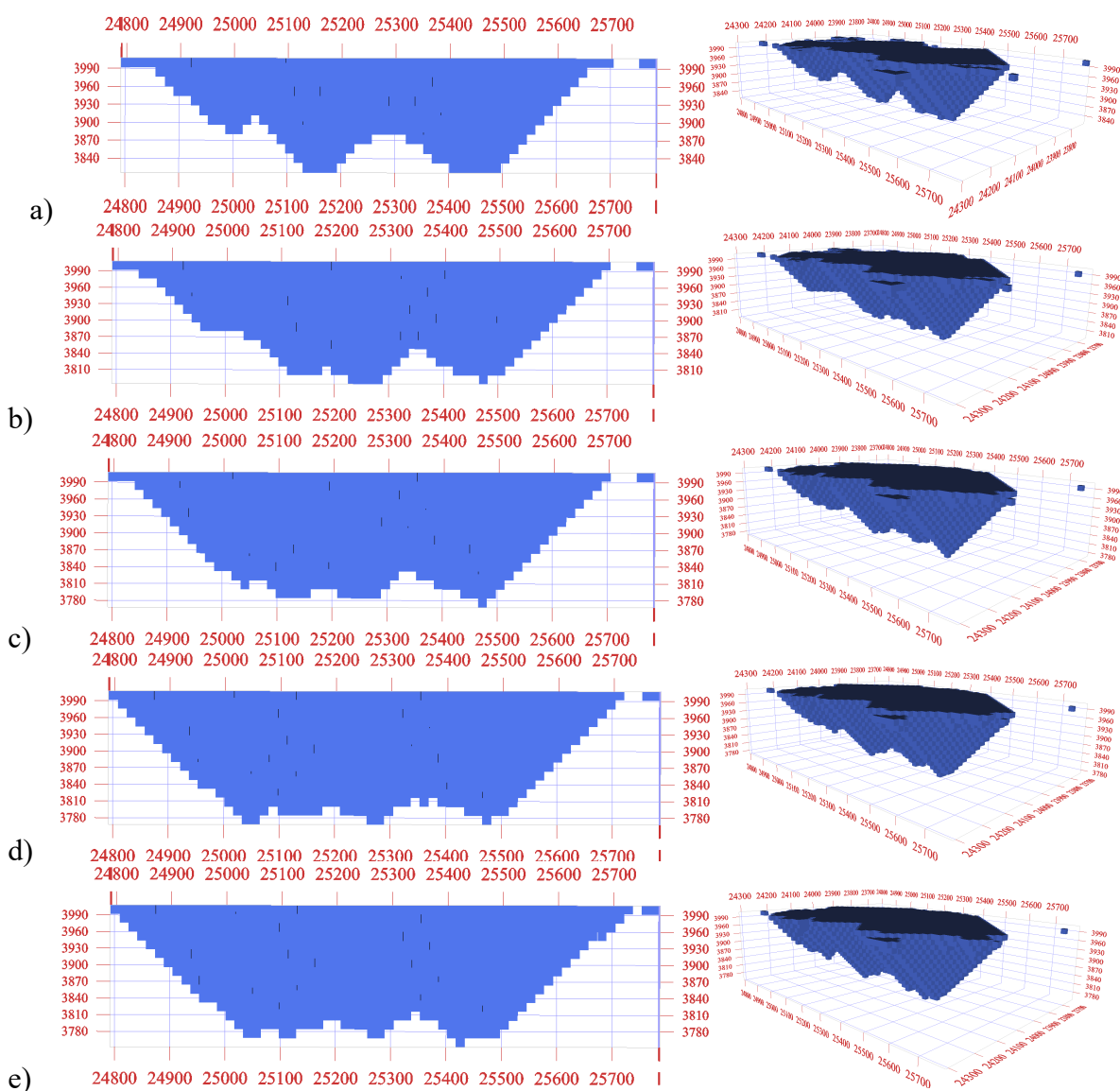


Figure 4.11. 2019 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The EBV allocation for 2019 demonstrates comparatively weaker economic conditions in comparison with 2017 and is governed by lower-value blocks. At $RF = 0.7 - 0.8$, only a highly limited and segmented core region exhibits positive values, primarily in the range of 20,000–80,000 USD, indicating limited economic viability. At $RF = 0.9$, the economic region becomes more apparent, but remains fragmented with values typically within the interval of 60,000–120,000 USD. At $RF = 1.0$, a continuous core is formed with EBV values generally attaining 80,000–140,000 USD. Notwithstanding that the extent is diminished and less pronounced than in more favorable periods. As the revenue factor increases to the range of 1.1 to 1.3, expansion takes place predominantly through the incorporation of marginal blocks valued at 40,000–100,000 USD, whereas high-value areas persist restricted and localized.



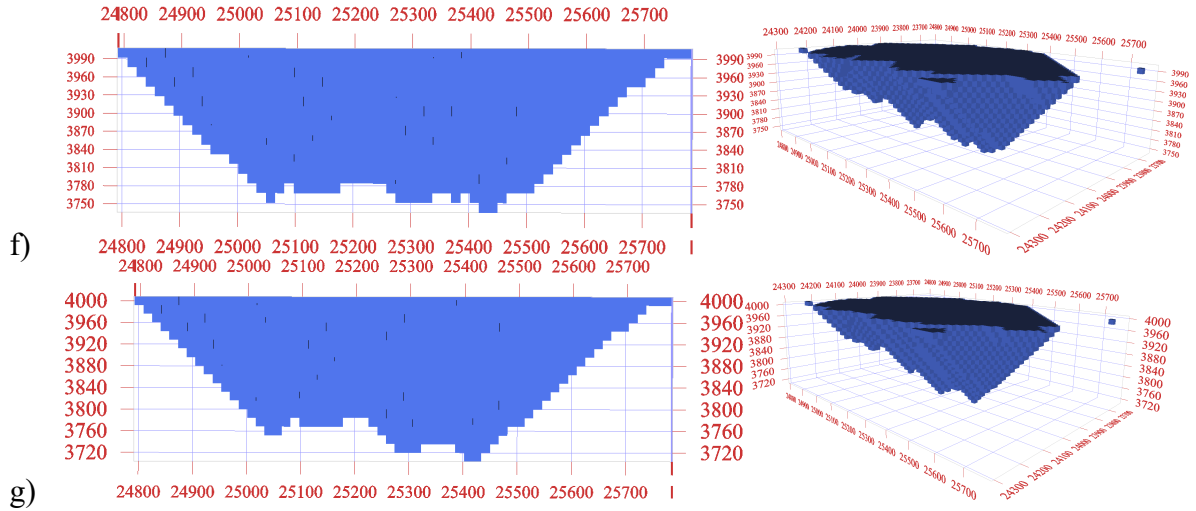


Figure 4.12. 2019 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The UPL findings for 2019 indicate a more restricted pit formation. It is consistent with reduced economic conditions in comparison with earlier years. At RF = 0.7, the pit is shallow and discontinuous, with limited horizontal extent and a confined base limited to the core region. Increasing the revenue factor to the range of 0.8 to 0.9 leads to a progressive increase in depth and enhanced consistency. However, the pit remains uneven with evident voids along the lower boundaries. At RF = 1.0, the pit achieves a consistent and stable structure, reaching depths of around 3770 – 3800 m elevation. However, it has a relatively diminished spatial extent and a less uniform base geometry. Subsequent rises to RF = 1.1 – 1.3 result in gradual enlargement mainly along the flanks and at depth. This smooths some inconsistencies but still preserves a comparatively confined form. In contrast to more favorable years, the expansion is more restrained and less evident, suggesting that a smaller number of additional blocks becomes financially feasible at higher RF values.

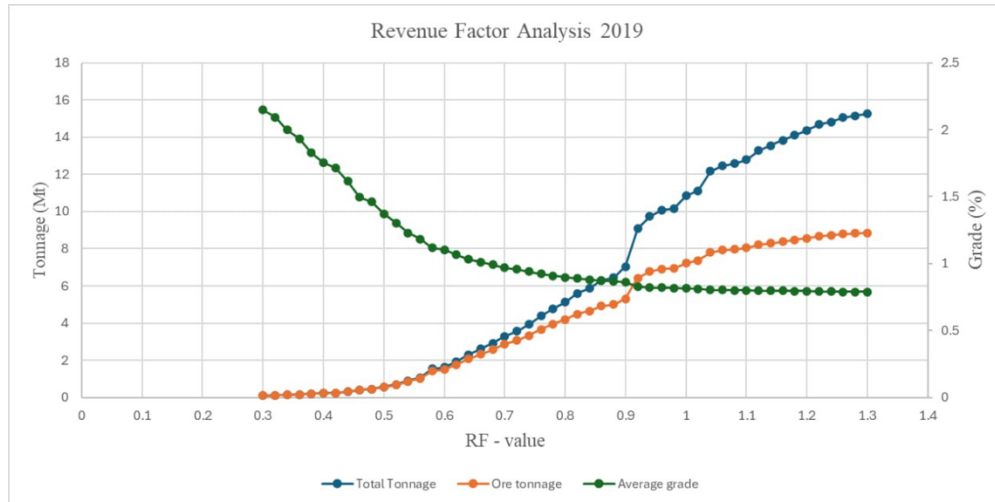


Figure 4.13. 2019 Revenue Factor Analysis

The RF analysis for 2019 illustrates a progressive shift from limited to economically viable pit conditions, with evident balances between tonnage and grade. Both total tonnage and ore tonnage gradually increase from minimal values, remaining around 3.5 Mt and 3 Mt at RF = 0.7, respectively. While the average grade is greatest at roughly 2.2-2.25%. As the revenue factor rises to 0.8 – 0.9, total tonnage increases consistently from about 6-7 Mt, and ore tonnage grows to approximately 5-6 Mt, while the average grade decreases significantly to around 0.75 – 0.8%. A key transition occurs at RF = 0.9 – 1.0, where total tonnage grows sharply from around 7 Mt to 11 Mt and ore tonnage reaches around 7.8 Mt, while the average grade becomes stable near 0.8%. Subsequent to this point, at RF = 1.1 -1.3, total tonnage persists to increase more progressively, attaining around 13 – 15.5 Mt, and ore tonnage rises to about 8 – 9 Mt, while the average grade declines marginally and stabilizes around 0.8 – 0.9%.

4.3.5 Analysis of Results for 2020

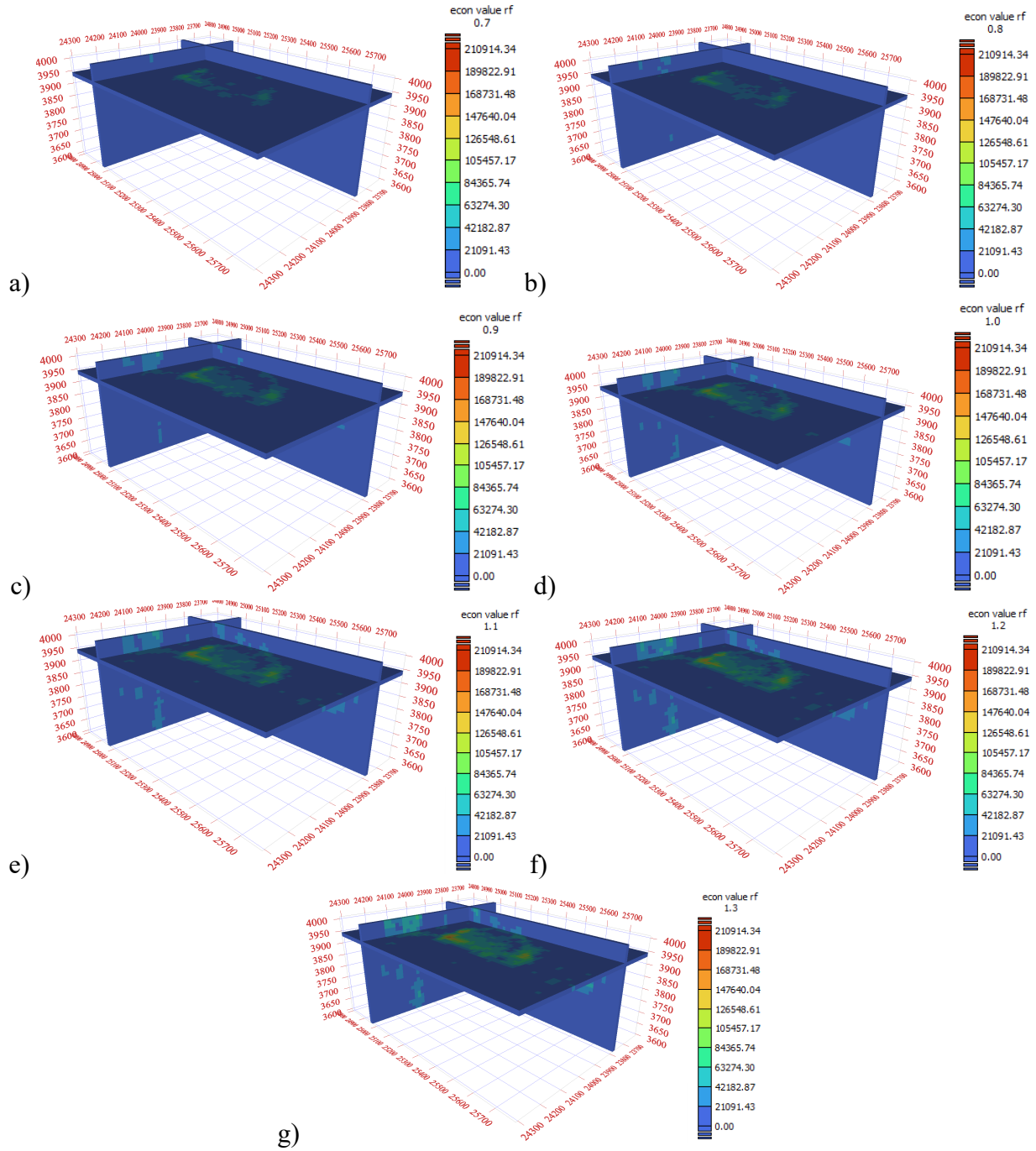


Figure 4.14. 2020 Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The EBV distribution for 2020 demonstrates the most pronounced economic outcome among all years, with a significantly greater share of high-value blocks. At a revenue factor in the range of 0.7 – 0.8, positive values are already more widespread than in earlier periods, with

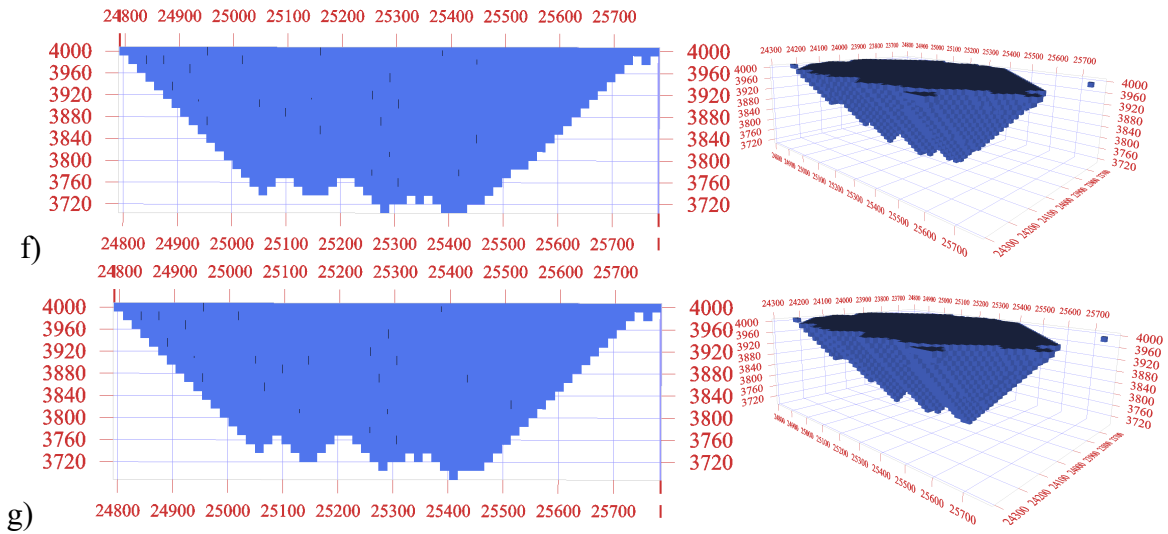


Figure 4.15. 2020 UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The UPL outcomes for 2020 illustrate the most expansive and effectively developed pit across all cases. It indicates improved economic circumstances and prior initiation of marginal blocks. At RF = 0.7, the pit shows a relatively wide surface extent, with a defined core depression. It suggests that economically feasible material exists even in conservative conditions. As RF increases to 0.8 – 0.9, the pit becomes deeper and extends laterally, creating a more continuous and smoother base, with a smaller number of discontinuities compared to earlier periods. At RF = 1.0, the pit achieves an optimal geometry, reaching roughly 3720 – 3750 m elevation, with a broad and consistent shape that maximizes both depth and horizontal extent. Increasing RF to 1.1 to 1.3 gives rise to considerable growth along the pit walls and at deeper levels. Its base evolves into more contiguous and extended, especially in the central and eastern sections. In contrast to previous periods, the additional material integrated at higher RF values merges seamlessly into the pit instead of appearing as isolated zones.

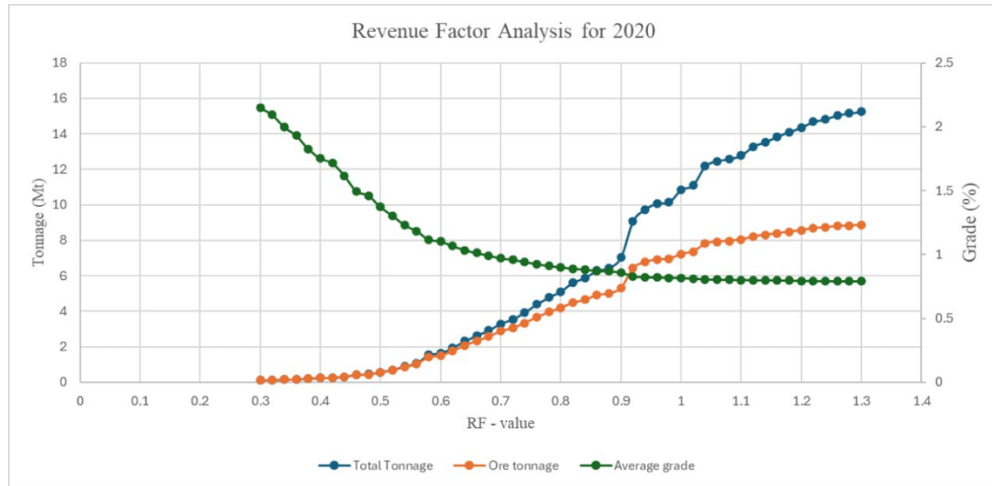
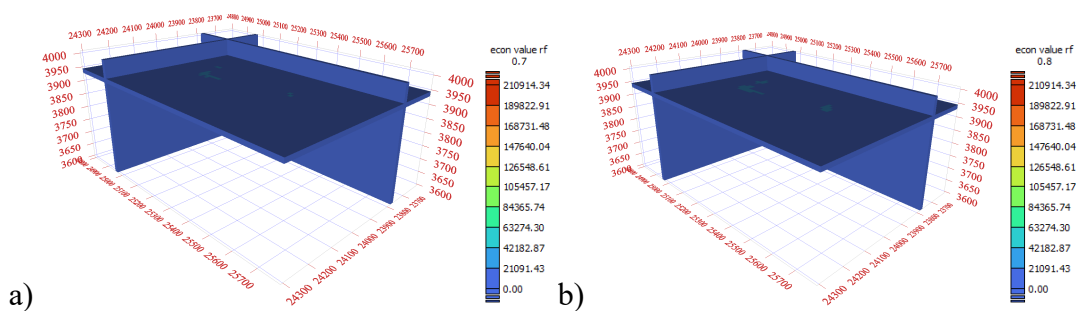


Figure 4.16. 2020 Revenue Factor Analysis

The RF analysis for 2020 demonstrates the most economically optimal conditions among all periods, with a significant increase in tonnage achieved at comparatively lower RF values. Overall, a similar pattern is observed with an increase in total tonnage and ore tonnage across the whole range of revenue factors throughout the five-year period. At $RF = 0.7$, the total tonnage equals around 3.7 Mt, and ore tonnage is about 3.5 Mt; at the same time, the average copper grade declines by approximately 1%. The highest peak of average grade is observed at approximately 2.3%. A significant shift takes place at $RF = 0.9 - 1.0$, where total tonnage rises rapidly from around 7 Mt to 11 Mt and ore tonnage attains around 5.8 Mt to 7.8 Mt, while the average grade stabilizes near 0.75-0.8%. Beyond this range of $RF = 1.1 - 1.3$, total tonnage continues to grow, reaching around 15-15.5 Mt, and ore tonnage grows to about 8.5-9 Mt, while the average grade stays relatively stable at ~0.8%.

4.3.6 Analysis of Results of Avalos and Ortiz (2020)



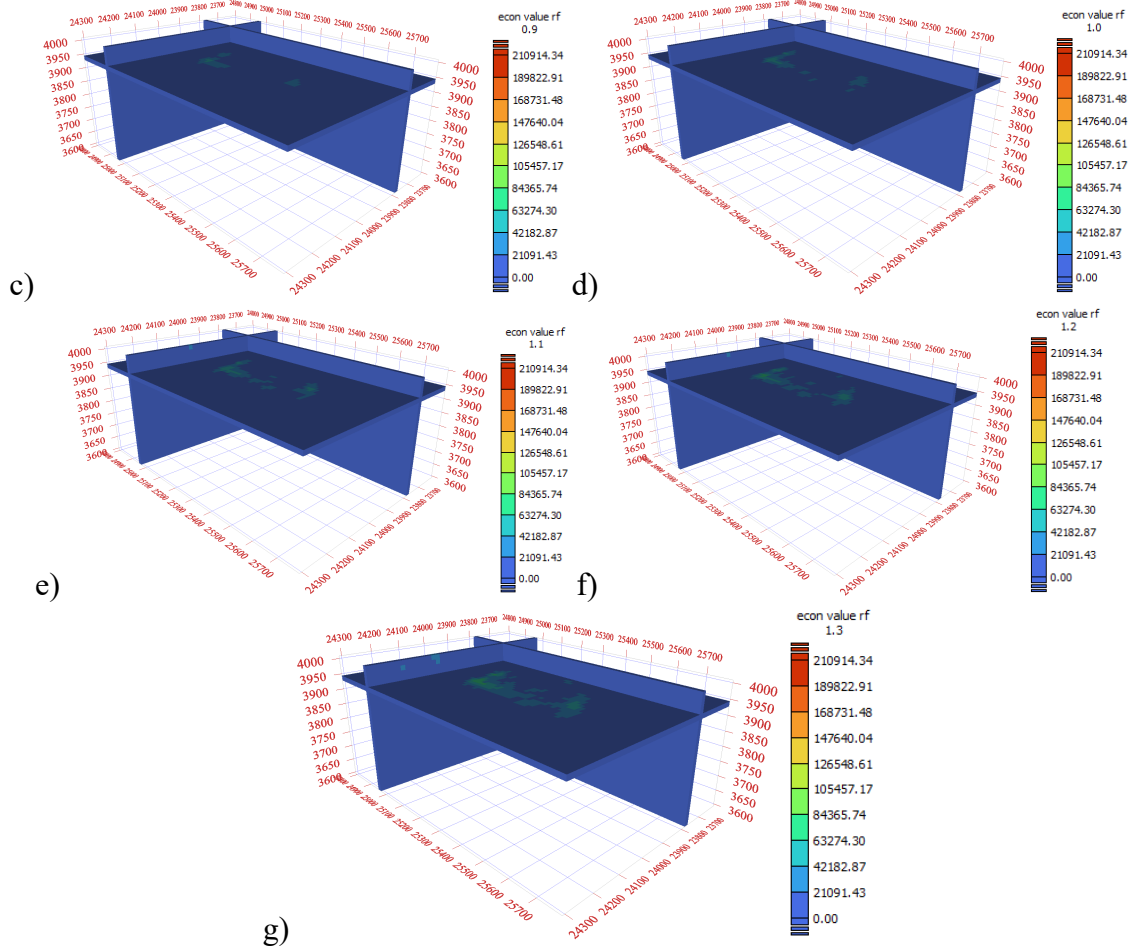
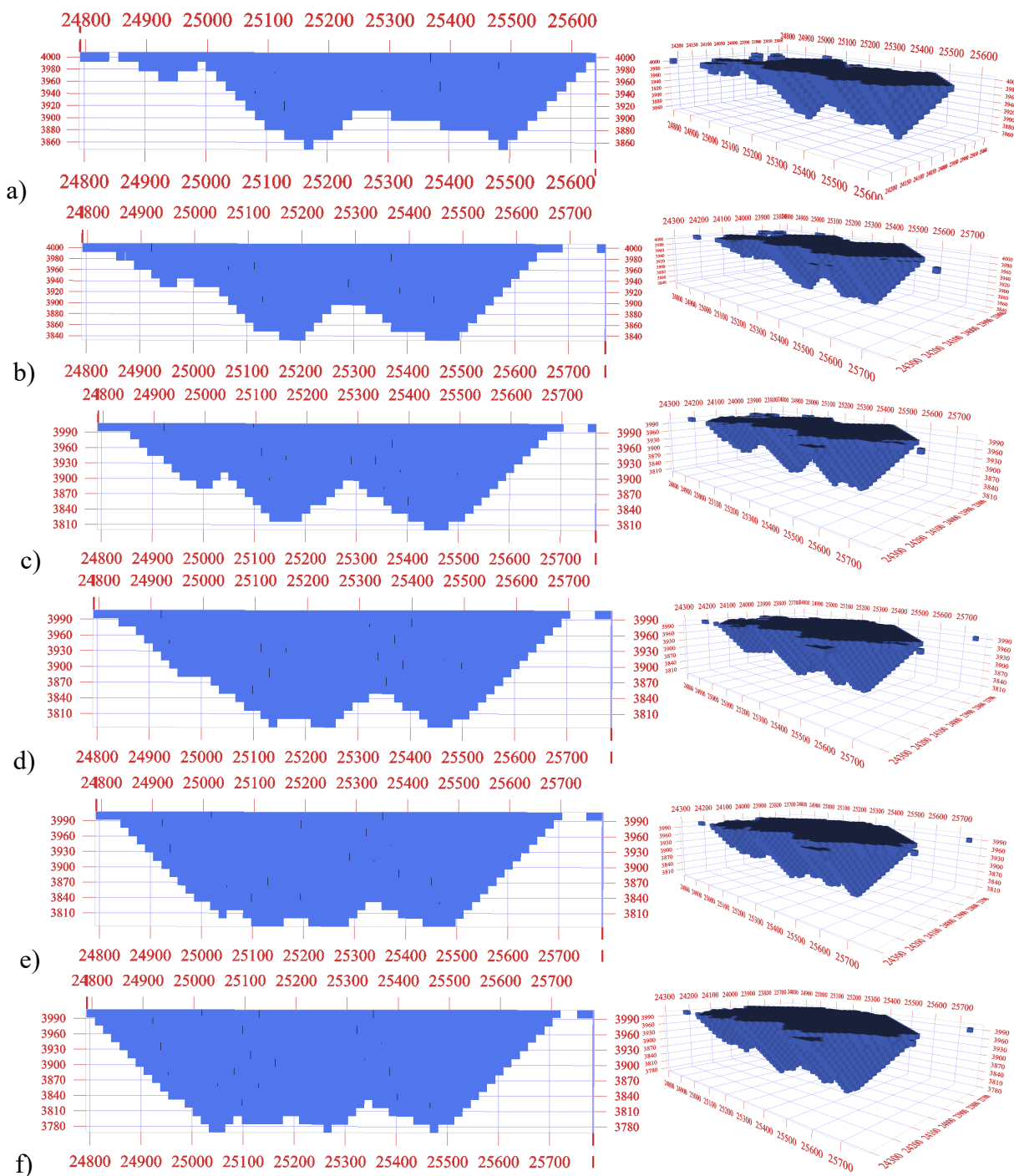


Figure 4.17. Avalos and Ortiz (2020), Economic block value with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The EBV results derived from the Avalos and Ortiz (2020) reference model indicate a more aggregated and conservative economic allocation compared to Kazakhstan's economic scenarios. It reflects the original economic assumptions applied in the initial study. Across all RF values from 0.7 to 1.3, EBV values range from 0 to around 210,000 USD. However, predominance of blocks persists within lower to moderate-value ranges (20,000-120,000 USD). At RF = 0.7 – 0.8, only a limited and fragmented core area demonstrates positive economic value, typically around 20,000-60,000 USD, suggesting constrained profitability under conservative conditions. As RF escalates to 0.9, a more continuous economic core starts to develop. Values increase to roughly 80,000-120,000 USD, although the spatial scope continues to be restricted. At RF = 1.0, the economy develops into a defined region, including the majority of profitable blocks clustered in the central

region. It reaches 100,000-140,000 USD, while high-value blocks exceeding this interval are limited. Additional rises to $RF = 1.1 - 1.3$ lead to gradual expansion, mainly including marginal blocks. Their values are under 100,000 USD, and there is no significant increase in high-value regions.



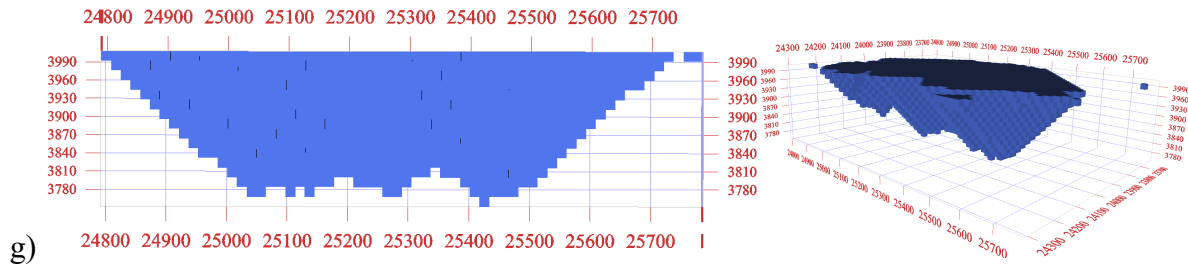


Figure 4.18. Avalos and Ortiz (2020) UPL with different RF values: a) RF = 0.7, b) RF = 0.8, c) RF = 0.9, d) RF = 1.0, e) RF = 1.1, f) RF = 1.2, g) RF = 1.3

The UPL results for the Avalos and Ortiz (2020) reference model indicate a stable but more irregular pit development compared to the assumed annual models based on Kazakhstan's settings. At RF = 0.7, the pit is shallow and fragmented, with material removal limited to a confined central zone between roughly 25,000-25,300 (Easting) and elevations exceeding 3860 m, indicating selective mining of solely the highest-value blocks. As RF escalates to 0.8-0.9, the pit expands laterally toward both flanks $\approx$ 24,900-25,000 and deepens locally to approximately 3840-3810 m. Although the geometry continues to be irregular due to discontinuous economic zones. At the base scenario RF = 1.0, the pit develops more uniformly, attaining depths near 3780-3810 m and expanding throughout most of the model width. Nevertheless, it maintains a non-uniform base by value fluctuation. Later increases to RF = 1.1 – 1.2 lead to significant vertical deepening to roughly 3780 – 3800 m and incorporation of low-grade blocks along the pit boundaries, especially toward the eastern side. At RF = 1.3, the ultimate pit boundary is realized, extending roughly 24,800-25,700 laterally and nearing minimum elevations of 3750-3780 m, with a more uniform and consistent shell.

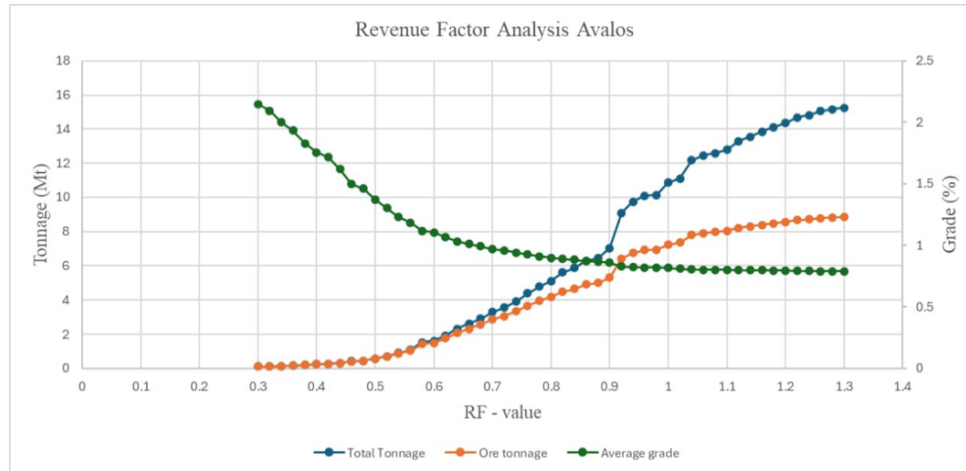


Figure 4.19. Avalos and Ortiz (2020) Revenue Factor Analysis

The RF analysis for the Avalos and Ortiz (2020) shows that at low RF values in the range 0.6 to 0.9, tonnage rises progressively, with total tonnage increasing from about 1.8-2 Mt to ~6.5 Mt and ore tonnage from ~1.8 Mt to ~5.5 Mt, while the average grade diminishes sharply from 1.1% to 0.9%. A noteworthy inflection arises around $RF \approx 0.9-1.0$, where total tonnage increases abruptly from ~ 7 Mt to 11 Mt. It indicates that a greater volume of low-grade material is incorporated. Beyond this, at $RF = 1.1-1.3$, a gradual increase is seen, reaching roughly 15 Mt total tonnage, and ~ 9 Mt of ore tonnage at the highest revenue factor. At the same time, the average grade becomes stable around 0.8%.

4.4 Comparison of reference case and KazMinerals Results ($RF = 1.0$)

To assess the impact of realistic economic conditions on pit optimization, findings acquired employing Kazakhstan-specific cost and price data for the period between 2016 and 2020, were compared with the reference case from Avalos and Ortiz (2020). The evaluation was performed at $RF = 1.0$, which represents the base ultimate pit in all scenarios.

Table 4.3 Total Tonnage ($RF = 1.0$)

Year	2016	2017	2018	2019	2020	Avalos
Total Tonnage (t)	15311462.4	18248089.6	14825574.4	14404915.2	17941913.6	10851942.4

Table 4.4 Ore Tonnage (RF = 1.0)

Year	2016	2017	2018	2019	2020	Avalos
Ore Tonnage (t)	8877772.8	9644544	8726016	8582246.4	9573990.4	7227084.8

Table 4.5 Average Grade (RF = 1.0)

Year	2016	2017	2018	2019	2020	Avalos
Average Grade	0.7885	0.7777	0.7910	0.7936	0.7785	0.8149

Table 4.6 Total Value (RF = 1.0)

Year	2016	2017	2018	2019	2020	Avalos
Total Value (USD)	120231396.2	172022821.1	118740837.1	108381286.6	182119247.2	48159705.76

To examine the reference case against the Kazakhstan scenarios comparatively, it is primarily essential to assess the preliminary economic inputs implemented in each model. In the Avalos and Ortiz (2020) reference case, the block economic evaluation was based on a copper price of 1.8 USD/lb, processing cost of 0.315 USD/lb, recovery of 80%, mining cost of 4.0 USD/t, and crushing-grinding cost of 10.0 USD/t. Alternatively, the Kazakhstan cases were analyzed by annual price and cost assumptions obtained from the KazMinerals dataset. The copper price fluctuated from 2.30 USD/lb in 2016 to 3.04 USD/lb in 2020, processing cost varied from 0.27 to 0.33 USD/lb, mining cost from 4.85 to 5.79 USD/t, and crushing cost from 8.08 to 9.65 USD/t. Compared with the original case, the Kazakhstan scenarios hence integrate higher copper prices with slightly higher mining costs, lower crushing costs, and, in most years, comparable or marginally lower processing costs, especially in 2016 and 2017. From an economic perspective, the increased revenue compensates for the higher costs, which results in improved block economic values and a lowered cutoff for separating ore from waste. This clarifies the reason Kazakhstan scenarios generate larger pits than the reference case. At RF = 1.0, total tonnage escalates from 10.85 Mt in the original case to between 14.40 Mt and 18.25 Mt in the Kazakhstan simulation cases, whereas ore tonnage grows from 7.23 Mt to 8.58-9.64 Mt. Simultaneously, average grade decreases insignificantly from 0.8149% in the reference instance to 0.777-0.794% in the Kazakhstan-specific models. It shows that additional tonnage is integrated with limited grade dilution. The more pronounced influence is detected in total pit value, which increases from 48.16

MUSD in the Avalos and Ortiz (2020) model to 108.38-182.12 MUSD under regional assumptions. These observations reveal that the larger pits and greater economic revenues are predominantly governed by the more favorable price-cost correlation. Nevertheless, it is expected to be highlighted that these Kazakhstan-based cost structures are still assumptions based on KazMinerals recordings, rather than accurate mine-specific operating costs. Therefore, it represents a scenario-based estimation of the extent to which Kazakhstan-relevant economic conditions influence UPL definition.

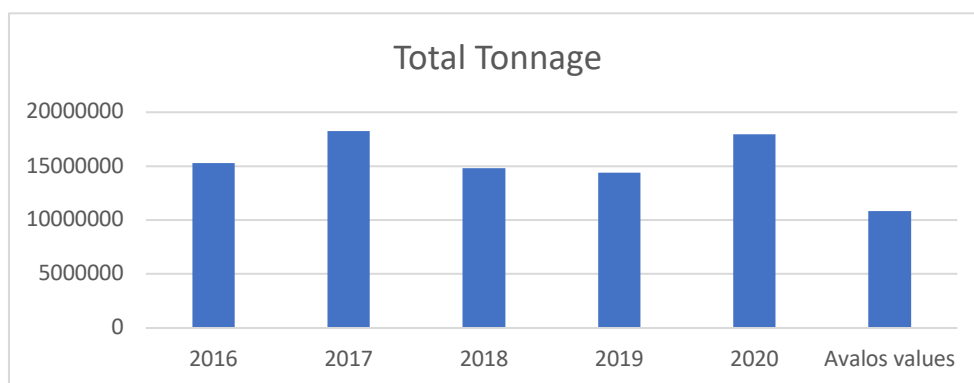


Figure 4.20. Comparison of Total Tonnage at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case

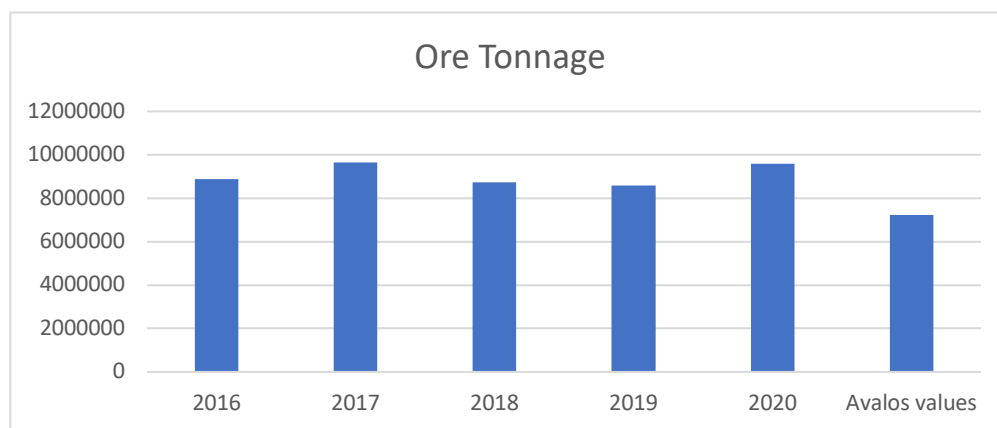


Figure 4.21. Comparison of Ore Tonnage at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case

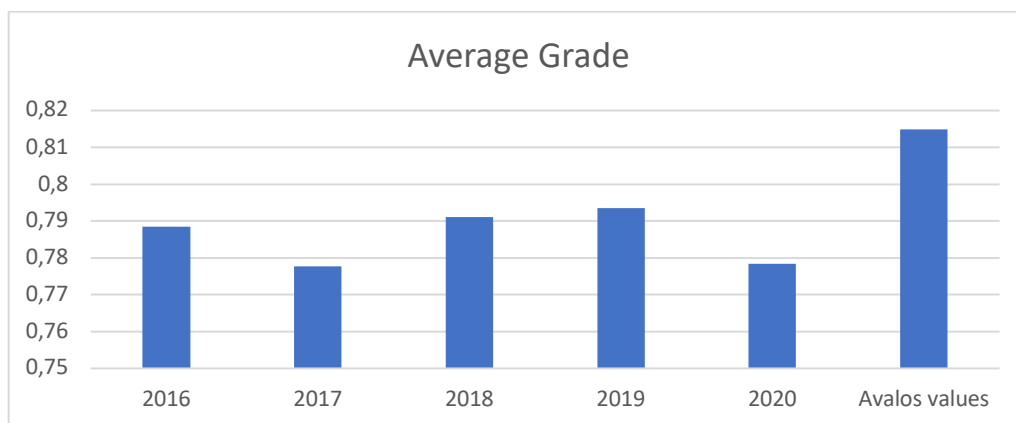


Figure 4.22. Comparison of Average Grade at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case

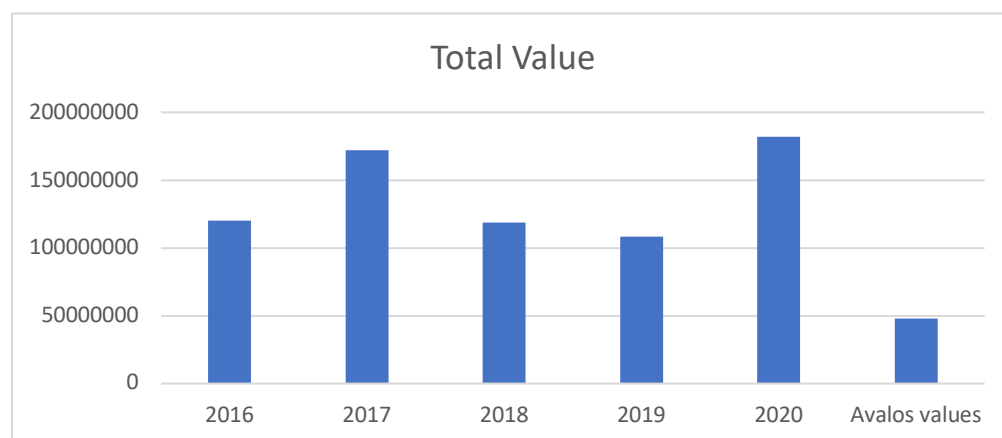


Figure 4.23. Comparison of Total Economic Value at RF = 1.0 for Kazakhstan scenarios (2016–2020) and Avalos and Ortiz (2020) reference case

The visual comparison demonstrates evident distinctions in the distribution and relative scale of the analyzed inputs across scenarios. Total tonnage shows significant variation, with maximum values arising in years associated with beneficial economic conditions, while the reference case stays lower. An equivalent trend is identified for ore tonnage, where Kazakhstan-based cases sustain an elevated recoverable ore across the entire period. In contrast, the average grade indicates minor fluctuation, staying within a narrow span. It implies that rises in tonnage are not influenced by substantial grade increases but rather by extended economic boundaries. The total value visualization reveals the most notable difference, with large amplification in a specific

period. It reflects sensitivity to price fluctuations and cost assumptions. Therefore, the comparison at $RF = 1.0$ shows that the primary impact of Kazakhstan-oriented economic parameters is not a significant increase in grade but an expansion of the economically mineable boundary and a notable growth in total project value.

5. Discussion

The findings verify that UPL determination is markedly governed by the economic assumptions applied in block assessment. Since the geological model, recovery, and slope constraints were fixed over all cases, the variations in pit size and value are assigned primarily to adjustments in copper price and operating cost variables. This reinforces the main argument of the thesis, that although pit optimization is mathematically specified by the pseudoflow/Lerchs-Grossmann approach, the final pit boundary is economically driven by the EBV model formulation.

The evaluation at $RF = 1.0$ delivers the most evident confirmation. Compared to the Avalos and Ortiz (2020) reference case, which yielded 10.85 Mt total tonnage, 7.23 Mt ore tonnage, 0.8149% average grade, and 48.16 MUSD total value, all Kazakhstan-based scenarios produced bigger pits and significantly higher total value. Total tonnage escalated to 14.40–18.25 Mt, ore tonnage to 8.58–9.64 Mt, and total value to 108.38–182.12 MUSD, whereas average grade minimally to 0.7777–0.7936% (Tables 4.3–4.6; Figures 4.20–4.23). This shows that the principal influence of Kazakhstan-specific economic data was not an enhancement in ore grade, but enlargement of the economically extractable pit limit.

Within the annual cases, the 2020 scenario delivered the most robust economic results, with 17.94 Mt total tonnage, 9.57 Mt ore tonnage, and 182.12 MUSD total value at $RF = 1.0$, whereas the 2017 scenario similarly generated an extensive pit with 18.25 Mt total tonnage and 172.02 MUSD. In comparison, 2018 and 2019 were lower performing, having total values of 118.74 and 108.38 MUSD, respectively. These discrepancies are consistent with the annual inputs presented in Table 4.2 and indicate that more beneficial price-cost dependencies yield wider profitable boundaries and higher project value.

The RF examination also reveals that pit expansion is not linear. In the annual RF charts (Figures 4.4, 4.7, 4.10, 4.13, and 4.16), the most evident shift happens near $RF = 0.9 - 1.0$, where total tonnage and ore tonnage rise rapidly while average grade levels off around 0.75–0.8%. This implies that a substantial volume of marginal material evolves into mineable merely upon a limit in economic parameters being achieved. Consequently, the outcomes explicitly respond to the thesis objectives by demonstrating the relationship between EBV composition, operating cost assumptions, and the final pit boundary within deterministic optimization.

Despite the approach's applicability to relative case analysis, a few methodological limitations should be recognized. The economic simulation in this research adopts a top-down approach, in which aggregated cost variables from KazMinerals data are allocated to the mining, processing, and crushing units. Although this method is applicable and relevant for scenario-based assessment, it simplifies the intrinsically complex composition of mining costs. In an ideal scenario, a bottom-up cost modeling approach derived from comprehensive operational variables, such as equipment, productivity, haulage distances, energy, electricity, and water consumption, and labor force distribution, would deliver a more precise and geographically specific cost evaluation. The present assumptions thus reflect generalized circumstances rather than definitive operational conditions. Furthermore, deterministic parameters were utilized for both Kazakhstan scenarios and the reference model. This was viewed methodologically as adequate for this research because it was structured as a controlled comparison of economic inputs under fixed technical conditions, not as a probabilistic prediction of future mine efficiency. The deterministic system thus acts here as a benchmark sensitivity tool for isolating the impact of local cost and price assumptions on pit geometry. This type of approach does not capture the volatility that characterizes actual mining systems, where inflation, exchange rates, fuel price changes, and operating inefficiencies can all influence the economic margin between ore and waste. Therefore, the optimized boundaries reflect economically reliable results only under the assumed variable sets, not across the full scope of uncertainty encountered in real mine planning.

Another limitation is the optimization based on undiscounted block values. While this is applicable to UPL formation in the Lerchs–Grossmann approach, it neglects the time dimension of mining economics. Production scheduling, capital timing, and discounting are not integrated. This means that a pit shell that is optimal in undiscounted conditions might not be optimal once

cash flow timing and capital allocation scheduling are evaluated. This shortcoming is pertinent in cases where contrasting scenarios that yield notably greater pits, since a component of their evident economic benefit may rely on when value is realized, instead of merely on how much value is present within the pit shell.

Ultimately, since the geological model, recovery, and geotechnical composition are derived from the Avalos and Ortiz reference case and preserved unchanged across the study, the results are most robust as a standardized analysis of economic sensitivity under constant technical conditions. These findings should not be extrapolated to all Kazakhstan ore deposits without evaluation. They may diverge in geometry, grade, continuity, metallurgical response, stripping conditions, and infrastructure environment.

These findings place the thesis within the literature gap determined in Section 2. Earlier research has shown that UPL optimization is mathematically rigorous but economically reliant on EBV parameters, while uncertainty evaluation in mine planning has concentrated more strongly on price and geology than on the internal structure of operating costs. The current results support that gap assessment by indicating that even within deterministic optimization, distinct cost compositions and price change pit geometry and overall project value. Therefore, the thesis contributes not by offering a novel optimization algorithm, but by illustrating the practical significance of region-specific economic inputs in the use of an existing optimization method.

6. Conclusions and Recommendations

6.1 Conclusion

This research was performed based on the modeled porphyry copper deposit of Avalos and Ortiz (2020). It represents a three-dimensional block model of $44 \times 62 \times 26$ blocks with block dimensions of $16 \times 16 \times 16$ m. The deposit shape, grade distribution, recovery, and slope constraints were preserved constant. Only economic variables were adjusted to examine the way Kazakhstan-based price and cost conditions influence UPL determination. This provided a structured foundation for comparing the reference model with the Kazakhstan-specific scenarios.

The research confirms that UPL evaluations are strongly sensitive to the economic assumptions incorporated in block assessment. Holding geological and geotechnical parameters

constant, changes in price and cost assumptions yielded predictable modifications in pit size, ore recovery, and overall financial value. These results suggest that the final pit is not merely a function of geology or geometry, but essentially driven by financial factors.

A principal finding is that region-specific economic simulation is essential for practical mine planning. The Kazakhstan scenarios produced systematically expanded pits and a larger project value compared to the reference case. This indicates that universal economic assumptions can underestimate the scale of economically recoverable material. Simultaneously, the comparatively minor fluctuation in average grade illustrates that pit expansion was obtained primarily by economic revaluation of marginal blocks instead of any alteration in the quality of mineralization.

The study also demonstrates the practical value of the deterministic pseudoflow approach as a relative planning technique. Within the scope of this thesis, the deterministic model was acceptable to demonstrate the first-order sensitivity of UPL to region-specific cost and price assumptions, even though more advanced stochastic methods would be needed to represent the full uncertainty of actual mining networks. By holding technical inputs constant and modifying only economic parameters, the analysis succeeded in isolating the effect of cost structure and price level in terms of pit optimization. In this regard, the thesis offers an explicit methodological foundation for analyzing how the regional economic context shapes strategic planning choices in open-pit mining.

Nonetheless, the observations are expected to be analyzed within the range of the applied assumptions. The cost structure adopted in this research is based on a top-down allocation obtained from a Kazakhstani copper company, and this constitutes a systematic approximation instead of a full operational costing model. Therefore, the outcomes should be interpreted as scenario-based interpretations into economic sensitivity, rather than as precise predictions of site-specific productivity.

6.2 Recommendations

Subsequent studies should concentrate on enhancing the economic accuracy of the modeling structure. The most crucial improvement would be the conversion from a top-down cost structure to a bottom-up costing procedure. It directly integrates operational factors such as haulage distances, equipment efficiency, energy, electricity, and water consumption, workforce, maintenance needs, and site facilities conditions.

Another recommendation is to integrate uncertainty explicitly into the optimization procedure. Since commodity prices, energy, and operating costs vary throughout time, stochastic techniques would deliver a more plausible foundation for assessing economic pit boundaries than fixed deterministic variables.

Additionally, research should expand the evaluation beyond undiscounted pit optimization. Combining the optimized shells with output scheduling and discounted cash flow or NPV-based assessment would strengthen the applied significance of the findings and deliver more solid ground for mine planning decisions.

Ultimately, the empirical base of the Kazakhstan scenarios should be extended. Employing cost data from various operations, commodities, and regions throughout Kazakhstan would enhance the inclusiveness of the results and enable a more resilient evaluation of how local economic conditions affect pit optimization.

Finally, this thesis work demonstrates that economically feasible input modeling is essential to reliable ultimate pit determination. Forthcoming planning investigations in Kazakhstan would benefit from more comprehensive probabilistic and operationally based economic simulations to reinforce stronger technical and investment choices.

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