

# Neural Network based robust control for dual axis reaction wheel inverted pendulum

**Yerzhan Rzagaliyev**, MSc student

**Thesis Supervisor:** Prof. Huseyin Atakan Varol

Department of Robotics

Nazarbayev University

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# Outline

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- Motivation
- Physical Setup
- System Dynamics
- System Identification
- Neural Network Implementations
- Simulations
- Experimental Results



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# Motivation

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- Industry 4.0

Internet of Things (IoT)

Big Data

Cloud computing

Augmented and virtual reality (AR/VR)

**Human-Robot interaction (HRI)**

Additive manufacturing (AM)



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# Motivation

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## Variable Impedance/Stiffness Actuators

- For safe HRI used new robots like variable impedance actuators (VIA) (Lasota et al, 2017)
- Highly nonlinear
- Decrease of positioning accuracy
- Vibrations

## Reaction wheel integration

- It has benefits to implement reaction wheel-integrated VSA (Baimyshev et al, 2016)
- Can boost the performance of the VSA



# Motivation

## Reaction wheel inverted pendulum

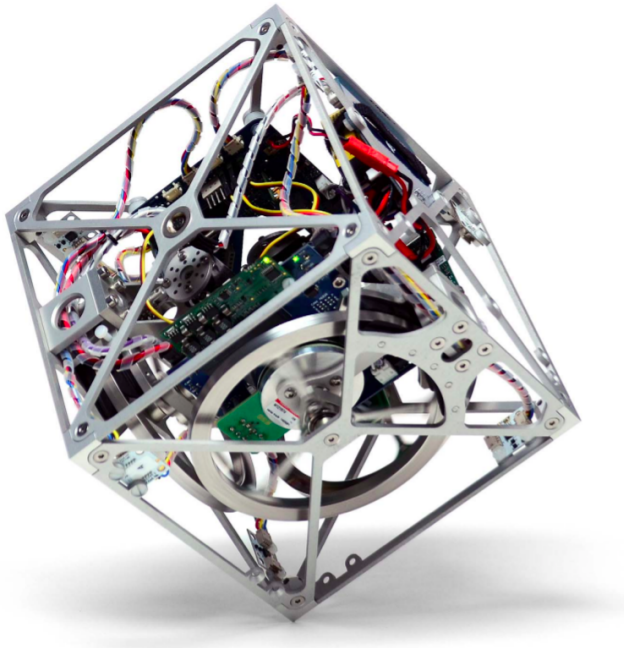


Figure 1. Cubli balancing on a corner (Muehlebach, D'Andrea, 2016)

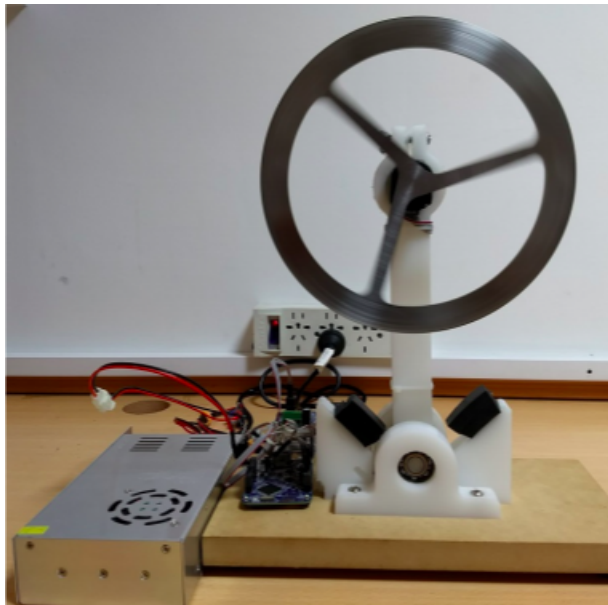


Figure 2. Self balancing single axis inverted pendulum (Belascuen, Aguilar, 2018)

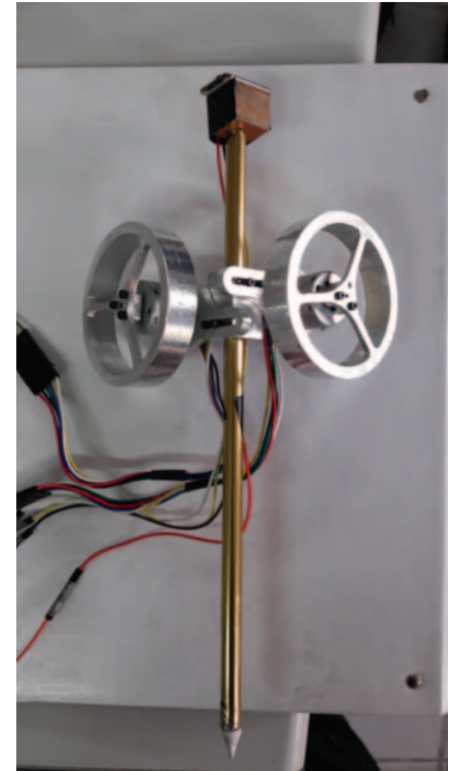


Figure 3. Self balancing dual axis pendulum (Turkmen et. Al, 2017)

# Design of the setup

## Lower Part

- 3D printed
- 8 springs
- Connected with universal joint
- 2 encoders

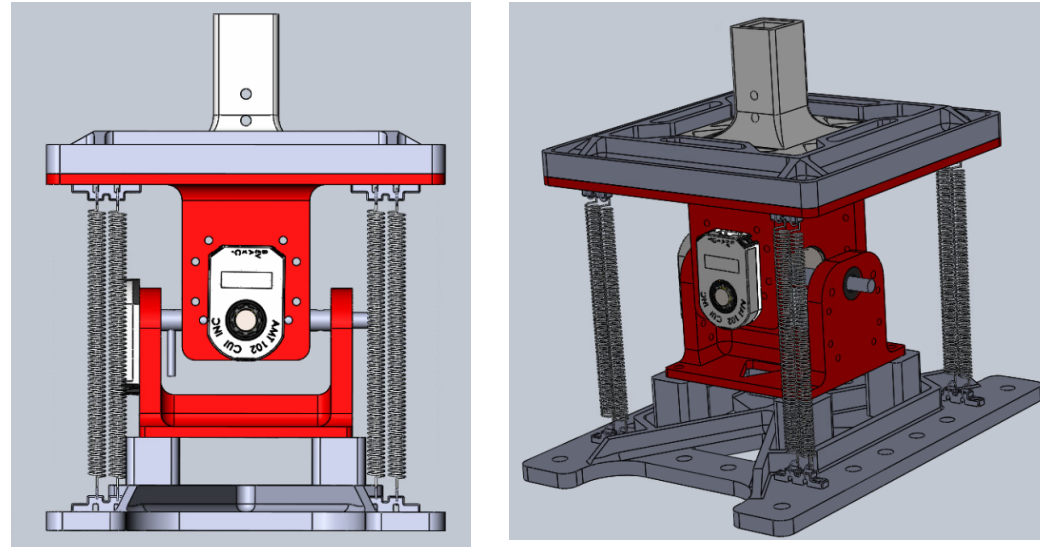


Figure 4. Lower part of the system

# Design of the setup

Upper part

- 3D printed
- Steel flywheel
  - Diameter – 8 cm
  - Mass – 160 g
- 2 encoders
- 2 Maxon motors
- 2 Vex gyroscopes
- 20 pieces
  - Mass – 50 g
  - Diameter – 15 mm
  - Thickness – 9 mm

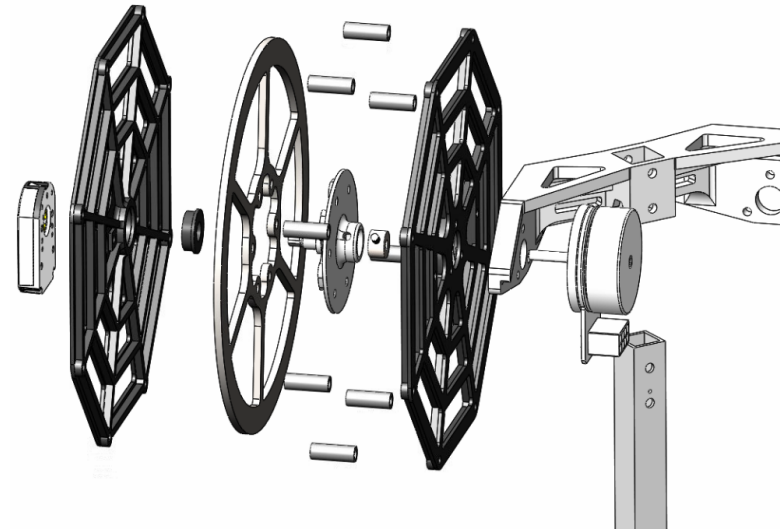


Figure 5. Upper part of the system

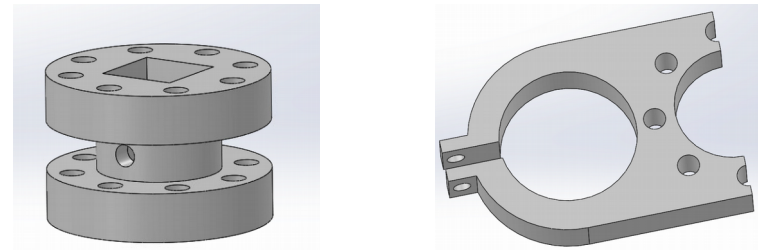
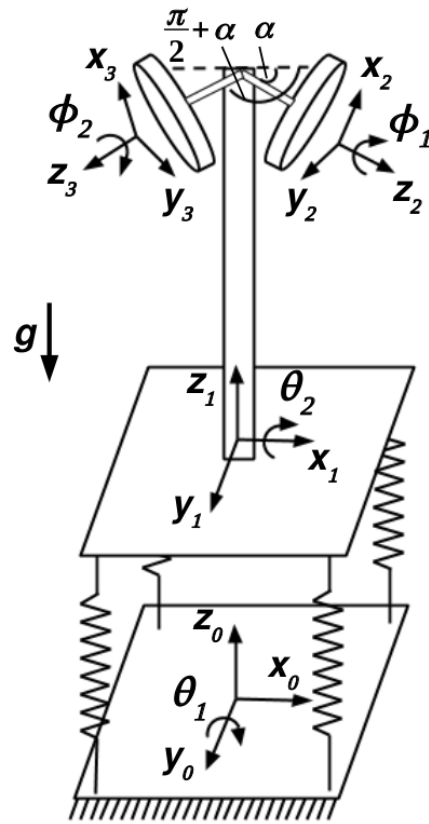
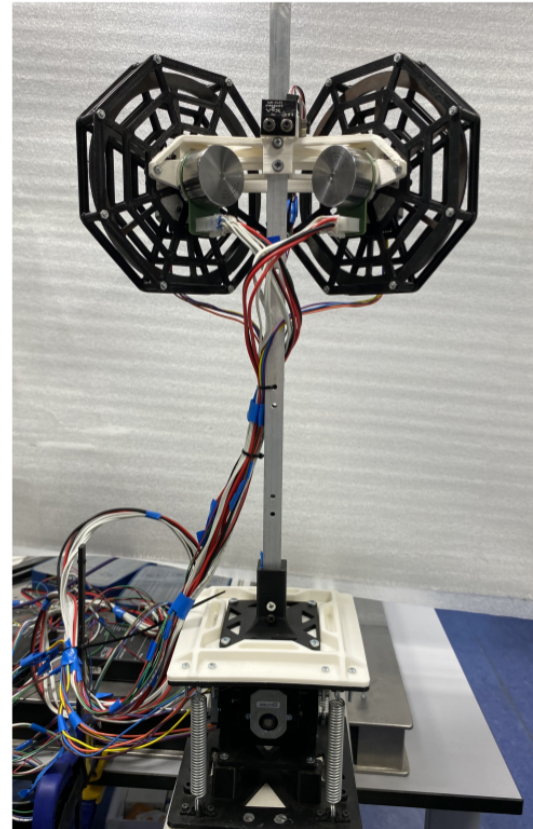


Figure 6. Mass holders

# Design of the setup



(a) Block diagram.



(b) Reaction wheel setup.

Figure 7. Experimental setup (Baimukashev et al, 2020)

# System Dynamics

## Kinematics

$${}^0_1R = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ 0 & 1 & 0 \\ -\sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix}$$

$${}^1_2R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_2) & \sin(\theta_2) \\ 0 & -\sin(\theta_2) & \cos(\theta_2) \end{bmatrix}$$

$$\hat{\omega}_p = {}^0_2 R ({}^1_2 R^{-1} [0 \ \dot{\theta}_1 \ 0]^T + [\dot{\theta}_2 \ 0 \ 0]^T)$$

$$\hat{\omega}_{\omega_1} = {}^0_2 R R_{\omega_1} [\dot{\phi}_1 \ 0 \ 0]^T$$

$$\hat{\omega}_{\omega_2} = {}^0_2 R R_{\omega_2} [\dot{\phi}_2 \ 0 \ 0]^T$$

## Lagrangian function

$$T = \frac{1}{2} \hat{\omega}_p^T I_1 \hat{\omega}_p + \frac{1}{2} \hat{\omega}_{\omega_1}^T I_{\omega_1} \hat{\omega}_{\omega_1} + \frac{1}{2} \hat{\omega}_{\omega_2}^T I_{\omega_2} \hat{\omega}_{\omega_2}$$

$$V = \bar{g}^T {}^0_2 R (m_p l_p + m_{\omega_1} l_{\omega_1} + m_{\omega_2} l_{\omega_2}) + \frac{1}{2} (k_1 (\theta_1 - \theta_{1i})^2 + k_2 (\theta_2 - \theta_{2i})^2)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = \tau$$

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau$$

$$\ddot{q} = M(q)^{-1} (\tau - C(q, \dot{q}) - G(q))$$



# System Dynamics

In order to control stabilization of the pendulum, we need to define all affecting forces and torques, and we have equation:

$$\tau = \begin{bmatrix} \cos(\theta_2)\cos(\alpha)(-K(i_{\omega_1} + i_{\omega_2}) + b_{\omega_1}\dot{\phi}_1 + b_{\omega_2}\dot{\phi}_2) - b_1\theta_1 \\ \sin(\alpha)(-K(i_{\omega_1} - i_{\omega_2}) + b_{\omega_1}\dot{\phi}_1 - b_{\omega_2}\dot{\phi}_2) - b_2\theta_1 \\ Ki_{\omega_1} - b_{\omega_1}\dot{\phi}_1 \\ Ki_{\omega_2} - b_{\omega_1}\dot{\phi}_1 \end{bmatrix}$$

By deriving dynamics from Lagrangian function, we have our state-space representation as:

$$\dot{x} = f(x, u) = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ (M(q)^{-1}(\tau - C(q, \dot{q}) - G(q)))^T \end{bmatrix}$$

# System Identification

In Solidworks

Mass – 1.49 kg

Center of mass (in z axis) – 31 cm

Moments of inertia:

- $I_{xx} = 0.19 \text{ kg}\cdot\text{m}^2$
- $I_{yy} = 0.20 \text{ kg}\cdot\text{m}^2$
- $I_{zz} = 0.01 \text{ kg}\cdot\text{m}^2$

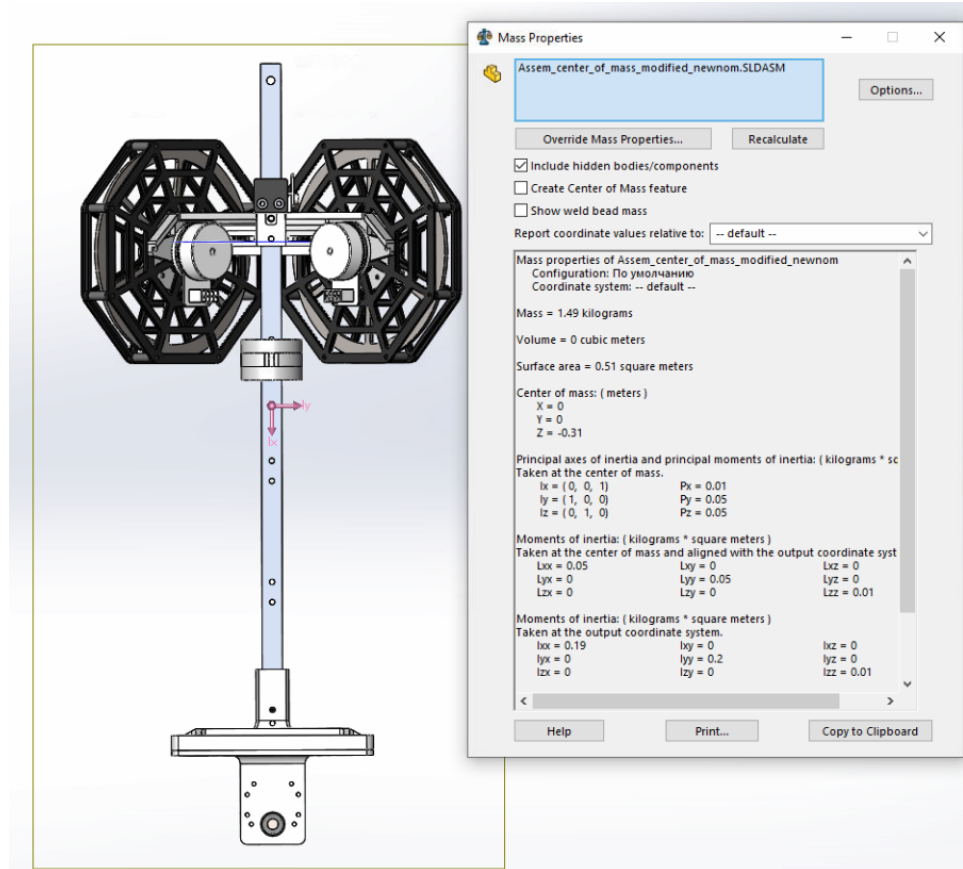


Figure 8. Mechanical properties

# System Identification

In MATLAB

Spring constant

- $k_1$  – 12.5 N·m/rad
- $k_2$  – 6.9 N·m/rad

Pendulum friction coefficient

- $b_1$  – 0.09 N·m·s/rad
- $b_2$  – 0.12 N·m·s/rad

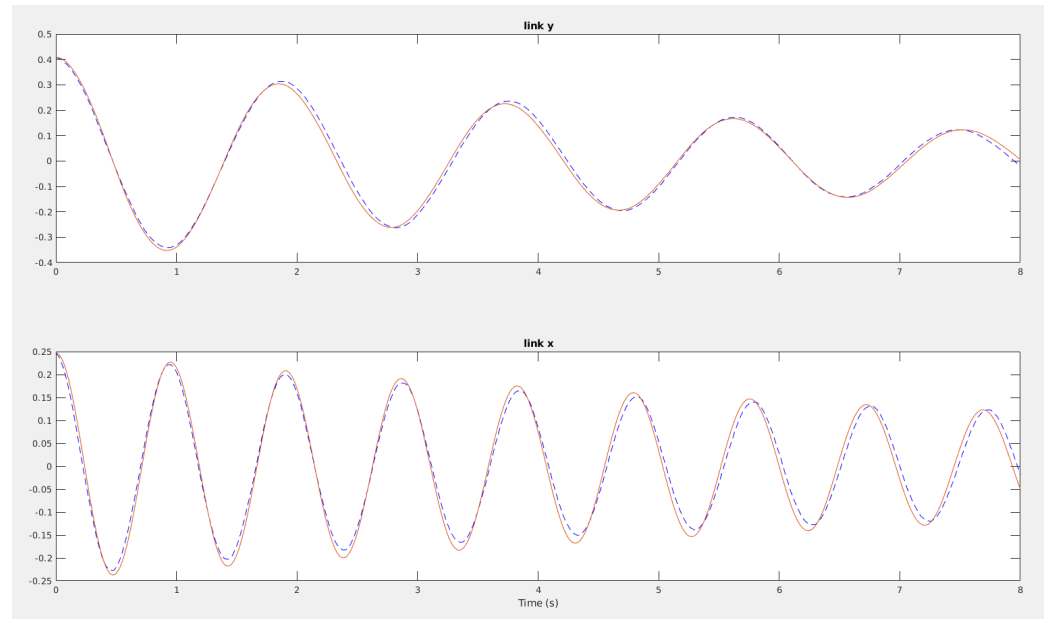


Figure 9. Defining rotational spring constant and pendulum friction coefficient. Real world result (blue dashed line) vs simulated result (red line)

# System Identification

Parameter	Symbol	Value	Unit
Mass of the whole pendulum	$m_p$	1.49	kg
Center of mass along z-axis	$l_p$	0.31	m
Rotational spring constant around x-axis	$k_1$	12.5	N · m/rad
Rotational spring constant around y-axis	$k_2$	6.9	N · m/rad
Friction coefficient on x-axis of the pendulum	$b_1$	0.09	N · m · s/rad
Friction coefficient on y-axis of the pendulum	$b_2$	0.12	N · m · s/rad
Friction coefficient of the reaction wheel	$b_{\omega 1}$	1.11E-04	N · m · s/rad
Friction coefficient of the reaction wheel	$b_{\omega 2}$	9.40E-05	N · m · s/rad
Motor torque constant	K	0.13	N · m/A
Moment of inertia of the pendulum	$I_p$	0.19	kg · m <sup>2</sup>
Moments of inertia of the reaction wheels	$I_1 = I_2$	7.53E-05	kg · m <sup>2</sup>

Table 1. Nominal parameters of the system

# Neural Network Implementations

## Optimal control problem (OCP) trajectories

- MATMPC (Chen et al, 2019)
- 8000 trajectories, after filtering 5300 clean
- 15000 trajectories, after filtering 8000 clean
- Mass – 0 to 30 %, spring constants – -20 to 0 %, friction coefficients – -20 to 20 %
- For testing 500 trajectories for each (nominal and changed)

## Nominal NN (FNN)

- 3 layers fully connected
- 128 neurons
- ReLU
- RMSprop – 0.0008
- 4800 nominal parameters trajectories
- DGX-2 100 epochs

## Robust NN (RNN)

- 2 layers fully connected + 2 layers Long Short-Term Memory (LSTM)
- 128 neurons
- ReLU
- RMSprop – 0.0008
- 2000 nominal + 7000 changed trajectories
- DGX-2 100 epochs

	Nominal	Changed
FNN based nominal NN	2	56
	2	68
RNN based nominal NN	3	56
	13	70

Table 2. The comparison of FNN and RNN for the nominal NN for 100 trajectories (failure rate)



# Simulations

Different initial conditions for simulating the NNs

500 trajectories with six random states:

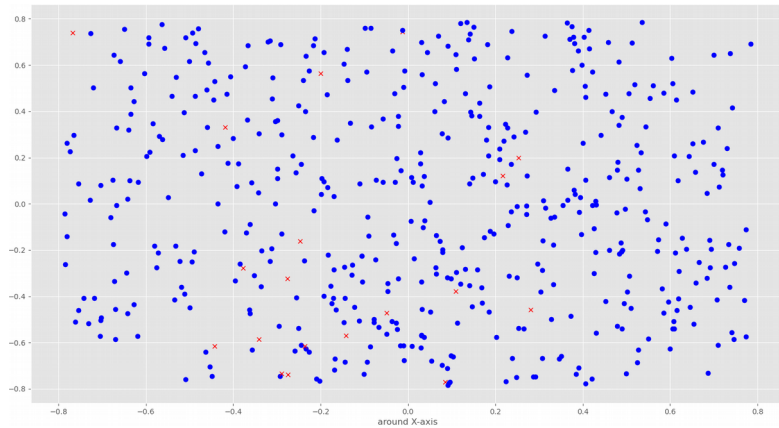
- Angular positions –  $\theta_1$  and  $\theta_2$
- Angular velocities –  $\theta'_1$  and  $\theta'_2$
- Wheel velocities –  $\omega_1$  and  $\omega_2$

		Init_data 500 traj
Nominal NN	Nominal parameters	478 (95.6 %)
	Changed parameters	233 (46.6 %)
Robust NN	Nominal parameters	482 (96.4 %)
	Changed parameters	420 (84 %)

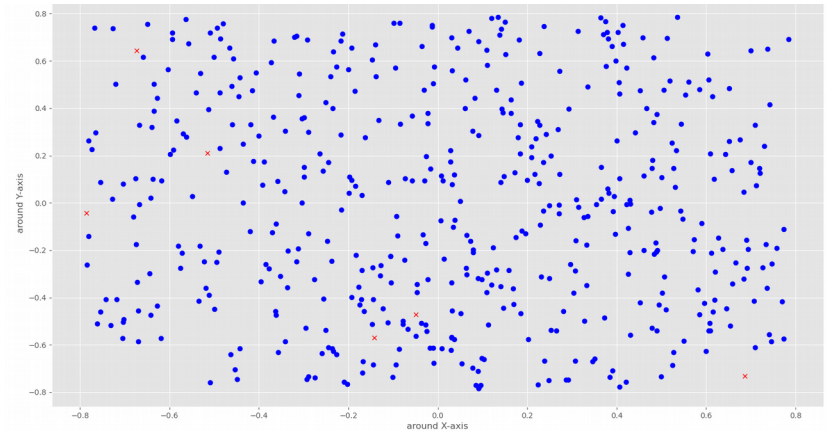
Table 3. The success rate Nominal NN and Robust NN for 500 random trajectories



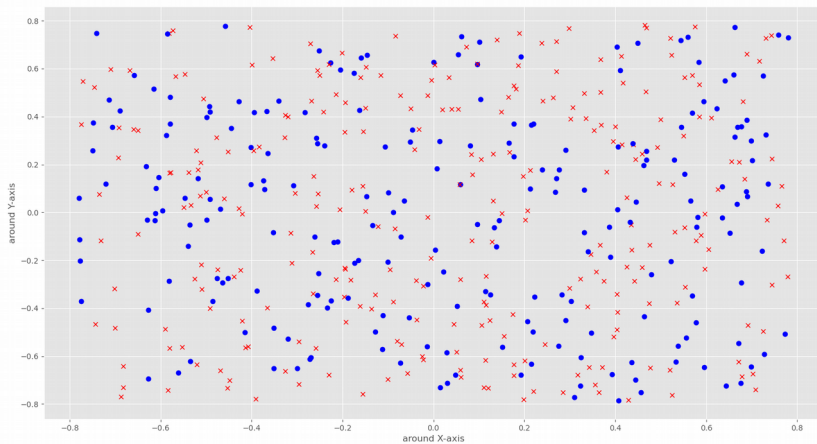
# Simulations



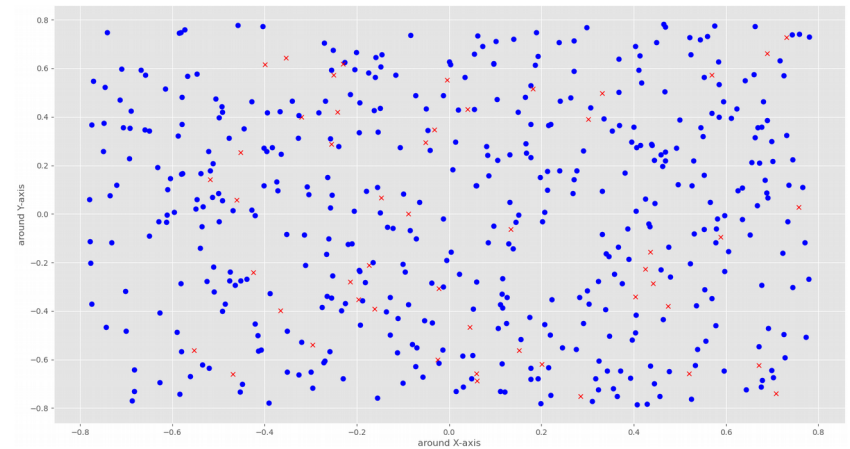
Nominal parameters, nominal NN



Nominal parameters, robust NN



Changed parameters, nominal NN



Changed parameters, robust NN

Figure 10. Simulation results (red dot – fail, blue dot – success)

# Simulations

## Nominal parameters:

- Nominal NN (478)
  - Mean – 1.16
  - Std – 0.16
- Robust NN (482)
  - Mean – 1.28
  - Std – 0.20
- 

## Changed parameters:

- Nominal NN (233)
  - Mean – 1.18
  - Std – 0.20
- Robust NN (420)
  - Mean – 1.29
  - Std – 0.21

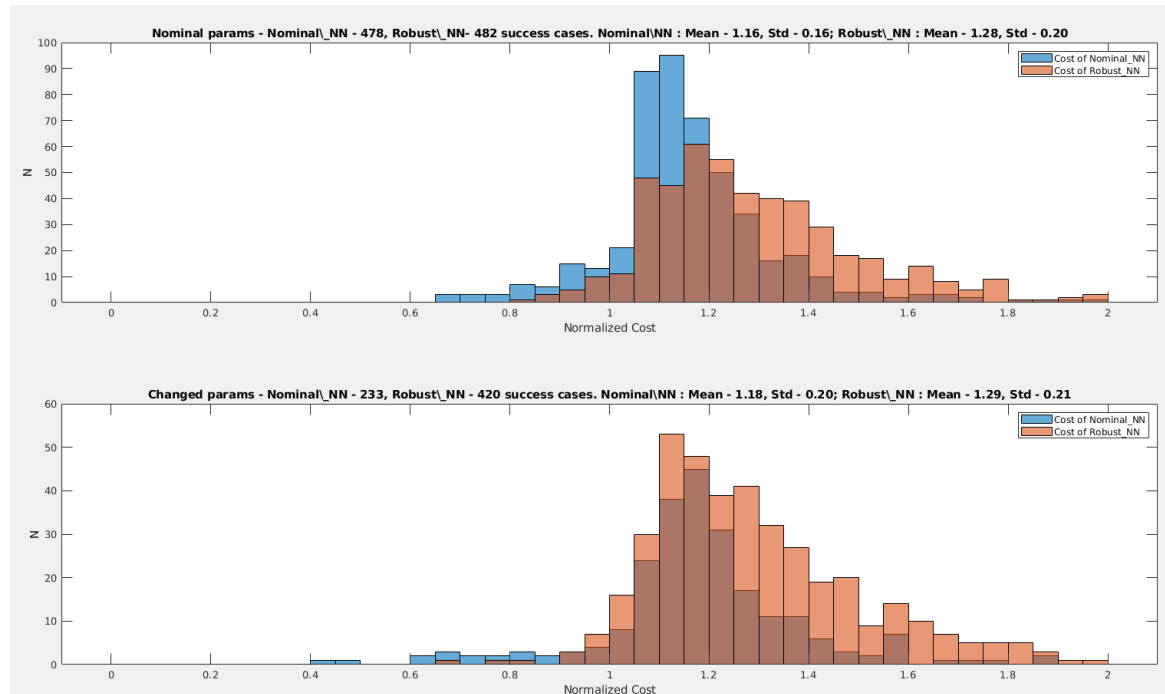


Figure 11. Histogram of costs of nominal NN and robust NN

# Simulations

- Average of system parameters on x and y axes
- Mass – from 0 to 30 %
- Spring constants – from -20 to 0 %
- Pendulum friction coefficients – from -20 to 20 %

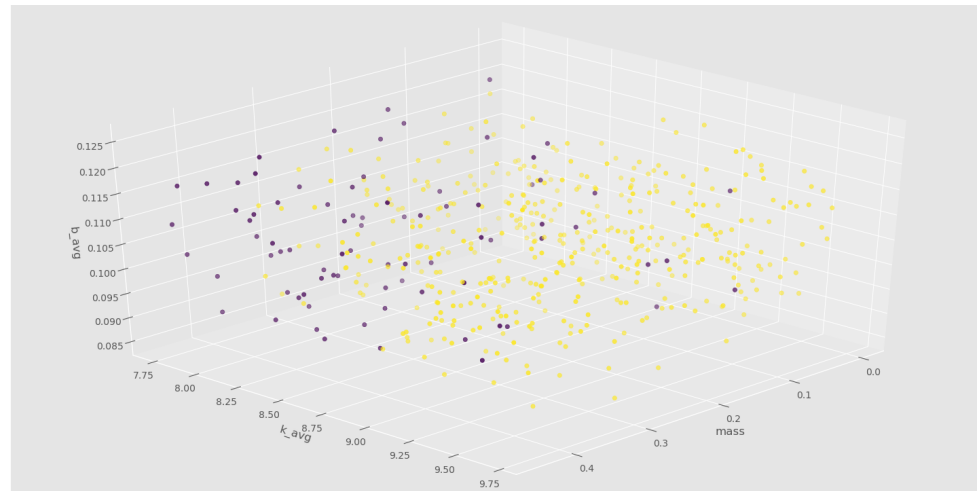
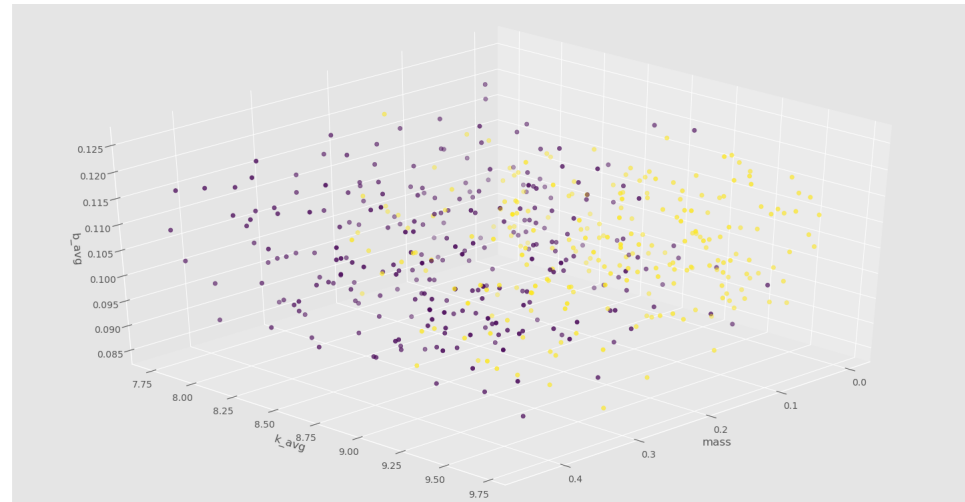


Figure 12. 3D plot of parameter simulation. (dark blue – fail, yellow – success)

# Experiments

	Mass (grams)			
	0	100	200	300
Nominal NN	10	6	4	0
Robust NN	10	9	5	0

Table 4. The success rates of Nominal NN and Robust NN for 10 different trajectories

	Mass (grams)			
	0	100	200	300
Nominal NN (x1e-4)	21.65	26.27	28.64	29.46
Robust NN (x1e-4)	21.99	22.83	26.31	27.32

Table 5. Average cost of Nominal NN and Robust NN for 10 different trajectories.

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# Future work and Conclusion

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- Conduct experiments with:
  - Putting balanced mass
  - Changed springs
  - Changed pendulum friction coefficient
- Reaction wheel integration improves VSA/VIA robots positioning accuracy
- Robust NN can deal with parameter uncertainties and has more accuracy comparing with nominal NN (~40 % more)



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# Thank you!

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NAZARBAYEV  
UNIVERSITY



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