GEOSTATISTICAL MODELLING OF GOLD GRADES BY CO-KRIGING-BASED APPROACH FOR PRESERVING THE OUTLYING VALUES

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ABSTRACT

Geostatistical modeling of Gold grade (Au) is challenging since the presence of outliers makes the distribution long-tailed and impacts significantly the process of mineral resource evaluation, the mine design and financial optimization. Capping is a widely used technique consisting of truncating the data to some top-cut grade. However, this procedure is likely to omit the most important part of a deposit that probably is economically considerable. In this research, a co-kriging-based approach is applied in a gold deposit to preserve the upper quantile of the Au distribution while improving the precision of the estimation. The rationale of this idea is to divide the grade of Au into: truncated grade, a weighted indicator above the top-cut grade and a zero-mean residual. After this decomposition, the co-kriging is able to jointly estimate the truncated grade and the indicator. The benefit of this approach is to provide unbiased grade estimation and choosing the optimum top-cut value while avoiding the outlying values for spatial continuity calculation and implementing the spatial prediction.

Keywords: Gold deposits, Co-kriging, Geostatistical modeling, Geostatistics , Open Pit Mine Design.

INTRODUCTION

Geostatistical modeling of geo-related attributes have been widely used in mining industry for mineral resource estimation and evaluation (Journel and Huijbregts, 1978; David, 1988; Krige, 1999). However, some variables showing the long-tailed distribution which makes the process of modeling challenging. In particular case, precious metals such as gold contain some extreme values that should be treated before any spatial continuity analysis and block modeling (Krige and Magri, 1982; Armstrong, 1984, Paravarzar et al. 2014). Ignoring those values make bias in the process of producing the block model and impact the economical consideration of a mining project. To tackle this issue, some methodologies proposed to reduce the influence of those extreme values (Journel and Arik, 1988; Parker, 1991; Arik, 1992; Costa, 2003; Machado et al., 2011, 2012). A straightforward method is to cap the high grades and reduce them to a certain value (Sinclair and Blackwell, 2002; Rossi and Deutsch, 2014). This technique is accepted in conventional international standards for reporting mineral resources and ore reserves (SAMREC, 2007; JORC, 2012). However, reset the high values ignore remarkable part of the dataset provided by bore holes or blast holes and is likely to advocate a bias in the block modeling (Maleki et al., 2014). Furthermore, there is no strong mathematical or geological concept behind detection of treating the extreme values (Sinclair and Blackwell, 2002). Rivoirard et al. (2013) proposed a methodology based on dividing the dataset into truncated, indicator and residual, for which the variogram analysis is more robust since the new variables do not keep high values. The estimation process is then based on co-kriging of truncated grade and indicator. The aim of this research is twofold: a) presentation the theory of capping and top-cut methodologies; b) resource estimation and quantification of recovery functions by two approaches and show the capability of decomposition approach (top-cut model) in comparison with capping through a real gold deposit.

METHODOLY

Capping

The high values in the long-tailed distribution are usually interpreted as outliers. Those values should be treated before estimation and mineral resource modeling in order to reduce their impact in the further analysis of a mining project. A convenient method termed capping is to detect the outlier value and then reduce other values higher than the defined outlier to the outlier itself. Choosing the optimum outlier value is somehow questionable. However, there are some statistical tools to investigate it (Rossi and Deutsch, 2014). Those tools are related to the global distributions (e.g. probability plot) which help to identify the extreme values (Parker, 1991). The procedure for capping is:

1- Choose an outlier value
2- Reset the high values higher than that outlier to the outlier itself
3- Variogram analysis of the capped variable and fit a theoretical variogram model
4- Implement simple or ordinary kriging based on the decision of stationary of the random function
5- Further analysis of the obtained block model for resource estimation

Top-cut model

Spatial interpolation of long-tailed distribution by combination of top-cut model and truncating the original variable was first introduced by Rivoirard et al. 2013. In this methodology, it is of interest to preserve the influence of high values by defining an equivalent measures identified by an indicator value. The steps are as follow:

1- Choose an optimal top-cut value.
2- Divide the original variable into truncated, indicator and residuals.
3- Variogram analysis of the new variables: in this case, direct and cross-variogram are required for measure the co-spatial continuity of truncated and indicator values. Linear model of coregionalization (Journel and Huijbregts, 1978) can be used to fit a positive semidefinitive condition in deriving the sill matrices. Since two variables in this research are considered, the three experimental direct and cross-variograms should be computed along the pre-specified anisotropy (Chiles and Delfiner, 2012; Wakernagel, 2003). For residuals, it is necessary only calculate the direct variogram.
4- Apply co-kriging for jointly deterministic modeling the truncated grade and the indicator. Use the kriging for separately estimation of the residual. Variance-covariance matrices in co-kriging and kriging systems are constructed on the basis of spatial continuity models so obtained from item 3. Simple or ordinary kriging can be used in both cases.
5- Back-transform the estimated values (truncated, indicator and residuals) in each block into the original variable (backward of item 2)
6- Further analysis of the back-transformed values for resource estimation

Therefore, the first method is dealing with only the block modeling based on capped values, while the top-cut model also considers the extreme values for estimation by defining an indicator value without loss of accuracy. In the following, it is of interest to compare these two methods through a gold deposit.

CASE STUDY

Presentation of the dataset

The dataset consists of 2544 samples composited in 1m length belonging to a gold deposit located in Australia. Location map of the samples and borehole locations can be seen in figure 1. The drilling pattern is regular with approximately 10 m spacing toward north. Statistical parameters calculated revealed that the gold grade shows a heavy-tailed distribution (Figure. 2; above). Consideration of high values indicates that they are not erroneous data and should not be discarded (Table 1). In this study, it is intended to apply above mentioned methodologies: capping and top-cup model for underlying gold grade estimation and compare the results for mineral resource estimation.

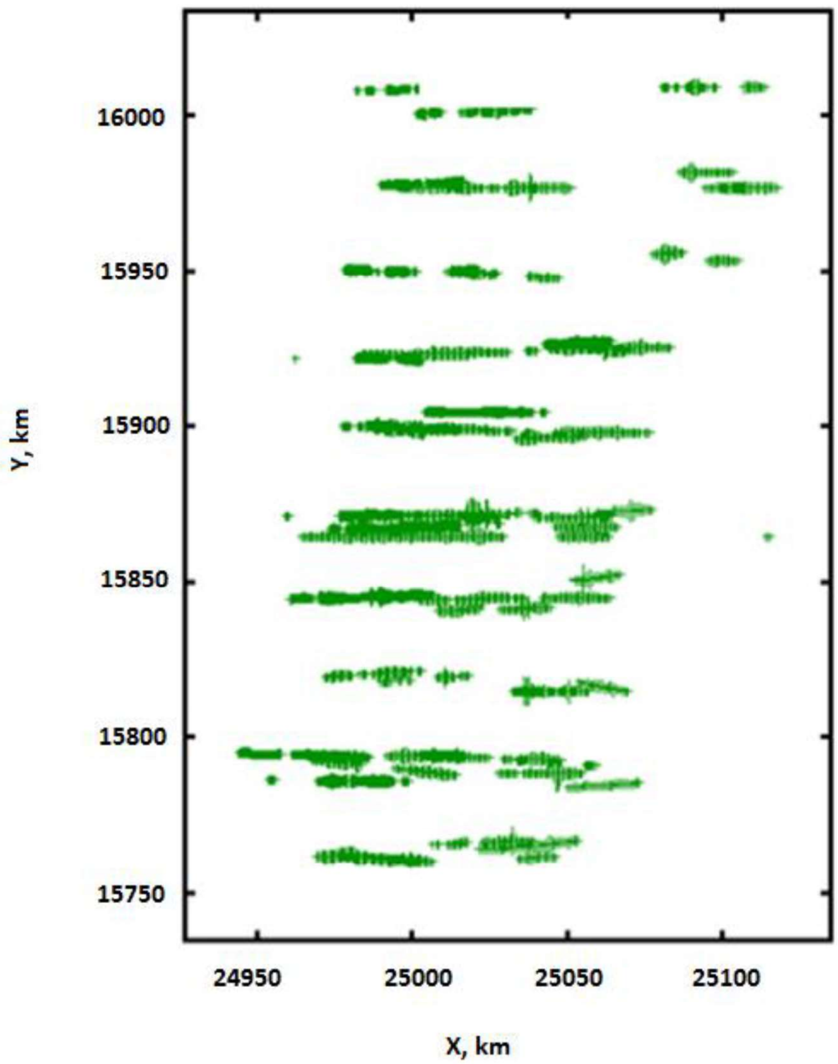


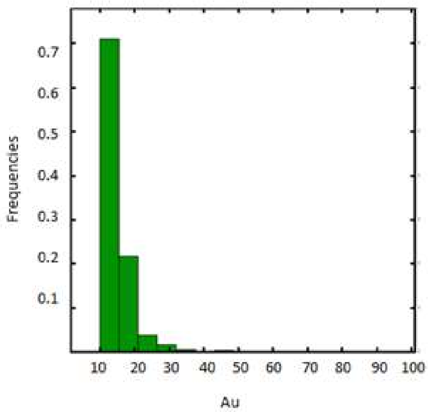
Figure 1 - Location map of the borehole data, green crosses are the sample locations

Table 1: statistical parameter calculated over the original gold grade

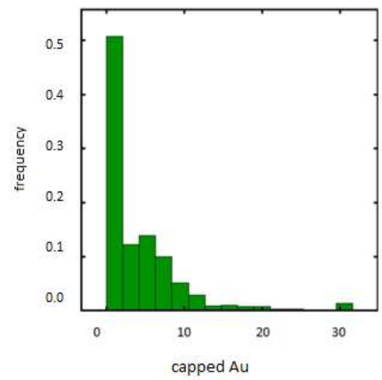
Variable	Mean	St. Deviation	Variance
Original grade	4.16	6.98	48.78

Choosing the outlier value

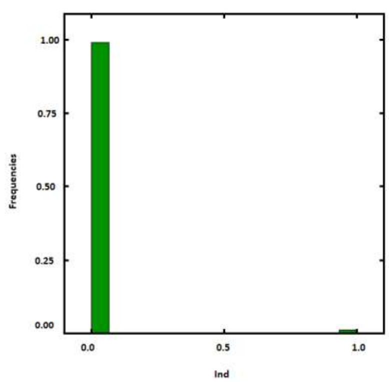
There are some techniques to detect the suitable outlier value required for both capping and top-cut model. Maleki et al. 2014 following Rivoirard et al. 2013 proposed to choose alternative thresholds as the candidate outlier values and compute the ratios between the indicator direct and cross-variograms alongside with successive thresholds. The first threshold that its ratio is approximately constant shows the minimal acceptance value for choosing the top-cut and capping value. However, in this research, for the sake of simplicity, the threshold “30” for gold is defined according to visual inspection of histogram (figure 1). The dataset is ready to be capped and truncated following above methods. Figure (2) shows the histogram of gold capped variable and furthermore the truncated values therewith the indicator and residual.



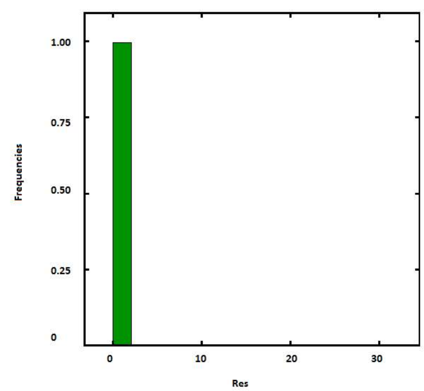
Original Grade



Capped Value



Indicator



Residual

Figure 2 - Histogram of treated values after indicating the outlier value associated with indicator and residual

Variogram modeling

The direct variogram for capped and residuals, direct and cross-variograms of truncated values and indicators are computed. The proper linear model of coregionalization is fitted with three spherical structures and nugget effect. The continuity is considered to be onmi-directional and isotopic (Figure 3).

Variogram formula for truncated and residual values, accordingly:

Truncated gold values:

$\gamma(h)=5.11\,Nugget + 25.39\ Sph\ (7.4m, 7.4m, 7.4m) + 1.542\ Sph\ (29.6m, 29.6m, 29.6m)$ (1)

Residual values:

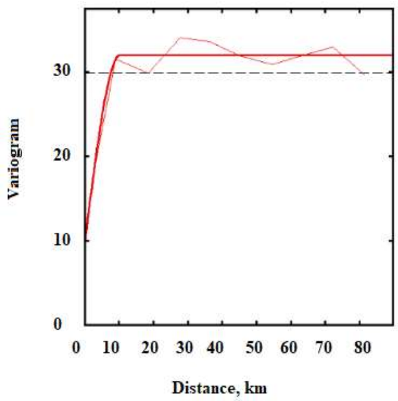
$\gamma(h)=0.874\,Nugget + 0.614\ Sph\ (7.4m, 7.4m, 7.4m)$ (2)

Cross variogram of truncated and indicator values:

$$\begin{pmatrix} \gamma_{Au\ trunc} & \gamma_{Au\ trunc-Ind} \\ \gamma_{Ind-Au\ trunc} & \gamma_{Ind} \end{pmatrix} = \quad (3)$$

$$= \begin{pmatrix} 9.6640 & 0.1378 \\ 0.1378 & 0.0067 \end{pmatrix} nugget$$

$$+ \begin{pmatrix} 22.32 & 0.2452 \\ 0.2452 & 0.0071 \end{pmatrix} Sph(10.28km, 10.28km, 10.28\ km)$$



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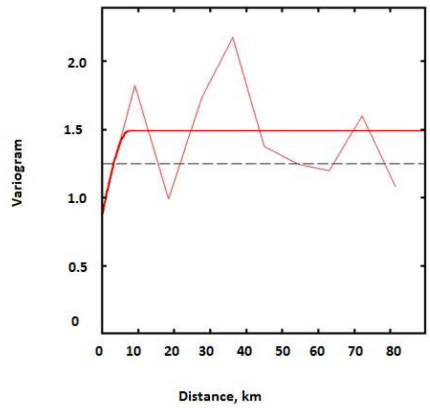

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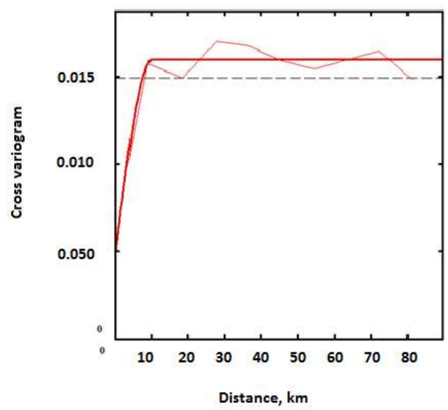


c

Figure 3 – a) Direct variogram of the truncated values; b) direct variogram of the residual values; c) cross variogram ot the estimated truncated value

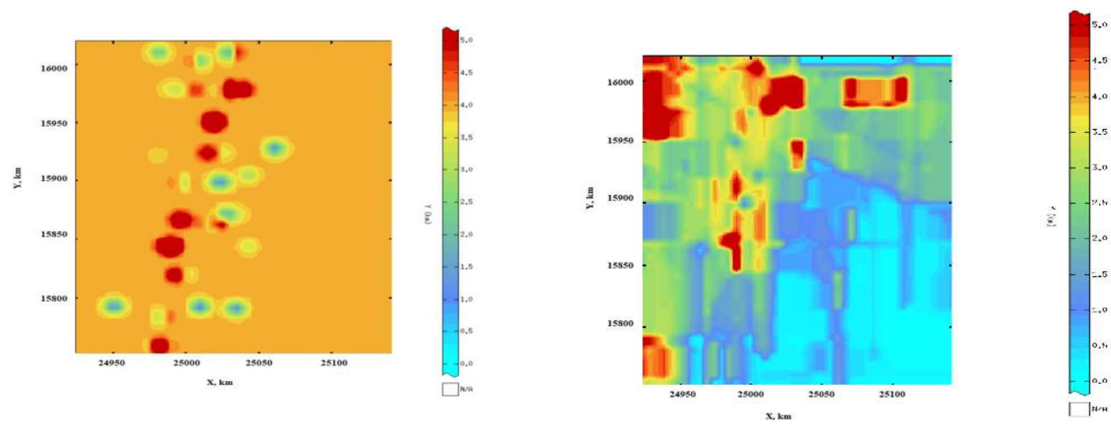
Spatial modeling

According to the previous models of spatial continuity so obtained, estimation of gold grade by two methodologies are implemented and compared:

1- Kriging: simple kriging has been applied for estimation of capped values and residual (capped method)

2- Co-kriging: simple cokriging has been applied for co-estimation of truncated value and indicator (top-cut model)

In both case, moving neighborhood is considered to select a part of sample location for estimation and co-estimation. In this method, the neighborhood surrounding the block is divided into sub-sectors and from each part, the pre-identified number of conditioning data are taken into account for (co)-estimating that block. This neighborhood is then moved from one block to another block in order to cover all region (Chiles and Delfiner, 2012). To do so, the search ellipsoid is selected as 800 m, higher than range of variogram; the number of participating points for estimation is minimum 4 and maximum 8 per sector. After co-kriging of truncated values and residual, it is necessary to back-transform the estimated values to the original space. Figure 4 shows the map obtained from two approaches. Top-cut model better reproduces the variability of high values while the capping is failed to estimate the reliable variability of gold grade.



a) Capping method b) Top-cut model

Figure 4: the estimation maps obtained from two methodologies; the high grade values are more distributed reasonably by top-cut model

The differences between the two approaches can be assessed globally, by calculating the mean grade above cut-off (Table 2 and Figure 5). It is seen that, for all the cut-offs, the traditional approach (kriging of capped grades) yields biased estimates in higher cut-offs while in lower cut-offs, this difference is not very distinguishable in comparison with the top-cut model.

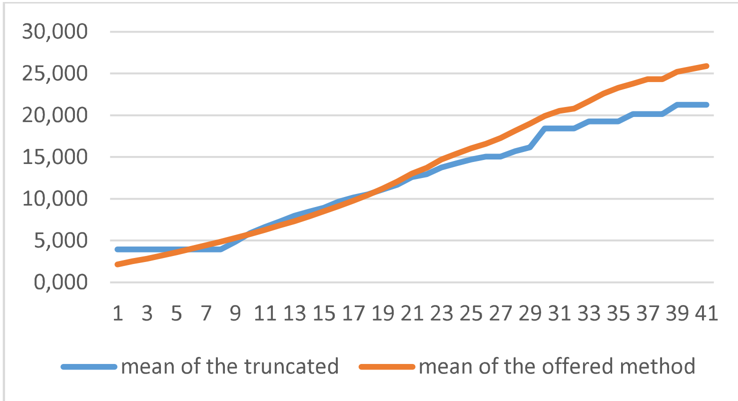


Figure 5 - The mean grade above cut-off

Table 2 - Statistical visualization of mean grade above cut-offs

Cut-off (g/T)	**Mean grade above cut-off (g/t)**	
	Truncated gold	According to the approached method
0.0	3.934	2.128
0.5	3.934	2.532
1.0	3.934	2.839
1.5	3.936	3.206
2.0	3.938	3.597
2.5	3.941	4.013
5	6.609	6.296
7.5	9.665	9.095
10	12.594	13.033
15	18.410	20.512
20	21.260	25.881

CONCLUSIONS

Geostatistical estimation methodologies are very applicable for reliable resource estimation and ore reserve evaluation based on international standards. The input data for such a spatial modeling obtained from exploratory drillholes. Presence of high values make the statistical analysis of the underlying grade non-robust and advocates one to use treating methodology to alleviate the influence of those values. In this study, two common and widespread used methodologies based on capping and truncation of high values are employed for resource modeling in a gold deposit with heavy-tail distribution. Comparison of results show that since top-cut model preserve the influence of high values in the dataset by contribution of indicator data, it is more reliable comparing to capping that only reduces the high values to a certain threshold and omit the extreme values.

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REFERENECES

Chile` s, J.P., Delfiner, P. (2012). Geostatistics: modeling spatial uncertainty, Second edition. Wiley, New York.

Maleki, M., Madani, N., & Emery, X. (2014). Capping and kriging grades with long-tailed distributions. *The Journal of the Southern African Institute of Mining and Metallurgy,* 114, 255–263.

Paravarzar, S., Madani, N., Maghsoudi, A. & Afzal, P. (2014). Testing the exactitude of estimation methods in the presence of outliers: An accounting for robust kriging. *Iranian Journal of Earth Sciences,* vol. 6, no. 1, pp. 24-30.

Parker HM. (1991). Statistical treatment of outlier data in epithermal gold deposit reserve estimation. Math Geol 23:125–199

Rivoirard, J., Demange, C., Freulon, X., Lécureuil, A. and Bellot, N. (2012). A Top-Cut Model for Deposits with Heavy-Tailed Grade Distribution. *Mathematical Geosciences*, 45(8), pp.967-982.

Rossi, M. E., Deutsch, C. V. (2014). Mineral resource estimation. New York: Springer.

Wackernagel, H. (2003). Multivariate Geostatistics — an Introduction with Applications. Springer, Berlin, 387pp.

Journel, A. G., & Huijbregts, C. J. (1978). Mining geostatistics. Academic press.

David, M. (1988). Handbook of applied advanced geostatistical ore reserve estimation. Developments in Geomathematics 6.

Krige, D. G. (1999). Essential basic concepts in mining geostatistics and their links with geology and classical statistics. South African Journal of Geology, 102, 147-152.

Krige, D. G., & Magri, E. J. (1982). Studies of the effects of outliers and data transformation on variogram estimates for a base metal and a gold ore body. Journal of the International Association for Mathematical Geology, 14(6), 557-564.

Armstrong, M. (1984). Common problems seen in variograms. Journal of the International Association for Mathematical Geology, 16(3), 305-313.

Paravarzar, S., Madani, N., Maghsoudi, A., & Afzal, P. (2014). Testing the Exactitude of Estimation Methods in the Presence of Outliers: An accounting for Robust Kriging.

Journel, A. G., & Arik, A. (1988, March). Dealing with outlier high grade data in precious metals deposits. In Proceedings of the First Canadian Conference on Computer Applications in the Mineral Industry (pp. 161-171). Balkema, Rotterdam.

Arik, A., & Kim, Y. C. (1992). Outlier restricted kriging: a new kriging algorithm for handling of outlier high grade data in ore reserve estimation. In Proceedings of the 23rd International Symposium on the Application of Computers and Operations Research in the Minerals Industry (pp. 181-187). Society of Mining Engineers, Littleton, CO.

Costa, J. F. (2003). Reducing the impact of outliers in ore reserves estimation. Mathematical geology, 35(3), 323-345.

Machado, R. S., Armony, M., Costa, J. F. C. L., Koppe, J. C., Baafi, E. Y., Porter, I., & Kininmonth, R. J. (2011). Field parametric geostatistics-geostatistical rigorous solutions dealing with extreme grades during resource estimation. In Proceedings of the 35th International Symposium on the Application of Computers and Operations Research in the Minerals Industry (pp. 191-201). Australasian Institute of Mining and Metallurgy, Melbourne.

Machado, R. S., Armony, M., & Costa, J. F. C. L. (2012). Field Parametric Geostatistics—A Rigorous Theory to Solve Problems of Highly Skewed Distributions. In Geostatistics Oslo 2012 (pp. 383-395). Springer, Dordrecht.

Sinclair, A. J., & Blackwell, G. H. (2002). Applied mineral inventory estimation. Cambridge University Press.