GEOSTASTISTICAL MODELLING OF GEOMETALURGICAL VARIABLES TROUGH TURNING BANDS APPROACH

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ABSTRACT

Geometallurgical variables have a significant impact in downstream activities of mining projects. Reliable 3D spatial modelling of these variables play an important role in mine planning and mineral processing, in which it can maximize overall viability of the project. This interdisciplinary paradigm involves geology, geostatistics, mineral processing and metallurgy, needs enhanced techniques to model these variables. In some circumstances, these geometallurgical responses demonstrate a good intrinsic correlation that motivates one to use co-estimation or co-simulation approaches. The latter allows to reproduce that dependency characteristic in the final model. Among others, total and soluble copper grades as two important geometallurgical variables in copper deposits show some complex interrelationship characteristics. The reason is that the total copper grade in the material processed by heap leaching is not in complete agreement with the expected recovered grade and one can see a significant variation in different oxide minerals. In such cases, the current approaches of probabilistic modelling such as independent simulation gives poor results. In this paper, turning band simulation methodology in combination with minimum/maximum autocorrelation factor (MAF) used to reproduce this kind of behaviour and compare with those results obtained from conventional co-simulation approach.

KEYWORDS

Geometallurgical modelling, Geostatistics, multi-Gaussian distribution, Co-simulation, Turning bands, Geomatallurgical variables.

INTRODUCTION

Geometallurgical mapping allows the integration of metallurgical responses of a deposit into 3D block models for the purpose of mine planning activities. Considering these parameters into resource modeling complements traditional geology and grade-based attributes, enabling a more comprehensive approach to the economic maximization of mineral production through better mine scheduling, planning and reduced associated risk and uncertainty (Macfarlane and Williams, 2014). Most of the time, geostatistical algorithms are applied for producing the high resolution of geometallurgical variables (Brissette et al. 2014; Deutsch et al. 2014; Tolosana-Delgado et al. 2015). However, in some circumstances such as oxide copper deposits, the complexity of these random functions requires consideration of enhanced geostatistical techniques. For instance, soluble copper is a fraction of total copper grade that recovered by heap leaching in these type of deposits (Emery, 2012; Hosseini and Asghari, 2015). Two difficulties often arise for joint spatial modeling of these two geometallurgical variables. The first difficulty for geostatistical simulation of total and soluble copper grades is inequality constraint, as soluble copper grade is always less than or equal to total copper grade. Conventional co-simulation approaches are not sufficient to reproduce such a crucial condition. To overcome this impediment, several avenues have been suggested (Mallet 1980; Dubrule and Kostov 1986; Leuangthong and Deutsch 2003; Emery, 2012). Emery et al. 2004 proposed change to the variables free of inequality constraint. In this context, the data should be converted to a new space and then after co-simulation, back-transferred to the original space. The second difficulty commonly met in practice corresponds to derive the theoretical cross-variogram structure for co-simulation. Such an inference can be implemented by linear model of coregionalization (Journel and Huijbregts 1978). Fitting this function to the experimental direct and cross-variograms is somehow demanding (Goovaerts 1993; Leuangthong and Deutsch, 2003). One alternative is based on transformation of converted cross-correlated variables into the factors that have no spatial interrelationship continuity. Principal component analysis and minimum/maximum autocorrelation factors (MAF) are the methods that eliminate the use of cross-variograms between correlated variables by decorrelation techniques (Bandarian et al. 2008; Swither and Green, 1984). The aim of this study is twofold: 1) following Emery et al. 2004, change of variables to the new variables free of inequality constraint has been applied for a dataset composed of total and soluble copper grades in a porphyry copper deposit (converting the total and soluble copper grades to total copper grade and solubility ratio, respectively); 2) apply MAF factorization methodology for joint simulation of the underlying converted variables; 3) comparing the proposed methodology (via MAF) with conventional co-simulation.

METHODOLOGY

Performance of MAF

Minimum/maximum autocorrelation factor (MAF) is a decomposition approach in geostatistical context that was first coined by Switzer and Green (1984) for image analysis. In this technique, it is of interest to convert the k spatial cross-correlated variables $Y(u) = \{Y_1(u), \dots, Y_k(u)\}$ to k uncorrelated factors (orthogonal) $\tau(u) = \{\tau_1(u), \dots, \tau_k(u)\}$ through the linear transformation based on principal component analysis (PCA).

The MAF can be classified into two main families. In first family, model-driven MAF, the transformation matrix for obtaining the orthogonal factors $\tau(u)$, is based on the fitting the linear model of coregionalization (LMC) on the direct and cross-variograms calculated from the primary variables $Y(u)$ (Vergas-Guzman and Dimitrakopoulos, 2003). However, a critical hypothesis in construction of this version is its restriction to the number of structures in LMC fitted model in order to ensure the independency between the factors for all the lag separations (Tran et al., 2006; Bandarian et al., 2008). In addition, the procedure for inferring a proper LMC model is somehow tedious (Davis and Greenes, 1983; Suro-Perez and Journel, 1991). An alternative can be the second family, the data-driven MAF, which originally, proposed by Switzer and Green (1984). The factors in the latter case are obtained without the requirement to fit a linear model of coregionalization, in which the transformation matrix is computed directly from input data through two successive PCA decompositions (Rondon, 2012; Desbarats and Dimitrakopoulos, 2000; Desbaratas, 2001). In this study, the following steps are presented to obtain two data-driven MAF factors:

1. Transform the original variables to normal score values with a mean of zero and variance one $N(0,1)$: this can be implemented by normal score transformation methodologies such as Gaussian anamorphosis (Rivoirard, 1994) or quantiles-based approach (Deutsch and Journel, 1998).

$$Z(u) = G^{-1}(F(Y(u))) \qquad (1)$$

   where $G^{-1}(.)$ is standard normal cumulative distribution function, $F(.)$ is the cumulative distribution function of the original variable $Y(u)$ and $Z(u)$ is the normal score value.

2. Compute the experimental variance-covariance matrix at lag 0: since we are dealing with normal score values, this matrix is identical to the sample correlation matrix. In the case of two variables, this matrix (V) is as:

$$V = Corr\{Z(u), Z(u)\} = \begin{bmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{bmatrix} \qquad (2)$$

   where the principal diagonal element equals one which is identical to the total variance, upper and lower diagonal elements $\rho_{12}(0)$ and $\rho_{21}(0)$ equal the linear correlation coefficient between two normal score variables $Z_1(u)$ and $Z_2(u)$, respectively.

3. Perform the spectral decomposition of above matrix (V) to derive the orthonormal eigenvectors matrix (M_1), associated with the underlying diagonal eigenvalues matrix (E_1), such that:

$$V = M_1 E_1 M_1^T \qquad (3)$$

   It is necessary to check that the entries of E are in decreasing order.

4. Calculate the PCA transformations at locations u by:

$$PCA(u) = E_1^{-1/2} M_1 Z(u) \qquad (4)$$

   where PCA(u) are the scores with normal standard distribution due to the priori multivariate Gaussian assumption.

5. Choose a proper nonzero lag distance h and calculate the sample covariance and cross-covariance matrices $\widehat{\Lambda}_{PCA}(h)$ over the PCA scores, so its related spectral decomposition with diagonal eigenvalues matrix (E_2) and orthonormal eigenvectors matrix (M_1) is:

$$\widehat{\Lambda}_{PCA}(h) = M_2 E_2 M_2^T \qquad (5)$$

   It is worth mentioning that since the PCA scores are normal values, the variance-covariance matrix is identical to correlogram matrix.

6. Finally, the MAF factors at location u can be derived:

$$\tau(u) = M_2 PCA(u) \qquad (6)$$

   The back transformation is performed through the inverse of matrix M_2:

$$Z(u) = M_2^{-1}\tau(u) \quad (7)$$

Joint Simulation with MAF

In this study, it is of interest to apply MAF transformation on the cross-correlated variables combined with independent geostatistical simulation. The method allows one to independently simulate the factors without the need to fit a linear model of coregionalization while all direct and cross-covariances have been taken into account. Nevertheless, prior to this paradigm, the approach based on changing the original variables to the new variables free of inequality constraint has been employed to mitigate the impediment of modeling the total and soluble copper grades (proper explanation to do so is presented in subsequent section). Joint simulation with MAF is then performed over the converted variables in order to show the capability of this combined method. First of all, the conventional co-simulation is utilized to multivariate spatial modeling the two cross-correlated converted variables. Second, through the same converted variables, the proposed algorithm (joint simulation with MAF) is used in combination with independent simulation. Finally, the results of first and second steps are validated and compared to verify the relevancy of the proposed algorithm to multivariate spatial modeling of cross-correlated variables. The simulation algorithm in both steps can be on the basis of any Gaussian simulation approach. The first step is very straightforward as thoroughly explained in Madani and Ortiz, 2017. For the second step, following algorithm is proposed:

1. Convert the original cross-correlated variables to the new variables free of inequality constraint
2. Transform the declustered converted variables into normal score data (Gaussian random field with mean 0 and variance 1) (Eq. 1)
3. Transform the normal score data into orthogonal MAF factors (Eq. 6).
4. Calculate the experimental variograms for each MAF factor
5. Independent Gaussian simulation of MAF factors
6. Back-transformation of the simulation results (realizations) into normal score space (Eq. 7)
7. Back-transformation of the normal score realizations into the original space in order to restitute the intrinsic cross-correlation

APPLICATION TO A REAL CASE STUDY

The dataset is composed of 3866 samples obtained from blast holes belonging to a porphyry copper deposit. The dataset is homotopic, means that the data is available for each variable at all sampling points (Wackernagel, 2013). Total copper (tCu) and soluble copper grades (sCu) have been measured at sampling locations (the pattern is illustrated in Figure. 1). In the following subsection, the original values are multiplied by a constant scale factor in order to preserve the confidentiality of the dataset. Table 1 shows the statistical parameters obtained after applying the cell declustering (Goovaerts, 1997). The scarcity of data in some regions makes the sampling patter irregular and statistical parameters possibly biased. The idea of declustering is to account for the weights of each location by cell-based technique to correct the pseudo skewness in the global distribution of the dataset (Deutsch and Journel, 1998).

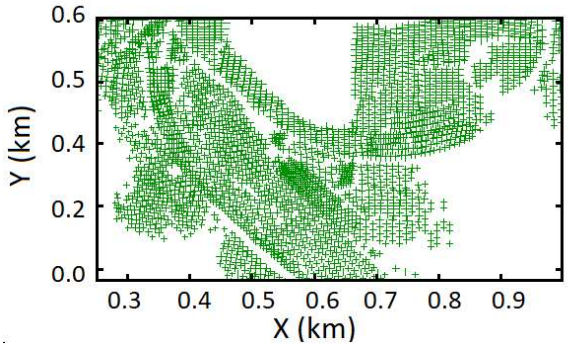

Figure 1 – Base map for the location of samples in the plane (the coordinates are local)

Table 1. Statistical properties of tCU and sCu

Variable	Original mean	Declustered mean	Max values	Min values
tCu(%)	0.83	0.87	4.76	0.11
sCu(%)	0.65	0.68	4.55	0.59

Multivariate analysis between these two variables by computing the linear correlation coefficient $\rho = 0.97$ explicitly indicates that those random attributes are highly correlated. This positive correlation implies that total copper grade increases as the soluble copper grade increases and vice versa. Therefore, this characteristic motivates one to jointly

estimate or simulate the total and soluble copper grades in the specified domain. However, as explained the soluble copper grade is always less than or equal to the total copper grade (inequality constraint) (Figure 2), in which it makes difficult the process of co-simulation by conventional methodologies. In this study, these two original variables have been changed to other two variables free of inequality constraint. In this context, the solubility ratio (SR) can be calculated through dividing the soluble copper grade by total copper grade. As this value has no unit, it can be reported as percentage:

$$\mathrm{SR}(\%) = \frac{\mathrm{sCu}}{\mathrm{tCu}} * 100 \qquad (8)$$

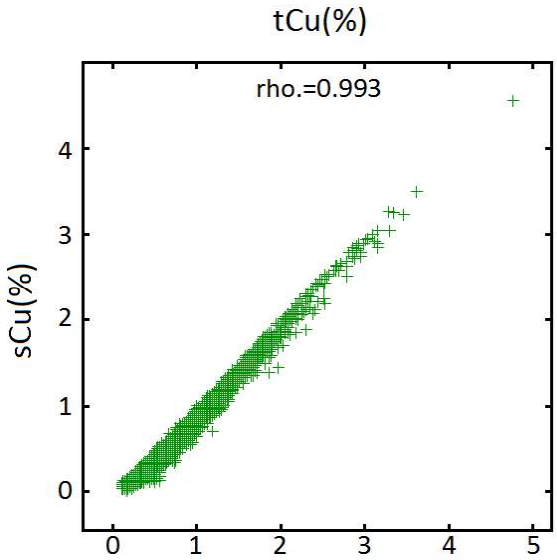

Figure 2 – scatterplot between total and soluble copper grades

Conventional co-simulation

After convert the cross-correlated variables (tCu and sCu) into the variables free of inequality constraint (tCu and SR), following the steps explained in above section, it is of interest, first co-simulate the total copper grade and solubility ratio by conventional Gaussian algorithm. All the Gaussian simulation methodologies can be applied. However, turning bands co-simulation (Emery, 2008) has been employed in this study because of its versatility and straightforwardness (Pravarzar et al. 2015). Prior to the modeling, the converted variables need to be transformed to the normal score values (Figure 3). This transformation guarantees that the values have standard Gaussian distribution $N(0,1)$ and can be used in Gaussian simulation algorithms (Figure 4, right). It is worth mentioning that, the scatter plot of normal scored original data without conversion to the new variables do not convey any multi-Gaussianity assumption, for which it restricts using the Gaussian simulation (Figure 4, left). Corroboration of multi-Gaussian assumption can be visually checked by elliptical shapes appear in scatterplot of two converted Gaussian random fields.

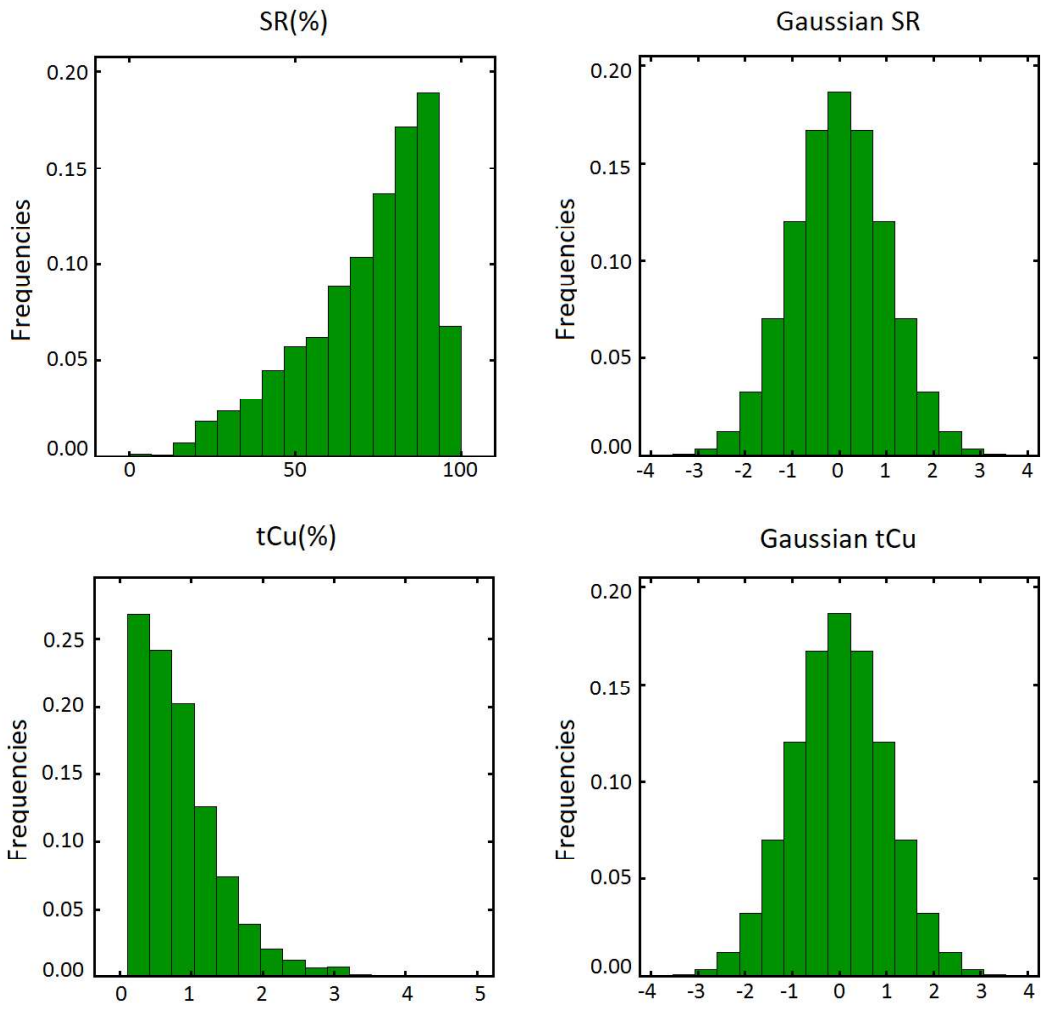

Figure 3 – normal score transformation (left: declustered histogram of the original data and right: histogram of the normal score values

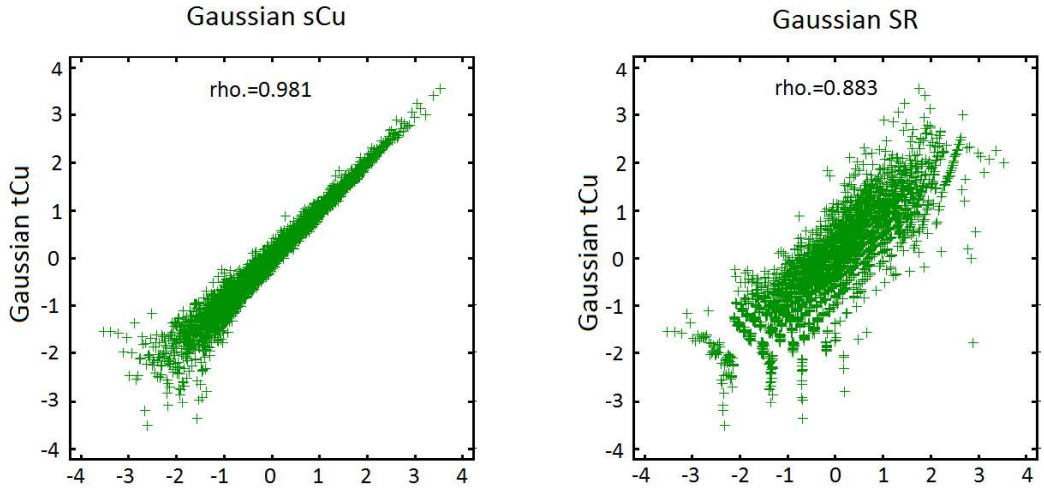

Figure 4: scatter plot of the data before and after conversion

Co-simulation methodology requires that the spatial continuity is primarily modeled by calculation of direct and cross-variograms of the transformed variables. The experimental direct and cross-variograms should then be fitted by manual or semi-automatic fitting approaches. Since, no anisotropy detected in the region, the fitted model (LMC) consists of nugget effect and two omni-directional spherical structures:

$$\gamma_{\mathrm{tCu}}(\mathrm{h}) = 0.27 + 0.32\mathrm{Sph}(51.97) + 0.41\mathrm{Sph}(252.88)$$

$$\gamma_{\mathrm{SR}}(\mathrm{h}) = 0.31 + 0.33\mathrm{Sph}(51.97) + 0.41\mathrm{Sph}(252.88)$$

$$\gamma_{\mathrm{tCu-SR}}(\mathrm{h}) = 0.20 + 0.30\mathrm{Sph}(51.97) + 0.37\mathrm{Sph}(252.88) \quad (9)$$

Having the theoretical fitted spatial continuity, simulation is then performed on a regular grid with dimension of 25 m × 10 m ×0.5 m. Simple co-kriging is utilized for the process of conditioning to the hard data (sampling locations). The proposed approaches can be substituted for ordinary co-kriging where the uncertainty is significant in mean value of the random fields (Emery, 2012). The neighborhood is moving with conditioning to 10 surrounding data characterized by isotropic distance equal 170 m derived from the variogram analysis. The number of lines for turning bands should be as large as possible (Emery 2008). Henceforth, it is set to 1000 lines for elimination of stripping effects and the number of realizations is considered to be 100.

Joint simulation with MAF transformation

The MAF factors are derived from the same normal score values ($\mathrm{NS_{tCu}}$ and $\mathrm{NS_{SR}}$) explained in conventional c-simulation section. To do so, since in this study, data-driven MAF is used, it is necessary to consider an arbitrary lag separation distance except 0 for calculation of cross-correlation matrix. So, 10m is chosen based on sampling spacing and range of spatial continuity. In order to check whether the factors are spatially decorrelated, cross-correlogram has been plotted. This shows that the spatial correlation between two factors τ_1 and τ_2 is close to zero (Figure 5). The histogram of the factors can also be checked whether they follow $N(0,1)$. Figure 6 shows that the global distributions of the factors follow a standard normal distribution.

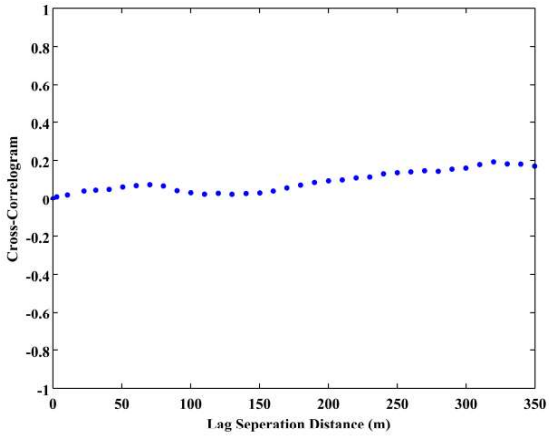

Figure 5 – Cross-correlogram between MAF factors using the data-driven approach

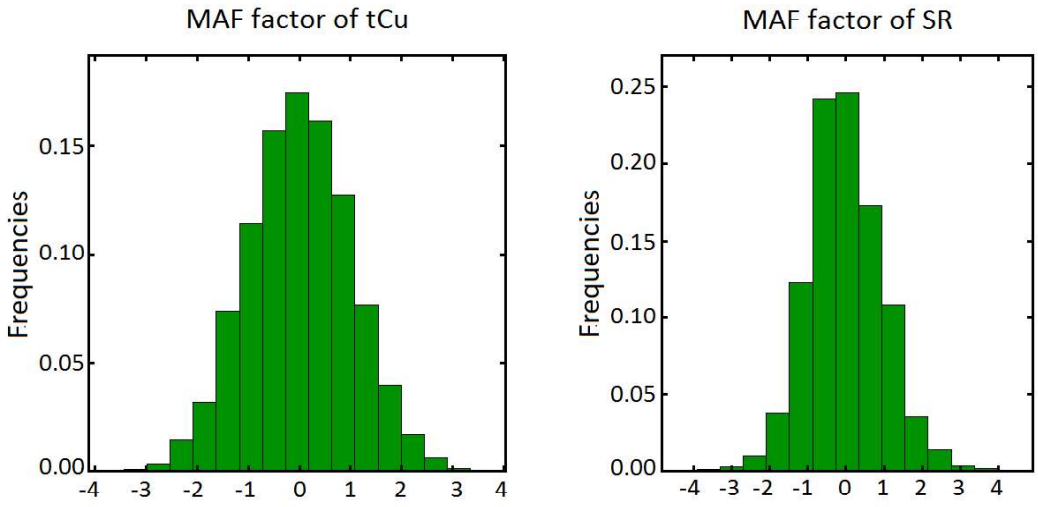

Figure 6 – Histogram of factors

Once the spatial decorrelation of factors is validated, the experimental direct variogram for each factor is computed and fitted:

$$\gamma_{\tau_1}(h) = 0.28 + 0.31Sph(51.97) + 0.42Sph(252.88)$$

$$\gamma_{\tau_2}(h) = 0.52 + 0.22Sph(51.97) + 0.27Sph(252.88) \qquad (10)$$

The fitted model to each factor is nugget associated with two spherical structures. The spatial continuity direction is omni-directional, same as the previous normal score variables (Figure 7).

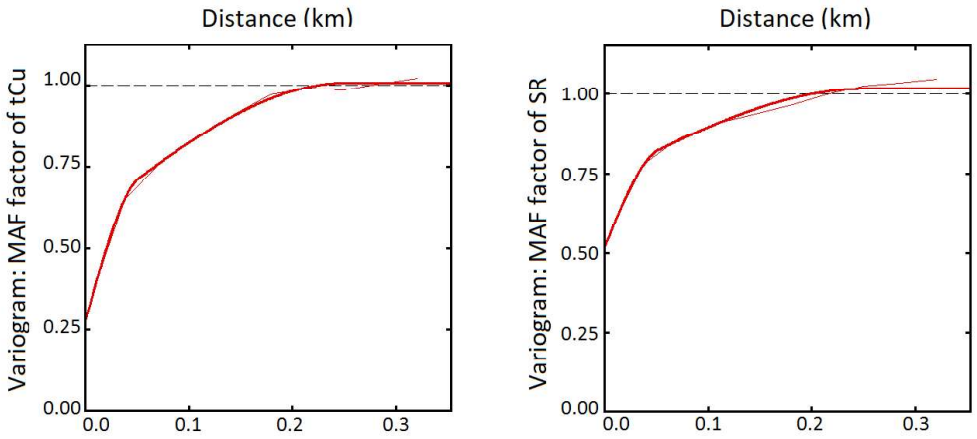

Figure 7 – Variogram analysis for each factor

Having the fitted variogram structures over the factor τ_1 and τ_2, the turning bands simulation algorithm (Emery and Lantuejoul, 2006) is employed to generate 100 realizations conditioned to two random fields of interest (factors). The size and dimension for independent simulation are the same as mentioned in section above. After simulation, it is necessary to back-transform the realization within two successive steps. The first back-transformation is from factor to normal score values; the second is from normal score values to total copper grade and solubility ratio. Since the aim of this research is to jointly simulate the total and soluble copper grades, the realizations so obtained are then back-converted by Eq. 8 to desired space. The maps of the average of 100 realizations produced from both methods: conventional cosimulation and joint simulation with MAF are illustrated in Figure 8 and 9.

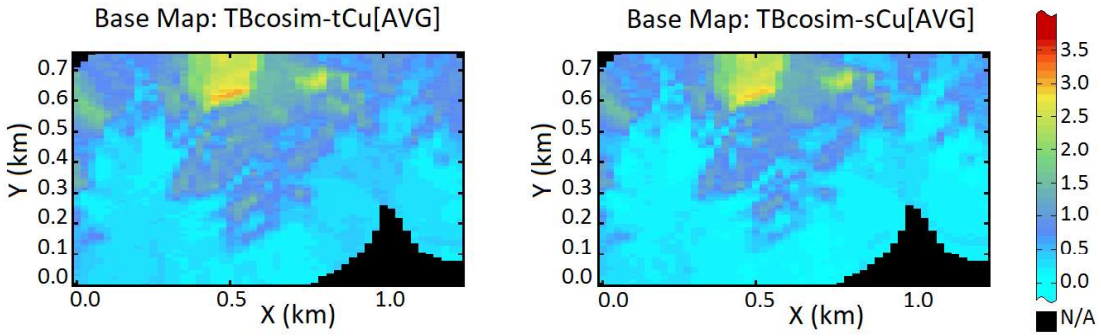

Figure 8 – Maps of simulated total and soluble copper grades for the average by co-simulation approach

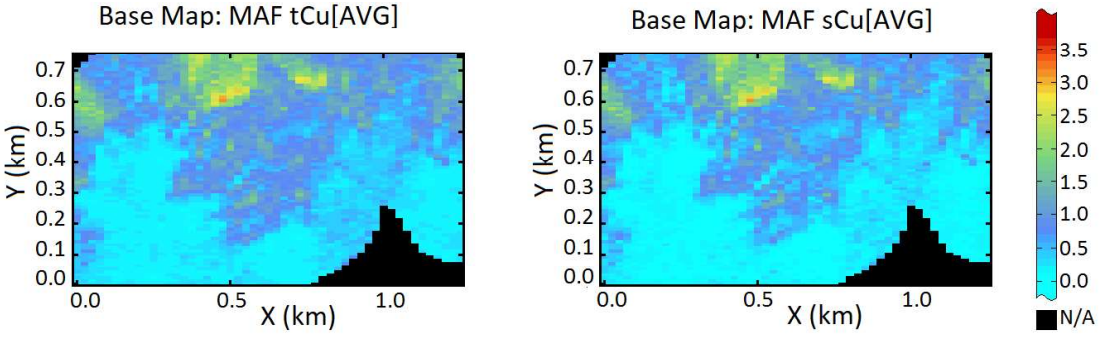

Figure 9 – Maps of simulated total and soluble copper grades for the average of realizations by MAF transformation approach

RESULTS AND DISCUSSIAN

The correlation coefficient is calculated for both results among total and soluble copper grades. The average correlation computed from 100 realizations obtained from MAF transformation and conventional cosimulation ($\rho_{MAF} = 0.994 \; and \; \rho_{Co-sim} = 0.992$) comparing to the declustered original correlation coefficient ($\rho_{tCu-sCu} = 0.993$) indicates that although the LMC is sufficient for re-establishing the data correlation, but the proposed algorithm (joint simulation with MAF) is also capable of restituting the desired correlation and can be applied to multivariate spatial modeling in the case when there is an inequality constraint among the variables. Figure 10 shows the scatter plot among these two variables. The shape of cross-correlation (dependency) between total and soluble copper grades and inequality constraint are satisfactorily reproduced. These results imply that the proposed algorithm does not show loss of accuracy.

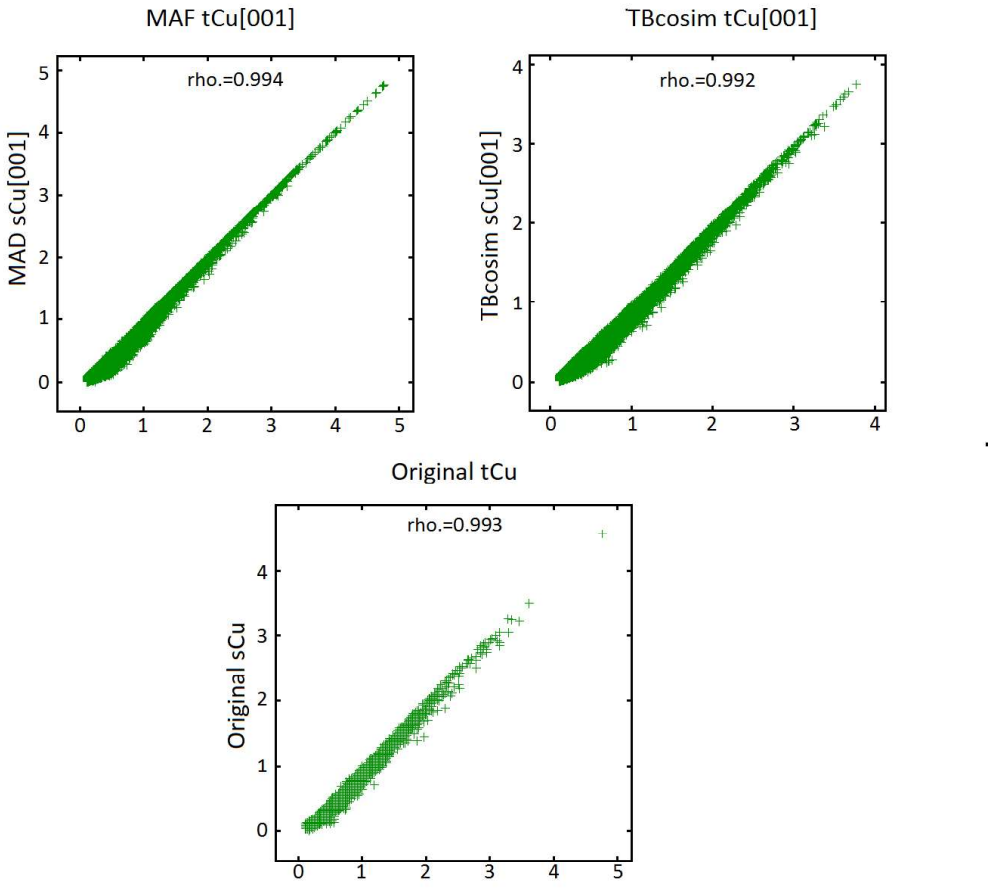

Figure 10 – The scatter plot of tCu(%) and sCu(%) obtained by MAF transformation (left-up); conventional so-simulation (right-up); original dataset (center-down)

CONCLUSIONS

Geostatistical modeling has been widely used in spatial modeling of geometallurgical variables. However, since those variables inherent complex characteristics, one needs to consider the improved methodologies. In this study, a new algorithm is proposed to jointly simulate the total and soluble copper grades with inequality constraint. In order to show the versatility of the presented technique, the results are compared with those obtained from conventional co-simulation that is based on inferring the linear model of coregionalization. The results approved that the correlation coefficients between total and soluble copper for both algorithms (conventional co-simulation and proposed algorithm) are reproduced extremely well, and multivariate complex interrelation of variables is regenerated without any deviation from the primary inequality constraint. The results of the proposed algorithm for joint simulation with MAF shows also the similar results as in co-simulation. Consequently, the use of LMC can be eliminated in order to build simpler geometallurgical modeling through factorization approach, which requires less computational time and resources.

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