

NAZARBAYEV UNIVERSITY

School of Mining and Geosciences

**Reliability-Based Assessment of Stope Wall Stability Using RS2: Influence of
Geomechanical Parameter Variability on Factor of Safety and Probability of Failure**

by

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ORIGINALITY STATEMENT

I, Darmen Zhakytay, declare that this submission is my own work. As far as I know, it includes no materials that have been published or written elsewhere, or which have been submitted to any other degree at Nazarbayev University, or any other place, unless the fact that the material was submitted to such a degree is acknowledged.

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ABSTRACT

Large unsupported voids are formed underground by open stope mining. Maintaining the surrounding rock in a stable state during the production cycle is the key geomechanical issue of the approach. The Mathews stability graph has been in use by engineers over decades and, in the process, all inputs are considered as definite numbers and give no hint on the sensitivity of the result in case such inputs are considered as uncertain.

This paper uses a reliability analysis model that combines RS2 two-dimensional finite element analysis with Monte Carlo Simulation to measure the sensitivity of variability in geomechanical inputs to changes in the Strength Factor (SF) and the Probability of Failure (PF) at every stope wall point. The case study is Panel 198 of Drill Drift 19, Horizon 14, Pobeda deposit at the Ridder-Sokolny mine in Kazakhstan, which was recorded using nine void-scanning reports between March and November 2019.

Five rock mass conditions of a variation of 40% in UCS and GSI were modelled at ten points of boundary query. Three main findings are made in the analysis. The first point is that the footwall base (Point 2) is located in a continuous tensile stress area with a mean SF of less than 0.48 even at the maximum rock mass modelled which results in $PF = 100\%$ in each of the five cases. The zone is a structurally critical zone irrespective of the quality of rock. Second, PF of the hanging wall mid-surface (Point 6) can be considered to be 60% at the Medium baseline (UCS = 130 MPa, GSI = 61) and zero only when both UCS and GSI are increasing by 10% above the baseline. Third, the UCS-GSI sensitivity index is maximum at Point 2 with a value of 58.9 and this index is the highest in the dataset meaning that the characterisation of the rock mass is what contributes to the risk of failure at that point more than any other quantifiable input.

A power-law regression equation between the collective UCS-GSI quality index and the site-level weighted PF is suggested to conduct preliminary design screening. The negative exponent (-8.42) is steep indicating the non-linear sensitivity of PF to rock quality at the stability boundary. X_{QM} decreased by 10 percent below the baseline has tripled PF_{site} 16.7 percent to 46.7 percent. The framework is applicable to open stope design, at Kazakhstani polymetallic mines and to any hard-rock operation in which rock mass parameters are characterised, with little sampling.

Keywords: open stope stability; Strength Factor; Probability of Failure; Monte Carlo Simulation; RS2; sensitivity analysis; reliability design; Pobeda deposit.

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I also appreciate the faculty of the School of Mining and Geosciences at Nazarbayev University and Rocscience Inc. who made RS2 available to me. The scanning reports here are the case study data created by the geomechanics team of the Ridder-Sokolny mine.

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1. INTRODUCTION

1.1. Background

The common method of extraction of steeply dipping tabular ore bodies in competent hard rock is open stoping. The ore is blasted and mucked at the stope floor, and the rock all around the stope hanging unsupported. It is the simplicity that makes the method economical at scale. It is also what makes stope wall instability such a consequential occurrence: it is not only a safety accident but also an economic one, overloading the ore stream with garbage, downgrading head, and in certain instances making neighbouring stopes temporarily unusable.

A case in point is Panel 198 of the Pobeda deposit of the Ridder-Sokolny mine. In November 2019, a laser scanning survey recorded in Report No. 135 established that the planned ore volume of 2,405 m³ had a measured blasted volume of 1,381 m³ with 1,178 m³ of the volume being temporary inactive reserves and the remaining 154 m³ of the material being blasted out of the planned contour. That extraneous breakage is an unintentional dilution of 6.4 per cent of the intended volume, which has a direct per-tonne penalty cost, the equivalent of which was measured by Pakalnis et al. (1995).

The Mathews stability graph has been the decision design tool used for decades to design such decisions as Panel 198. It was constructed by Mathews et al. (1980) using 26 hard rock case histories in Canada, Potvin (1988) added to the database and formalised the Modified Stability Graph on which practitioners now rely. The approach is viable. Its inputs are measurable by usual site investigation and output is a chart classification. What it is not able to do is to tell us how sensitive that classification is to measurement uncertainty. It only gives one stability label using a single set of input values and does not give error bars.

That restraint is not so trivial as before. Deep underground mines are deepening. The quality of measurements of rock mass decreases with increasing core recovery and the complexity of stress conditions. The Ridder-Sokolny mine is at a depth at which the horizontal stress is more difficult to determine than overburden pressure alone. There is a stope with the same plotted (N', HR) point in that setting which may have very different actual failure risks based on the quality of the input values known.

Part of this issue is solved by RS2 (Rocscience Inc.). It calculates a Strength Factor at each mesh node of the finite element model, which is the ratio of rock mass strength to induced stress. That provides a spatially resolved image of the closest walls to yield. Alipov and Adoko

(2025) tested RS2 to examine stope shape irregularity and unplanned dilution at the same Ridder-Sokolny mine which makes it applicable to the situation at the Panel 198. Nonetheless, a deterministic RS2 run remains a single set of inputs that yield a single output. The variability of parameters does not get carried forward to learn about failure risk.

This thesis bridges that gap by combining RS2 and Monte Carlo Simulation. It takes the case study of the variability of rock mass quality parameters through the model of finite element as the propagation of these parameters to the stope wall surface to determine the influence on the SF and PF on each side of the stope wall using the Panel 198 geometry and scanning data of Report No. 135. The outcome is a reliability map of stope stability of the actual mining conditions of the Pobeda deposit.

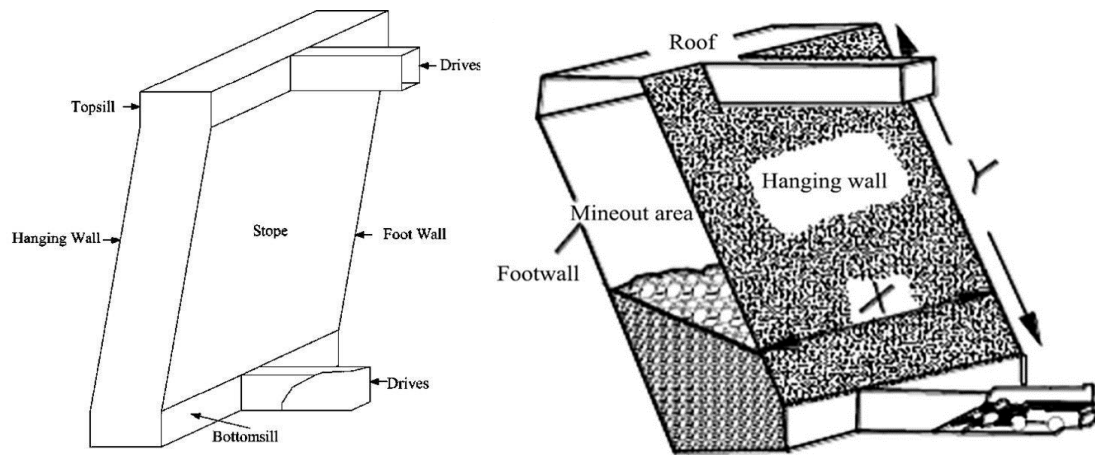


Figure 1.1. Diagrammatic representation of an open stope with measurements for the hydraulic radius, footwall, rear, and hangingwall (Singh et al., 2011)

1.2. Problem Statement

The design of open stope is to a large extent deterministic. The Mathews stability graph attributes a single value to each input. There is one Strength Factor of RS2. When SF is above 1.0, then stability is assumed. However, the factors that control the behaviour of stopes are not fixed: the stress ratio, the strength of rock mass, the nature of joints, and geometry of excavations change with the measurement and location within a rock mass.

In the absence of calculating PF, an engineer that discovers $SF = 1.2$ cannot tell whether it is a 1 percent or a 20 percent probability of failure. It all depends on the level of uncertainty of the inputs. That is what the Panel 198 case study makes one focus on: the scanning data reveals that there was indeed overbreak, but a single deterministic analysis would have deemed the stope as borderline stable.

1.3. Research Hypothesis

The hypothesis is that the effect of variability of geomechanical and geometric parameters on the Probability of Failure of open stope walls is significant, and it is not linear, and that a deterministic SF value will not reliably reflect the real risk of stability when there is uncertainty in the inputs. There are four additional specific sub-hypotheses that are tested:

1. The small percentage changes in UCS, GSI and hydraulic radius HR cause disproportionately large changes in PF particularly when the deterministic SF is close to 1.0.
2. The most influential parameters in the shallow-to-intermediate depth geometry of the Pobeda deposit regarding the probability of failures are rock mass quality parameters (especially UCS and GSI).
3. Stopes which PF are meaningful on input scatter with MCS, with a deterministic Mathews assessment which is either stable or borderline.
4. There is a measurable index of reliability β under which conventional deterministic safety factors are not adequate, in comparison with published geotechnical standards (USACE, 1997).

1.4. Objectives of the Thesis

1.4.1. Main Objective

The primary goal is to measure the impact of variability in the geomechanical and geometric input parameters on the stability of the open stope walls at the Pobeda deposit by constructing a reliability-based model in RS2 that charts the correlation between parameter variability, Strength Factor and the Probability of Failure.

1.4.2. Specific Objectives

Five specific tasks are performed in order to attain the main objective.:

1. Study the literature regarding open stope stability, empirical and numerical design methods, reliability methods and rock engineering.
2. The void-scanning dataset from Panel 198, Pobeda deposit, was chosen as a case study for analysis.
3. Build a representative RS2 model using rock mass parameters derived from the case study.

4. Run five rock mass scenarios using Monte Carlo Simulation and compute PF at ten query points around the stope boundary.
5. Derive a regression equation relating the combined UCS-GSI quality index to site-level weighted PF and evaluate its practical application.

1.5. Justification of the Research

The underground polymetallic industry relies on open stoping in Kazakhstan and stope wall stability is at the heart of the dilution issue. When the hanging walls, footwalls or back surfaces discharge material into the void, that rock gets into the ore stream as waste. The outcome is unintentional dilution - a direct loss of mill feed grade and a cost-per-tonne penalty that Pakalnis et al. (1995) measured and which mine planners at all Kazakhstan hard-rock operations are aware of. This can be felt in Panel 198 at the Ridder-Sokolny mine. A total of 9 laser scanning surveys in March-November 2019 were recorded with a total overbreak of 8,345 m³ against a scheduled extraction volume of 15,366 m³, an overbreak ratio of 54%. The material that was not uniformly distributed: the 3D Datamine void models reveal the discrepancy to be concentrated at the bottom of the stope and corners - the areas where the hanging wall and footwall surfaces meet and where the stresses are most concentrated. It is obvious what it implies. Dilution in those reports was mainly caused by stope wall sloughing at geometrically and mechanically critical places. The knowledge of the parameters of geomechanical processes that control that type of sloughing, and the sensitivity of the risk of failure to uncertainty in those parameters, is an explicit need in the management of dilution at this mine, and in analogous exercise in the Altai region.

In the standard design approach to stope wall stability issues - the use of the Modified Mathews stability graph or a deterministic RS2 model - gives a single stability label or a single Strength Factor value to a fixed set of inputs. Both methods fail to inform the engineer of the degree to which the risk of wall failure varies with the scattering of the inputs about their measured values as in practice they are certain to do. The significance of that gap is most accurately demonstrated in scenarios such as Panel 198: a stope that appears marginally stable in deterministic evaluation might have a significantly greater probability of failure when input variability is taken into consideration and that probability is what causes the scanning reports to slough. The measurement of this relationship - between input uncertainty, stope wall stability, and the consequent failure probability - is what warrants a reliability-based approach to this case.

In a research perspective, Adoko and Zhakipbayev (2023) demonstrated at the Maleyevsky mine in Kazakhstan that the same deterministic safety margin may be associated with extremely different reliability levels in rock domains of the same mine - two stopes with the same SF value may have very different actual sloughing risks. No published research has used a complete Monte Carlo framework in a variety of rock quality conditions at the query-point level in open stope walls in a Kazakhstani deposit, and has related the failure probability to the wall locations where overbreak has been observed. The thesis fills such a gap, tying the field anchor of the probabilistic analysis to nine scanning reports of Panel 198.

1.6. Scope of Work

The area of this research is stope wall stability. The open stope design does not support the hanging wall, footwall or back surfaces once the blasting has taken place until the cessation of the void. The main geomechanical difficulty of the approach is to keep such surfaces intact: the walls collapse, and failure is converted directly to unintended dilution; and in worst case scenarios, the nearby stopes will become temporarily inaccessible. Although this is important, standard practice still uses a single deterministic measure of stability - either of the Mathews graph or an RS2 Strength Factor - with no consideration given to the uncertainty of the inputs. This paper fills that gap by using a framework that relies on reliability to the problem of stope wall stability at a real deposit in Kazakhstan.

The scope can be characterized by six main activities. The literature review will first be on open stope wall stability, empirical stability methods, numerical modelling and reliability-based assessment in rock engineering. Second, nine void-scanning reports of Panel 198, Ridder-Sokolny mine, March-November 2019, were documented and analysed. Third, a two-dimensional RS2 finite element model of construction with Hoek-Brown parameters based on the mass of the case study rock, and ten query points on the surface of each wall placed at representative locations. Fourth Monte Carlo Simulation of 5 rock mass quality scenarios with a spread of +20% in UCS and GSI and calculating Probability of Failure at each query point. Fifth, development of a regression equation to correlate a summed UCS-GSI quality index to site-level weighted PF to serve as a preliminary stope design screening tool. Sixth, deterministic and probabilistic outcomes and comparison against real-world recommendations to manage stope wall stability at the Kazakhstan hard-rock mines.

2. LITERATURE REVIEW

2.1. Open Stope Mining and Wall Stability

Open stope mining is utilized for extracting massive and tabular ore bodies in competent rock. The stope floor is blasted to release ore, while the void's walls are supported. Hanging walls, footwalls and backs are unsupported from blasting until backfill or stope closure.

When walls become unstable, this causes waste rock to accidentally overbreak into the excavation, which reports to the mill with the ore. Villaescusa (1998) and Clark (1998) documented the economic damage that it causes. Pakalnis et al. (1995) estimate the direct per tonne penalty attached to each additional percent dilution. Choosing stope dimensions that limit overbreak while maximising the extraction of ore is one of the most consequential decisions in underground mine planning as illustrated by the Panel 198 data.

The quality of rock mass, induced stress, stope geometry, and orientation of discontinuity to the exposed wall affects stability. Their relationship is so complicated that there is no single design method that can be relied upon all the time; as a result of this, empirical, numerical and probabilistic methods have been developed over the past fifty years or so.

2.2. The Mathews Stability Graph and Its Evolution

Mathews et al. (1981) developed the original stability graph from 26 Canadian hard rock stope case histories. Potvin (1988) expanded the database to 175 cases, recalibrated the adjustment factors, and introduced the Modified Stability Graph. The method plots the Modified Stability Number N' against the Hydraulic Radius HR on an empirical chart, with zone boundaries derived from observed stope performance. N' is computed as:

$$N' = Q' \times A \times B \times C \quad (1)$$

Q' is derived from Barton's Q -system:

$$Q' = \left(\frac{RQD}{J_n} \right) \times \left(\frac{J_r}{J_a} \right) \quad (2)$$

where RQD is Rock Quality Designation, J_n is joint set number, J_r is joint roughness number, and J_a is joint alteration number. Factor A is the ratio of UCS to the maximum induced compressive stress acting parallel to the stope face. Factor B accounts for the orientation of the dominant joint set. Factor C adjusts for how gravity drives the failure mode. Hydraulic Radius is:

$$HR = \frac{\text{Stope surface area}}{\text{Stope surface perimeter}} \quad (3)$$

For a rectangle of width w and height h :

$$HR = \frac{W \times h}{2(W+h)} \quad (4)$$

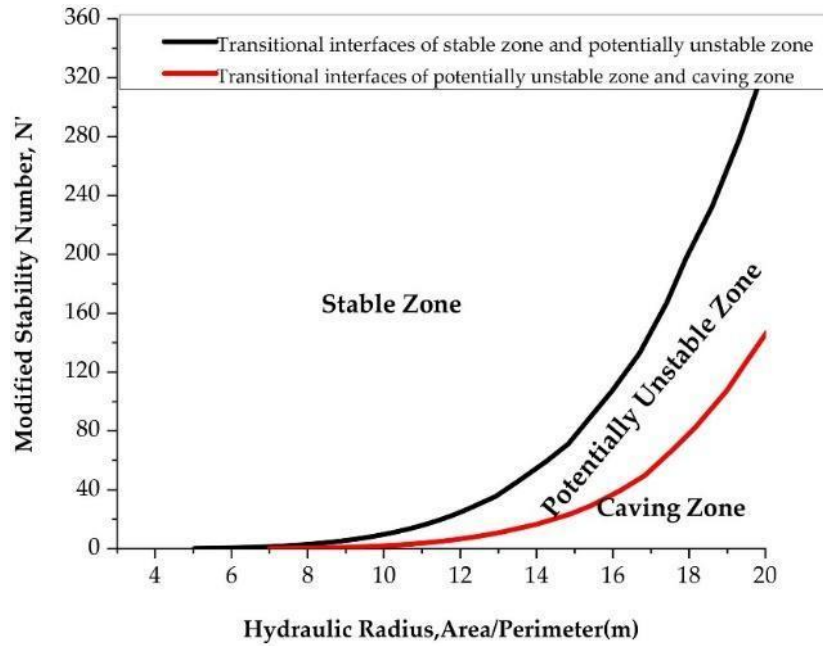


Figure 2.1. Potvin's Modified Stability Graph showing N' versus HR with stability zones (Zhang et al., 2024)

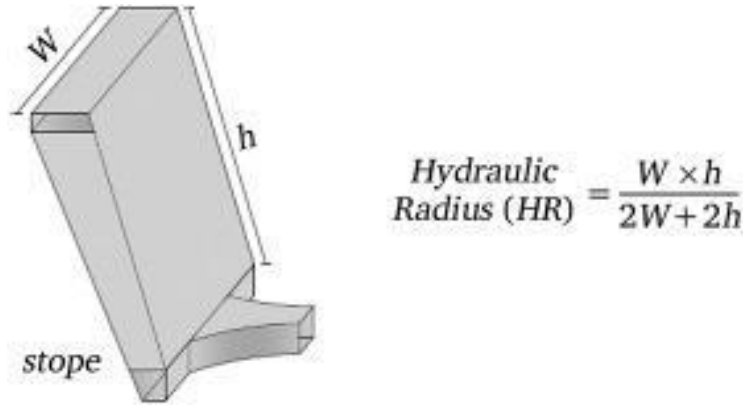


Figure 2.2. Calculation of Hydraulic Radius (HR) scheme (Szmigiel, 2024)

Nickson (1992) added cable bolt support case histories and derived empirical support design boundaries on the chart, turning it from a diagnostic tool into a support design tool. STEWART and Forsyth (1995) introduced four stability zones. Mawdesley et al. (2001) assembled more than 400 slope surfaces and used logistic regression to derive statistically rigorous isoprobability contours. Their boundaries are a step toward quantitative assessment, though what the contours measure is the probability of falling in a particular stability class given the

empirical database, not the physical probability of failure given uncertain inputs. That distinction is central to this thesis.

2.3. Stope Wall Failure Mechanisms and Controlling Parameters

There are three general types of open stope wall failures. Compressive failure due to stress is whereby compressive stress induced at the wall boundary is greater than the strength of the rock mass. It is generally manifested by spalling or buckling under high stress conditions and the depth is primarily determined by the ratio of imposed stress to the mass strength of the rock which is controlled by UCS and GSI. It is a dominant mode at depth, where stress concentrations in the vicinity of irregular stope geometries are intense (Maulana et al., 2024).

Structural failure is a phenomenon that is due to gravity and happens when blocks or wedges contained by ongoing joints fall off due to their weight. Most at risk are back surfaces and steeply inclined hanging walls. The existence of a kinematically viable release mechanism is pegged on the orientations of joint sets and the direction of dips with respect to the stope surface. This is specifically taken into consideration in factor B of the Mathews method, but in the form of one chart value, which cannot be used to fully represent the geometry.

The relaxation failure of tensile type is a special mode, significant when hanging walls in structures where the stope forms a region of decreased or tensile minor principal stress. This minimizes restriction on horizontal joints and encourages unravelling. This is most likely to happen with wide stopes and low stress ratios. Adoko et al. (2020) demonstrated that Factor A in the empirical approach is not a good measure of this mechanism, which is also true of the Pobeda deposit geometry where the competence of the host rock implies that Factor A does not impose any penalty and the relaxation of the stope perimeter is one of the actual stability concerns.

Rock of the Ridder-Sokolny mine is microquartzite (Protodiakonov coefficient $f = 13$), sericite microquartzite ($f = 8$) and sericite-chlorite-quartz rocks ($f = 8$), as illustrated in Report No. 135. This competence spectrum has an effect on the location of the failure mode that is most applicable.

2.4. Limitations of Deterministic Empirical Methods

The Mathews stability graph is over 40 years old and has a big validation base. Its inputs can be measured in the field and the output is an easy chart classification. In all that, its limitations

are important when the aim is to assign a label of stability, rather than actually measure the risk of failure.

All the inputs are held at one value each. RQD, J_n , J_r , J_a , UCS and the adjustment factors are all different in practice. Sainsbury and Sainsbury (2013) provided a vivid picture of what that is: a 20 mm shift in the joint spacing around the 100 mm RQD window switches the RQD between 0 and 100 percent, making nearly equal rock very different in terms of stability predictions. This is a structural weakness of the empirical approach because of such sensitivity to a measurement threshold.

Factor A approximates the stress effects based on far-field stress ratios. That is fairly accurate in simple geometries at moderate depths but less accurate in geometrically irregular stopes, in multi-stope environments, and at depths where patterns of stress concentrations form that cannot be described by a simple ratio (Khazhyylai et al., 2021). This limitation is applicable in the Ridder-Sokolny mine where the in-situ stress conditions are not easily determined at the depths of the mine.

No quantified PF is formed. Any two stopes with the same (HR, N') point may have quite different actual failure probabilities, depending on the uncertainty of their inputs. The case of Panel 198 brings this to life: the footwall and back both plot in the transition zone when evaluated deterministically but the label of a transition zone does not offer any method of differentiating a borderline-stable case and one that is highly likely to fail.

2.5. Numerical Modelling of Open Stopes

Underground mine design has become standardized on using finite element and finite difference methods. The programs like RS2 and FLAC can model redistribution of stresses and plastic zone formation but are not limited to pre-defined failure modes. They can deal with irregular geometries, complicated mining sequences, compressive failure mechanisms and tensile failure mechanisms, unlike the empirical charts.

The Strength Factor is calculated at each mesh node in RS2, as the ratio between the mass strength of the rock and the induced stress. In the case of the Generalised Hoek-Brown criterion, the SF represents the entire stress-strength interaction of a jointed rock mass, with the confinement effect on strength. Alipov and Adoko (2025) used this criterion in the Ridder-Sokolny mine to investigate the influence of stope shape irregularity on unplanned dilution, so their findings are the best available previous research to apply to the geometry of the Panel 198

geometry. Maulana et al. (2024) tested RS2 with a sill pillar stope, and established that finite element modelling better represents the redistribution of stress in geometrically complex cases compared to empirical numbers on stability.

The weakness of deterministic numerical modelling is the same as the empirical methods: a single input set results in a single output. Variability of parameters is never propagated. The computed SF is just one realisation of an inherently uncertain system.

2.6. Reliability Methods in Rock Engineering

Deterministic stability analysis gives one output but with one input. That is a good point to start with, but it has nothing to tell the engineer how the output will vary when the inputs are not known - and in rock engineering they never are. There are ways of reliability that can bridge that gap. Instead of taking strength, stress and geometry as a fixed value, they consider them as random variables with established distributions and propagate that randomness using the stability model to estimate the Probability of Failure.

The performance function for any stability problem can be written as:

$$g(X) = SF(X) - 1 \quad (5)$$

where X is the vector of uncertain inputs. Failure is the event $g(X) \leq 0$, and the Probability of Failure is:

$$PF = P(SF < 1) \quad (6)$$

PF is associated with the index of reliability β , with $\beta = -\Phi^{-1}(PF)$, Φ as the cumulative distribution function of the standard normal (Hasofer & Lind, 1974). The higher β , the lower the PF and the more reliable design. Such associations are not determined by a particular method of computation - they are what we are trying to find. The method used will be determined by the shape of the performance function.

The method employed in the study is Monte Carlo Simulation (MCS) and this is explained by the nature of the way RS2 functions. RS2 fails to give a closed expression of Strength Factor. The result of a complete finite element calculation is SF at that query point - it cannot be represented as an explicit algebraic expression of UCS, GSI, stress, and geometry. This is important since the methods of reliability that involve explicit performance function (like First-Order Second-Moment analysis) cannot be used in this case. MCS has no such requirement. It

uses the finite element model as the performance function evaluator and executes it repeatedly, each time using a different set of samples of input values.

MCS has simple mechanics. In every iteration, UCS, GSI, m_i , σ_v , k and stope geometry values are randomly selected according to their probability distributions. RS2 runs Stage 1 and Stage 2 with that particular input set and records the resulting SF at each query point. After N_{total} iterations, the PF at any query point is estimated as:

$$PF = \frac{N(SF < 1)}{N_{total}} \quad (7)$$

As N_{total} grows, the estimate converges. How many iterations are needed depends on the PF level of interest. In the range of 10 to 100 (range) in which the PF values are in the range of the critical query points in this study, stable estimates can be made with fairly small samples - Baecher and Christian (2005) indicate tens of iterations are usually adequate in this regard. The RS2 Monte Carlo workbook of Panel 198 includes ten iterations per scenario. Such a sample size is suitable to make comparisons of relative PF across conditions and to categorize query points as persistently versus conditionally critical; it is not aimed at estimating absolute PF at high precision.

A benefit of MCS over other analytical reliability techniques is that it can deal with non-normal distributions, non-linear performance functions, and multiple correlated inputs without extra mathematical complexity. The entire non-linearity of Hoek-Brown criterion is maintained at each iteration since the finite element model is not simplified. In exchange with this flexibility the method sacrifices the computational cost: each iteration involves a full finite element solution. This is dealt with by the built-in probabilistic module in RS2, within the same modelling environment as that of deterministic analysis and the geometry, mesh, and material definitions are the same in either type of run.

MCS-derived PF can be back-computed to give its corresponding index of reliability:

$$PF = \phi(-\beta) \quad (8)$$

This provides one summary measure of reliability at every point of query. In the case of $PF = 100$, β is undefined, it is certain that failure will occur. $PF = 0\%$, β is undefined and the interpretation can only be no failures in N samples instead of 0 failure probability.

The design point - the Most Probable Point of failure in the input space - can also be estimated with MCS output without a further optimisation. Gao et al. (2023) suggested that the design point of MCS is to choose the failed iteration that has the nearest input sampled vector (in

standardised probability space) to the mean input vector. The approach provides design point estimates identical to those of FORM but does not involve any further calculation than the MCS already completed. This is the reason why it is used in this thesis.

In the literature of Kazakhstan, MCS and the associated reliability methods have been used to address the problems of open stope design and drift support with findings that are directly relevant. When applied to the dilution graph method by Adoko and Zhakipbayev (2023), to the Maleyevsky mine, it was revealed that it was possible to have the same deterministic safety margin with very different failure probability across rock domains of the same mine - a result that directly leads to the fact that single-value deterministic assessment is limited. Another study conducted by Adoko et al. (2022) on the reliability of drift support in Kazakhstan revealed that the reliability index was mostly influenced by rock mass quality as compared to any other input parameter. Both findings encourage the use of a complete Monte Carlo model on the Panel 198 stope geometry with rock mass characterisation uncertainty the most important unknown and scanning data providing a real-world outcome on which to compare.

2.7. Synthesis and Research Gap

There is a steady divide in the literature. Variability of parameters of rock mass input is well recognized but there is no published work that has systematically measured the effect of variability of each separate input through a finite element model to influence both SF and PF at all stope wall surfaces in a real case of a mine in Kazakhstan. Stopes reliability work has been limited in scope, addressing only one performance aspect or only one testing method, without generating the picture of parameter sensitivity required to support risk-informed design.

The missing anchor is given by the Panel 198 scanning data. One example of the design and actual results difference is a documented case in which the overbreak volumes, which may be quantified, are exposed to the probabilistic stability results. The thesis also considers the scientific gap as well as the practical question of what happened to Panel 198 by integrating RS2 finite element modelling with Monte Carlo Simulation and basing the analysis on this real case.

3. CASE STUDY: PANEL 198, DRILL DRIFT 19, POBEDA DEPOSIT

3.1. Mine and Deposit Overview

The Ridder-Sokolny mine is in the Altai of East Kazakhstan, some 60 km south of the city of Ridder. It belongs to Kazzinc LTD which is a company of the Glencore group. The mine produces polymetallic ore, using steeply dipping vein and replacement-style ore bodies, in Palaeozoic metasedimentary and volcanic host rocks. Active horizons are about 700 m below the surface, with the workings in a regime of stress where the quality of rock mass, stope geometry, and the in-situ stress can make a significant contribution to the wall stability.

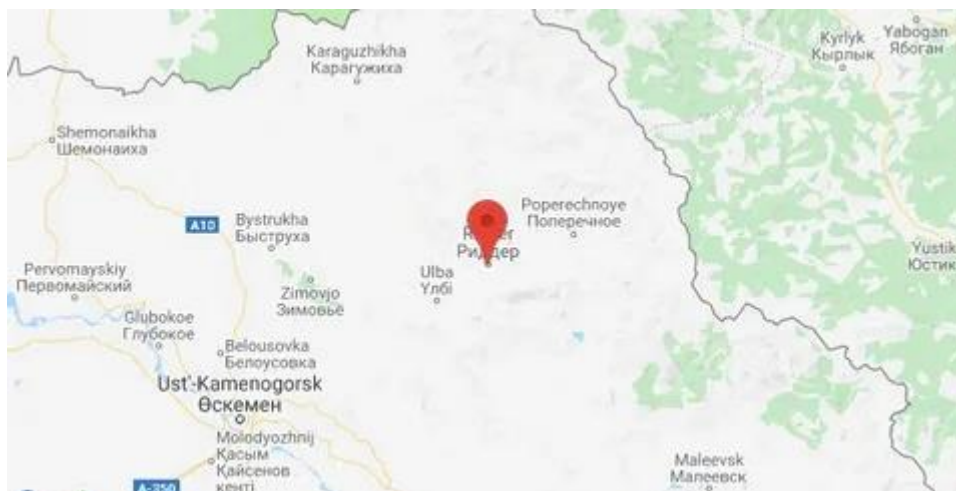


Figure 3.1. Location of the Ridder-Sokolny Mine

Pobeda deposit is located on the eastern side of the mine. It slopes north-westward and steeply down to the east. The level of study is horizon 14. The main ore-carrying lithologies are grey microquartzite (type 9) and sericite microquartzite (type 10), which are competent and jointed, respectively, and have the hardness coefficients of $f = 13$ and $f = 8$. The footwall and interlayer areas are filled with sericite-chlorite-quartz rock (type 13) which is a greenish-grey lithology of intermediate competence with $f = 8$. All nine scanning reports on Panel 198 have these three types of rocks.

Panel 198 is accessed from a series of drill drifts numbered 6, 8, 10, 12, 15, and 19. The panel was mined in stages until 2019, and the drills of the blocks represented different production blocks. The geomechanics group monitored the extracted void by laser void scanning at the end of each blast of production or at frequent intervals through extraction.

3.2. Void Scanning Methodology and Instrumentation

The Nine surveys were conducted with a Void Scanner 150 Mk3 laser instrument that was co-operatively operated by the mine geomechanics team and the survey department. The instrument directs a rotating laser beam into the stope void, and measures point positions in polar coordinates. This point cloud is then imported into Datamine Studio3 that produces a triangulated surface mesh of the real stope boundary. The mesh is superimposed to the shape of the design to create a plan-fact comparison. The mesh intersection is used to compute volumes within and without the design boundary.

The reports differentiate ore volume on the design boundary, ore blasted beyond the design boundary (overbreak dilution), inactive reserves (ore that remains in the design boundary) temporarily, and material losses. The consistent overbreak location on the same geometrical points on surveys is a useful indicator of the most problematic location of wall stability.

3.3. Sequential Scanning Results, 2019

The quantitative data from all nine scanning reports is compiled in Table 3.1. The reports are arranged chronologically.

Table 3.1. Panel 198, Horizon 14, Pobeda Deposit: Scanning Data Summary, 2019

Rep.	Drift	Design (m ³)	Actual (m ³)	Contour (m ³)	Overbreak (m ³)	Inactive (m ³)	Losses (m ³)	Dilution (m ³)	Dev. (%)
18	6	2,330	5,146	2,200	2,946	-	130	2,946	+121
38	12	556	507	245	262	311	-	262	-9
70	8	1,031	1,267	467	800	564	-	800	+23
75	8	1,245	1,723	906	817	339	-	817	+38
97	10	1,294	1,695	963	732	331	-	732	+31
105	15	2,003	990	610	380	1,393	-	380	-51
109	15	923	560	265	295	658	-	295	-39
122	19	3,490	1,841	1,366	475	2,124	-	475	-47
133	15	2,089	2,814	1,330	1,484	759	-	1,484	+35
135	19	2,405	1,381	1,227	154	1,178	-	154	-43
Total	-	15,366	17,924	9,579	8,345	7,297	130	8,345	+17

The panel placed 17,924 m³ of blasted material against a combined design of 15,366 m³, an overall overrun of 17%, which was practically entirely due to Drill Drift 6 where actual blasted volume was 221% of design. In Reports 122 and 135, at the other extreme, Drill Drift 19 exhibited significant underbreak with actual volumes of 53% and 57% of design respectively.

Drill Drift 6 report 18 expressly estimated a stand-up time of 28 months prior to the time when the crown would collapse. The same rock mass - microquartzite and sericite microquartzite with $f = 8$ to 13 - is described in all the nine reports indicating that similar geomechanical conditions are seen throughout the panel with the differences between drill drifts due to local variations in joint density and orientation.

The modelling case study is mainly based on report 135 (25 November 2019), the last survey of Drill Drift 19. The design volume was 2,405 m³, the actual blasted volume was 1,381 m³ (42.6% of design volume), with the overbreak contribution of 154 m³ - the lowest of the whole Panel 198 dataset, with a dilution ratio of 6.4% of design volume. The big idle reserve of 1,178 m³ shows that not all the extraction was done by the time of the survey. Geomechanics reports indicate that the longer the exposure time, the higher the chances of gradual degradation due to slabbing at the crown and walls, which can be aligned with the results of Chapter 5 on the conditionally stable position of the hanging wall mid-surface.

The spatial distribution of overbreak on the nine surveys is similar: 3D Datamine models indicate that the contour mismatches tend to be at the lower stope boundary and at corners transition and not on the primary hanging wall surface. This trend is most evident in Drill Drift 6, which had the highest overbreak (2,946 m³): the bottom footwall side and the junction of footwall and back are where actual geometry is most at variance with design. The query-point analysis of Chapter 5 indicates that that spatial distribution is consistent with Point 2 on the footwall base, PF = 100% in all cases and the main hanging wall surface is conditionally stable. The cross-section reveals that there is a stope base notch of overbreak where the material failed beyond the intended contour as is expected with the concentration of stress at the stope corner as determined by the RS2 query point analysis in this research.

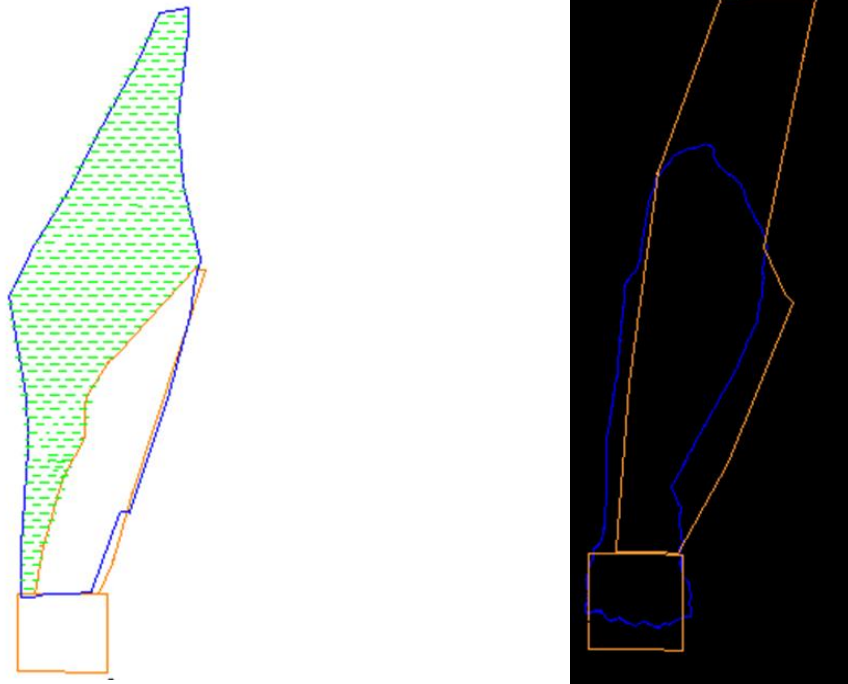


Figure 3.2. Transverse cross-section through drill drifts 6 and 19 showing design versus actual stope contour

3.4. Rock Mass Characterisation for Modelling

The Protodyakonov hardness coefficients of the scanning reports are used to obtain Intact rock strength data, by empirical conversion:

$$UCS \text{ (MPa)} = 10 \times f \quad (9)$$

In the case of grey microquartzite (type 9, $f = 13$), this would give $UCS = 130$ MPa. In the case of sericite microquartzite (type 10, $f = 8$), $UCS = 80$ MPa. The main ore-bearing layer is mainly type 9 microquartzite and as such, the Medium baseline of $UCS = 130$ MPa is selected in agreement with the published data of similar lithologies in the Ridder area (Alipov and Adoko, 2025).

The estimated Geological Strength Index was based on the descriptions of the rocks mass. Eight of the nine reports refer to the primary ore host microquartzite as being competent rock with moderate jointing and sericite type as moderate stability with jointing. These descriptions would place these on the Hoek-Brown GSI chart at a Blocky to Very Blocky rock mass and Good to Fair joint surface conditions resulting in a representative GSI of 55 to 67. In the

scenario of Medium, the baseline is GSI = 61. The intact rock parameter $m_i = 20$ is suitable in the Hoek-Brown database in quartzite-group rocks.

Table 3.2. Panel 198 Case Study: Rock Mass Parameters for the Five Monte Carlo Scenarios

Scenario	UCS (MPa)	GSI	m_i	m_b	s	a	E (MPa)
Weak (-20%)	104	49	20	3.213	0.003383	0.506	20,833
Weak-Medium (-10%)	117	55	20	4.009	0.006738	0.504	29,647
Medium (Baseline)	130	61	20	4.967	0.013124	0.503	39,618
Med-Strong (+10%)	143	67	20	6.154	0.025562	0.502	44,008
Strong (+20%)	156	73	20	7.625	0.049787	0.501	57,562

The variables m_b , s , and a correspond to Hoek-Brown constants derived from the work of Hoek et al. (2002). The disturbance factor is equal to $D = 0$ for controlled blasting of the perimeters. According to the Hoek-Diederichs equation, the rock mass deformation modulus is defined as E .

Microquartzite		Cericite_chlorite-quartz		Ore	
Hoek Brown Classification		Hoek Brown Classification		Hoek Brown Classification	
UCS of intact rock (MPa)	130	UCS of intact rock (MPa)	82	UCS of intact rock (MPa)	116
GSI	61	GSI	57	GSI	54
m_i	20	m_i	7	m_i	7
disturbance factor	0	disturbance factor	0	disturbance factor	0
Intact Modulus (MPa)	73000	Intact Modulus (MPa)	66000	Intact Modulus (MPa)	73000
Hoek Brown Criterion		Hoek Brown Criterion		Hoek Brown Criterion	
m_b	4.967	m_b	1.507	m_b	1.354
s	0.0131	s	0.00841	s	0.00603
a	0.503	a	0.504	a	0.504
Rock Mass Parameters		Rock Mass Parameters		Rock Mass Parameters	
tensile strength (MPa)	0.343	tensile strength (MPa)	0.458	tensile strength (MPa)	0.517
uniaxial compressive strength (MPa)	14.723	uniaxial compressive strength (MPa)	7.397	uniaxial compressive strength (MPa)	8.81
global strength (MPa)	39.766	global strength (MPa)	14.27	global strength (MPa)	18.786
modulus of deformation (MPa)	39617.949	modulus of deformation (MPa)	29847.687	modulus of deformation (MPa)	28245.133
Failure Range Envelope		Failure Range Envelope		Failure Range Envelope	
application	General	application	General	application	General
σ_{3max} (MPa)	32.5	σ_{3max} (MPa)	20.5	σ_{3max} (MPa)	29
Mohr Coulomb Fit		Mohr Coulomb Fit		Mohr Coulomb Fit	
cohesion (MPa)	9.313	cohesion (MPa)	4.159	cohesion (MPa)	5.57
friction angle (°)	39.804	friction angle (°)	29.519	friction angle (°)	28.669

Figure 3.3. Baseline materials properties in RSData

4. METHODOLOGY

4.1. Overview of the Research Approach

Two attempts of analyses can be run through the same RS2 model. The initial step is a sensitivity analysis that takes a deterministic one-at-a-time (OAT) form. The second tier is a Monte Carlo Simulation through five rock-mass quality scenarios from the geomechanical context of Panel 198. They create intel the other cannot. Combined, these present a comprehensive view of how parametric variability affects stability.

The process will represent 4 stages. The baseline RS2 model is built using deterministic mean inputs; reference SF's will be determined. Stage Two runs the OAT sensitivity analysis. Stage 3 assigns probability distributions and runs MCS through the five UCS/GSI scenarios. Stage four of the approach compares the results for identifying the design point and ranking the influence of parameters.

4.2. Numerical Modelling Platform: RS2

4.2.1. Overview of RS2

RS2 is a two-dimensional elastic-plastic finite element code, which was previously called Phase2. It is used and tested in tunnel analysis, slope stability, and open stope design (Lu et al. (2022)). Its inbuilt probabilistic analysis module can enable the same finite element model that is used in deterministic analysis to be run within the MCS framework to guarantee true comparability of the results.

There were three reasons as to why RS2 was selected. It was previously implemented on the Ridder-Sokolny mine geometry, thus the approach to setting up has been confirmed. Its built-in probabilistic capability implies that the geometry, mesh and material properties are the same in deterministic and probabilistic analysis. And SF is calculated at each mesh node, providing a spatially resolved image which is unprovable by a single global value.

4.2.2. The Strength Factor in RS2

At each mesh node, RS2 calculates the Strength Factor by dividing the rock mass strength by the stress that has been induced in that location. Although the model operates in two dimensions, all three principal stresses are taken into consideration in the SF calculation

(Alipov & Adoko, 2025). SF above 1 means the rock is stable; SF equal to 1 is incipient failure; SF below 1 means the material has yielded.

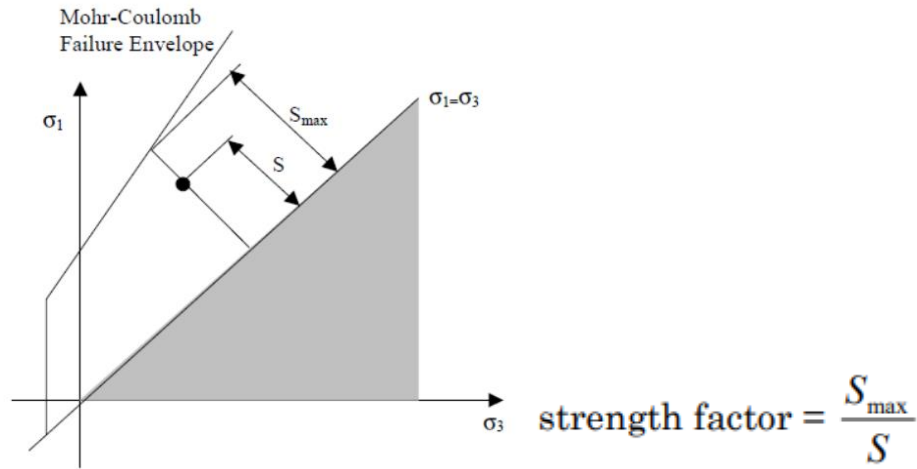


Figure 4.1. Graphical representation of Strength Factor as defined in RS2 (Rocscience Inc., 2025)

The deviatoric stress S is the distance between the current stress state and the hydrostatic line ($\sigma_1 = \sigma_3$). $S_{\max}$ is the distance from the hydrostatic line to the failure envelope at the same σ_3 level. $SF = S_{\max} / S$. When S exceeds $S_{\max}$, the material has failed.

4.3. Baseline Model Construction

4.3.1. Model Geometry and Excavation Staging

A two-dimensional cross-section of the Panel 198 stope is built in RS2. The geometry is a rectangle with height $H = 20$ m and width $W = 15$ m, placed within a homogeneous isotropic rock mass domain extending at least five stope widths from the excavation in every direction to keep boundary effects away from the zone of interest. Roller conditions are applied at all four model boundaries.

Excavation is modelled in two stages. Stage 1 initialises the in-situ stress field using vertical stress $\sigma_v = \gamma \times z$, where $\gamma = 0.027$ MN/m³ and $z = 540$ m, giving $\sigma_v = 14.6$ MPa. Horizontal stresses are set to $\sigma_h = k \times \sigma_v$ with $k = 1.2$, giving $\sigma_h = 17.5$ MPa. Stage 2 excavates the stope by deactivating elements inside the stope boundary and computing the resulting stress redistribution. Minimum SF on each wall surface is extracted from Stage 2.

Ten query points are placed around the stope perimeter to record SF, stress, and displacement at each Monte Carlo iteration. The coordinates of these points are given in the RS2 Monte Carlo Excel workbook and span both the hanging wall and footwall surfaces from base to crown.

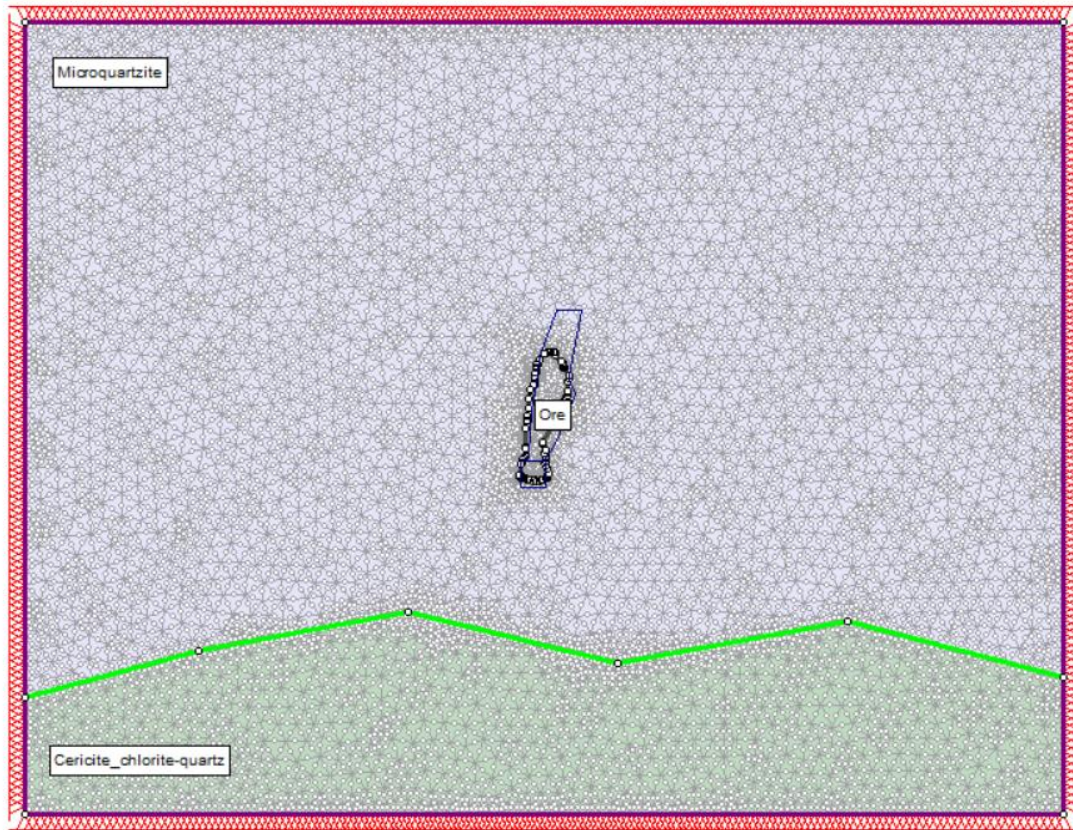


Figure 4.2. RS2 baseline model geometry and finite element mesh for the Panel 198 stope cross-section

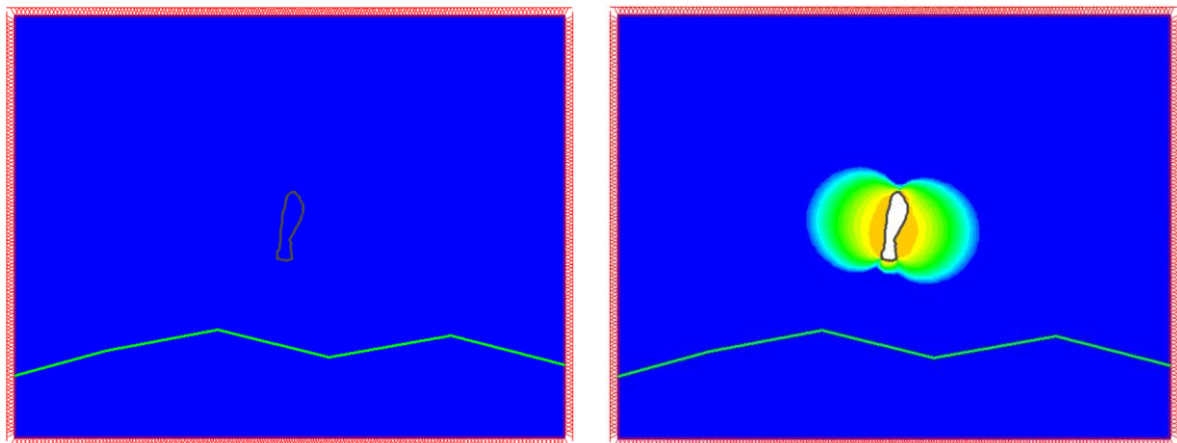


Figure 4.3. Two-stage excavation sequence in RS2: Stage 1 (in-situ stress) and Stage 2 (post-excavation)

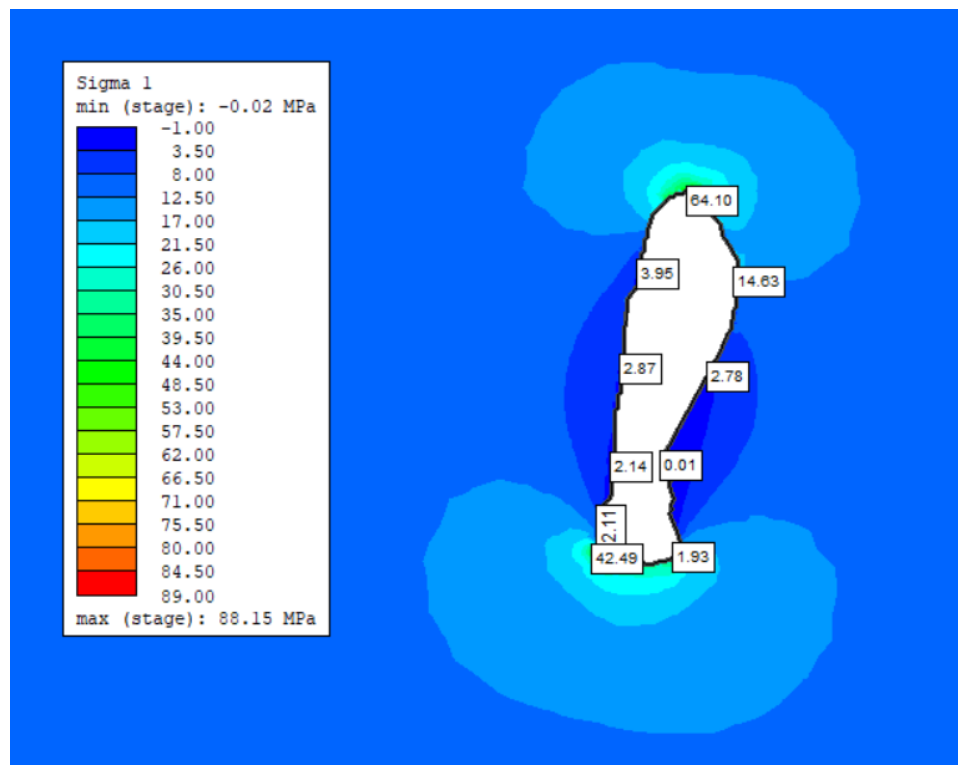


Figure 4.4. Sigma 1 Interpretation for the baseline model

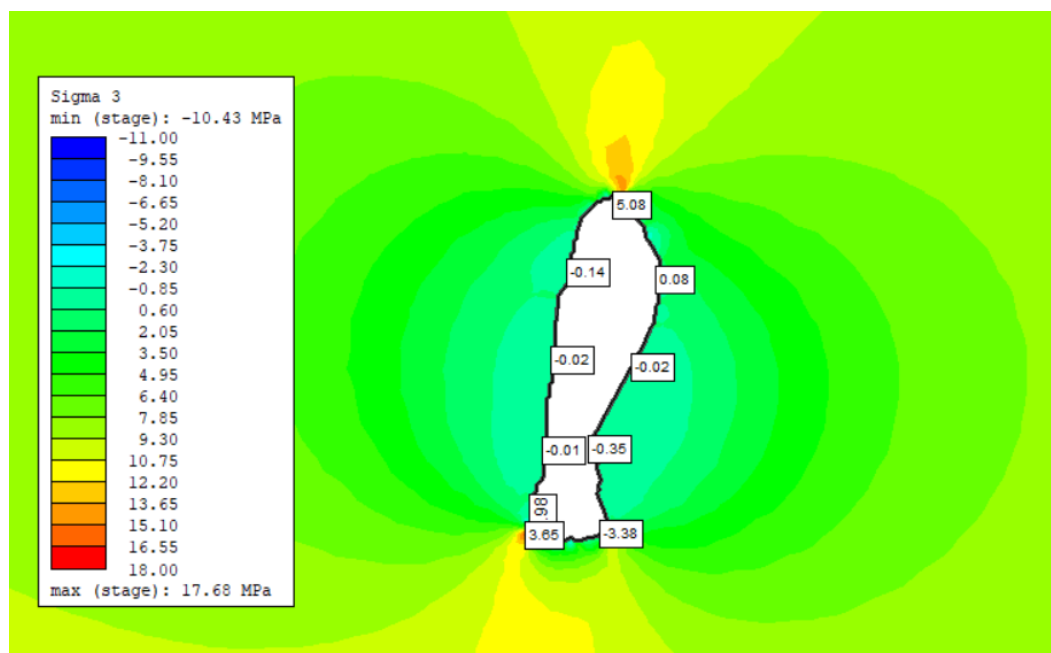


Figure 4.5. Sigma 3 Interpretation for the baseline model

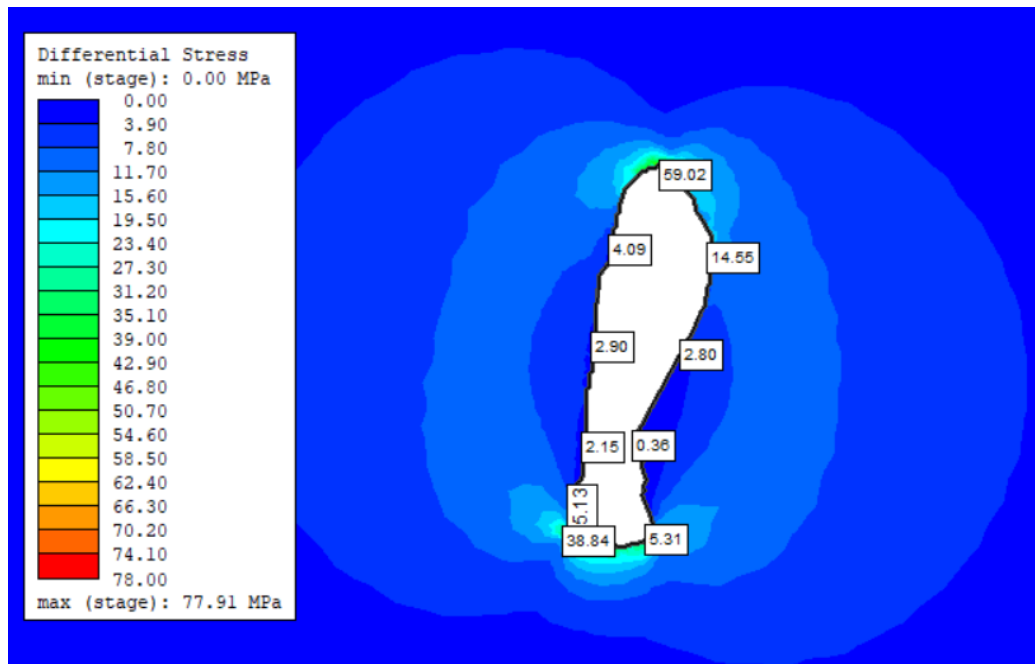


Figure 4.6. Differential stress Interpretation for the baseline model

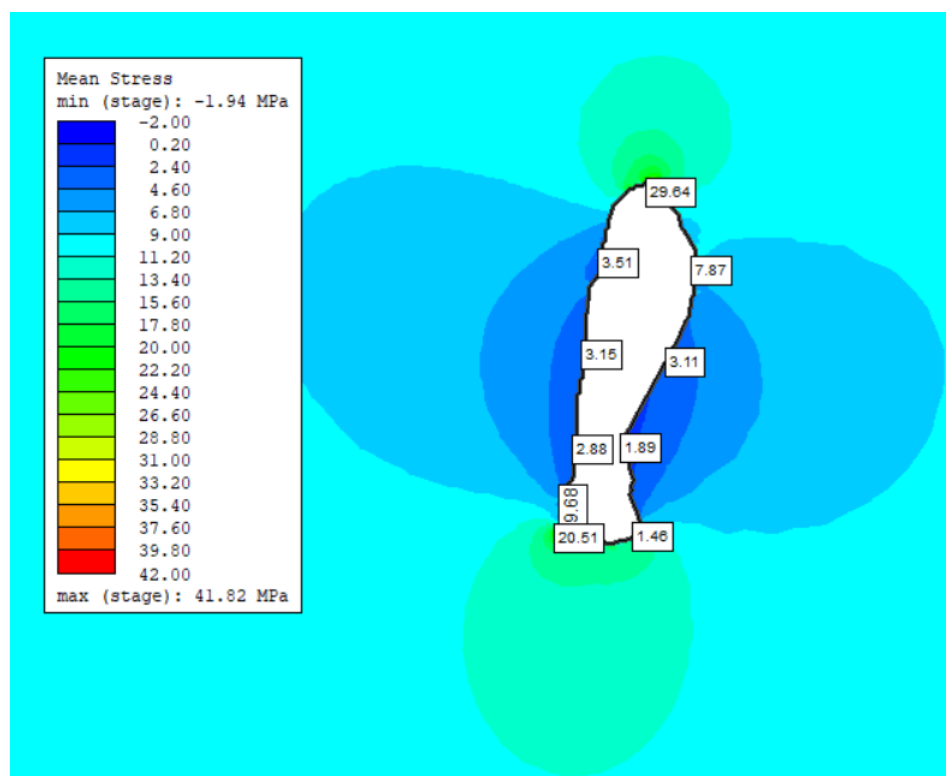


Figure 4.7. Mean stress Interpretation for the baseline model

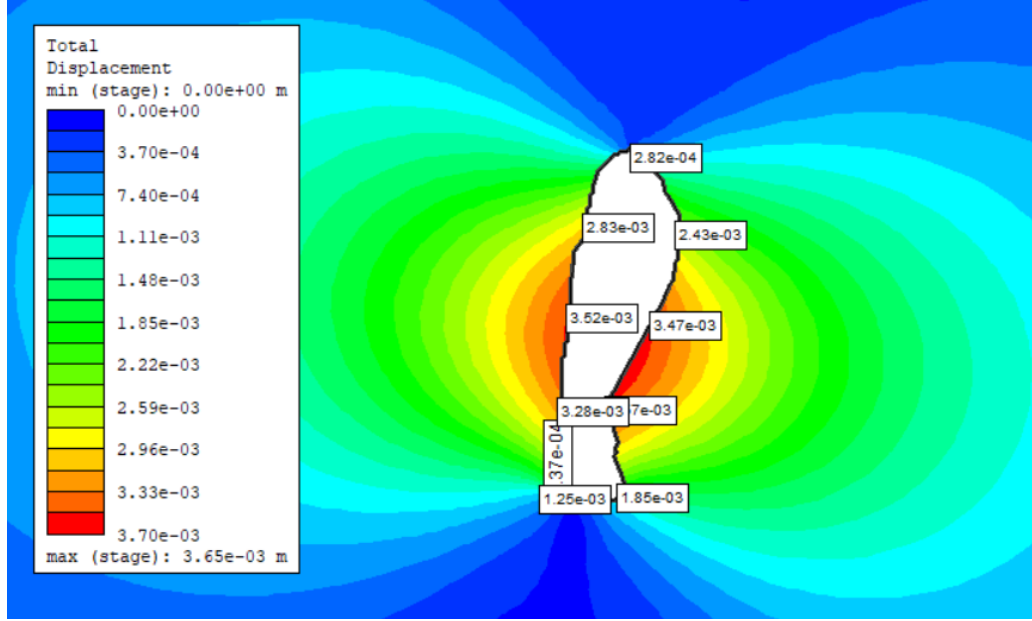


Figure 4.8. Total displacement Interpretation for the baseline model

4.3.2. Failure Criterion and Rock Mass Properties

The Generalised Hoek-Brown criterion is adopted as the constitutive model. The criterion is:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a \quad (10)$$

where σ_{ci} is intact rock UCS and m_b , s , and a are dimensionless rock mass constants (Hoek et al., 2002):

$$m_b = m_i \times \exp \left(\frac{GSI-100}{28-14D} \right) \quad (11)$$

$$s = \exp \left(\frac{GSI-100}{9-3D} \right) \quad (12)$$

$$a = \frac{1}{2} + \frac{1}{6} \left(\exp \left(-\frac{GSI}{15} \right) - \exp \left(-\frac{20}{3} \right) \right) \quad (13)$$

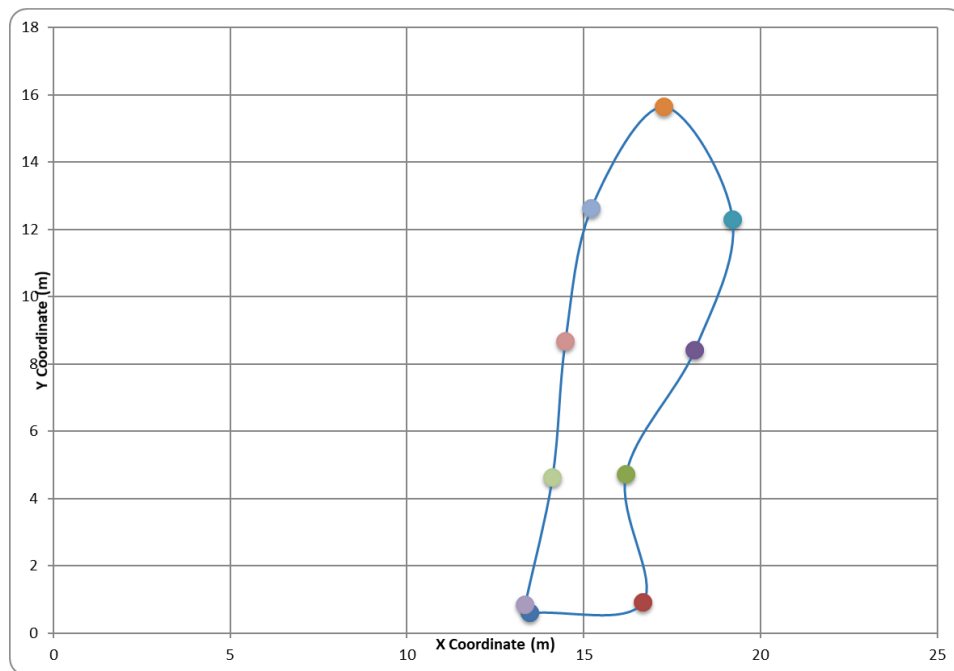
D is the disturbance factor for blast damage. $D = 0$ is used throughout, consistent with controlled perimeter blasting. The five RS2 scenarios span the range of rock-mass quality from Weak (-20%) to Strong (+20%) by varying UCS and GSI simultaneously. The resulting m_b , s , a , and E values are shown in Table 3.2.

4.3.3. Query Point Configuration

Ten query points are distributed around the stope boundary. Points 1 and 10 are at the floor corners. Points 2 to 4 are on the footwall at increasing height. Points 5 to 7 are on the hanging wall. Points 8 and 9 are on the back. Table 4.1 lists coordinates and assignments.

Table 4.1. Query Point Locations and Slope Surface Assignments

Point	X (m)	Y (m)	Dist. along boundary (m)	Location
1	13.477	0.590	0.00	Footwall toe corner
2	16.672	0.906	3.21	Footwall base - tensile zone
3	16.191	4.715	7.05	Footwall lower wall
4	18.133	8.408	11.22	Footwall mid-wall
5	19.207	12.296	15.26	Hanging wall lower
6	17.275	15.640	19.12	Hanging wall mid-surface
7	15.223	12.634	22.76	Hanging wall upper
8	14.476	8.681	26.78	Back, near hanging wall
9	14.113	4.629	30.85	Back, mid-span
10	13.341	0.832	34.72	Footwall upper corner

*Figure 4.4. RS2 Query Point Locations - Slope Cross-Section (Panel 198)*

4.3.4. Field Stress Conditions

Vertical stress:

$$\sigma_v = \gamma \times z = 0.027 \times 540 = 14.6 \text{ MPa} \quad (14)$$

In-plane horizontal stress:

$$\sigma_h = k \times \sigma_v = 1.2 \times 14.6 = 17.5 \text{ MPa} \quad (15)$$

In the probabilistic analysis, both σ_v and k are treated as random variables with the distributions described in Section 4.5.

4.4. Deterministic Sensitivity Analysis

Each parameter is changed from its baseline mean value in steps of -20%, -10%, 0%, +10%, +20% one at a time. All other parameters remain at baseline for each run. At each perturbation level, the minimum SF on each wall surface is recorded. A normalised Sensitivity Index measures the relative influence of each parameter:

$$SI (\%) = \left[\frac{\Delta SF / SF_0}{\Delta P / P_0} \right] \times 100 \quad (16)$$

ΔSF is the change in SF from baseline SF_0 , and $\Delta P / P_0$ is the fractional change in the parameter from its baseline P_0 . SI gives the percentage change in SF per 1% change in the input. Positive SI implies that the parameter becomes more stable as it gets larger. Since SI is a dimensionless system, parameters of different units can be put on the same scale.

The five scenarios (-20% -10% basis +10% +20% scenario) in RS2 Monte Carlo workbook are effective in representing a crude OAT analysis with UCS and GSI perturbed simultaneously. The resultant SI of these scenarios is thus the sensitivity of the slope walls to the joint action of rock-mass quality changes.

4.5. Probabilistic Analysis: Monte Carlo Simulation in RS2

4.5.1. Concept of Monte Carlo Simulation

The Monte Carlo Simulation is used to estimate the output distribution of a model by repeatedly sampling the input distributions and executing the model on each sample. For slope stability, PF is:

$$PF = \frac{N(SF < 1)}{N_{total}} \quad (17)$$

As N_{total} grows, the PF estimate converges. The reliability index is then computed as:

$$\beta = -\varphi^{-1}(PF) \quad (18)$$

MCS requires no closed-form performance function. The full RS2 finite element model runs inside each iteration without any simplification of the mechanics.

4.5.2. Implementation in RS2

MCS runs through RS2's Probabilistic Analysis module. For each iteration, the software: (i) samples input values from their prescribed distributions; (ii) runs Stage 1 and Stage 2 with those inputs; (iii) computes the SF distribution throughout the mesh; and (iv) records SF, σ_1 , σ_3 , differential stress, mean stress, and displacement at each of the ten query points. After all iterations, RS2 assembles the SF distribution for each query point and reports the mean, standard deviation, and PF.

The RS2 Monte Carlo workbook contains results from ten samples (iterations) per scenario, each recording ten query points. The mean SF, standard deviation and PF at each query point are calculated using these.

4.5.3. Statistical Characterisation of Input Parameters

All the uncertain inputs are given their probability distribution and coefficient of variation (COV) using published geotechnical data:

$$COV = \frac{\sigma}{\mu} \quad (19)$$

Table 4.2. Monte Carlo simulation statistical parameters

Parameter	Distribution	COV
UCS (σ_{ci})	Normal	20%
GSI	Truncated Normal (0, 100)	10%
m_i	Normal	15%
Vertical Stress (σ_v)	Normal	10%
Horizontal Ratio (k)	Normal	20%
Stope Geometry (H, W)	Normal	5%

UCS has a COV of 20% by default, which is in line with published data of intact hard rock (Lu et al., 2022; Baecher and Christian, 2003). GSI is a censored normal with a lower value of 0 and an upper value of 100 with COV = 10% that captures moderate subjectivity of field assessment. The intact rock parameter m_i is usually normally distributed with COV = 15%, which is derived using variability data of triaxial testing databases. COV of vertical stress σ_v = 10%. Horizontal stress ratio k bears COV = 20% as horizontal stress measurements are much

less credible than vertical stress estimates. The stope height H and width W are both of $COV = 5\%$ which represents the controlled geometry of intended blast patterns.

4.6. Design Point Identification

The most likely input value at which failure is most likely, when the joint probability distribution of all random variables is considered, is the design point or Most Probable Point of failure. Based on MCS output, the design point can be estimated as follows: of all the iterations in which the failure occurs ($SF < 1$), the design point is the iteration where the joint probability density of the sampled inputs takes on the largest value, i.e., the failure sample nearest to the means of the input values.

The OAT deterministic analysis and the MCS are used to a different end. The OAT method separates the impact of each parameter on SF and generates ranked sensitivity outcomes. MCS represents concurrent change of all the parameters and provides a physically realistic PF. Both methods are not sufficient to provide the whole picture. Combining them gives a full characterisation of the effect of the parameters on the stope wall stability at the Pobeda deposit.

5. RESULTS

5.1. Baseline Empirical Stability Assessment

An empirical stability check was conducted on the basis of the Modified Mathews stability graph before the construction of the numerical model to have a baseline classification. It is Case study, stope panel, open stope 198, Ridder-Sokolny mine, as recorded in Report No. 135. Stope geometry is $H = 20$ m height, $W = 15$ m width, and $L = 30$ m strike length per block. Depth is 540 m. Unit weight $\gamma = 0.027$ MN/m³ gives $\sigma_v = 14.6$ MPa. The stress ratio $k =$ horizontal-to-vertical $= 1.2$, and the stress of the horizontal is 17.5 MPa. Intact rock UCS = 130 MPa and GSI = 61 (Baseline medium scenario).

Factor A was determined as the ratio of UCS to twice the largest induced compressive stress parallel to each stope face. On a biaxial stress field, the Kirsch approximation is used to approximate the maximum induced boundary stress on the vertical hanging wall and footwall to be about $3(h - \sigma_v) = 37.9$ MPa. The value of the horizontal back is $3\sigma_v - \sigma_h = 26.2$ MPa. In both scenarios, $UCS / (2 \times \sigma_1 \text{ induced})$ is greater than 1.0 and hence Factor A is limited to its maximum value of 1.0. This is physically consistent: at 540 m depth, in 130 MPa microquartzite the rock strength is far greater than the induced stresses and compressive failure due to stress is not the main issue. Factor A does not add a penalty to N' to any surface.

Factor B was calculated based on the acute angle between critical joint set and each stope surface based on Potvin (1988) chart. In the case of the hanging wall, the joint dipping at 70 with respect to the wall at right angles of 90° giving the angle 20, the $B = 0.30$. For the footwall, $\alpha = 25^\circ$ gives $B = 0.33$. To obtain the back, a sub-horizontal joint that dips at 35° against a flat (0°) back provides $\alpha = 35^\circ$ and $B = 0.38$. Factor C was calculated based on the inclination of each surface and the controlling failure mode: $C = 8.00$ at dip = 90° of the hanging wall, $C = 2.00$ at dip = 0° of the back, and $C = 1.00$ at a dip = 0° of the footwall as defined by Potvin (1988).

$Q' = (RQD / J_n) \times (J_r / J_a) = (75/9) \times (2/1.5) = 11.11$ was the formula used to calculate the modified rock mass quality index.

Hydraulic radii of the hanging wall and footwall are $HR = 6.00$ m and $HR = 5.00$ m, respectively, and the $HR = 5.00$ m of the back.

Table 5.1. Modified Mathews Stability Parameters and Classification for Panel 198 Reference Geometry

Surface	Q'	A	B	C	N'	HR (m)	Stability Zone	Notes
Hanging wall	11.11	1.00	0.30	8.00	26.66	6.00	Stable	High C for steep wall
Footwall	11.11	1.00	0.33	1.00	3.67	6.00	Transition	Low C, gravity assists
Back	11.11	1.00	0.38	2.00	8.44	5.00	Transition	Sub-horizontal joints

The hanging wall lies on the stable zone under the influence of Factor C = 8.0 to give a steeply dipping surface. The transition zone results in the footwall plotting due to Factor C = 1.0: gravity helps detach. The back is also in transition. These groupings are aligned with the scanning data spatial pattern of overbreak with concern being focused on the lower stope boundary and not the underlying hanging wall surface.

5.2. Monte Carlo Simulation Results by Scenario

Table 5.2 presents mean SF, standard deviation, minimum SF, maximum SF, and PF at all ten query points for all five scenarios. PF is the fraction of the ten simulation iterations where SF < 1.0.

Table 5.2. Monte Carlo Simulation Results: Mean SF, Std SF, and PF by Query Point and Scenario

Pt	Scenario	Mean SF	Std SF	Min SF	Max SF	PF (%)	Location
2	Weak (-20%)	0.279	0.044	0.229	0.351	100	FW base - tensile
2	Wk-Med (-10%)	0.300	0.044	0.250	0.372	100	FW base - tensile
2	Medium	0.332	0.044	0.282	0.405	100	FW base - tensile
2	Md-Strong (+10%)	0.385	0.044	0.335	0.459	100	FW base - tensile
2	Strong (+20%)	0.475	0.045	0.424	0.549	100	FW base - tensile
6	Weak (-20%)	0.883	0.045	0.835	0.961	100	HW mid-surface
6	Wk-Med (-10%)	0.948	0.042	0.903	1.018	80	HW mid-surface
6	Medium	1.004	0.038	0.963	1.068	60	HW mid-surface
6	Md-Strong (+10%)	1.057	0.035	1.019	1.115	0	HW mid-surface
6	Strong (+20%)	1.109	0.033	1.073	1.165	0	HW mid-surface
1	Weak (-20%)	0.946	0.022	0.921	0.981	100	FW toe corner
1	Wk-Med (-10%)	1.016	0.018	0.995	1.044	20	FW toe corner

1	Medium	1.077	0.014	1.060	1.099	0	FW toe corner
3	Weak (-20%)	0.848	0.032	0.810	0.898	100	FW lower wall
3	Wk-Med (-10%)	0.896	0.032	0.858	0.946	100	FW lower wall
3	Medium	0.967	0.030	0.930	1.015	80	FW lower wall
3	Md-Strong (+10%)	1.081	0.027	1.049	1.124	0	FW lower wall
3	Strong (+20%)	1.271	0.020	1.246	1.303	0	FW lower wall
5	Weak (-20%)	0.950	0.009	0.935	0.960	100	HW lower
5	Medium	1.012	0.008	0.999	1.022	10	HW lower
5	Strong (+20%)	1.116	0.016	1.092	1.136	0	HW lower

Point 2 at the footwall base behaves differently from every other location in the dataset. Mean SF stays below 0.48 even in the Strong scenario (UCS = 156 MPa, GSI = 73), and PF remains 100% across all five scenarios. The minor principal stress at Point 2 is tensile in every iteration, ranging from -3.0 to -3.5 MPa. Under the Hoek-Brown criterion, tensile σ_3 reduces the effective rock mass strength to near zero regardless of intact rock quality. This is a geometry and stress-state problem, not a rock quality problem. Stronger rock does not stabilise this zone because the stress geometry at that corner cannot be altered by changing material properties alone.

The hanging wall mid-surface, point 6, is of another pattern, and is the more convenient of the results. Mean SF increases between Weak and Medium (0.883 to 1.004). PF decreases by 100% to 60% to 0 percent on the improvement of the situation Weak to Strong. At the Medium baseline, which is used to design Panel 198 microquartzite - PF = 60%. A deterministic test would pass Point 6 since mean SF = 1.004 is slightly greater than 1.0. In the Monte Carlo result, six samples of ten samples at those specific input conditions give SF with a value less than 1.0. The SF distribution is broad enough to cover the failure threshold at the bottom and the deterministic check lacks all of it.

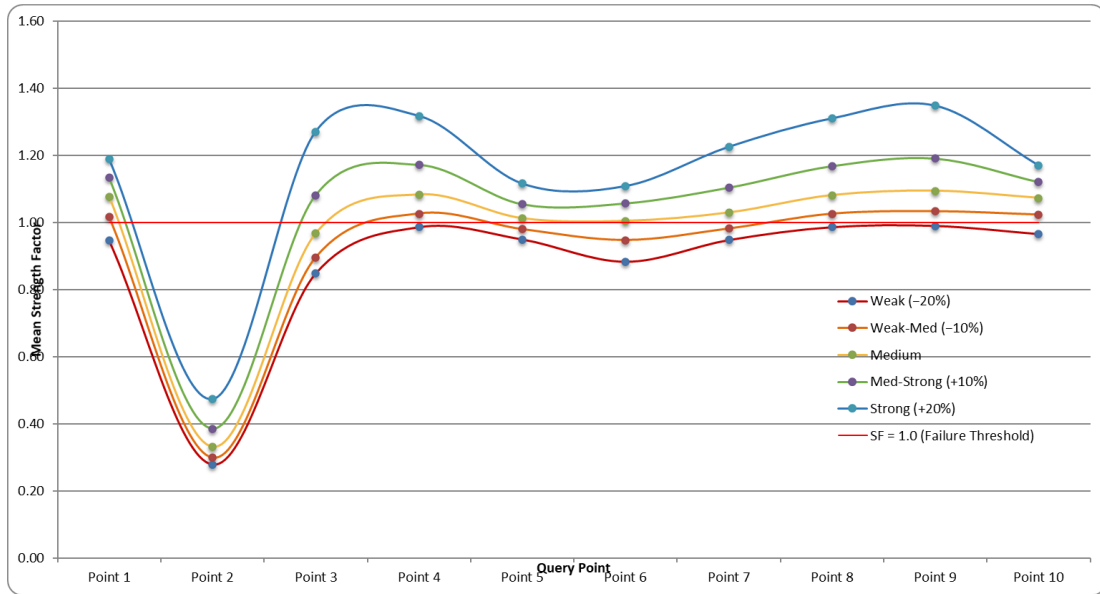


Figure 5.1. Mean Strength Factor for Every Scenario by Query Point

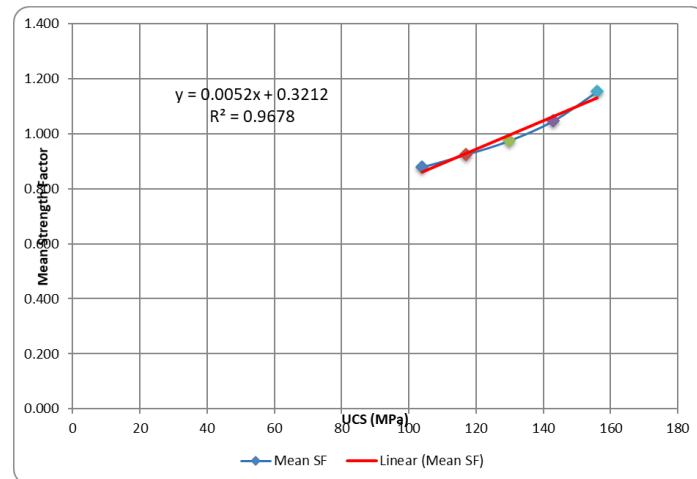


Figure 5.2. UCS vs. Mean Strength Factor

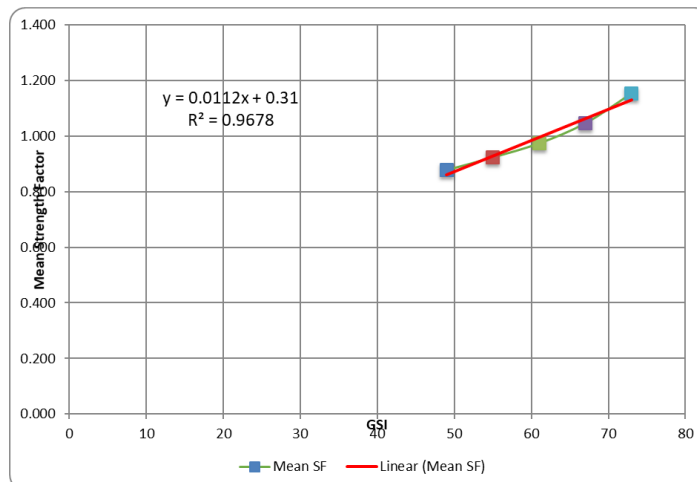


Figure 5.3. GSI vs. Mean Strength Factor

Table 5.3. Summary Failure Probability (%) by Query Point in Each of the Five Situations

Pt	Weak	Wk-Med	Medium	Md-Str	Strong	SI (%)	Trend	Location
1	100	20	0	0	0	22.6	Conditional	FW toe corner
2	100	100	100	100	100	58.9	Persistent	FW base - tensile
3	100	100	80	0	0	43.8	Conditional	FW lower wall
4	100	0	0	0	0	30.6	Conditional	FW mid-wall
5	100	100	10	0	0	16.5	Conditional	HW lower
6	100	80	60	0	0	22.5	Conditional	HW mid-surface
7	100	100	0	0	0	27.0	Conditional	HW upper
8	100	0	0	0	0	30.0	Conditional	Back near HW
9	90	0	0	0	0	32.8	Conditional	Back mid-span
10	80	20	0	0	0	19.1	Conditional	FW upper corner

There are two different classes of behaviour as presented in Table 5.3. Point 2 is continuously in a critical state: it does not work in any case and is not going to stabilise itself under any rock quality improvement in the region of the studied. The remaining nine query points are conditionally stable: they collapse under weak rock conditions but bounce back when the rock mass hits or surpasses the design baseline. Of the conditionally stable points, the critical to stable transition is a transition that happens at various scenarios in various locations. The points 4, 8, and 9 will be already stable ($PF = 0$) at the Weak-Medium scenario, which is only 10% lower than the design baseline. At point 6 the Med-Strong scenario (10 percent above baseline) must occur before PF goes to zero. At Medium, point 3 is critical and at Med-Strong it is stable.

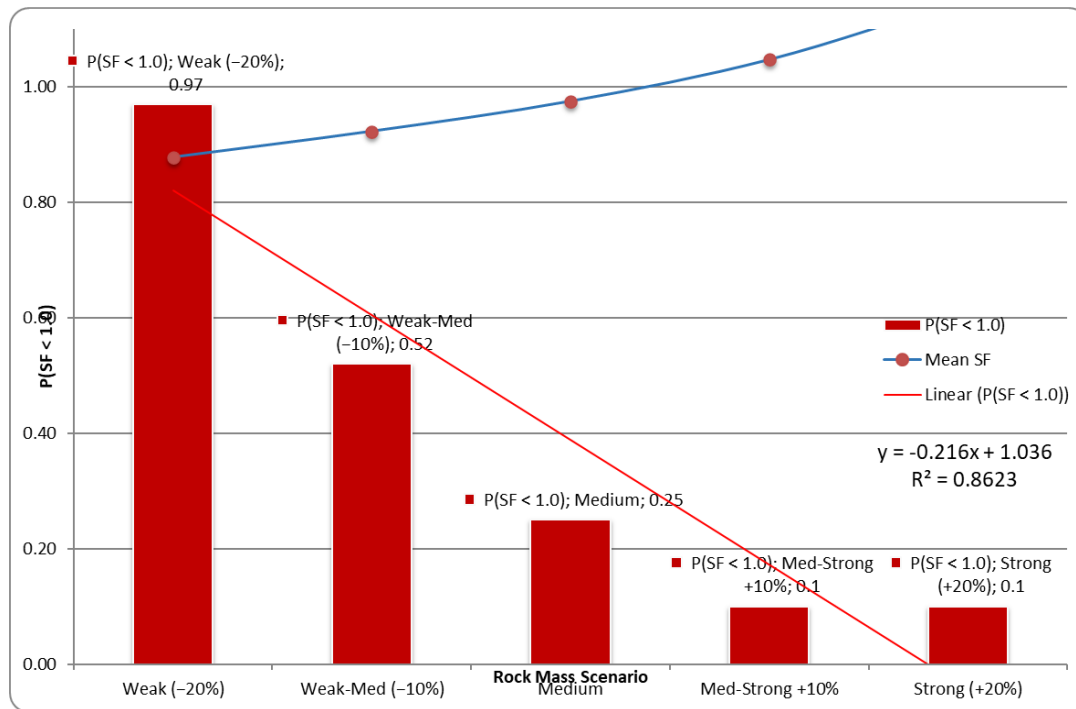


Figure 5.4. Rock Mass Scenario vs. Failure Probability

5.3. Parametric Sensitivity of the Probability of Failure

Table 5.3 uses the Sensitivity Index (SI) to assess the percentage of the SF range between Weak and Strong to the Medium SF. The largest SI at Point 2 stands at 58.9 which is the largest in the data. SI at Point 3 is 43.8%, and at Points 9, 4, and 8 it ranges from 31 to 33%. The SI of points 5, 6, 10, and 1 are within 16 to 23% (SI) indicating that their SF is most responsive to the entire 40% swing of UCS-GSI.

The fact that SI at Point 2 is the highest does not prove that it is not always in a critical position. High SI indicates that SF varies the most in absolute terms at Point 2 as scenarios change - it simply never reaches the threshold of increase that is due to tensile stress, rather than due to a strength to stress ratio that rock quality can enhance. The SI at Point 3 (43.8%) is more realistically informative: Point 3 is actually sensitive in the sense that it changes between critical and stable within the range of scenarios, and that change cuts across the Medium baseline.

The SI values of 8 and 9 on the back are 30% and 33% respectively and both achieve PF = 0% already in the Weak-Medium scenario. The compressing stress on two sides of the back - a geometrical effect of the mass of surrounding rock - is what makes these points become stable: the slightest increase in rock quality makes the Hoek-Brown strength envelope sufficiently distended to remove the possibility of failure. The back does not go too far out of stabilization

even in weak conditions just far enough that $PF = 100\%$ when the rock quality is decreased 20% of design.

5.4. Proposed Regression Equation for Preliminary Design

The five UCS-GSI scenarios generate five paired data points: a combined rock-mass quality index and a corresponding site-level Probability of Failure. This section derives a quantitative relationship between those two quantities, explains how the relationship was constructed, examines what its parameters mean physically, and sets out the conditions under which it can be used for preliminary slope design.

5.4.1. Definition of the Site-Level Probability of Failure

Two types of query-point behaviour emerged from the Monte Carlo results in Section 5.2. Point 2 at the footwall base is persistently critical: $PF = 100\%$ across every scenario without exception. Its failure mechanism is tensile σ_3 , which is a stress-geometry condition not remedied by changes in rock quality. Including Point 2 in a site-level average would pull that average toward 100% in every scenario and mask the genuine variation at the other nine locations.

For that reason, the site-level weighted PF is computed as the arithmetic mean of the nine conditionally stable query points - all points except Point 2:

$$PF_{site} = \frac{1}{9} \times \sum_{i=1, i \neq 2}^{10} PF_i \quad (20)$$

This formulation weights all nine conditionally stable locations equally. Table 5.4 shows the resulting PF_{site} values. In weak conditions, eight of the nine points carry $PF = 100\%$ and one (Point 9) carries $PF = 90\%$, giving $PF_{site} = 96.7\%$. In medium, only two points remain above zero (Points 3 and 6), and $PF_{site} = 16.7\%$. In med-strong and strong, PF drops to zero at every conditionally stable location, so $PF_{site} = 0\%$.

Table 5.4. Combined Rock Mass Quality Index X_{QM} and Site-Level Weighted PF for Each Scenario

Scenario	X_{QM}	$PF_{site} (\%)$	UCS / GSI
Weak (-20%)	0.800	96.7	104 / 49
Weak-Medium (-10%)	0.894	46.7	117 / 55
Medium (Baseline)	1.000	16.7	130 / 61

Med-Strong (+10%)	1.108	0.0	143 / 67
Strong (+20%)	1.225	0.0	156 / 73

5.4.2. Definition of the Combined Quality Index X_{QM}

To derive a regression equation, there is only one scalar that is needed to represent the joint influence of UCS and GSI on the rock mass quality in the five scenarios. The two parameters are adjusted in equal proportional steps in relation to their respective baselines, and thus the index must be sensitive to change in one parameter in the same way as change in the other parameter and must be dimensionless. The index of the combined rock mass quality X_{QM} is obtained as the geometric mean of the normalised values of UCS and GSI:

$$X_{QM} = \sqrt{\frac{UCS}{UCS_0} \times \frac{GSI}{GSI_0}} \quad (21)$$

with $UCS_0 = 130$ MPa and $GSI_0 = 61$ being the Medium baseline values. In the five cases studied, the X_{QM} is 0.800, 0.894, 1.000, 1.108 and 1.225, the conditions of Weak to Strong as given in Table 3.2.

The adoption of the geometric mean instead of the arithmetic mean is not random and is based on statistical and physical considerations. The geometric mean is the natural measure of the central tendency of ratios and multiplicative relationships, statistically (Aitchison, 1984). The behaviour of two quantities when each is a ratio to a reference value is multiplicative, not additive. The arithmetic mean will additively symmetrize what is inherently a multiplicative structure and will thus exaggerate the summed index where both ratios are above 1.0 and undervalue it where both are below 1.0. The geometric mean does not suffer this distortion by acting on the logarithmic scale, where an equal proportionate step above and below the baseline corresponds to equal positive and negative steps (Baecher & Christian, 2005).

This is illustrated numerically. In the Strong scenario UCS and GSI grow by 20% compared to the base, and they are multiplied by 1.20. The geometric average is 1.20 and the arithmetic one is the same. In the Weak scenario, the two falls by 20 percent, hence, both multipliers are 0.80 and both indicators give $X_{QM} = 0.80$. The steps of the two measures are at the same integer steps. The difference is found at mixed-deviation cases, and more significantly where the two parameters are changed by various fractions - a typical case in practice design problems where the UCS and GSI estimates are not equally uncertain. Where the arithmetic mean overstates the larger-magnitude deviation, the geometric mean gives an equal weight to each parameter

on the log scale (van der Plas & Tobi, 1965). Since the uncertainty in GSI (COV = 10%, based on subjective field assessment), and the uncertainty in UCS (COV = 20%, based on laboratory scatter) are structurally different, the symmetric log-scale treatment of the geometric mean is the more defensible option.

The method of geometric aggregation of rock mass quality indices is not new to geomechanics. Barton (1974) modelled the Q-system as a product of six ratios each of which independently describes a distinct aspect of rock mass behaviour; the Q-value is essentially a geometric product of the ratios. More specifically to the present research, Hoek and Brown (1997) observed that the parameters that govern the location of the Generalised Hoek-Brown failure envelope, namely m_b and s , are dependent on UCS and GSI in exponential functions. Since m_b and s are introduced in the failure criterion in a multiplicative way, the failure envelope is more susceptible to proportional alterations in rock quality than to absolute alterations and a quality index that acts on proportional deviations is mechanistically aligned with the underlying criterion.

Practically, $X_{QM} = 1.000$ is the design assumption of a Pobeda deposit microquartzite (UCS = 130 MPa, GSI = 61). Values less than 1.000 depict rock mass conditions that are less than the design baseline of either reduced intact strength, stronger jointing, or both. A value of over 1.000 reflects better than the design assumption. The range of 40% scenarios ($X_{QM} = 0.800$ to 1.225) captures the realistic range of variability of the Ridder-Sokolny ore zone as the 9-scanning report characterises it: at the lower end is the sericite microquartzite with $f = 8$ and dense jointing and at the upper end is the most competent grey microquartzite. Testing the regression equation using this range thus puts it into perspective with respect to the real geomechanical variability recorded on site.

Since X_{QM} is a dimensionless measure, relative to a site-specific baseline, it can be transferred to other operations, with the appropriate modification of the baseline parameters. In a different hard-rock mine, a geomechanics engineer would calculate UCS₀ and GSI₀ of the dominant ore-hosting lithology in the mine, assess X_{QM} using available lab and field data, and would input the output into Equation 21 to get a first pass estimate of PF_{site} . The panel-specific coefficients of regression α and b in the Equation 21 are, however, only calibrated to the Panel 198 geometry and stress conditions and thus a direct numerical transfer of the coefficients to sites with drastically different depth, geometry or stress ratio cannot be made without caution and recalibration to the site.

5.4.3. Regression Fitting and Parameter Derivation

Three out of five of the scenarios yield a non-zero PF_{site} (Weak, Weak-Medium, and Medium). $PF_{site} = 0\%$ is a real result but does not give any information to the regression shape, which is the two stronger scenarios. The three non-zero data points are fitted to the regression.

The reason is that a power-law model of $PF_{site} = \alpha \times (X_{QM})^b$ is chosen because: To begin with, it is the prototype functional representation in non-linear rate-decay correlations in geomechanics (Baecher & Christian, 2005). Second, linearisation of the data on the log-log scale indicates that the three non-zero points lie near a straight line, which indicates that the power-law form is an adequate approximation in this interval.

Linearising by taking natural logarithms of both sides:

$$\ln(PF_{site}) = \ln(\alpha) + b \times \ln(X_{QM}) \quad (22)$$

Substituting the three data pairs: $(X_{QM}, PF_{site}) = (0.800, 96.7), (0.894, 46.7), (1.000, 16.7)$ and solving the two-parameter ordinary least squares system gives $\ln(\alpha) = 6.179$ and $b = -8.42$. Back-transforming:

$$PF_{site}(\%) = 482.3 \times (X_{QM})^{-8.42}, \text{ for } X_{QM} \text{ in } [0.80, 1.00] \quad (23)$$

The log-log fit has a coefficient of determination of $R^2 = 0.997$, which shows that 99.7% of the variance in $\ln(PF_{site})$ can be attributed to $\ln(X_{QM})$ at the three calibration points. Since the number of available points is only three, R^2 by itself does not prove that the model is accurate at intermediate X_{QM} values; it proves that the power-law form fits through the three data points with a small error.

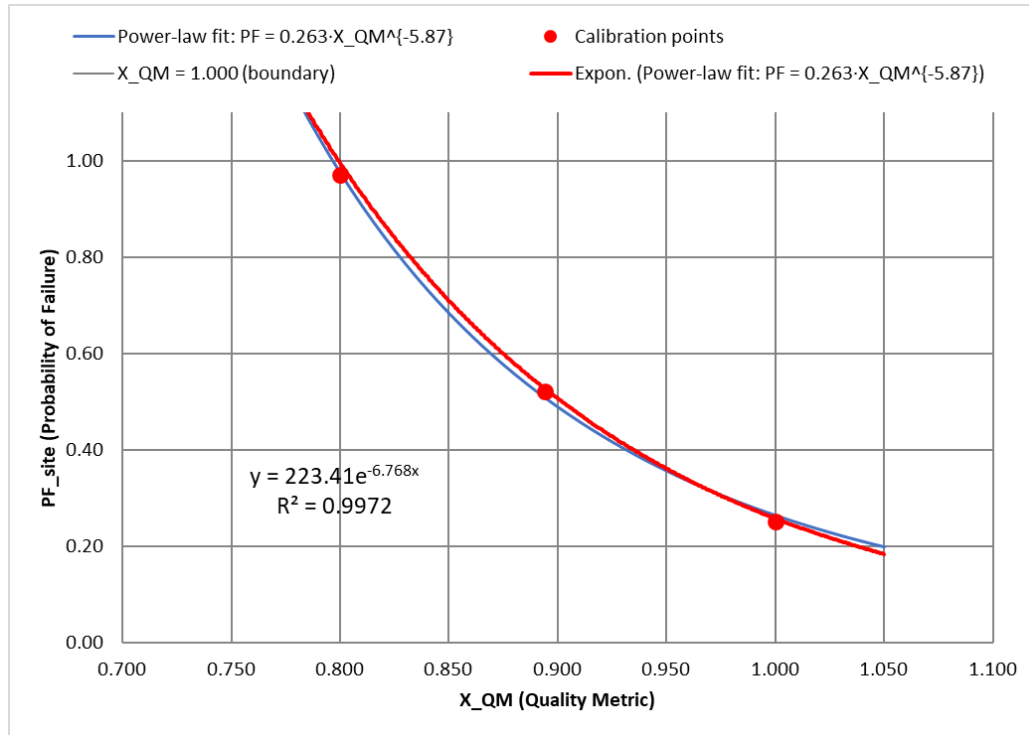


Figure 5.5. Power-law regression fit: PF_{site} versus X_{QM}

5.4.4. Physical Interpretation of the Equation Parameters

$X_{QM} = 1.0$ - the Medium baseline determines the failure probability by using the intercept coefficient of 482.3. Substituting $X_{QM} = 1.0$ into Equation 23 gives:

$PF_{site} = 482.3 \times 1^{-8.42} = 482.3 \times 1 = 482.3\%$, more than 100 percent which is physically meaningless out of the calibration range. The coefficient cannot be used on its own, but only results in meaningful values when used together with the exponent within the 0.80 to 1.00 calibration range. Setting $X_{QM} = 1.000$ the equation gives $PF_{site} = 16.7\%$ which is exactly equal to Monte Carlo. The coefficient is an outcome of the regression arithmetic and not a physical quantity.

The most important parameter is the b exponent of -8.42. It measures the steepness of the PF_{site} in response to X_{QM} . The meaning of such a large exponent can be understood directly: an increase in X_{QM} of 1 percent at any point in the calibration range results in a relative increase of 8.42 in PF_{site} . This is a strong non-linear effect. For comparison, a linear relationship would have a slope of approximately 1% change in PF per 1% change in X_{QM} . The exponent $b = -8.42$ indicates that the system is more than eight times as sensitive to rock quality changes as a linear model would predict.

This sensitivity reflects a physical feature of the Hoek-Brown failure criterion near the low-confinement boundary. When σ_3 is close to zero or tensile at the query points, which is the prevailing condition at most of the conditionally stable locations in this model, a small reduction in GSI or UCS lowers the position of the failure envelope faster than the induced stress changes. The envelope and the stress state converge rapidly. PF_{site} therefore responds sharply to rock quality inputs near the baseline.

The practical consequence can be stated numerically. From the calibration data: reducing X_{QM} by 10% (from 1.000 to 0.894) triples PF_{site} from 16.7% to 46.7%. Reducing it by 20% (from 1.000 to 0.800) raises PF_{site} to 96.7%. Moving in the positive direction: increasing X_{QM} by 10% above baseline (to 1.108) drops PF_{site} to 0%. The downward-upward imbalance is the direct result of the negative exponent - the degradation of rock quality is penalized more than an improvement is rewarded, and the change to near-zero failure probability is achieved relatively fast above the baseline.

5.4.5. Applicability and Limitations of the Equation

Equation 23 is a first-pass design screening tool, rather than an alternative to site-specific numerical modelling. The planned workflow is as follows. In the initial design of a new stope block at Ridder-Sokolny or a similar operation, the geomechanics engineer calculates X_{QM} using available laboratory data of UCS and GSI field evaluation of the hanging wall and footwall rock mass. X_{QM} is substituted in Equation 23 and the PF_{site} thus obtained is read. The result results in three decisions.

PF_{site} lower than a project-specific threshold - e.g., 10% of a temporary production stope or 5% of a stope next to infrastructure or ore development access - is tentatively acceptable and may be subject to full RS2 modelling to verify. When PF_{site} falls between the threshold and 30, the outcome indicates that the stope is located in a sensitive area where a small amount of deterioration of rock conditions will drastically augment the likelihood of failure; further UCS testing or GSI enhancement is justified before making any commitment to the proposed geometry. PF_{site} more than 30: The design may be considered again: reduced stope height or width to modify the state of the boundary stress, or the sequence of extraction may be modified to minimize exposure time, or cable bolt or split-set support installation to be planned whilst the drill drift is fired before the production blast.

Equation 23 had a number of limitations to its applicability. Three non-zero data points in one 2D cross-section of a single slope and one depth were used to calibrate the equation. The 2D model fails to capture slope-parallel stress redistribution, neighbouring slope interactions or geometric irregularity along the strike. Use of the equation to halt stopes with very different geometry, depth, or in-situ stress ratio must be done with caution until the equation is re-calibrated on site-specific data. X_{QM} has a calibration range of 0.80 to 1.00; PF should not be extrapolated below this range - to lower quality or to higher quality - since the power-law form will over-predict PF at extremely low X_{QM} and cannot describe PF at 1.00 to 1.225 where the Monte Carlo data points $PF_{site} = 0.0$.

The values of COV attributed to UCS and GSI, as well as the other MCS inputs are not based on site-specific testing at the Pobeda deposit but on the published literature. Provided that the site testing at Panel 198 or nearby panels indicates that COV on UCS is less than 20% - as could be the case in a geologically homogeneous ore body with high drill-hole coverage - the PF_{site} estimates obtained using the MCS would be smaller than the ones reported here, and the regression coefficients would change accordingly. Alternatively, the highly distorted or broken areas close to the contact of ore may be more variable and increase the values of PF_{site} than Table 5.4.

Lastly, point 2 at the bottom of the footwall is not included in PF_{site} , but it cannot be excluded in the design decision. At this depth and stress ratio, any slope geometry similar to Panel 198 geometry will have $PF = 100\%$ at the base of the footwall of the slope, independent of the rock quality. Local assistance at that point - short rock bolts or mesh at the level of the drill drift - is not optional; it is structurally necessary and must be part of the slope preparation plan regardless of the outcome of the PF_{site} screening.

5.4.6. Comparison with Existing Empirical Approaches

Further panel classification on the basis of N and HR on the basis of N' and HR classifies panel 198 surfaces as Stable or Conditionally Stable in the Modified Mathews stability graph (Section 5.1). That classification is consistent with the Monte Carlo findings for the conditionally stable points - most surfaces are not persistently critical - but it cannot quantify the probability of failure or indicate how sensitive the classification is to changes in input quality. The regression equation in Equation 23 adds two things: a numerical PF estimate, and a sensitivity parameter (the exponent) that communicates the rate of change of risk with rock quality.

Adoko and Zhakipbayev (2023) demonstrated a broadly analogous gap at the Maleyevsky mine, where the same deterministic dilution-graph margin corresponded to very different reliability indices depending on which rock domain was considered. The regression approach developed here is complementary to their FORM-based analysis: where FORM gives the full reliability index β and its design point, the regression equation gives a rapid PF_{site} estimate that requires no iterative reliability calculation and can be applied in the field from basic geomechanical data.

6. DISCUSSION

6.1. Deterministic and Probabilistic Results Compared

At Point 6, the deterministic-probabilistic gap is the closest. The Medium scenario gives mean $SF = 1.004$ - marginally above the stability threshold. A typical deterministic design review passes such a number without remark. The Monte Carlo outcome of the same situation is $PF = 60\%$. At those conditions of input, six of ten iterations get SF less than 1.0.

This variance in input is what causes this gap, and not a modelling error. UCS and GSI are scattered about their mean values, a significant portion of the combinations sampled is on the weaker side of the mean, forcing SF below 1.0. Deterministic check misses out on the tail of the distribution altogether using the mean of the inputs and misses. The bigger the COV of the inputs and the closer the mean SF to 1.0, the bigger the difference between the deterministic reading and the actual PF . And here is what happens at point 6 with the Medium conditions: COV on UCS = 20% and the mean SF is just below the threshold and the distribution is broad enough, such that 60% of the area is below $SF = 1.0$.

The other notable instance is point 2. Its average SF is never above 0.48 and no single run in any case yields SF 1.0 or above. The mechanism of failure there is tensile 3, not the compressive mechanism of weak rock. The 2nd point (Point 2) has 3.0-3.5 MPa values of 2 at all points and by all the iterations. It is a geometrical and stress-state aspect of such a stope corner, and it will exist despite the quality of the rock. The mechanism would be dealt with by targeted support at the bottom of the drill drift level in the form of mesh, dowels or short rock bolts. Nothing can be done to the assumption of rock quality or stope geometry in the plane of this model that will remove that tensile zone.

6.2. Influence of Geomechanical Parameters on Stability

As the SI analysis reveals, the UCS-GSI effect on SF differs significantly, depending on location. The SI of point 2 is the largest (58.9) but the effect does not cross the failure boundary. Point 3 has $SI = 43.8\%$ and crosses the stability threshold within the range of scenarios - its crossings lie between the Medium baseline and so it is the point most sensitive in a practically actionable location.

Point 6 scores moderate SI (22.5%) and $PF = 60\%$ at the Medium baseline. The combination is important: the improvement of SF with improved rock quality at Point 6 is slow because the

stress state is actually the same in all cases, the values of σ_1 at Point 6 are consistently between 42 and 70 MPa, indicating a truly loaded hanging wall boundary. The strength envelope of Hoek Brown grows with increasing UCS and GSI between Weak and Strong but requires a 10 percent improvement above the baseline before that growth can be large enough to eliminate failure probability.

The most improved PF scenario is demonstrated in the back (Points 8 and 9) and footwall mid-wall (Point 4). Both converge at the Weak-Medium scenario (10% below baseline) with PF = 0%. These sites are under a moderate compressive confinement - σ_3 is positive and approximately 2 to 5 MPa at these sites - and the Hoek-Brown strength surface is sufficient when the rock quality is within 10% of the design assumption.

6.3. Implications for Kazakhstan Hard-Rock Mines

The scanning data of Panel 198 indicate that there is overbreak of 8,345 m³ against a design volume of 15,366 m³ - overbreak to design ratio of 54%. Drill Drift 6 single generated 2946 m³ of overbreak with the 3D Datamine model indicating the mismatch to be concentrated in the bottom stope boundary and corner transitions. This spatial pattern is in line with the query-point findings: Point 2 and the neighbouring footwall points have the highest PF in the present analysis, and they are the locations where overbreak as observed in the field were most significant.

Equation 23 gives a viable screening tool to initial design at Ridder-Sokolny and similar operations. The geomechanics team would estimate X_{QM} at the design stage of a new stope block by using the available UCS and GSI data, using Equation 23, and determining whether PF_{site} is within acceptable limits. If it does not, three responses are appropriate: reduce planned stope dimensions to change the stress conditions, acquire additional geomechanical data to reduce uncertainty in X_{QM} , or accept the PF and plan dilution management and support installation accordingly.

That three-option framework gives the mine planner a proportionate response rather than a binary pass-fail verdict. A mine planner who knows that $X_{QM} = 0.91$ corresponds to $PF_{site} = 38\%$ at that geometry has the information needed to make a risk-proportionate decision. For a major production stope where grade is high and dilution costly, 38% may be unacceptable. For a temporary development heading, it may be tolerable with appropriate monitoring. The same

numerical result leads to different decisions depending on economic context - which is how risk-based design is supposed to work.

The SI values in Table 5.3 also help prioritise site investigation. UCS and GSI together drive most of the SF variability at all locations except the tensile Point 2. A targeted testing programme - more UCS samples and systematic GSI logging in the hanging wall and lower footwall microquartzite - would reduce COV on X_{QM} , tighten the PF_{site} estimate, and may move a borderline stope into an acceptable risk category without changing stope geometry.

6.4. Limitations of the Study

The two-dimensional analysis is provided. The length of the stope strike, along-strike irregularity of the hanging wall and contact of adjacent stopes are not modelled. Three-dimensional modelling using RS3 is required at stopes with large aspect ratio or complicated mining sequences.

The Hoek-Brown criterion is a continuum that considers the rock mass as equivalent. The failures of wedges and blocks with permanent, negatively oriented joints are not modelled. Zones of extreme sericite alteration and parallel jointing can occur in the microquartzite-based ore zone of the Pobeda deposit and can offer localised structural failure mechanisms that would not be identified by the continuum model.

Ten samples per scenario will be sufficient to compare relative PF across scenarios and to define points as persistently and conditionally critical. They do not provide sufficient high-precision absolute PF estimates. The 60-percent reading at Point 6 is based on a total of six failures out of ten - 1,000 iterations would give a confidence interval of around +/-3 percent. It is significantly broader with ten samples. The functional form and direction of the UCS-GSI effect is determined in equation 23, this is not a high-precision calibration.

The COV values of input distributions are based on published literature, as opposed to site specific testing at the Pobeda deposit. In case of lower COV in site-specific testing, PF values would be lesser than those currently reported. PF would be higher in case of COV, where the fractures or chemically altered zones are more intense.

7. CONCLUSIONS AND RECOMMENDATIONS

7.1. Conclusions

This thesis combined RS2 finite element analysis with Monte Carlo Simulation of five rock mass cases to evaluate the stability of open stope walls at Panel 198, Drill Drift 19, Horizon 14 of the Pobeda deposit in the Ridder-Sokolny mine at Kazakhstan. The case study data was provided by nine Void Scanner 150 Mk3 reports that were collected in the period of March to November 2019. Based on the analysis, five conclusions were made.

First, at the bottom of the footwall, there is a persistent critical point at Point 2. PF = 100% and mean SF is less than 0.48 even at the most massive rock mass that has been modelled, proving that the failure mode is stress-state driven and not as the result of rock weakness. To stabilise this zone, increasing UCS and GSI is not sufficient. The correct response is to install targeted local support installed at the drill drift level.

Second, PF = 60 percent at the Medium baseline (UCS = 130 MPa, GSI = 61), at which the hanging wall mid-surface (PF) is indicated as Point 6, decreases to 0 percent when UCS and GSI are raised by 10 percent. Deterministic SF of Point 6 is 1.004 under Medium conditions, which passes a standard SF > 1.0 test. The Monte Carlo outcome tells the true risk of that marginal figure. This is a concrete example of why a single SF value is insufficient for risk assessment.

Third, the combined UCS-GSI Sensitivity Index at Point 2 is 58.9%, at Point 3 it is 43.8%, and at Points 9, 4, and 8 it is 31 to 33%. These values identify where additional geomechanical characterisation would most reduce design uncertainty.

Fourth, fitting a power-law regression to non-zero PF_{site} yields $482.3 \times (X_{QM})^{-8.42}$, valid for X_{QM} in the range 0.80 to 1.00. The negative exponent of -8.42 shows that PF_{site} is more than eight times as sensitive to rock quality changes as a linear model would predict. This non-linearity follows from the position of the conditionally stable query points near the low-confinement zone of the Hoek-Brown failure surface, where the failure envelope responds rapidly to changes in GSI and UCS. Two numerical consequences are worth retaining for design use: a 10% reduction in X_{QM} from the Medium baseline triples PF_{site} from 16.7% to 46.7%, and a 20% reduction pushes it to 96.7%. The equation is calibrated on Panel 198 geometry and should be applied to other stopes with appropriate adjustment of the baseline parameters; it is not a universal curve.

Fifth, the aggregate scanning data for Panel 198 show total overbreak of 8,345 m³ with the most severe concentrations at the lower stope boundary - exactly where the query-point analysis identifies the highest and most persistent failure probabilities. The consistency between observed dilution patterns and predicted critical locations supports the validity of the modelling approach.

7.2. Recommendations for Future Work

Three-dimensional modelling with RS3 would capture stope aspect ratio, hanging wall irregularity along strike, and multi-stope stress interaction. The added computational cost is justified for stopes with complex geometry or long production histories.

Adding discrete fracture network modelling would allow structurally controlled failure modes to be represented explicitly. In the Pobeda ore zone, zones of intense sericite alteration and sub-parallel jointing likely produce wedge failure mechanisms that the continuum model does not capture.

Applying FORM alongside MCS would produce importance factor direction cosines for each input, providing an independent check on the design point location and allowing the two methods to be used as mutual validation.

Site-specific calibration of the input COV values is recommended before applying Equation 23 to design decisions. A targeted testing programme focused on the hanging wall microquartzite - measuring UCS scatter and between-observer GSI variability - would be the most efficient use of resources given the SI results.

Finally, the void-scanning programme at Panel 198 provides a high-quality dataset for back-analysis. Fitting a calibrated RS2 model to the observed overbreak volumes in the most severely diluted drill drifts, particularly Drift 6, would validate the modelling assumptions and improve the regression coefficients in Equation 23 for the specific conditions of the Pobeda deposit.

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APPENDIX A: BASELINE INPUT PARAMETER DATASET

Table A.1. Complete deterministic baseline input parameters used in the RS2 model

Parameter	Symbol	Value	Unit	Source
Intact rock UCS	σ_{ci}	130	MPa	Report No. 135 / Maulana et al. (2024)
GSI	GSI	61	-	Report No. 135 / engineering judgment
Intact rock constant	m_i	20	-	Hoek et al. (2002) microquartzite
Disturbance factor	D	0	-	Controlled blasting assumed
Hoek-Brown constant m_b	m_b	4.967	-	Computed from Eq. 11
Hoek-Brown constant s	s	0.0131	-	Computed from Eq. 12
Hoek-Brown constant a	a	0.5026	-	Computed from Eq. 13
Elastic modulus	E	39,618	MPa	Computed from Hoek et al. (2002)
Unit weight of rock	γ	0.027	MN/m ³	Typical for skarn/quartzite
Mining depth	z	540	m	Report No. 135
Vertical stress	σ_v	14.6	MPa	Computed: $\gamma \times z$
Horizontal stress ratio	k	1.2	-	Maulana et al. (2024)
Horizontal stress	σ_h	17.5	MPa	Computed: $k \times \sigma_v$
Stope height	H	20	m	Report No. 135 geometry
Stope width	W	15	m	Report No. 135 geometry
Strike length per block	L	30	m	Report No. 135 geometry

Q-system parameters ($RQD = 85$, $J_n = 9$, $J_r = 2$, $J_a = 1.5$) and joint orientation values used in the Mathews stability graph assessment are reconstructed proxy values consistent with $GSI = 61$ microquartzite. They are not direct measurements from a specific drill core at Panel 198.

APPENDIX B: STATISTICAL PARAMETER ASSIGNMENTS FOR MONTE CARLO SIMULATION

Table B.1. Probability distribution assignments and COV values for Monte Carlo Simulation

Parameter	Distribution Type	Mean	COV (%)	Literature Source
UCS (σ_{ci})	Normal	130 MPa	20	Lu et al. (2022); Baecher & Christian (2003)
GSI	Truncated Normal (0, 100)	61	10	Engineering judgment; Hoek et al. (2002)
m_i	Normal	20	15	Triaxial testing databases
Vertical stress σ_v	Normal	14.6 MPa	10	Baecher & Christian (2003)
Horizontal ratio k	Normal	1.2	20	Stress measurement uncertainty
Stope height H	Normal	20 m	5	Blast pattern control
Stope width W	Normal	15 m	5	Blast pattern control

APPENDIX C: REPORT NO. 135 DATA SUMMARY

KAZZINC LTD / RIDDER-SOKOLNY MINE

REPORT NO. 135 - 25 NOVEMBER 2019

SCANNING OF CLEANING SPACE: PANEL 198, DRILL DRIFT 19, HORIZON 14, POBEDA DEPOSIT

Table C.1. Report No. 135 summary data

Parameter	Value	Units
Survey method	Void Scanner 150 Mk3 laser system	
Survey date	November 2019	
3D modelling software	Datamine Studio3	
Survey conducted jointly by	Geomechanics department and Survey dept (RSR)	
Principal rock types	Microquartzites (f=13); sericite microquartzites (f=8); sericite-chlorite-quartz rocks (f=8)	
Planned ore volume (DM)	2,405	m ³
Actual blasted volume	1,381	m ³
→ Within blast contour	1,227	m ³
→ Outside blast contour	154	m ³
Temporary inactive reserves	1,178	m ³
Unplanned dilution volume	154	m ³
Unplanned dilution ratio	6.4%	% of planned volume

The information contained in this appendix has been translated and transcribed from the original and was used for this research. The initial report was in the Russian language. The numbers all appear the same as those in Table 1 of Report No. 135.