

TARGET-SHAPE IDENTIFICATION FOR RADAR APPLICATIONS

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**Submitted in fulfillment of the requirements
for the degree of Master of Science
in Mechanical & Aerospace Engineering**



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April 2026

DECLARATION

I hereby, declare that this manuscript, entitled “*Target-Shape Identification for Radar Applications*”, is the result of my own work except for quotations and citations, which have been duly acknowledged.

I also declare that, to the best of my knowledge and belief, it has not been previously or concurrently submitted, in whole or in part, for any other degree or diploma at Nazarbayev University or any other national or international institution.



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Date: 11.04.2026

Abstract

The interaction of electromagnetic (EM) waves is vital for applications like radar-based target shape identification. However, determining the shape of flying objects remains a challenge, especially in the case of modern military planes with complex geometries. This is mainly due to the ill-posed nature of the Inverse Scattering Problems (ISPs). This research introduces a novel approach for shape reconstruction based on the scattered EM field around the target. It addresses limitations of conventional radar imaging methods like poor accuracy caused by environmental noise, limited signal information, high computational time, and complexity. The proposed method implements an Iso-Geometric Boundary Element Method (IGABEM) for geometry representation as well as scattered EM field approximations. Optimization algorithms in MATLAB are used for target shape reconstruction by minimizing the deviation between the actual scattered signal and the one coming from candidate geometries. The proposed approach improves radar technologies and can contribute to military threat assessment and air traffic control applications.

Keywords: *electromagnetic waves, electromagnetic scattering, shape reconstruction, Inverse Scattering Problem, Iso-geometric Boundary Element Method.*

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List of Abbreviations & Symbols

EM	Electromagnetic
ISP	Inverse Scattering Problem
IGA	Iso-Geometric Analysis
BEM	Boundary Element Method
IGABEM	Iso-Geometric Analysis enabled Boundary Element Method
NURBS	Non-Uniform Rational B-Spline
DoF	Degree of Freedom
GA	Genetic Algorithm
BIE	Boundary Integral Equation
SCP	Shape Controlling Point
A	Area
λ	Wavelength
α	Illumination Angle
σ	Surface Electric Conductivity
k_0	Wavenumber
η_0	Wave Impedance
E	Electric Field
J	Electrical Current Flowing per Unit Area
E_{back}	Background Electric Field
E_{scat}	Scattered Electric Field
G	Green's Function
Ω	Domain
$\partial\Omega$	Target's Boundary Geometry
$H_0^{(2)}$	Zeroth Order Hankel Function of the Second Kind
P_{scat}	Scattered Power per Unit Length in the Z-axis
r	NURBS Curve
b_i	Control Points
$R_{i,k}$	Rational Basis Functions

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Chapter 1 – Introduction

When electromagnetic (EM) waves interact with objects, they can be transmitted, absorbed, or scattered in different directions with a change in their intensity. Among these interactions, EM scattering plays a vital role in applications like radar-based target shape identification [1], [2]. Inverse Scattering Problems (ISPs) determine the shape and other physical properties of the scatterer based on scattering data produced by that object. However, determining the shape of flying objects remains a challenge, especially in the case of airplanes with complex geometries. This is mainly due to the ill-posed nature and high noise sensitivity of ISPs. Small uncertainties in the data may lead to large errors in the obtained calculations [3]. This hinders the accurate and reliable results identification, especially when facing the scenario in which noise is unavoidable. In the case of this research (indicative example in Fig. 1.1), the scattering signals not only stem from the flying object, but also from ground and other objects on the terrain, like trees and settlements. Typically, noise filtering techniques are necessary to isolate the target aircraft's EM scattering signal from the environmental noise.

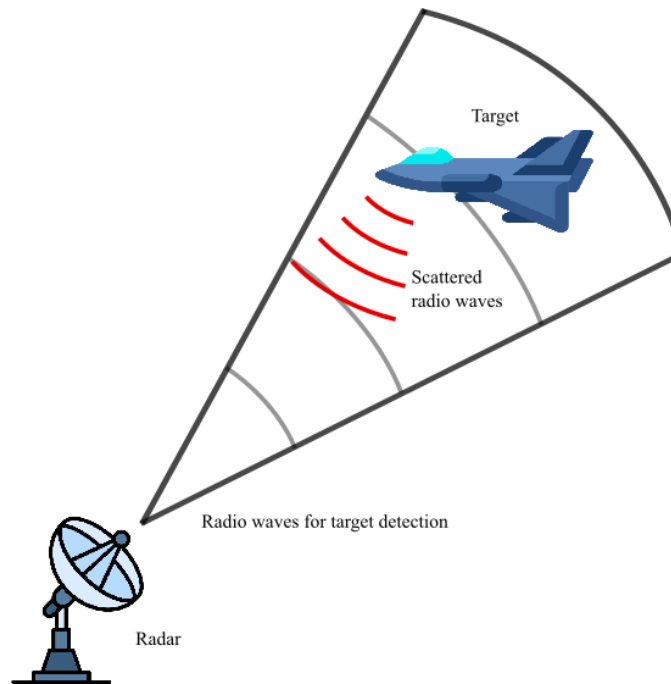


Figure 1.1: Problem visualization.

Despite these challenges, shape determination is crucial in military applications, since it enables fast aircraft model identification, threat assessment, and strategic response. Previously,

shape determination was done only on relatively small unmanned aerial vehicles, but not on military planes. Shape determination via EM scattering can also be used in civilian applications, ensuring public safety, by air traffic surveillance and control.

To overcome limitations of current target shape identification methods, this research proposes a novel reconstruction framework that employs the Iso-Geometric Boundary Element Method (IGABEM). IGABEM proved to be a reliable tool for engineering analysis in unbounded domains as well as scattered EM field calculation; see [4], [5], [6]. In this work, MATLAB will serve as the implementation framework due to its extensive numerical toolboxes, particularly local and global optimization algorithms as well as packages for geometry representations. These tools will be used in shape reconstruction by minimizing the difference between actual signals and calculated response of candidate geometries. The proposed framework addresses limitations of conventional radar imaging methods like poor accuracy caused by limited signal information, environmental noise, motion-dependence, and complexity.

Chapter 2 – Literature Review

2.1. IGABEM for electromagnetic wave interactions

The Iso-Geometric Boundary Element Method (IGABEM) is a numerical approach apt for solving partial differential equations in unbounded domains with no analytical solutions. Originally, Iso-Geometric analysis (IGA) was developed to connect Computer Aided Design models with Finite Element Analysis solvers by using NURBS basis functions for both geometry and analysis. The Boundary Element Method (BEM) is a technique for solving problems with boundary integral equations, reducing dimensionality and computational complexity. IGA combined with BEM made it possible to solve boundary integral equations more efficiently [4].

IGABEM typically represents geometric entities by employing a NURBS representation although this choice is not an integral part of the method and other representations can be used. In its core, IGA uses the same basis functions for the analysis and representation of the geometry, resulting in automated and more efficient computations [7]. Authors in [6] studied the role of geometry representation in IGABEM and proposed a novel physics-informed parametrization approach which is applicable to all problems typically treated with BEM approaches. In their work, IGABEM was formulated for the case of an exterior flow problem in an unbounded domain with the velocity potential being approximated with NURBS. The proposed physics-informed parametrization enables automatic refinement in complex regions based on the variation of the field quantity, i.e., velocity potential in this case. This led to relatively high accuracy with fewer degrees of freedom (DoFs).

One of the key applications of IGABEM is solving Maxwell's equations for EM wave interactions via boundary integral equations. Authors in [5] studied how shape optimization can affect the EM field concentration with the employment of IGABEM. They developed an optimization framework for shaping nanotubes that are optimal for EM field concentration. IGABEM provides an accurate estimation of the electric field on the nanotube's boundaries which in turn allows for a sufficiently accurate calculation of the field in the nanotube's interior. The solver developed in MATLAB was demonstrated to be computationally efficient and extremely accurate when compared with analytical solutions for trivial geometries. These results are

significant for photonic structures and EM applications such as receiver antennas. Comparatively, [8] developed an IGABEM framework for EM scattering analysis of dielectric objects based on Neural Networks. Significant reduction in scattering was achieved through shape optimization of dielectric object improving its stealth characteristics. NURBS-based geometry construction improves solver's efficiency and flexibility, while keeping its accuracy.

IGABEM for EM waves interaction may include various optimizers and programming environments. In [5], the authors utilize MATLAB software package with its genetic algorithm (GA) implementation being used to optimize the nanotube's shapes and get into the vicinity of the global maximum. Further, a gradient-based optimizer was fed with GA's results as starting point to maximize the chances of reaching the global maximum. Similarly, in [4] GA's results were used as initial data for local optimizers, but their solver was programmed in C++, allowing faster execution.

Based on pertinent literature, we see that IGABEM has been widely applied to simulate EM concentration and scattering. Success in these fields makes IGABEM application to problems involving inverse scattering a promising approach.

2.2. Inverse problem solving in electromagnetic scattering

Radar imaging finds numerous applications in military and aerospace. It plays a vital role in target identification and works using electromagnetic (EM) waves. EM waves' propagation and scattering are employed for shape identification of the objects that are located afar [9]. One of the challenges of traditional solving methods is the ISP, which determines the shape and other physical properties of the scatterer based on scattering data produced by that object.

To improve existing conventional scattering models, [9] focuses on relevant mathematical formulation and computation techniques. Accuracy can be improved through the upgrade of scattering approximations and accounting for nonlinear effects. Enhanced scattering models reduce the noise in the image and increase their resolution simultaneously. This also makes new image reconstruction algorithms applicable for the detection of complex-shaped objects. On the other side, [10] attempts to increase the stability of ISP solutions and overcome challenges like local minima by developing robust numerical algorithms. The authors propose Recursive

Linearization Method (RLM) as an approach to solve ISPs. Moreover, it was shown that multiple signal frequencies help in achieving unique outcomes and reduce noise effects. Authors also mention the possibility of implementing hybrid methods, where direct imaging approaches and optimizers are combined, to generate better initial guesses.

Solving ISPs is one of the major prerequisites for the identification of an object's shape and its material properties, when using scattered EM waves. In real world applications, radar systems often utilize this method to detect the target and reconstruct its shape.

2.3. Inverse scattering problems solved by optimization algorithms

Solving ISPs with global optimization algorithms and meta-heuristics is becoming increasingly popular. Since ISPs are often nonlinear with numerous local minima, gradient-free optimization methods like subspace-based optimization, particle swarm-based optimization and genetic algorithms are used to solve them [11], [12], [13], [14]. Compared to gradient-based algorithms where noise is amplified in derivatives, gradient-free algorithms are suitable for cases where noise robustness is required due to employment of sampling, projections and stochastic explorations. Moreover, evolutionary algorithms like genetic or swarm-based algorithms use parallelization that speeds up the solving process which in turn balances the additional computational cost of a higher number of iterations [11], [14]. However, computational cost and overall efficiency of the optimization methods can be improved significantly, if they are combined with learned models that guide the optimizer [13], or parametric signal processing like the Matrix Pencil Method proposed in [11]. Overall, all global optimization algorithms proposed in pertinent literature are reported to be sufficiently accurate when solving ISPs. Recent advances also highlight the possibility to integrate machine learning models or physics-informed neural networks with optimization, which will further reduce the computational cost and widen the range of optimizers' applicability [13].

2.4. Conclusion

The existing literature highlighted current challenges of target shape reconstruction techniques used in radars. Mostly, linearization of the ISPs is performed while solving them. This

indeed simplifies the problem, but it sacrifices accuracy of the result, which is crucial in real-time applications. Shape reconstruction algorithms in radars have many limitations, which makes them costly and slow at the same time. To address these problems, the IGABEM approach can be used for geometry reconstruction and EM waves scattering data analysis. Computational advantages of IGABEM make it possible to speed up the existing ISPs solving while maintaining or even increasing accuracy. The reconstruction itself can be done using parameter fitting of optimization methods in MATLAB. Combination of global and local optimization methods can help in avoiding local minima.

Chapter 3 – Aims and Objectives

The main goal of this work is to develop and validate a novel computational framework for the shape identification of flying objects (mainly aircrafts) based on EM scattering data. The framework will integrate IGABEM with MATLAB's optimization algorithms to improve accuracy, robustness, and efficiency of ISPs' solutions. To achieve this aim, the research will pursue the following objectives:

- Review the existing literature related to simulation of EM waves interaction using IGABEM and/or ISPs solving using optimization algorithms.
- Develop a methodology that estimates EM scattering with an IGABEM approach. Formulate the IGABEM boundary integral equation and objective function.
- Use parametric models to construct valid geometries.
- Apply a hybrid approach using MATLAB's global and local optimization algorithms for shape identification.
- Evaluate the numerical accuracy of the proposed method by calculation of relative errors.
- Assess the robustness of the method against environmental noise, computational demand, and applicability to complex geometries.
- Indicate potential benefits of the framework for military and surveillance applications.

Chapter 4 – Methodology

4.1. Electromagnetic scattering

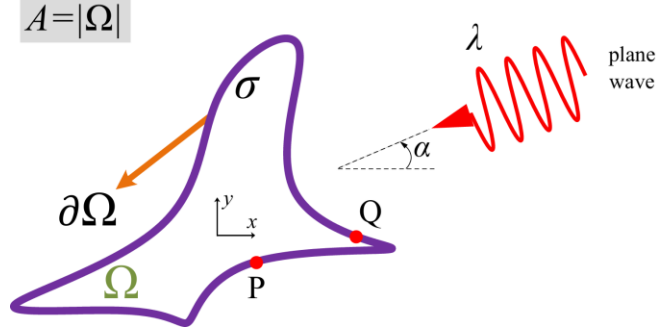


Figure 4.1: Setup of the problem: target with area A and surface conductivity σ illuminated by plane wave with wavelength λ at angle α .

We consider the problem where an unknown target with area A and surface conductivity σ is illuminated by plane EM wave with wavelength λ at angle α , as shown in Fig. 4.1. The resulting EM waves' behavior is calculated using Maxwell's equations [4], [5]. Specifically, we obtain the wave equation directly from Maxwell's equations:

$$[\nabla^2 + k_0^2]E(P) = ik_0\eta_0J(P) \quad (1)$$

where $E(P)$ denotes the electric field at a random point P in the domain, k_0 represents the wavenumber, η_0 is the wave impedance into vacuum, and $J(P)$ the amount of electrical current flowing per unit area. For the electric field at the boundary of the scatterer, we have:

$$E(P) = E_{back}(P) + E_{scat}(P), \quad (2)$$

where $E_{back}(P)$ is the background electric field (e.g., coming from the radar) and $E_{scat}(P)$ is the scattered part computed via:

$$E_{scat}(P) = -ik_0\eta_0\sigma \oint_{\partial\Omega} G(P, Q)E(Q) dl_Q, \quad (3)$$

where σ is a surface electric conductivity of the aircraft's surface material. For simplicity, it is assumed here that conductivity is constant along the plane's geometry $\partial\Omega$. $G(P, Q)$ is the Green's function for the problem at hand and is given by:

$$G(P, Q) = -\frac{i}{4} H_0^{(2)}(k_0 \|P - Q\|), \quad (4)$$

with $H_0^{(2)}$ being the zeroth order Hankel function of the second kind.

We obtain the corresponding boundary integral equation (BIE) by combining Eqs. (2) and (3):

$$E(P) + ik_0 \eta_0 \sigma \oint_{\partial\Omega} G(P, Q) E(Q) dl_Q = E_{back}(P) \quad (5)$$

The BIE is constructed so we only need to calculate the EM field on the boundary of the target's body. After computing $E(Q)$, i.e., the electric field at the boundary, we may use the same BIE to compute $E(P)$ at any point P in the domain.

In our case we consider a plane EM wave for which E_{back} is given by:

$$E_{back}(P) = e^{-ik_0(x \cos \theta + y \sin \theta)} \quad (6)$$

where θ corresponds to the angle between the horizontal axis and wave propagation direction.

Considering (r, ϕ) and (r_0, ϕ_0) to be cylindrical coordinates of P and Q respectively, Eq. (4) can be rewritten as:

$$G(r, \phi, r_0, \phi_0) = -\frac{i}{4} H_0^{(2)} k_0 \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi - \phi_0)} \quad (7)$$

According to [15], Hankel function can be expanded like:

$$H_v^{(2)}(z) \sim \sqrt{\frac{2}{\pi z}} e^{-i(z - \frac{1}{2}v\pi - \frac{1}{4}\pi)}, \quad (-2\pi < \arg z < \pi). \quad (8)$$

Now, Eq. (7) takes following form after application of Newton's approximation:

$$G(r, \phi, r_0, \phi_0) \approx -\frac{i}{4} \sqrt{\frac{2i}{\pi}} \frac{e^{-ik_0 r}}{\sqrt{k_0 r}} e^{-ik_0 r_0 \cos(\phi - \phi_0)} \quad (9)$$

The z component of the scattered electric field can be found as:

$$E_z(r_P, \phi_P) = -ik_0 \sigma \eta_0 \oint_{\partial\Omega} G(r_P, \phi_P, r_Q, \phi_Q) E(r_Q, \phi_Q) dl_Q \stackrel{(9)}{\Rightarrow}$$

$$E_z(r_P, \phi_P) \approx -\frac{k_0 \sigma \eta_0 \sqrt{2i} e_0^{-ikr_P}}{4\sqrt{\pi} \sqrt{k_0 r_P}} U(\phi_P) \quad (10)$$

where $U(\phi_P)$ is voltage-like quantity and given by:

$$U(\phi_P) = \oint_{\delta\Omega} e^{-ik_0 r_Q \cos(\phi_P - \phi_Q)} E(r_Q, \phi_Q) dl_Q \quad (11)$$

With application of Poynting's Theorem for the field in Eq. (3), the scattered power per unit length in the z-axis becomes:

$$P_{scat} = \frac{k_0}{16\pi\eta_0} |\sigma\eta_0|^2 \int_0^{2\pi} |U(\phi)|^2 d\phi \quad (12)$$

4.2. IGABEM Formulation

The geometry of the target will be represented with NURBS which is a typical choice in IGA approaches. Furthermore, an appropriate parametric model will be used to ensure that the generated geometric instances do not have self-intersections, corners or other unwanted discontinuities; a similar approach was used in [4], [5], [6]. The EM field on the surface of the targeted object is then expressed using the same representation (basis functions) which are used in the employed geometry. This is the core requirement of an IGA approach and enables the automatic (no need of further domain discretization) and efficient solution of the resulting linear system. As shown in pertinent literature, this approach secures an accurate approximation of the field quantities with a relatively low number of DoFs.

The target's boundary geometry can be described using the following NURBS curve:

$$r(t) = \sum_{i=0}^n R_{i,k}(t) b_i, \quad t \in I = [t_{k-1}, t_{n-1}] \quad (13)$$

where $r(t)$ produces the points on the object's boundary, b_i are the curve's control points and $\{R_{i,k}(t)\}$ are the corresponding rational basis functions that ensure the continuity of the spline.

IGA approaches assume that the field quantity, i.e., the electric field E in our case, can be accurately expressed if the same basis functions are employed, and therefore:

$$E(Q) = \sum_{i=0}^n R_{i,k}(t)e_i, \quad t \in I, \quad (14)$$

with e_i being the unknown control values. If we now plug Eq. (14) into Eq. (5), we get the IGABEM BIE:

$$\sum_{i=0}^n R_{i,k}(t)e_i + ik_0\eta_0\sigma \sum_{i=0}^n e_i \int_I G(t,\tau)R_{i,k}(\tau)\|\dot{\mathbf{r}}(\tau)\|d\tau = E_{back}(t) \quad (15)$$

H-refinement (using knot insertions) produces a sequence of nested finite-dimensional spline spaces $\mathcal{S}_k(\mathcal{J}^{(0)}) = \mathcal{S}_k(\mathcal{J}) \subset \mathcal{S}_k(\mathcal{J}^{(1)}) \subset \dots \subset \mathcal{S}_k(\mathcal{J}^{(m)})$, with $m \in \mathbb{N}^*$ denoting the number of knots inserted in $\mathcal{J} = \mathcal{J}^{(0)}$ (the so-called knotvector), which can be used to achieve a more accurate approximation of $E(Q)$ while maintaining the original geometry intact:

$$\sum_{i=0}^{n+m} e_i R_{i,k}^{(m)}(t) + ik_0\sigma\eta_0 \sum_{i=0}^{n+m} e_i \int_I G(t,\tau)R_{i,k}^{(m)}(\tau)\|\dot{\mathbf{r}}(\tau)\|d\tau = E_{back}(t) \quad (16)$$

Although various methods can be used to solve Eq. (16), we employ a collocation method that uses the Greville abscissae for securing a low condition number for the matrix in the resulting linear system which improves the achieved accuracy. Hence, the BIE in Eq. (16) is converted into a system of linear equations corresponding to each of the selected collocation points. Greville abscissae are knot averages defined as $\hat{t}_j = \frac{1}{k-1}(t_{i+1} + t_{i+2} + \dots + t_{i+k-1})$, and they are equal, in number, to the number of basis functions. They guarantee collocation close to the maximum of the corresponding basis function and therefore help in maintaining a good condition number for the matrix in the resulting linear system. Therefore, by collocating at the Greville abscissae Eq. (16) yields the following linear system:

$$\sum_{i=0}^{n+m} e_i R_{i,k}^{(m)}(\hat{t}_j) + ik_0\sigma\eta_0 \sum_{i=0}^{n+m} e_i \int_I G(\hat{t}_j,\tau)R_{i,k}^{(m)}(\tau)\|\dot{\mathbf{r}}(\tau)\|d\tau = E_{back}(\hat{t}_j) \quad (17)$$

where $j = 0, 1, \dots, n+m$. From this linear system $\{e\}_{i=0}^{n+m}$ can be found and plugged into Eq. (14) to calculate the electric field on the boundary of the target. Now, Eq. (11) can be rewritten in the following way:

$$U(\phi) = \int_I e^{-ik_0 \|\mathbf{r}(\tau)\| \cos(\phi - \phi(\tau))} \tilde{\mathbf{E}}(\tau) \|\dot{\mathbf{r}}(\tau)\| d\tau \quad (18)$$

where $\phi(\tau) = \arctan(\frac{y(\tau)}{x(\tau)})$. This can be further used in Eq. (12) to calculate the scattering power per unit length in the z-axis.

4.3. Inverse problem formulation

In the scope of this work, the unknowns are the shape, size, and material of the target object. To get more information about the targeted object, EM waves are sent by a multi-frequency radar. We assume here 5 EM waves with varying wavelengths $\lambda_i = \frac{\sqrt{A}}{0.25(i+1)}$, normalized with respect to the characteristic area of the target A , that hit the object. Additionally, a limited part of the scattered signals becomes available to our receiver. If α is the illumination angle, we assume that we can only receive the scattered signals in $[\alpha - \xi; \alpha + \xi]$, where ξ is a relatively small angle which can be changed depending on the problem at hand. In this work, three signal ranges of 90, 60 and 30 degrees will be considered, meaning that ξ will assume the value of 45, 30 and 15 degrees, respectively. In the default setup, the EM waves hit the target from the left side, which corresponds to $\alpha = 180^\circ$.

Assuming now that the shape s is encoded by an appropriate parametric vector $\mathbf{v}$ and σ is the unknown electric conductivity, the ISP can be solved using the following minimization problem:

$$\begin{aligned} \text{find } \sigma^*, \mathbf{v}^*: \quad & f(\sigma^*, \mathbf{v}^*) = \min_{\sigma, \mathbf{v}} \sum_{i=1}^5 \int_{\alpha-\xi}^{\alpha+\xi} \left(U_i(\phi) - U_{s_i}(\phi, \mathbf{v}, \sigma) \right)^2 d\phi \\ \text{subject to:} \quad & \mathbf{v} \in \mathcal{V} \subset \mathbb{R}^n, \\ & \sigma \in \mathcal{C} \subset \mathbb{C}, \\ & \text{with given } \lambda_i \text{ and } U_i(\phi), i = 1, \dots, 5, \end{aligned} \quad (19)$$

where n is the number of parameters used to determine the shape of the target and $U_i(\phi)$ correspond to actual i^{th} scattering signal when hit by the i^{th} EM wave. The optimizer is searching for the values of σ and the shape-controlling parametric vector $\mathbf{v}$ of the boundary $\partial\Omega_s$, so that

$U_s(\phi) \approx U(\phi)$ and consequently $\partial\Omega_s \approx \partial\Omega$, while the area $A_s(\mathbf{v}) = |\Omega_s|$. L^2 -norms between available and calculated scattering profiles are minimized to successfully reconstruct target's geometry.

4.4. Parametric model

If curve control points are used directly as design variables in our optimization framework, it will possibly result in complex high-dimensional optimization problems as it will require a lot of constraints to secure construction of valid shapes [5]. Thus, an equal-sector-based parametric model that constrains placement of shape controlling points (SCPs), shown as red points in Fig. 4.2, in the corresponding sector was used to generate valid (non-self-intersecting) geometries. The design vector $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$, $\mathbf{v} \in \mathcal{V} = [0, 1]^n$, with $n = 2m$, is used to define the number and position of the m SCPs. Specifically, the polar coordinates of their positions are computed as:

$$\mathbf{q}_i = \left[r_i \cos\left(\phi_i + \frac{2\pi i}{m}\right), r_i \sin\left(\phi_i + \frac{2\pi i}{m}\right) \right]^T$$

$$\text{where} \quad r_i = v_i(r_{\max} - \epsilon) + \epsilon, \quad \phi_i = \frac{2\pi v_{i+m}}{m}, \quad (20)$$

$$\mathbf{v} \in [0, 1]^n, \text{ i.e., } v_i, v_{i+m} \in [0, 1], i = 0, 1, \dots, m-1,$$

where ϵ is small positive number to avoid SCP placement at the center and $r_{\max}$ is the maximum distance SCP can be placed from the center.

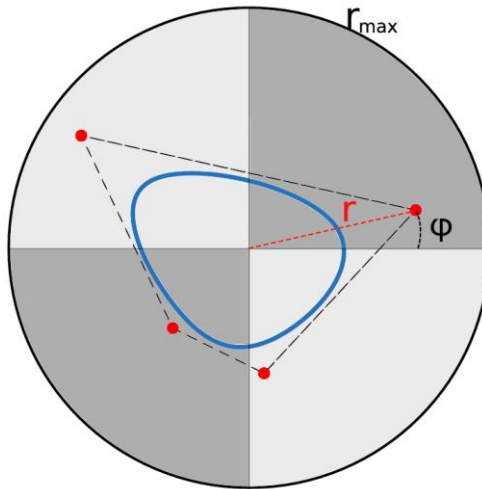


Figure 4.2: The equal-sector-based parametric model's construction.

4.5. Optimization in MATLAB

As was stated in previous sections, this work aims to solve the ISP via optimization. To do that, MATLAB's Optimization and Global Optimization toolboxes were combined:

- *fmincon()*: a gradient-based nonlinear programming optimizer supporting linear and nonlinear constraints [16];
- *patternsearch()*: a direct search global optimizer supporting linear and nonlinear constraints [17];

To ensure that the obtained solution is a global optimum while limiting the number of needed iterations, a combination of the two optimizers listed above was used to devise a hybrid optimization scheme. *patternsearch()* was used to identify the region of the global optimum and subsequently its result is supplied to *fmincon()* as the starting point to quickly reach the global optimum. This hybrid approach proved to be effective in solving optimization problems as shown in [4].

Chapter 5 – Results and Discussion

5.1. Methodological accuracy

The accuracy of the framework was evaluated via comparing the analytical and numerical solutions of the electric field on a benchmark geometry. A circle was used to compute the L^∞ and L^2 norms of the deviation between two solutions for increasing number of h- (knot insertion) and p-refinements (degree elevation of the corresponding rational polynomials). The geometry is exactly represented by the NURBS basis at all refinement levels, which is a key advantage of IGA over standard BEM. Fig. 5.1 demonstrates high convergence rates, consistent with results reported in pertinent literature [4], [5], [6]. The quintic representation reaches near machine-precision with fewer than 100 DoFs. The result validates the IGABEM numerical solution and justifies its use in the scope of this work.

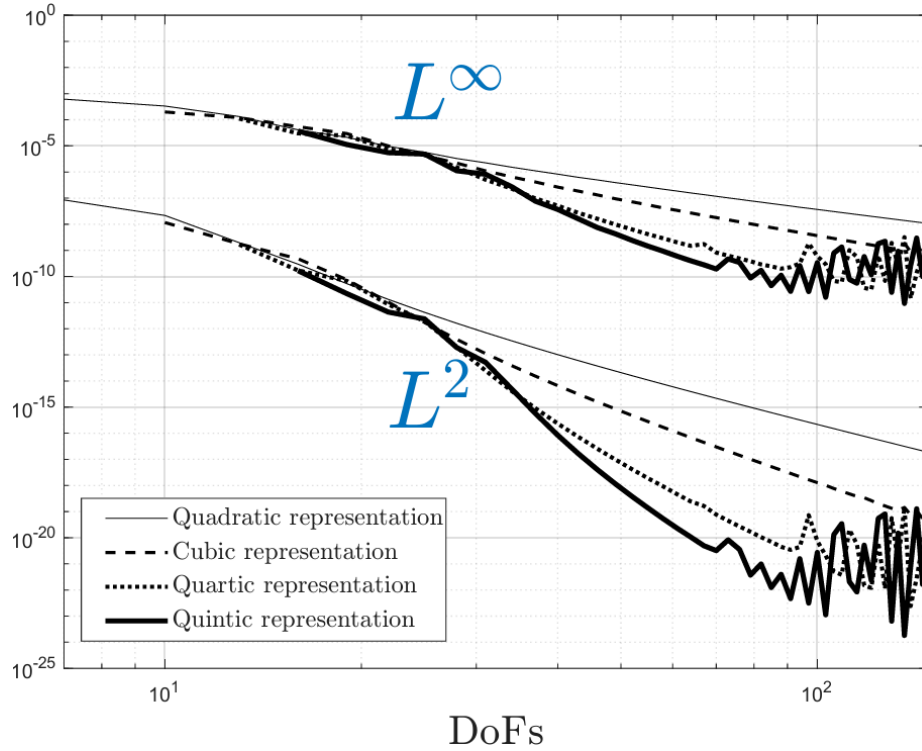
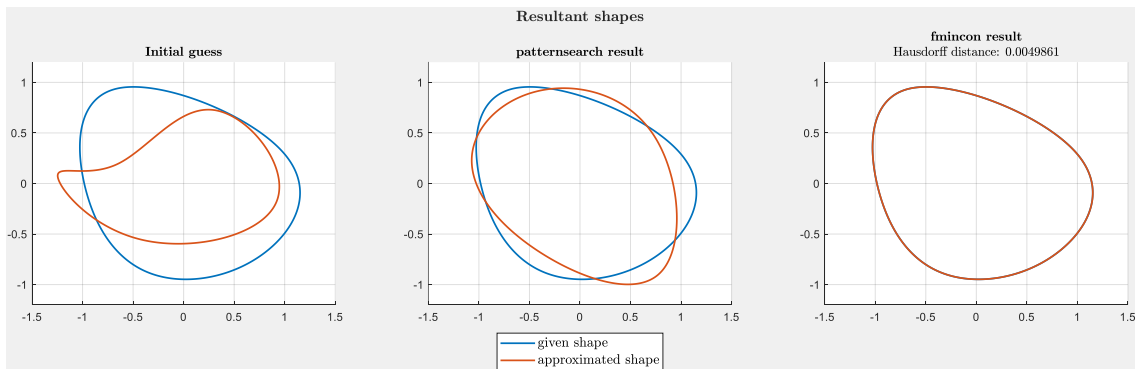


Figure 5.1: Difference between the analytical and numerical solutions of the electric field on the circular geometry.

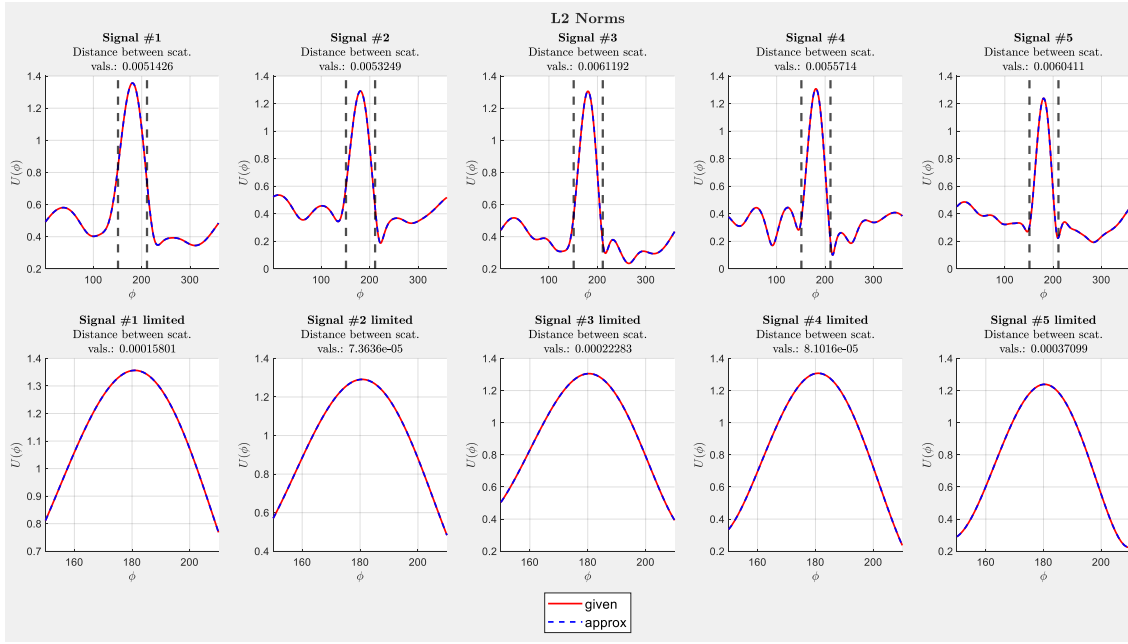
5.2. Shape reconstruction results

5.2.1. Framework testing

After the framework was fully developed, it was decided to evaluate it via shape reconstruction of the simple target shown with blue color on the top left plot in Fig. 5.2(a). As was stated in Section 4.3, we consider the inverse scattering problem of simultaneously recovering an unknown target geometry and its electric surface conductivity. A geometry with $\sigma = 0.001 - 0.010i$, $A = 2$ and 5 SCPs was chosen to perform the first test. For this case $\xi = 30$ degrees, meaning that 60 degrees (out of 360 degrees) of each signal is available and can be supplied to our framework to perform the shape reconstruction. In Fig. 5.2(a) we can see the shape approximation results with blue curve being the original target geometry and orange curve being the reconstructed geometry at each stage. Left panel shows the initial guess used as a starting point to the global optimizer, i.e., *patternsearch()*. Middle panel displays the resulting shape after *patternsearch()*, which is then fed to the local optimizer *fmincon()* as a starting point. Right panel illustrates the result from *fmincon()*, which is our final shape reconstruction result. As one can easily see, the shape was accurately identified with a final Hausdorff distance of $d_H = 0.005$ (between the actual and calculated shape). In Fig. 5.2(b) the scattered signal from the target (red-solid line) and the approximated geometry (blue-dashed line) are compared. The top plots show the full range of the scattered signal. The bottom plots display the available signal range that is enclosed by two black dashed vertical lines in the top plots. The signal lines are overlapping, which means that the electric surface conductivity was correctly identified ($\sigma_{approx} = \sigma$). To evaluate the accuracy of the target identification, L^2 -norms were utilized. Fig. A1 in the Appendix shows that the RMSE of L^2 -norms are $5.653e - 03$ and $2.118e - 04$ for the full range and limited range of the signals respectively.

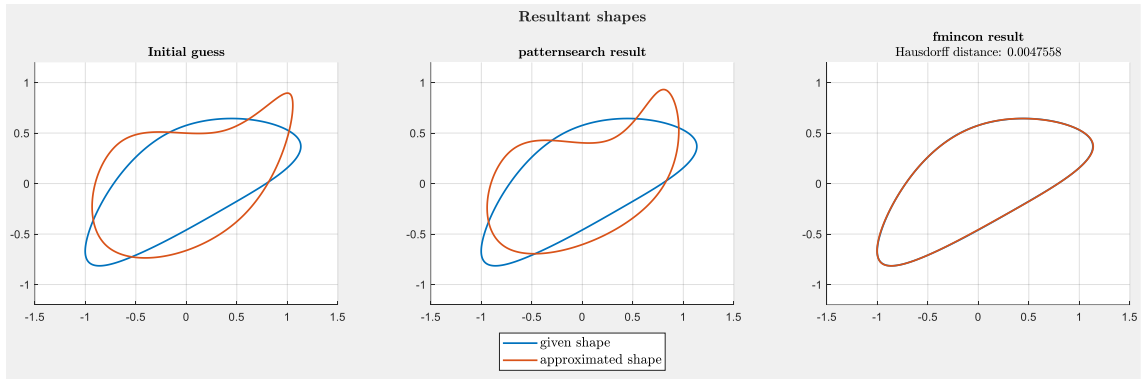


(a)

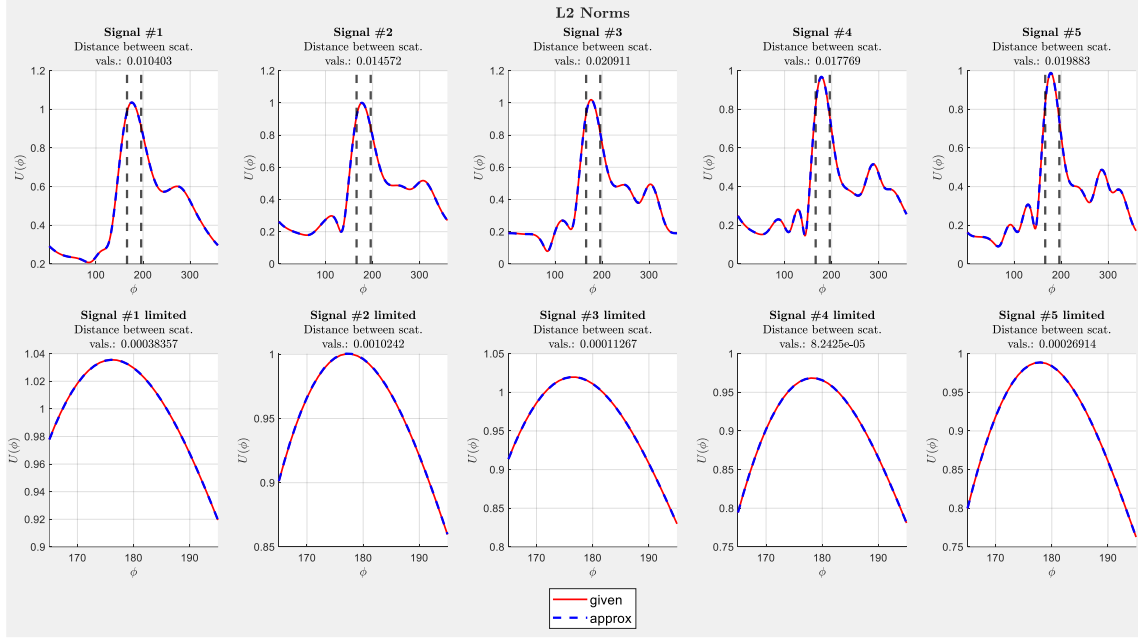


(b)

Figure 5.2: Approximation results for (a) shape and (b) scattering signals of the simple target with 5 SCPs and 60 degrees of scattering signal available.



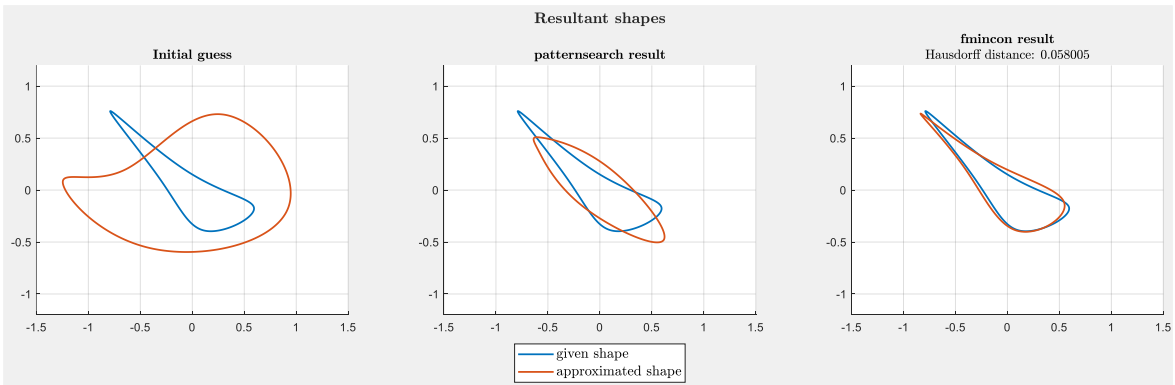
(a)



(b)

Figure 5.3: Approximation results for (a) shape and (b) scattering signals of the oval-like target with 5 SCPs and 30 degrees of scattering signal available.

For the next test, an oval-like geometry (blue-colored curve in Fig. 5.3(a)) was chosen with the same properties as in the first test ($\sigma = 0.001 - 0.010i$, $A = 2$ and 5 SCPs). However, the available scattering signal range was decreased to 30 degrees ($\xi = 15$ degrees). From the right panel of Fig. 5.3(a) the final shape reconstruction result seems to be accurate with Hausdorff distance $d_H = 0.005$. Fig. 5.3(b) shows that scattering signal lines overlapping, which is reasonable since the electric surface conductivity was correctly identified ($\sigma_{approx} = \sigma$). Fig. A2 in the Appendix shows that the RMSE of L^2 -norms are $1.714 - 02$ and $5.076e - 04$ for the full range and limited range of the signals respectively.



(a)

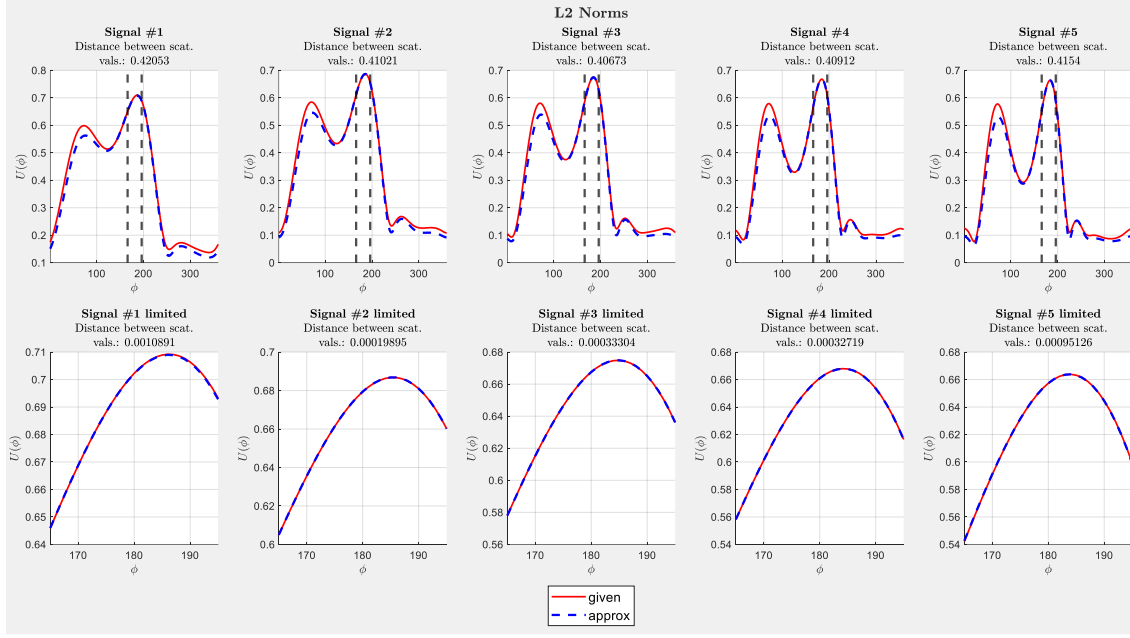
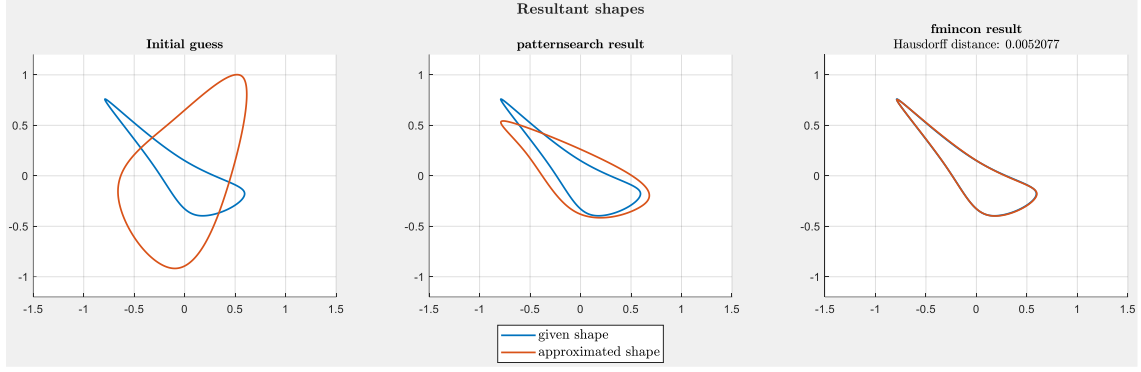
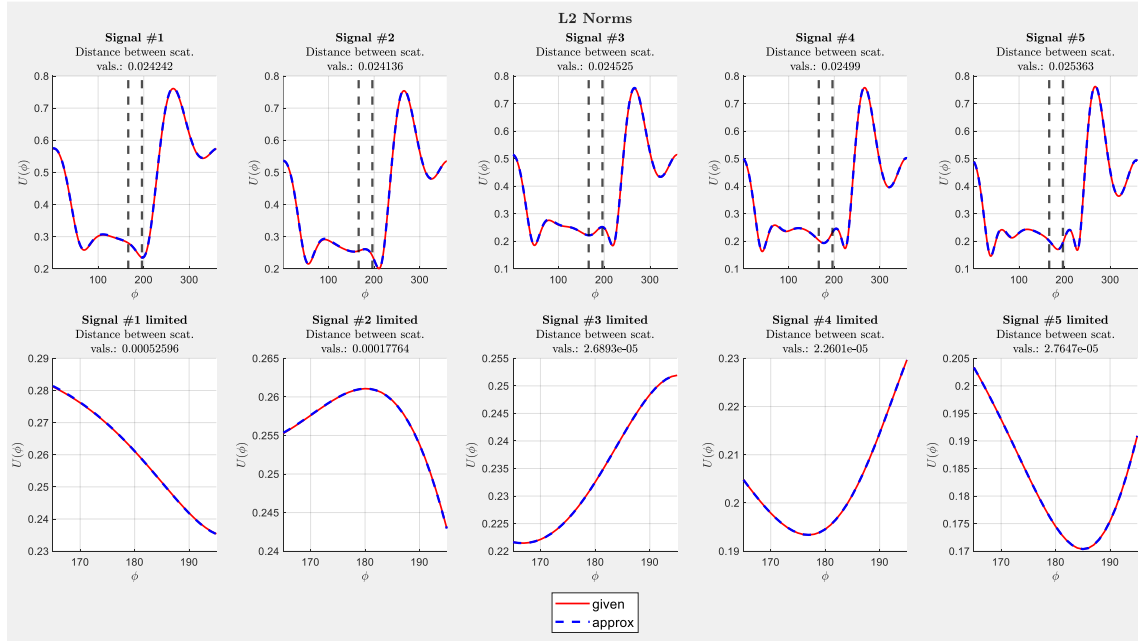


Figure 5.4: Approximation results for (a) shape and (b) scattering signals of the complex target with 5 SCPs and 30 degrees of scattering signal available.

A more complex geometry (Fig. 5.4(a)) was chosen to perform the third test. The properties of the target remained as in the previous two cases ($\sigma = 0.001 - 0.010i$, $A = 2$ and 5 SCPs). The available scattered signal range is also 30 degrees. Fig. 5.4(a) clearly demonstrates that reconstruction closely resembles the target's key geometrical features, with small differences and Hausdorff distance $d_H = 0.058$. At the same time, bottom plots of Fig. 5.4(b) show that approximated scattering signal is very close to the available signal (L^2 -norm = $6.854e - 4$). However, when we look at the whole signal displayed in top plots, noticeable deviations from the target's actual scattered signal can be observed (L^2 -norm = 0.412). The approximated surface conductivity is $\sigma_{approx} = 0.004 - 0.009i$. Such results can be explained by the very similar signals received for varying wavelengths. Having them very similar makes using multiple EM waves pointless. To demonstrate this, the incidence angles of the EM waves were changed.



(a)

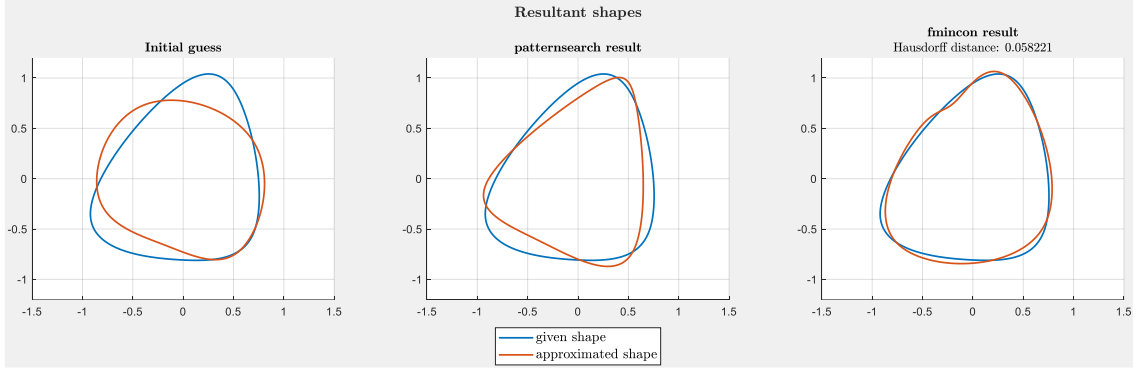


(b)

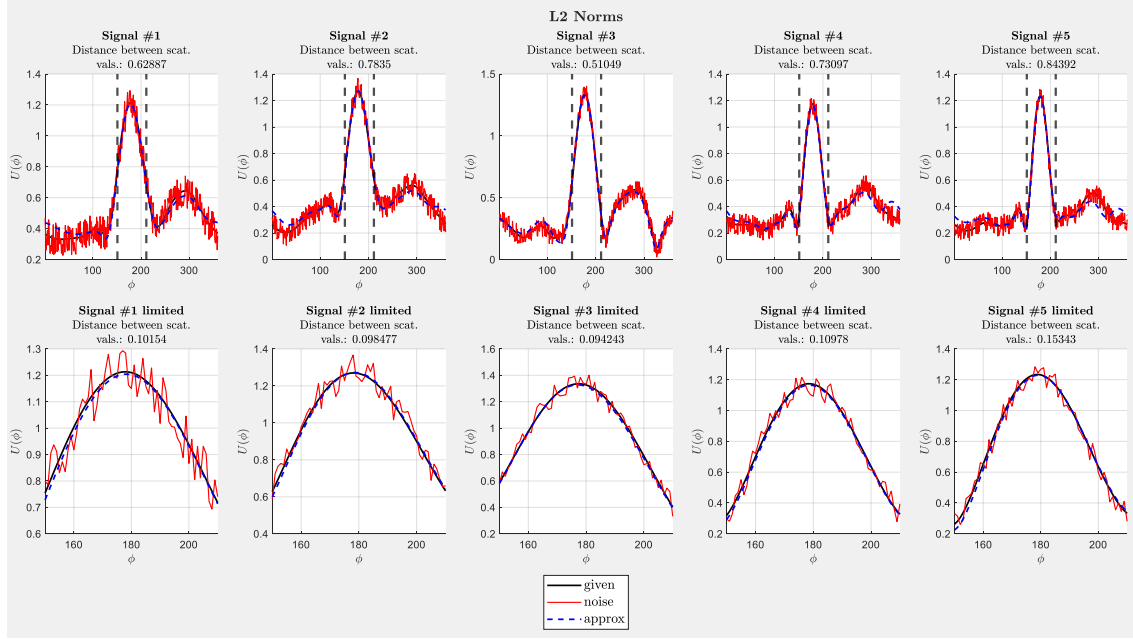
Figure 5.5: Approximation results for (a) shape and (b) scattering signals of the complex target with 5 SCPs, 30 degrees of scattering signal available and $\alpha = 90^\circ$.

In Section 4.3, illumination angle α was mentioned to be 180° by default problem setup. It was changed so that EM waves hit the target from the top ($\alpha = 90^\circ$) and shape reconstruction process for complex shape with 5 SCPs was executed again (as in Fig. 5.4). The resultant shape is shown in Fig. 5.5(a) with Hausdorff distance $d_H = 0.005$. From Fig. 5.5(b) it can be observed that full range scattering signals match. The electric surface conductivity was correctly identified ($\sigma_{approx} = \sigma$). Fig. A4 in the Appendix shows that the RMSE of L^2 -norms are $2.466e - 02$ and $2.491e - 04$ for the full range and limited range of the signals respectively. The success is directly

related to the fact that available signal portions are very different, giving our framework more information about the target. Which was our main goal when decreasing range of available signals.



(a)



(b)

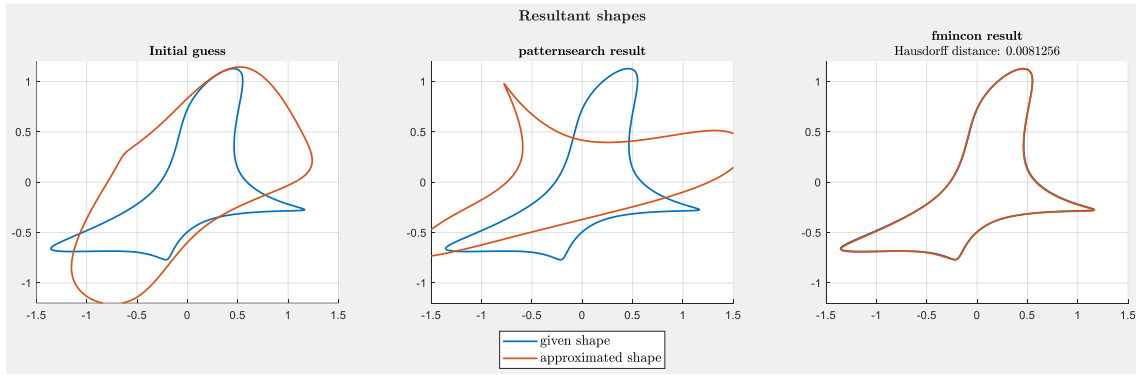
Figure 5.6: Approximation results for (a) shape and (b) scattering signals of the simple target with 5 SCPs and 60 degrees of scattering signal available with 20% noise level.

Next step was dedicated to bringing the framework closer to real life applications. To be precise, various levels of noise from 0 to 20% were introduced to the scattered signal data. This test was performed on a geometry with $\sigma = 0.001 - 0.010i$, $A = 2$ and 5 SCPs. 60 degrees of scattering signal available was considered. Fig. 5.6(a) displays the shape reconstruction result that has small deviations from the original shape with Hausdorff distance $d_H = 0.058$. The noisy signal

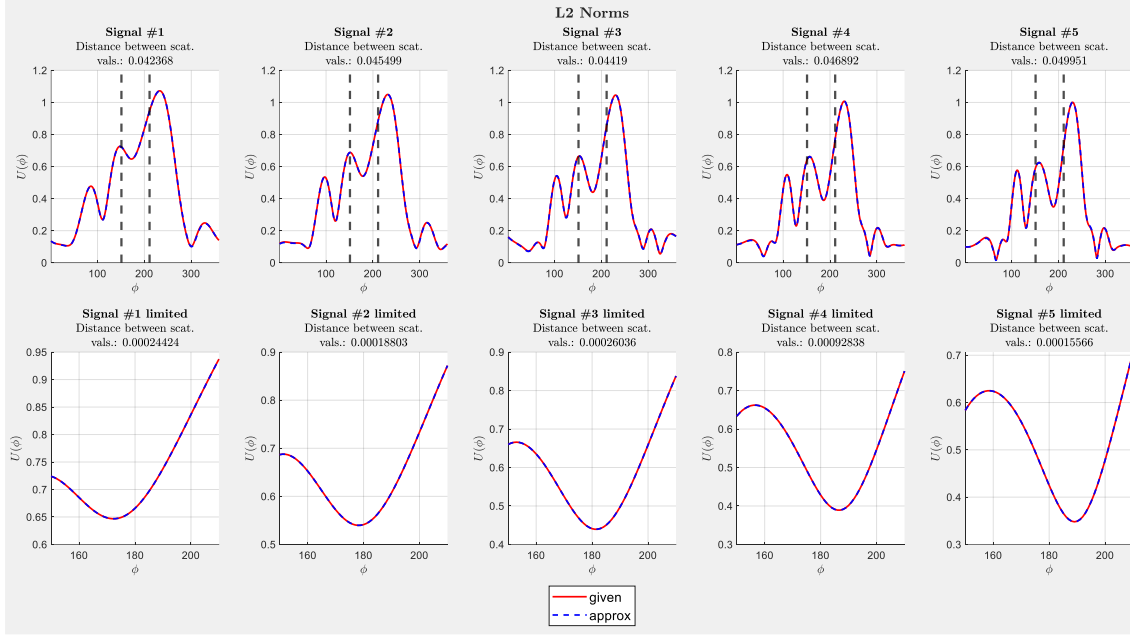
is shown with a red line in Fig. 5.6(b) and from the bottom plots we can see that the approximated signal follows closely (L^2 -norm = 0.114) the actual noisy signal excerpts, but from the full range plots, high deviations (L^2 -norm = 0.709) are noticed at the sides of the given signals data. The approximated electric surface conductivity was found to be 20% off in the real part, $\sigma_{approx} = 8e - 04 - 0.010i$.

5.2.2. Plane-like shape reconstructions

After several successful tests with relatively simple shapes, a target that bears some airplane features (several pointy edges) with 9 SPCs was selected for geometry reconstruction. The electric surface conductivity and area of the target was unchanged, $\sigma = 0.001 - 0.010i$, $A = 2$. Since the number of SCPs is much higher than previous cases, a wider range (60 degrees) of the scattering signal was considered. As can be seen in Fig. 5.7, both shape and signal reconstruction were successful with Hausdorff distance $d_H = 0.008$ and $\sigma_{approx} = 0.001 - 0.010i$. Best shape identification for this case was done at $\alpha = 135^\circ$.



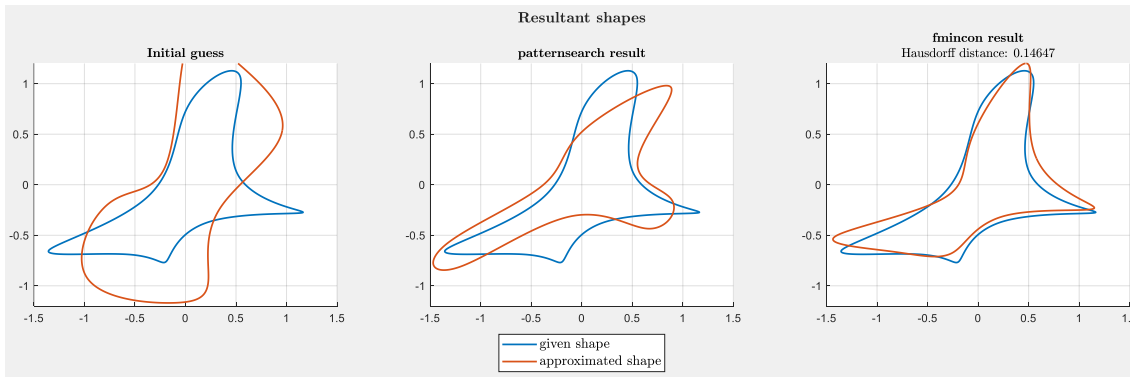
(a)



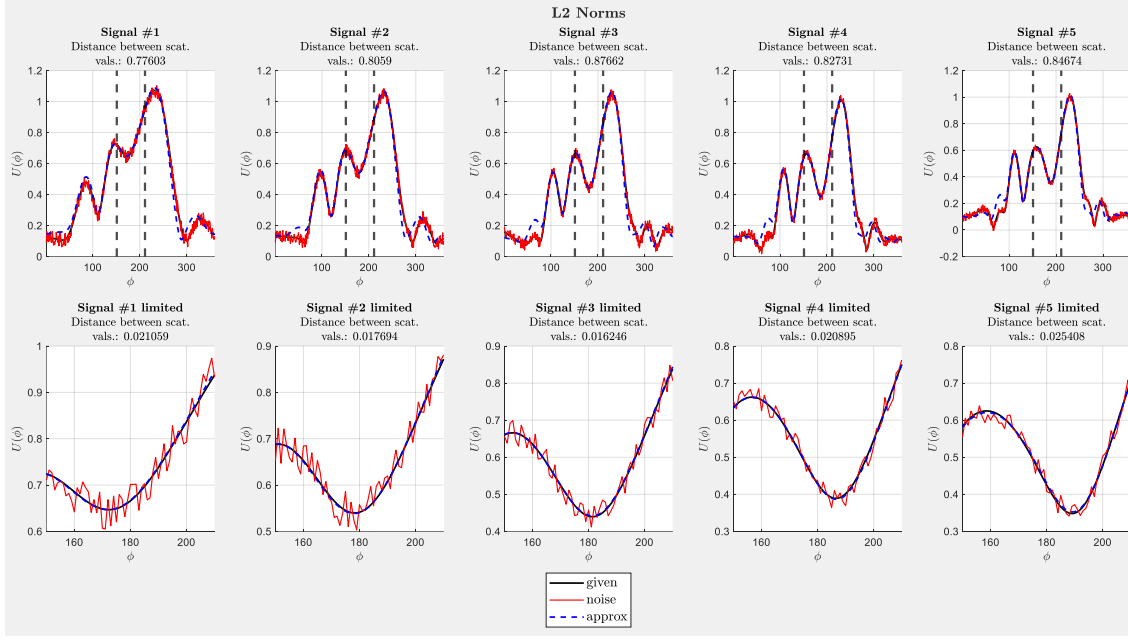
(b)

Figure 5.7: Approximation results for (a) shape and (b) scattering signals of the plane-like shaped target with 9 SCPs and 60 degrees of scattering signal available.

Fig. 5.8 below demonstrates the same case as Fig. 5.7, but with 10% noise added to the given signal. From Fig. 5.8(a) we see that the overall geometrical features were identified via shape identification and the Hausdorff distance was found to be $d_H = 0.146$. The limited signal portions were reconstructed with L^2 -norm = 0.021 despite high noise level. Nevertheless, full signal profiles have deviations at the sides (L^2 -norm = 0.827). The approximated electric surface conductivity is $\sigma_{approx} = 0.002 - 0.010i$, which is slightly off in real part compared with $\sigma = 0.001 - 0.010i$.



(a)



(b)

Figure 5.8: Approximation results for (a) shape and (b) scattering signals of the plane-like shaped target with 9 SCPs and 60 degrees of scattering signal available with 10% noise level.

As the next step, the number of SCPs was increased up to 10 and shape reconstructions were performed with new geometry given in Fig. 5.9(a). However, due to increased complexity, the shape approximations were failing consistently, meaning that the limit of the framework with the $\xi = 30$ degrees was reached. This led to a decision to widen the range of the available scattering signal to 90 degrees ($\xi = 45$ degrees). With more available scattering data framework successfully identified the target shape with $d_H = 0.017$ and $\sigma_{approx} = 0.001 - 0.010i$. Full signal profiles had minimal deviations with L^2 -norm = 0.014.

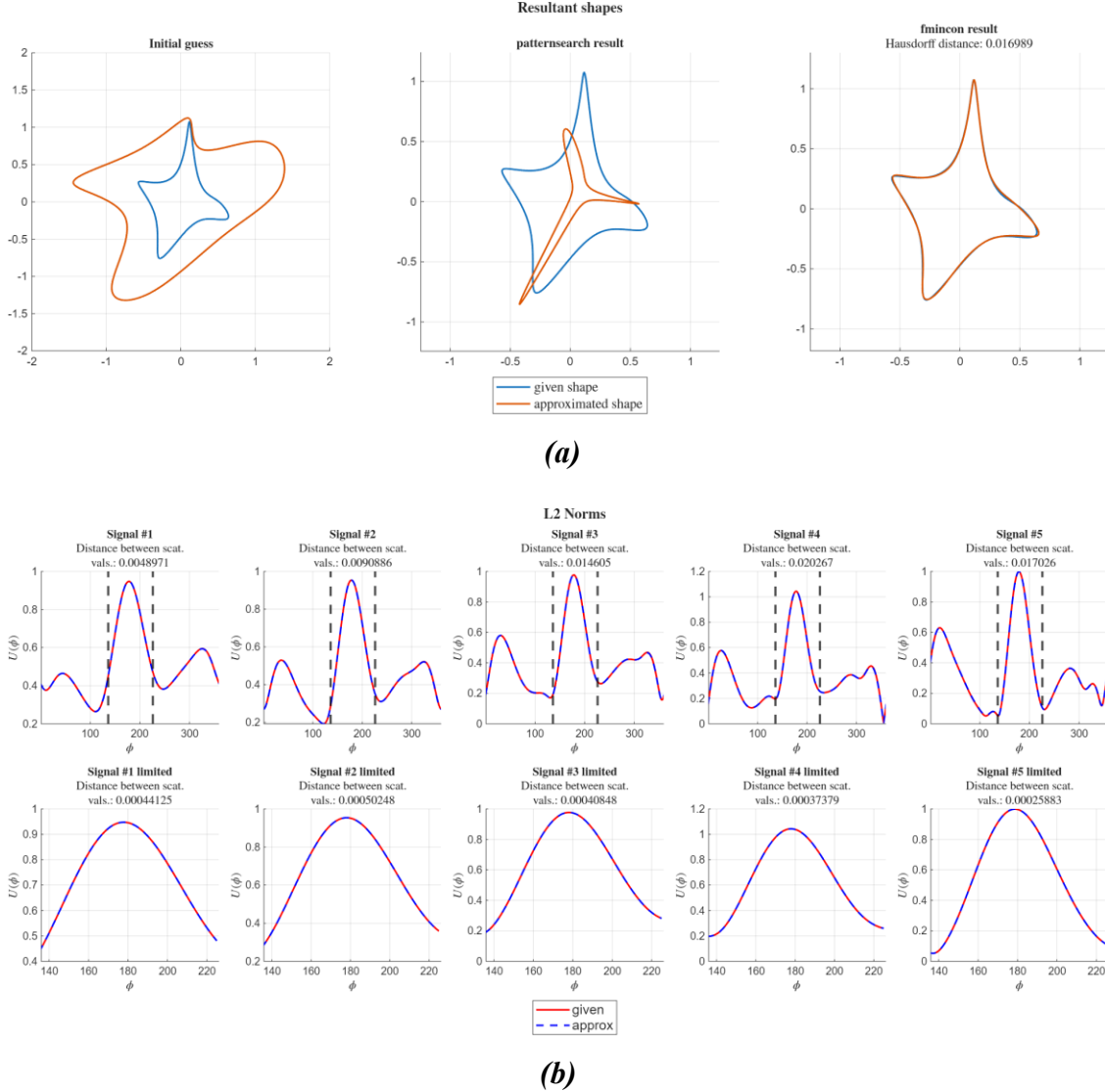


Figure 5.9: Approximation results for (a) shape and (b) scattering signals of the plane-like shaped target with 10 SCPs and 90 degrees of scattering signal available.

After, shape reconstruction was performed with 10% noise added to the scattering data. As a result, very similar shape (Fig. 5.10) was acquired with $d_H = 0.049$ and $\sigma_{approx} = 0.002 - 0.010i$. The available scattering profile was reconstructed accurately giving L^2 -norm = 0.049, but the deviation in the whole range of the signal is L^2 -norm = 0.528.

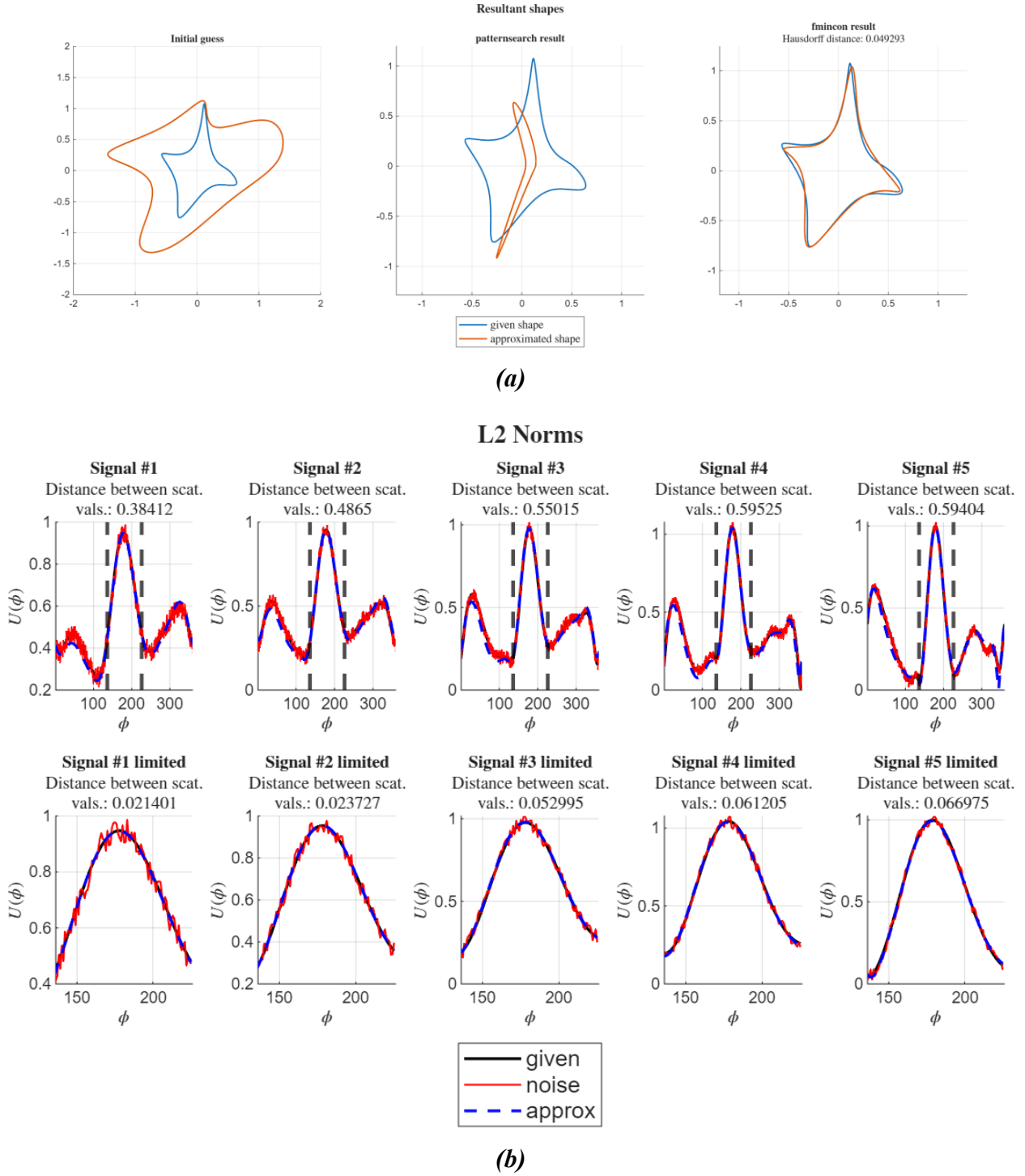


Figure 5.10: Approximation results for (a) shape and (b) scattering signals of the plane-like shaped target with 10 SCPs and 90 degrees of scattering signal available with 10% noise level.

5.2.3. SCPs and scattering signal complexity correlation

So far, the exact number of SCPs from the target geometry has been used to perform shape reconstruction. Nonetheless, it is unrealistic due to the target shape being unknown. A separate study was conducted to predict how many SCPs are needed to identify the target. This study focuses on the stochastic relationship between SCPs and scattering signal complexity. The complexity of the signal was evaluated by calculating its inflection points. As an example, the plane-like target was used from Section 5.1.2. On Fig. 5.11, target geometry with corresponding scattering signal complexity is shown. As can be seen, the target with 9 SCPs leads to 20 inflection points ($N_{IP} = 20$) in its full range signal.

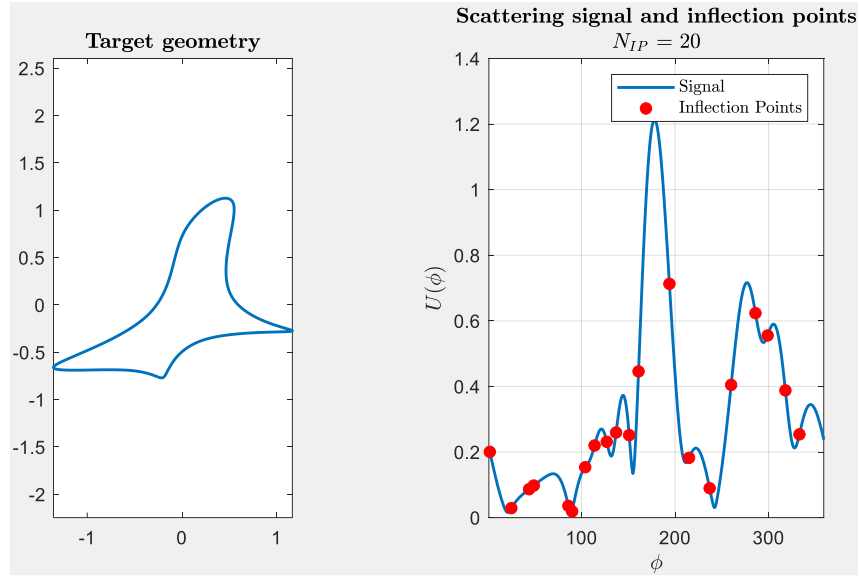


Figure 5.11. Target geometry with corresponding scattering signal complexity.

In total, seven different numbers of SCPs (ranging from 4 to 10) were evaluated and for each SCP count, signals from 200 randomly generated shapes were evaluated. To obtain statistical insights, the minimum, maximum, mean, median and standard deviation of inflection points were calculated for each SCPs number. The results of this study are summarized in Fig. 5.12. If we were to use the statistics obtained and predict the number of SCPs required to reconstruct plane-like geometry given in Fig. 5.11, then based on mean and standard deviation from 8 to 10 SCPs would be chosen. And highly likely, the middle number of candidate SCPs would be chosen, which is 9 SCPs, and it is same as the number of SCPs the target has.

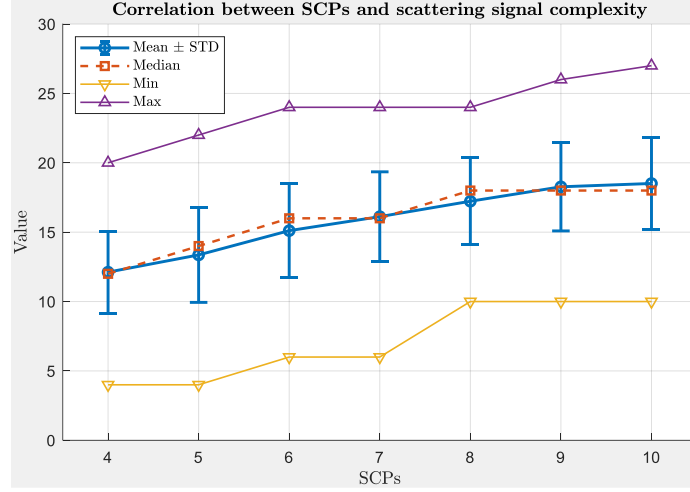


Figure 5.12. Correlation between SCPs and scattering signal complexity: summary statistics (Min, Max, Mean, Median, STD) for each number of SCPs.

5.3. Applications

The proposed shape identification framework has two main applications in military and civilian domains. In military contexts, precise shape reconstruction of aircraft geometry from scattered EM signals enables threat assessment and strategic response capabilities. This can be invaluable in cases that involve stealth or large-scale military planes, where traditional radars struggle due to limited signal coverage or noise coming from the environment. An accurate target determination could enhance defense systems in distinguishing between decoys and actual threats. In addition, this benefits optimization of interception trajectories and improving targeting accuracy.

In civilian settings, EM-based shape recognition can improve air traffic surveillance and safety management. For instance, airports monitoring systems could leverage reconstructed aircraft geometries to track non-cooperative or malfunctioning drones and detect unauthorized flights. Beyond aviation, the framework could also be used in sensing to detect vehicles in disaster zones with poor visibility. The development of such framework that is robust to noise and reliable with small available signal widens the applicability of radars in both security and safety.

5.4. Limitations

Limitations and constraints may arise from the proposed framework:

- Computational constraint: computationally intensive, especially for detailed aircraft geometries.
- Methodological constraint: the framework is limited by MATLAB computational and optimization toolboxes.
- Limited signal availability: it may lead to lower chances of the optimization algorithms converging to the global optimum. However, this is a design requirement since having access to the full signal is unrealistic in real-life applications.
- Angle sensitivity: our framework performs best when the available signal fractions differ across frequencies; further studies are needed in identifying proper angles.
- Noise sensitivity: signal noise in combination with its limited range will further affect optimization results. However, as with the previous limitation, noise is unavoidable since it comes from ground, terrain and surrounding objects.
- Lack of validation methods: validation of this work is based on analytical solutions and relative error calculation. No validation with real-life examples is within the scope of this work.

To overcome some of the challenges and weaknesses listed above, a powerful PC will be required to accommodate optimizations with heavy computations. MATLAB optimizers can enable parallel computing. This will ensure that time spent on each run is minimized. To avoid noise effects on small portions of signals' data global optimization algorithms like genetic algorithms can be used. Application of noise filtering techniques must be considered.

Transition to C++ can also be considered because it is a low-level language that gives full control over the data flow in the framework, which can decrease the computational times significantly. It also has a large number of libraries that offer a wide variety of optimization algorithms. For example, PAGMO library that was developed by the European Space Agency [18], includes a wide variety of appropriate optimizers, such as Improved Harmony Search (IHS),

Compass Search (CS), COBYLA (Constrained Optimization BY Linear Approximation) , and others.

5.5. Material/resource requirements

Here is the list of resources that were required for this thesis work:

1. *High Performance Computing cluster* for fast calculations of each optimization.
2. *MATLAB license* which gives access to:
 - 2.1.Optimization toolbox,
 - 2.2.Global Optimization toolbox,
 - 2.3.NURBS toolbox.
3. *Access to academic literature resources* (IEEE, Elsevier, Springer) for the proposed validation framework.

Chapter 6 – Conclusion

The main objective of this work was to develop as well as validate a novel computational framework, which is designed for shape identification of flying objects based on EM scattering. The framework combines IGABEM with MATLAB's optimization algorithms to solve the ISP of reconstructing an unknown target's geometry and surface electric conductivity from limited scattering signal readings. The aims stated in Chapter 3 have been addressed and the results presented demonstrate that the proposed approach is effective, accurate and robust under realistic operating conditions.

The numerical accuracy of the IGABEM solver was first validated by comparing its solution against analytical solution for a circular benchmark geometry. The L^∞ and L^2 error norms demonstrated high convergence rates consistent with those reported in the existing literature. This confirms the reliability of the IGABEM solver prior to its application in the shape reconstruction framework.

To test the developed framework simple and oval-like shapes were used as target geometries. As a result, the framework achieved accurate reconstructions with Hausdorff distances as low as $d_H = 0.005$ and correctly identified the electric surface conductivity in both cases. Further tests with more complex target geometries revealed the importance of the illumination angle of the radar's radio wave. The framework's robustness against environmental noise was also assessed by introducing up to 20% noise to the scattering data. While noise caused deviations in the reconstructed conductivity's real part and increased Hausdorff distances, the overall shape features were still successfully identified, demonstrating the practical viability of the approach.

Recognition of plane-like geometries with 9 and 10 SCPs showed the scalability of the developed framework to more complex targets. With 60 and 90 degrees of available signal respectively, both clean and noisy cases yielded accurate shape identifications. In addition, a separate study pinpointed a statistical correlation between the number of SCPs and the complexity of the scattering signal, which was quantified through counting of inflection points. This provides a practical tool for estimating the appropriate number of SCPs before running the full reconstruction, which is a necessary step given that the target's true shape is unknown in real applications.

The proposed framework has demonstrated applicability to both military and civilian areas. In military context, it enables fast aircraft model identification and threat assessment from limited radar data, which is particularly relevant for stealth targets and complex-geometry aircraft. In civilian settings, the framework can contribute to air traffic surveillance and the detection of non-cooperative or malfunctioning aerial vehicles.

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Appendices

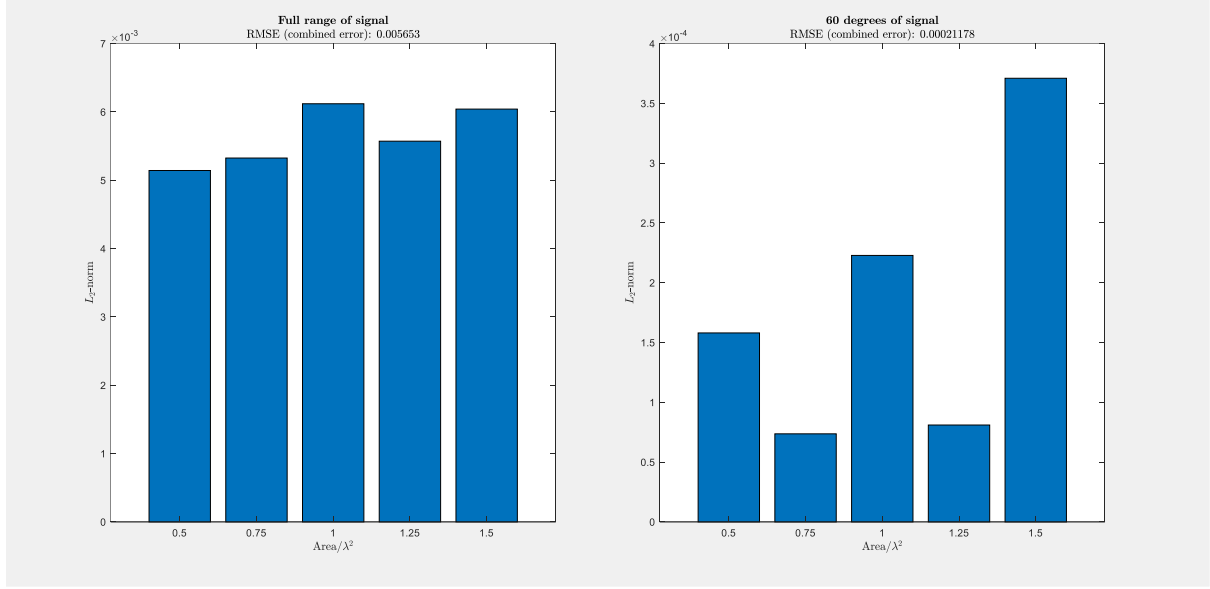


Figure A1: L2-norms for each scattering signal of the simple target with 5 SCPs in both cases, full range and 60 degrees.

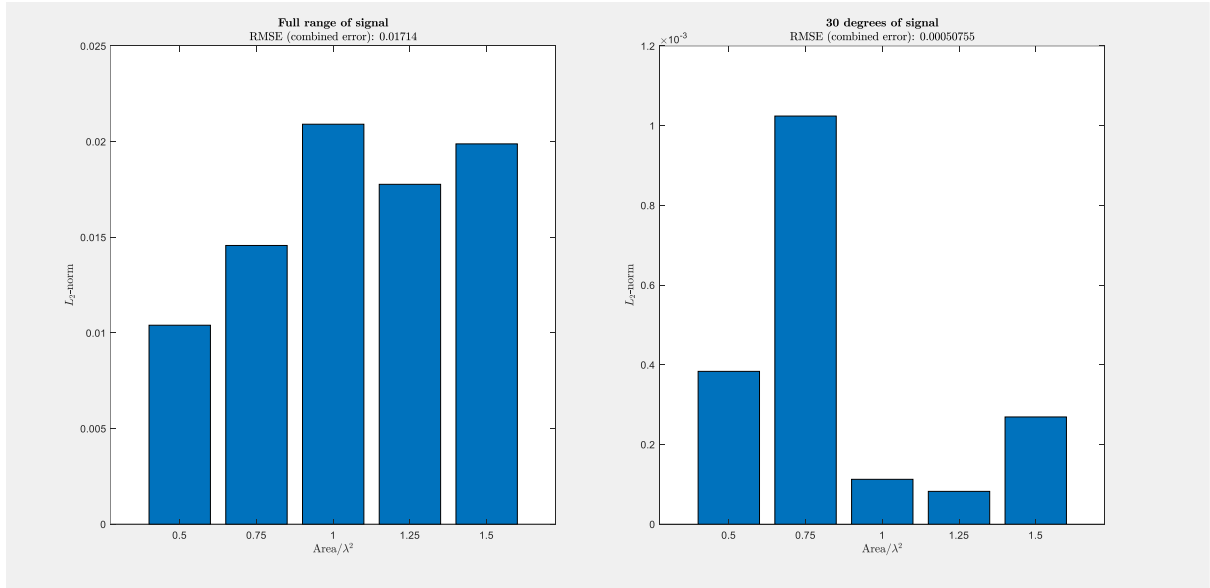


Figure A2: L2-norms for each scattering signal of the oval-like target with 5 SCPs in both cases, full range and 30 degrees.

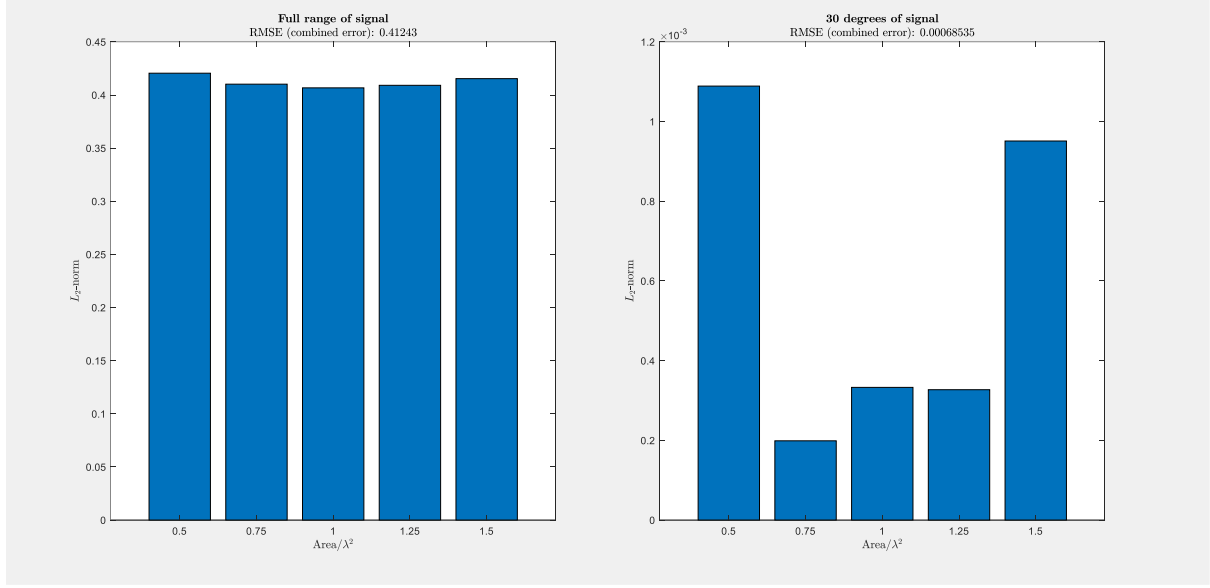


Figure A3: L_2 -norms for each scattering signal of the complex target with 5 SCPs in both cases, full range and 30 degrees.

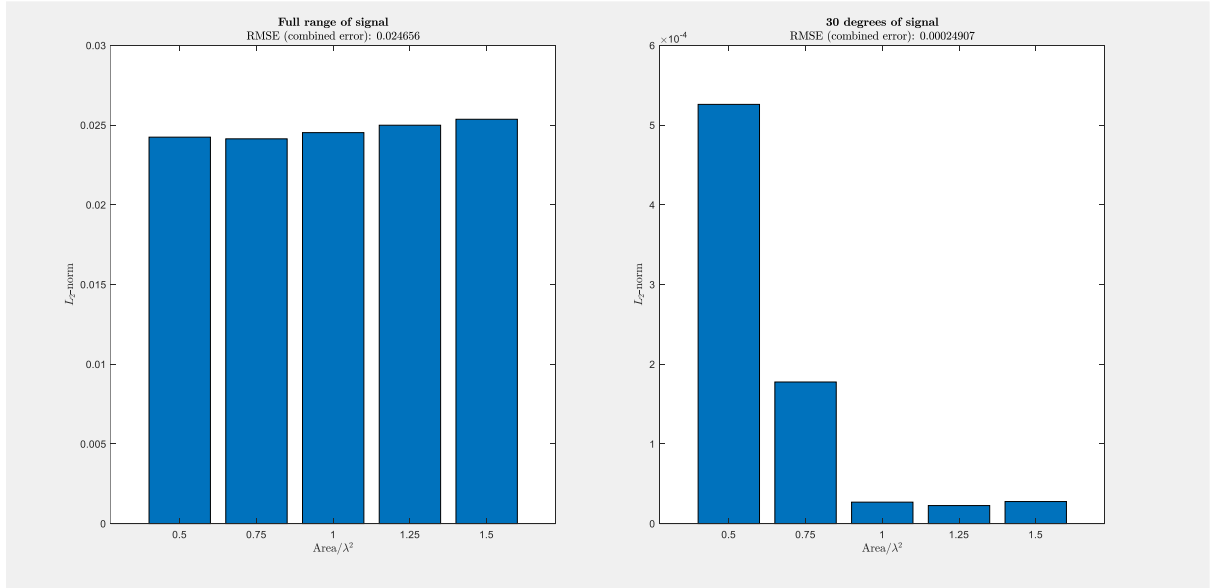


Figure A4: L_2 -norms for each scattering signal of the complex target with 5 SCPs and $\alpha = 90^\circ$ in both cases, full range and 30 degrees.

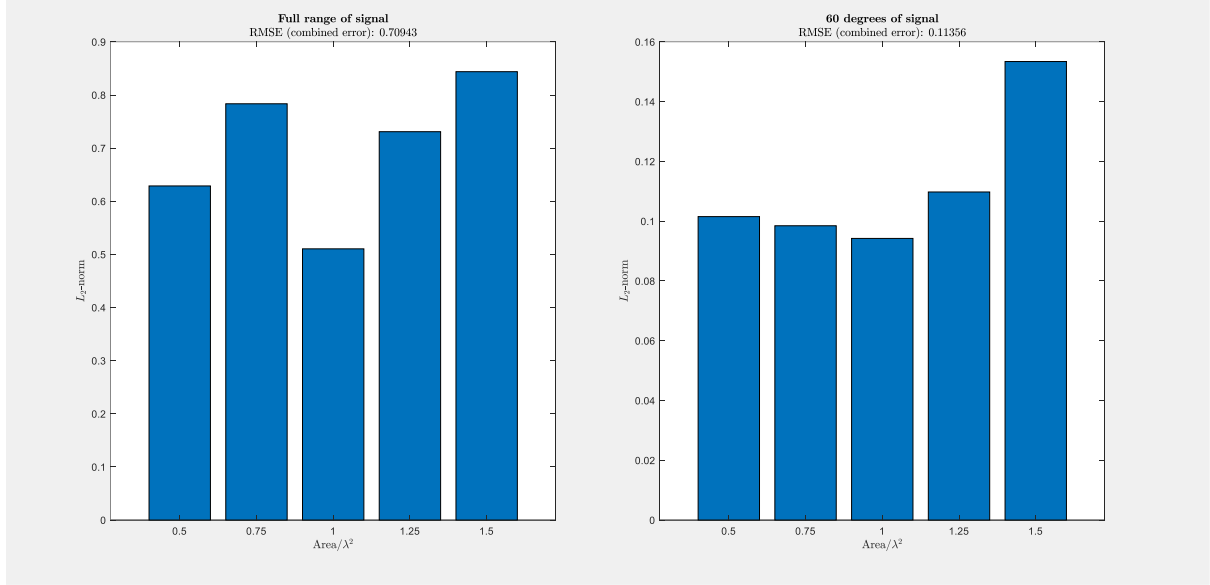


Figure A5: L_2 -norms for each scattering signal of the simple target with 5 SCPs and 20% noise level in signals' data in both cases, full range and 30 degrees.

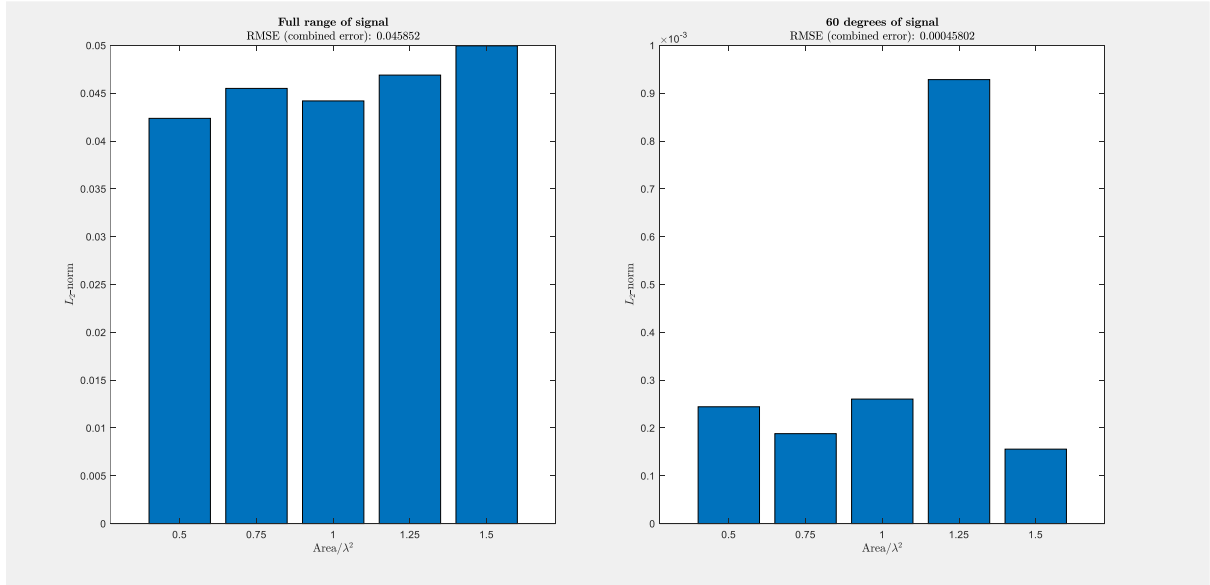


Figure A6: L_2 -norms for each scattering signal of the plane-like target with 9 SCPs in both cases, full range and 60 degrees.

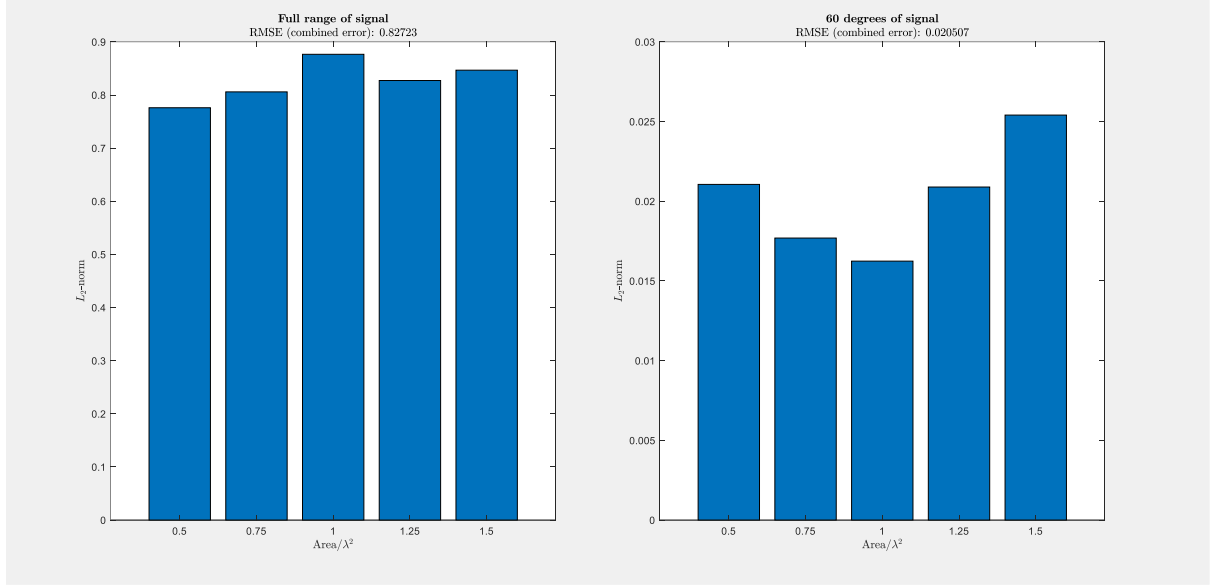


Figure A7: L_2 -norms for each scattering signal of the plane-like target with 9 SCPs and 10% noise level in signals' data in both cases, full range and 60 degrees.

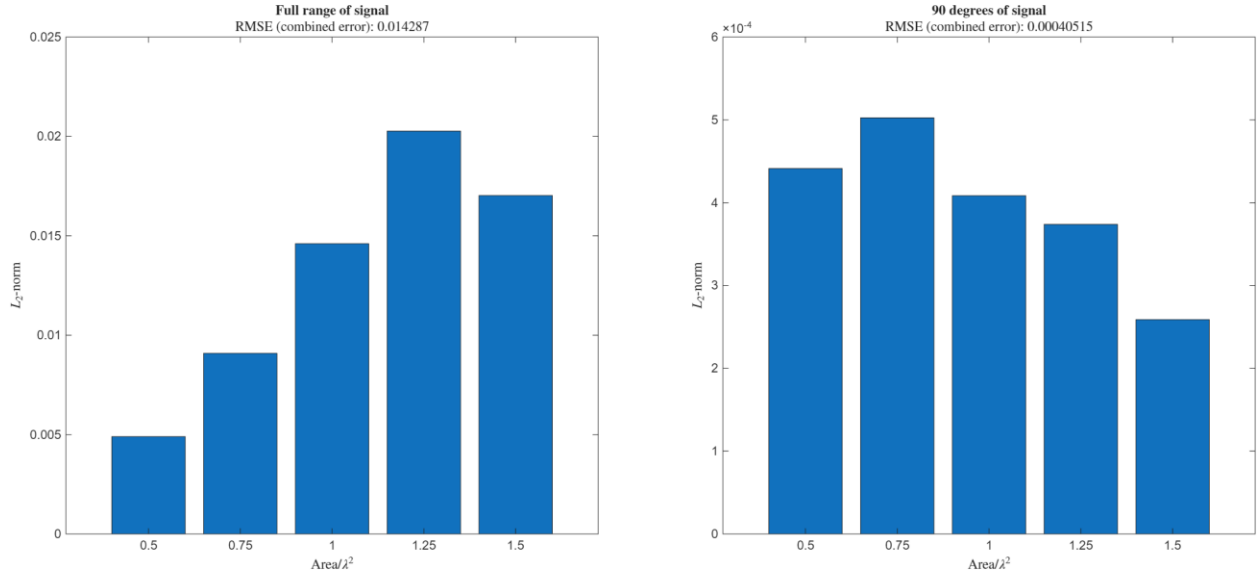


Figure A8: L_2 -norms for each scattering signal of the plane-like target with 10 SCPs in both cases, full range and 90 degrees.

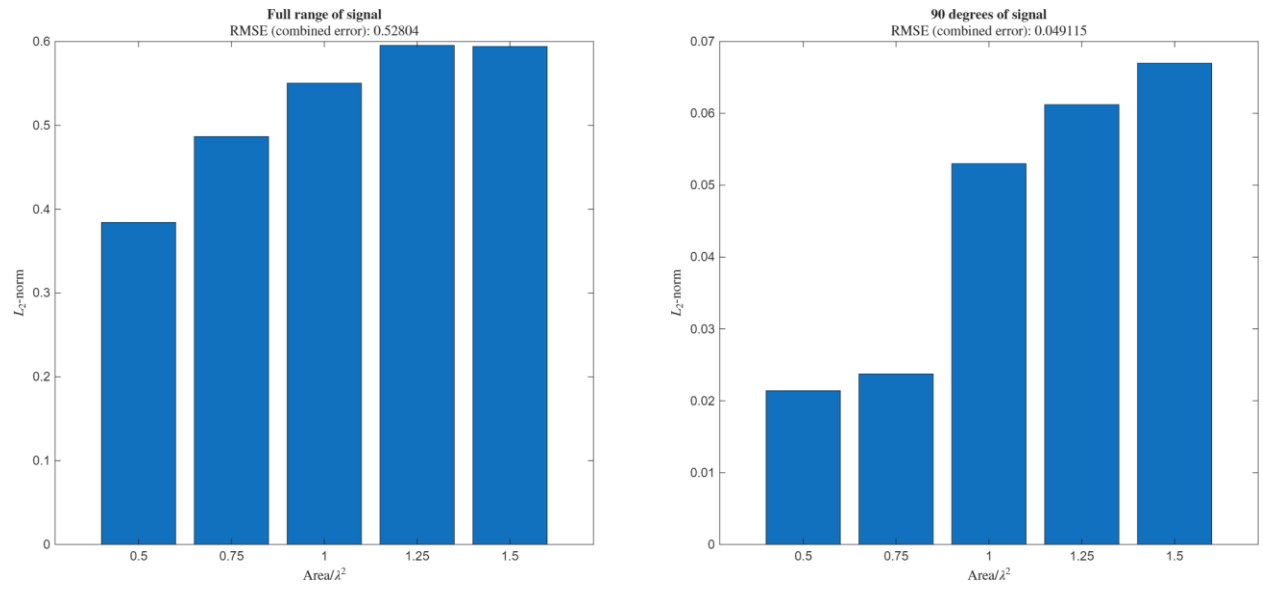


Figure A9: L_2 -norms for each scattering signal of the plane-like target with 10 SCPs and 10% noise level in signals' data in both cases, full range and 90 degrees.