Comparison of sequential Gaussian simulation and simple kriging for grade prediction

by

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ORIGINALITY STATEMENT

I, Talgat Kassymkan, hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or substantial proportions of material which have been accepted for the award of any other degree or diploma at Nazarbayev University or any other educational institution, except where due acknowledgement is made in the thesis.

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I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation and linguistic expression is acknowledged.

Signed on 01.04.2024

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ABSTRACT

Accurate prediction of grades plays an important role in the mining industry: differentiation of valuable ore and non-profitable waste material is a key step in mine planning. This paper delves into the comparison between sequential Gaussian simulation (SGS) and simple kriging methodologies concerning their efficacy in grade prediction and the classification of ore and waste materials. The study investigates the application of both systems to predict the ore grades within iron deposit. It investigates their abilities to accurately predict the spatial distribution of ore grades across varied geological formations. Furthermore, this research aims to ascertain whether SGS methods exhibit superior performance in classifying materials into ore and waste categories compared to traditional simple kriging systems. The findings of this study are expected to provide valuable insights into the strengths and limitations of SGS and simple kriging methods for grade prediction in mining operations. This comparative analysis aims to aid mining engineers and professionals in selecting the most effective methodology for optimizing resource delineation and decision-making processes in mining projects.

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1. INTRODUCTION

1.1 Overview

Identification of ore body shape, its grade distribution, ore/waste classification and evaluation of economic potential is a key part of resource exploration. Grade prediction and exploration processes should be done continuously even after the start of extraction. Geochemical and geospatial data obtained before and after the exploration process is updated on a regular basis to make decisions whether to expand and continue the mining process or to stop it due to the lack of economic profitability. Since each further step in mining requires more investments, these decisions should be based on more accurate information obtained from resource estimation (Sinclair, A. J., & Blackwell, G. H, 2002)

During the 80's of the previous century, many gold mining companies neglected grade continuity and mineralization of target location before the production decisions (Clow (1991) cited by Sinclair and Blackwell, 2002). Consequently, errors in estimations of grades and tonnages have led to early closures of numbers of mines. Thus, more accurate and detailed grade predictions require high attention to avoid economic losses. If geologic continuity (physical or geometric occurrence of geologic features) is more straightforward for longer distances, value (grade) continuity is much shorter than the dimensions of the geologic structure.

Nowadays, autocorrelation functions like semivariograms and correlograms are used to quantify value continuity in a given domain. They represent increasing average divergence of grade as the distance between samples is increased. Relative distances are used to create graphs which represent variations in continuity in different directions. Example of a such variogram is depicted below:

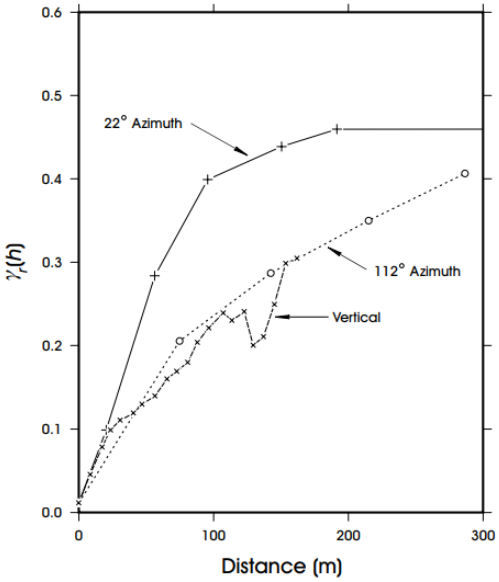

Figure 1. Experimental semivariograms (autocorrelation functions) for horizontal and vertical directions for Huckleberry porphyry copper deposit, British Columbia (Sinclair and Blackwell, 2002).

Uncertainty in resource estimation is considered to be an unavoidable trait (Hekmatnejad, 2017). Consequently, from a financial point of view it is crucial to diminish uncertainty as possible and obtain most accurate estimations in grade and tonnage calculations. These estimations are further used to quantify dilution (waste material which is not separated from ore material and sent to processing), mining costs and conditions. Dilution is also an inherent trait of mining operations due to the shape of the ore body, geology and scale of operations. Improvements in grade prediction can decrease the amount of dilution and directly affect the profitability of mining operations. Depending on the methods used in grade prediction, errors and under/overestimations may differ. Geostatistical methods like kriging are applied to resource estimation. Implementation of cut-off grade helps to differentiate between ore and waste material. Ordinary kriging requires special attention to the conditional unbiasedness of the predicted grades. In contrast, non-linear (disjunctive) kriging offers unbiased predictions and lowers uncertainty thanks to grade control (Hekmatnejad, 2017).

The purpose of this paper is to compare SGS and SK systems in grade prediction for a given case: iron deposit and see how estimated grades differ from original data. Implementing these predictions on real case studies and comparison of both is expected to give clear difference in two approaches and show accurate way of resource estimation.

1.2 Problem statement and motivation

In mining operations, accurate estimation of ore grades and effective classification of materials as ore or waste are fundamental challenges. Traditional linear kriging systems have been widely used for grade prediction and material classification, but their limitations in handling non-linear relationships and complex geological settings have prompted exploration into non-linear kriging methods. Accuracy and efficiency of different grade prediction systems and ore/waste classification is crucial in mining. This study aims to investigate and compare the performance of SGS and SK systems.

1.3 Objectives

The main objectives of this paper are:

- To evaluate the effectiveness of SGS and SK in predicting ore grades and classifying materials as ore or waste.
- To identify the strengths and limitations of both approaches and determine their suitability for different mining scenarios (if any).

Specific objective of this paper is to compare the accuracy and efficiency of SGS and SK in ore/waste classification based on grade estimations.

1.4 Importance

This paper's significance lies in its exploration of advanced kriging methodologies for grade prediction and material classification in mining contexts. By comparing the efficiency of both SGS and SK systems, this research seeks to offer valuable insights into selecting a better approach for accurate ore prediction and waste management in mining operations. The outcomes of this comparative analysis will aid mining professionals in making further feasibility decisions, optimizing resource handling, and improving the overall efficiency of mining processes.

2. LITERATURE REVIEW

2.1 Resource estimation (*depending on grade and volume*)

Since around 1945, statistical methods and terminology have been crucial in the characterization of ore (Sichel, 1952; Swanson, 1945, cited by Sinclair and Blackwell, 2002). These techniques have been widely used to evaluate quantitative numerical variables such as deposit characteristics or metal grades. These applications are mainly concerned with average values, variability or spread of values, probability distribution shapes (represented by histograms), autocorrelation, basic correlation analysis, relationships between variables, and various probabilistic statements related to these topics. These conventional statistical techniques are essential for understanding and summarizing mining data, especially when it comes to processes for estimating mineral inventories.

Sampling

Statisticians define a population as the complete set of characteristics under investigation, such as a mineral deposit. This population is described by variables (such as grades) which have certain measures (like mean and standard deviation) and a distinct pattern or distribution of all potential values (data items) around the mean, known as the probability density function or histogram. This describes the overall universe or deposit being analyzed. Deriving the parameters (characteristics) of the universe (deposit) from a sample of potential items is a common goal of statistical research (rock sample assays). There are two different interpretations of the word "sample". In the context of statistics, the sum of n individual values creates a sample (made up of n items) that represents the deposit. But when it comes to mining, a sample usually means a physical piece of rock material from which a representative subset is examined in order to calculate quantitative measures of its quality, like grades (Sinclair and Blackwell, 2002). Instead of being dispersed randomly across space, samples used in mining evaluations are frequently arranged in a variety of patterns, from somewhat regular to highly irregular two- or three-dimensional spatial data arrays. Although the volume or mass of rock in any given sample may vary, most databases strive for uniformity in support, or sample size and shape.

Weighting

The arithmetic average (m), which is calculated by adding up all n values and dividing the result by n, is the most widely used metric to measure central tendency, or the preferential clustering of values within a data set:

$$m = \frac{\Sigma x_i}{n} \quad (1)$$

Where: m=mean value, ∑x_i=sum of all n values, n=number of values;

However, in mineral resource estimation, a limited number of samples with different size and characteristics does not allow to use conventional mean calculations, thus *weighted mean* value must be considered (Sinclair and Blackwell, 2002). For example, imagine having two samples with copper grades 2.5% and 1.0%. Mean value of these samples would be:

$$m = \frac{2.5 + 1.0}{2} = 1.75\%$$

However, if the length of first sample is set to be 3 m and the second sample has length of 1 m, weighted mean would be:

$$m_w = \Sigma w_i * x = 2.5 * 3/4 + 1.0 * 1/4 = 2.125\% \quad (2)$$

And if consideration of densities (2.7 and 3.3 g/ml) are applied, weighted mean would be:

$$m_w = \Sigma w_i * x_i = \frac{2.5*3*2.7+1.0*1*3.3}{2.7*3+1*3.3} = 2.066\%$$

These considerations and calculations make weighted mean unbiased, thus more reliable data is obtained. To avoid biases of laboratories in the mining industry, usually two or more laboratories are used and duplicates of the same samples are sent to them to examine results. Then in resource estimation either paired or correlated data is used to reduce bias.

Median and mode

The value corresponding to the middle data item in an ordered data set (ordered from high to low values, or vice versa) is known as the median. This is another crucial indicator of central tendency, especially for non-symmetrically distributed data. It means that 50% of the values are higher than the median and 50% of the values are lower. The median is a more reliable estimator of central tendency than the mean for small numbers of items.

Modes are local peaks on a histogram; they are narrow class intervals of data that are more abundant than the data in both adjacent class intervals. The three measures of central tendency are generally distinct from one another, even though a mode can correspond to either the mean or the median values; in the case of a normal distribution, mode, median, and mean are equivalent. In addition, modes play a crucial role in indicating the potential existence of complex distributions comprising two or more subpopulations (referred to Sinclair, 1976, 1991). They are

also essential for comprehending and identifying outliers, particularly those with unusually high values

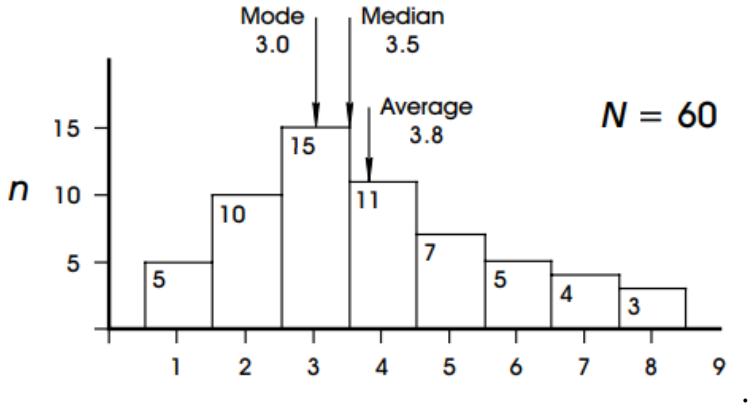

Figure 2. An example of a hypothetical data set's histogram showing how the means, medians, and modes can vary within a single data set. Each bar lists the numbers of data for each class. (Sinclair and Blackwell, 2002)

Dispersion

A measure of the divergence of data values is called dispersion. The range of data, or the difference between a data set's minimum and maximum values, is a straight-forward, although typically impractical, way to characterize dispersion. Due to its extreme sensitivity to the existence of a single extreme value, the range is typically inappropriate for characterizing dispersion. The variance, or s2, is a data set's most basic measure of dispersion.

$$S_2 = \frac{\Sigma(x_i - m)^2}{(n-1)} \quad (3)$$

where "x_i" is any data value, "m" is the mean of the data, and "n" is the number of data items. Degrees of freedom, or (n − 1), have their roots in statistical theory and are related to the sampling distribution of s2 (variance of a sample) as opposed to σ2 (variance of a population). When a small sample (n < 30) is used to characterize an entire population, the divisor (n − 1) is used to ensure that s2 is not biased. A variance is a "squared" number, meaning that when calculating it, there is no distinction between positive and negative differences. Since it is in the same units as the variable under consideration rather than variance, the square root of the variance, also known as the standard deviation, is the widely used practical measure of dispersion (Sinclair and Blackwell, 2002).

Covariance

The formula for covariance S_{xy} provides a quantitative measure of the systematic variations of two variables (x and y):

$$S_{xy} = \Sigma[(x_i - m_x)(y_i - m_y)]/n \quad (4)$$

where: m_x and m_y are the means of the two variables being compared.

The covariance is positive when high values of x are linked to high values of y and low values of x are linked to low values of y; a negative covariance is produced when low values of x are linked to high values of y, or vice versa. The covariance is zero when x and y are statistically independent, but this isn't always the case—two variables can be dependent on one another even though their covariance is zero (Sinclair and Blackwell, 2002).

Cut-off grade

The ore grade stands as a crucial factor in mining operations, significantly influencing the categorization of material into ore and waste sections. Typically, the kriging estimator is widely used to predict ore block grades. In conventional mining, if a parcel's estimated grade exceeds the cut-off grade, it's termed as ore; otherwise, it's marked as waste. However, an alternate method involves simultaneous consideration of parcel grades and the economic implications of their destination using simulation-based techniques. This study applies kriging, simulation-based methods, and profit/loss functions to a real-world case study for ore/waste classification based on initial exploration data. The results are then compared against actual blast hole sample data to validate these methods' effectiveness. The findings suggest that simulation-based methods exhibit superior performance and better alignment with real data, offering increased adaptability. (Mousavi, 2016)

2.1.1 Kriging

Kriging is a general term used to describe a variety of estimation techniques (block or punctual) that rely on minimizing estimation error, usually through the use of least-squares analysis. The phrase was created in honor of D. Krige by G. Matheron and P. Carlier. Matheron's geostatistical theory was derived from Krige's empirical work on reserve estimation in South African gold mines (Cressie 1990). Kriging is a globally unbiased (i.e., unbiased, on average, over the entire data range) estimation procedure, however there may be a considerable conditional bias in the kriging results.

The general term "kriging" refers to a number of specific techniques, such as probability kriging (PK), simple kriging (SK), ordinary kriging (OK), indicator kriging (IK), universal kriging (UK), and multiple indicator kriging (MIK). They are all predicated on the same general ideas: that a mathematical function that can be deduced from the realization (data) of a regionalized variable can model the autocorrelation of that variable and be used to aid in estimation (Sinclair and Blackwell, 2002).

As illustrated schematically in Fig. 10.1, the general problem to be solved by kriging is to provide the best estimate of an unknown point or block from a discrete data set (samples). There are eight data available in this example to estimate Block A, and it is implicitly assumed that using data from both inside and outside of Block A will enhance the estimation.

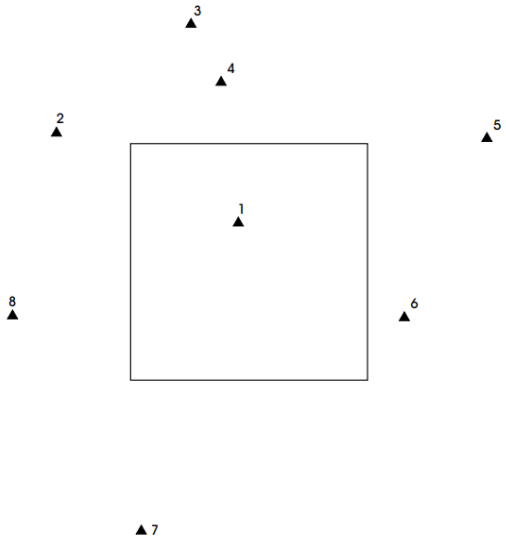

Figure 3. The general two-dimensional block estimation problem is to estimate the mean grade of a block by utilizing a subset of nearby and included data (Sinclair and Blackwell, 2002).

To provide the best estimate of Block A, it is conceivable that the eight data points could be weighted in a manner:

$$grade_A = w_1 s_1 + w_2 s_2 + w_3 s_3 + w_4 s_4 + w_5 s_5 + w_6 s_6 + w_7 s_7 + w_8 s_8 \quad (5)$$

If the variable is random, meaning all weights are equal, then simple data averaging is acceptable; however, if the variable is regionalized, meaning there is significant autocorrelation over distances greater than the sample to block spacings, then simple data averaging is not optimal. It is obvious that a nearby datum should be given more weight than a more distant datum when autocorrelation is significant, though it is unclear by how much.

2.1.1 Ordinary kriging

Linear kriging is a method used to forecast a regionalized parameter's value at any given point by employing a weighted average of known values of this parameter from nearby locations. In practical applications, this parameter is often viewed as a manifestation of a second-order stationary random field, characterized by a consistent mean and a recognized spatial correlation structure, typically modeled via an auto-covariance function or variogram. In the context of this, simple kriging (SK) operates under the assumption that the mean value is fixed, whereas ordinary kriging (OK) treats this mean value as an unknown, offering greater adaptability when the regionalized parameter maintains a locally constant mean but varies globally across space (Hekmatnejad, 2017).

Estimation variance of the ordinary kriging method is given by the following equation:

$$\sigma^2{}_e = E\{[Z - Z^*{}_k]^2\} \quad (6)$$

Where: Z is the true value, $Z^*{}_k$ is the kriging estimator.

This expression can be given in the form of the semivariogram as well:

$$\sigma^2{}_e = 2[\Sigma w_i \gamma(B, s_i)] - \Sigma\Sigma w_i w_j \gamma(s_i, s_j) - 2\,\bar{\Upsilon}(B, B) \quad (7)$$

where $w_i\gamma$ (B, s_i) is the weighted average semivariogram value between all data points and the block to be estimated, $w_i w_j \gamma(s_i, s_j)$ is the weighted average semivariogram value between all possible pairs of data, and $\bar{\Upsilon}$(B, B) is the average semivariogram value of all possible pairs of points within the block to be estimated.

With the restriction that the weights must add up to one (w_i= 1), equation 7 can be minimized. A new unknown, the Lagrange parameter, μ, is introduced into the system of equations by this constraint, which is introduced as an expression equivalent to zero in the minimizing procedure (referred to Isaaks and Srivastava (1989) by Sinclair and Blackwell, 2002). The final formulas that result from this process are referred to as the ordinary kriging system.

$$\begin{bmatrix} \gamma(s_1,s_1) & \gamma(s_1,s_2) & \dots & \gamma(s_1,s_n) & 1 \\ \gamma(s_2,s_1) & \gamma(s_2,s_2) & \dots & \gamma(s_2,s_n) & 1 \\ \gamma(s_3,s_1) & \gamma(s_3,s_2) & \dots & \gamma(s_3,s_n) & 1 \\ \gamma(s_n,s_1) & \gamma(s_n,s_2) & \dots & \gamma(s_n,s_n) & 1 \\ 1 & 1 & 1 & 0 & \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_n \\ \mu \end{bmatrix} = \begin{bmatrix} \gamma(s_1,B) \\ \gamma(s_2,B) \\ \gamma(s_3,B) \\ \gamma(s_n,B) \\ 1 \end{bmatrix} \quad (8)$$

Where: w_i is the sample weight that needs to be calculated, $\gamma\,(s_i, B)$ is the gamma value between a datum and the block that needs to be estimated and $\gamma\,(s_i, s_j)$ is the gamma value between any two data (Sinclair and Blackwell, 2002).

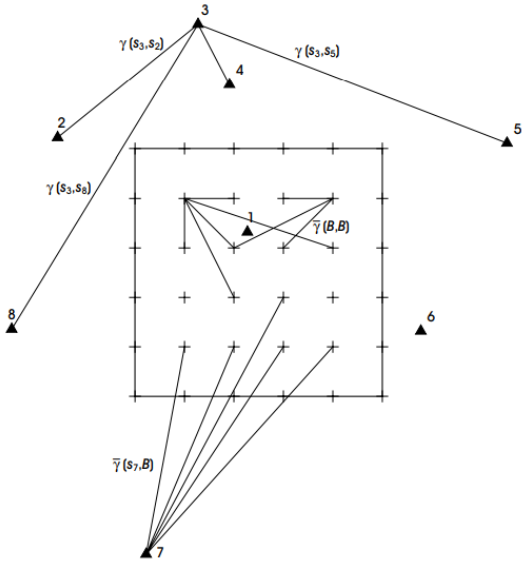

Figure 4. The Block A from Figure 3 and illustration of how the values are obtained for calculations in Equation 8 (Sinclair and Blackwell, 2002).

As a result, solution of these equations gives weights, so block/point grade estimate is derived from the equation below:

$$Z^*_k = \Sigma w_i s_i \quad (9)$$

The corresponding kriging variance is given by:

$$\sigma^2_k = \Sigma w_i \gamma(s_i, B) + \mu - \bar{\Upsilon}(B, B) \quad (10)$$

A set of kriging equations is usually impractical to implement by hand. Thankfully, a variety of software packages for a broad range of geostatistical computations, including different kriging techniques.

2.1.2 Simple kriging

According to Wackernagel (2003), the estimation of simple kriging is based on the following equations:

$$\hat{\gamma}(h) = \frac{1}{2n}\sum_{i=1}^{n}\big(Z(x_i) - Z(x_i + h)\big)^2 \quad (11)$$

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i) + [1 - \sum_{i=1}^{n} \lambda_i]\mu \quad (12)$$

Where: stationary mean (constant all over the domain) = μ. Due to the constant value of stationar mean, SK is sometimes referred to as “kriging with known mean” (Wackernagel, 2003). Greater value of $1 - \sum_{i=1}^{n} \lambda\_i$ indicates a poorly sampled region. According to Wackernagel (2003) and Webster and Oliver (2001), SK assumes second-order stationary, which is constant mean, variance, and covariance over the domain or region of interest. Ordinary kriging, which does not have a prior mean, is most frequently applied because such an assumption is often overly restrictive (Burrough and McDonnell 1998). It is an unbiased kriging algorithm that uses the mean of original samples to predict variables.

2.1.3 Sequential Gaussian simulation

The SGS algorithm uses simulated values as conditioning data after simulating nodes one after the other sequentially. The data are converted into Gaussian space since the SGS method requires the use of standard Gaussian values. Calculations and modeling are done on altered data variograms. To assess the nodes of the grid, a random path and a simulated grid must be defined. In each node, a Gaussian probability distribution is found based on the kriging mean and variance. It is required to select a random path in order to estimate at each node. In each node, a random value selected at random from a Gaussian probability distribution is referred to as the "simulated value" (Asghari and Amnieh, 2014).

2.2 Mine planning

Mining involves the extraction of naturally occurring substances from the earth with the aim of generating financial gain. Final target of any mining activity is generation of as much profit as possible with lowest initial investments. Extraction is done in different ways depending on several considerations: economic, geospatial, political, etc. The process of projecting prospective mine sites is called “Mine planning”. This stage is crucial, since once ore extraction starts, it is hard to change the mining system on-site. Depending on the nature of minerals, extracted materials can be classified into: metallic ores (ex. Iron, copper), nonmetallic minerals (ex. sand, gravel) and fossil fuels (coal) (Newman, 2010). Although volume calculations are important in all of them, grade prediction and ore/waste classification is crucial for metallic ore extractions. Moreover, metallic ore extraction has further division into underground and surface mining systems which imply different approaches in mine planning due to the wide range of techniques used in different systems. According to Newman (2010), mining has 5 stages: prospecting, exploration, development, exploitation and reclamation. Grade prediction can be considered as an attribute of the exploration stage, however mining companies imply it also in the exploitation stage to improve accuracy and preciseness of previous results.

2.2.1 Ore/waste classification

During the exploration phase, geologists assess the deposit's worth by drilling holes to gauge the mineral concentration and its distribution across the ore deposit. Methods like kriging (Krige 1951) and simulation techniques (Deutsch 2004) are employed for interpolation. These techniques generate tonnage-grade curves that illustrate the potential advantages of mining the ore deposit based on specific economic parameters. According to the economic parameters and processing plant characteristics “cut-off” grade is set to differentiate between valuable “ore” material and waste material which does not carry any economic value or does not contain metalliferous content. This step should be done prior to investments, design and planning of a mine site (Mousavi, 2016).

Because exploration drilling is expensive, there are limited drill holes spaced far apart. Furthermore, due to the intricate and fluctuating characteristics of mineral deposits, there's inherent uncertainty in estimated grades. Addressing this uncertainty is crucial to avoid underestimation or overestimation. Suddenly sending an ore parcel to a waste area results in significant financial loss. On the other hand, processing a waste parcel lowers process efficiency, reducing recovery rates and consuming substantial energy resources, thus leading to financial loss. That is why utilizing the most effective method for distinguishing between ore and waste materials can improve planning and reduce the risks linked with mining activities. In this paper, it is associated with reducing uncertainty in the kriging process and creation of a proper three dimensional shape of orebody to proper understanding of mine site.

3. METHODOLOGY

Initial dataset consists of 2230 iron ore grade values in a given area. To conduct kriging and simulation proper EDA and variogram analysis was done. Moreover, to properly compare the two methods of estimation, for both systems point and block estimation were to be applied: point simulation gives statistical analysis and block simulation gives grade-tonnage analysis. All of the work was done in ISATIS.neo software.

3.1 EDA

Histogram in Figure 5 depicts original raw data consisting of 2230 samples.

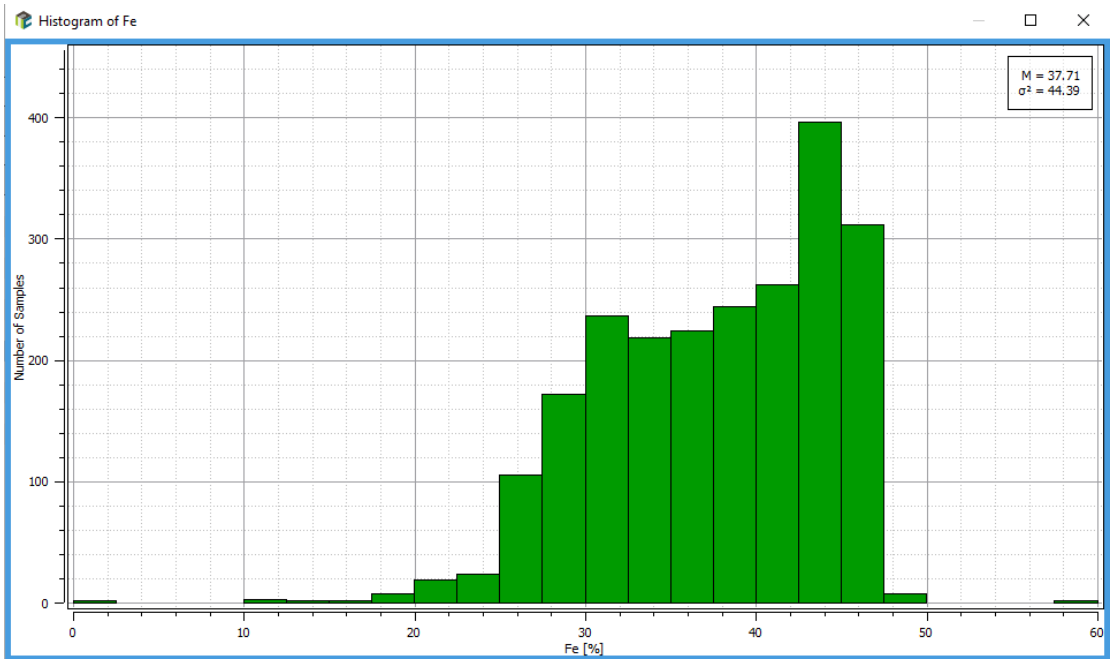

Figure 5. Histogram of original raw data

Since original data was good, no capping was applied for simple kriging, however for Gaussian simulation, this dataset was changed to Gaussian variables.

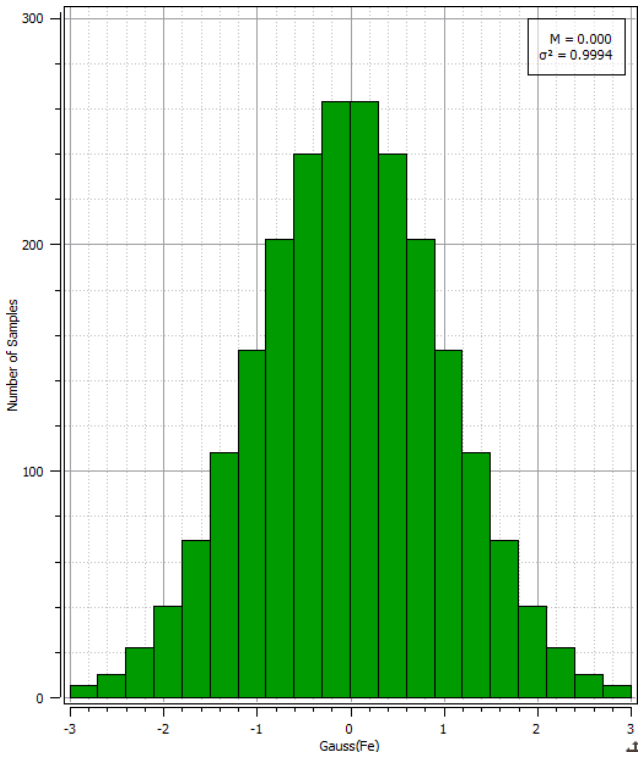

Figure 6. Gaussian variable of original dataset

3.2 Variogram analysis

Statistical parameters of the dataset can be seen in variogram analysis, thus two datasets were sent to variogram analysis. For both cases, it was decided to use multidirectional calculation mode (more reliable). By adjusting lag value and maximum distance, the most suitable two scenarios were chosen for further analysis. Suitability of the variogram is seen from alignment of points over the lines in the following figures:

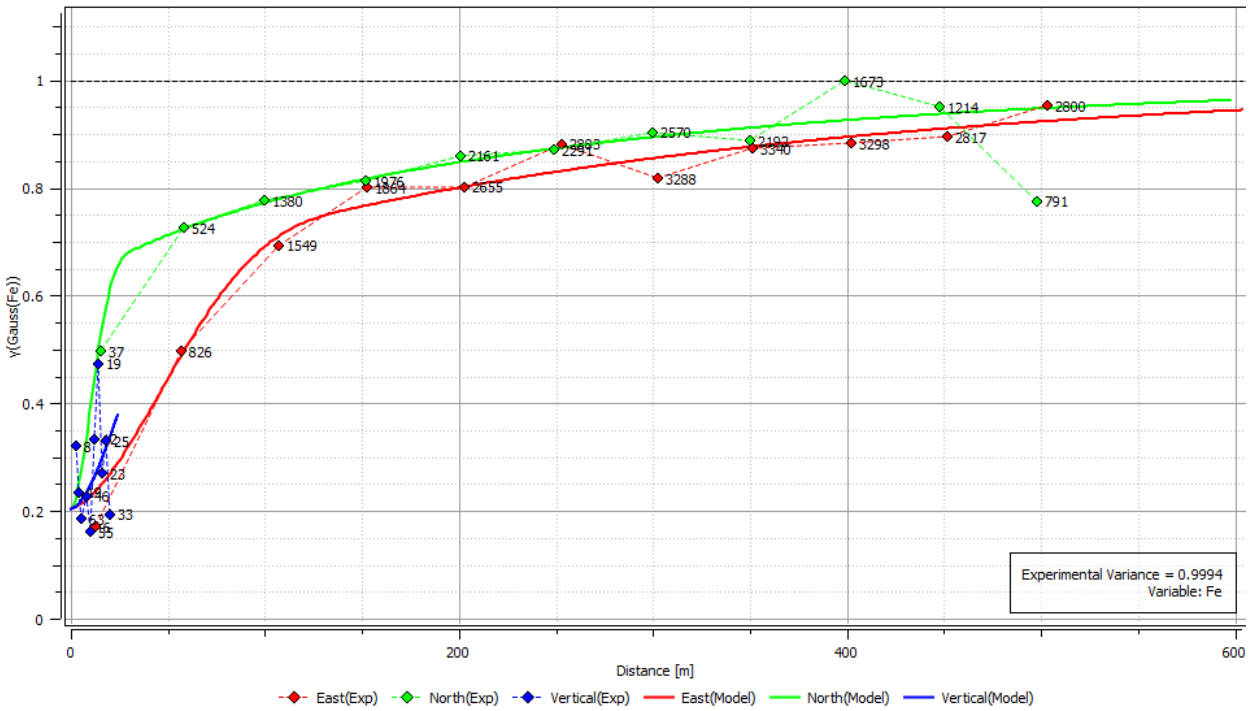

Figure 7. Variogram for Gaussian simulation (multidirectional)

Following formula describes variogram of Gaussian simulation:

$$\gamma(h) = 9.04\ nugget + 18.54\ cub(155.315\ m, 33.756\ m, 86.855\ m) + 3.43\ \exp(1006.13\ m, 187.001\ m, 1006.13\ m) + 13.38\ \exp\ (916.165\ m, 849.144\ m, 1006.13\ m)$$

According to the formula above, total sill for SGS is 44.39 and nugget is 9.04.

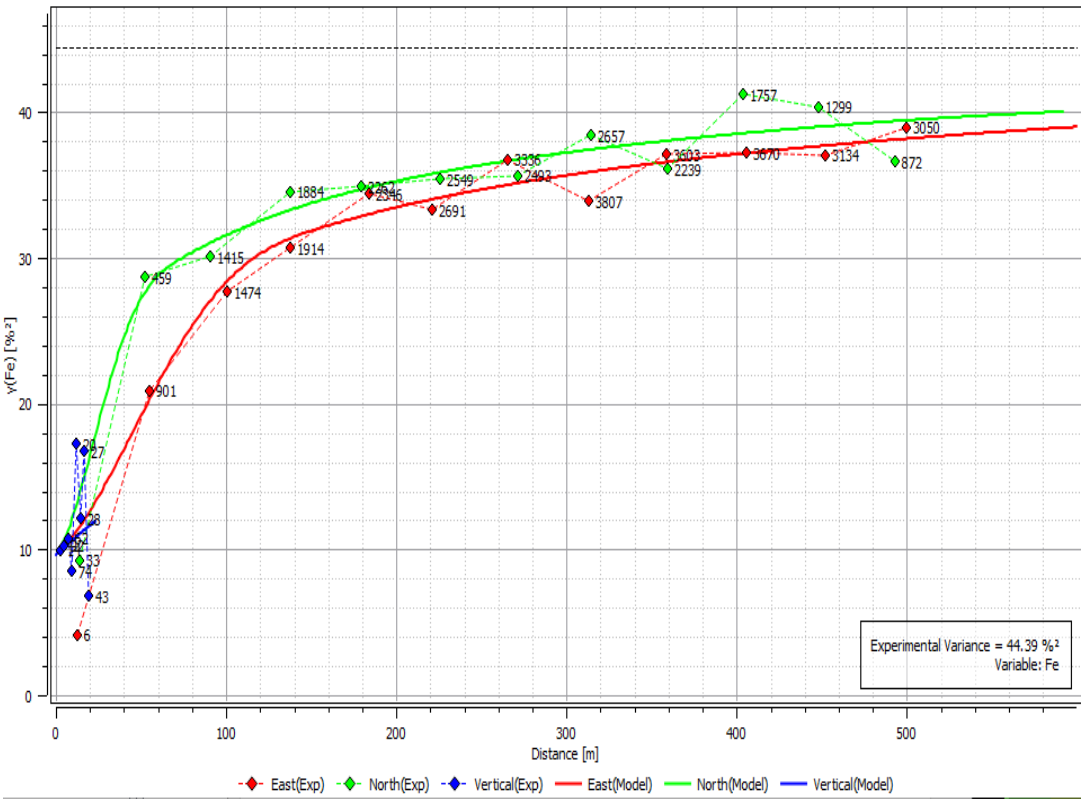

Figure 8. Variogram for SK (multidirectional)

Following formula describes variogram of Simple Kriging:

$$\gamma(h) = 9.65\, nugget + 13.02\, cub(163.167\ m, 76.285\ m, 999.481\ m) + 6.63\, \text{sph}(999.481\ m, 753.096\ m, 409.37\ m) + 11.19\, \exp\,(384.249\ m, 265.19\ m, 417.929\ m)$$

According to the formula above, total sill for SK is 40.49 and nugget is 9.65.

3.3 Kriging

Before any estimation it is necessary to create a grid over the area which is to be estimated. Thus, from the original dataset distribution grid with dimensions of 2145x1095x105 meters and one block dimensions of 15x15x15 were created.

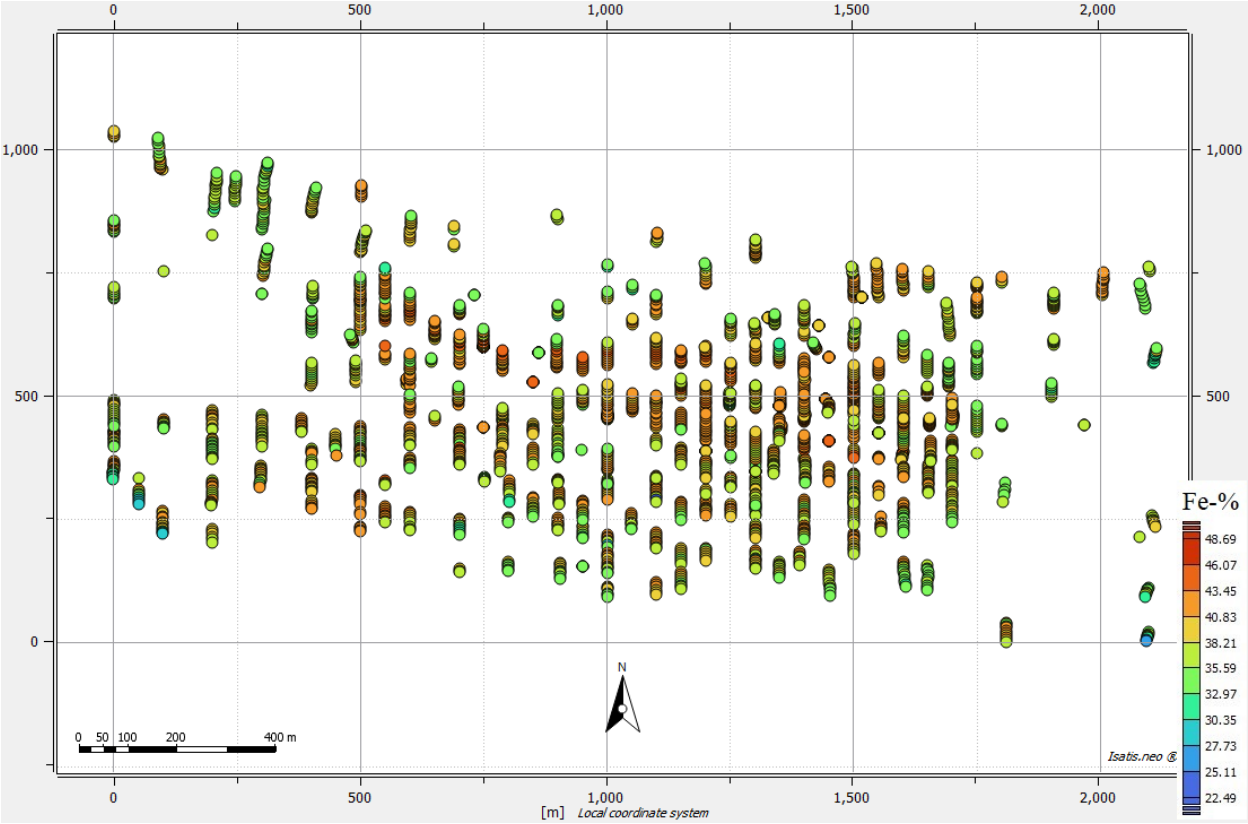

Figure 9. Distribution of initial borehole data in 2D (top view)

Then using a geostatistical set obtained from variogram analysis was used to conduct simple kriging with two calculation modes: point and block.

3.4 Sequential Gaussian simulation

Same grid from 3.3 Kriging was used in sequential Gaussian simulation to create 100 realizations. In this case also point and block kriging were calculated, discretization steps number for block kriging was set to 2.

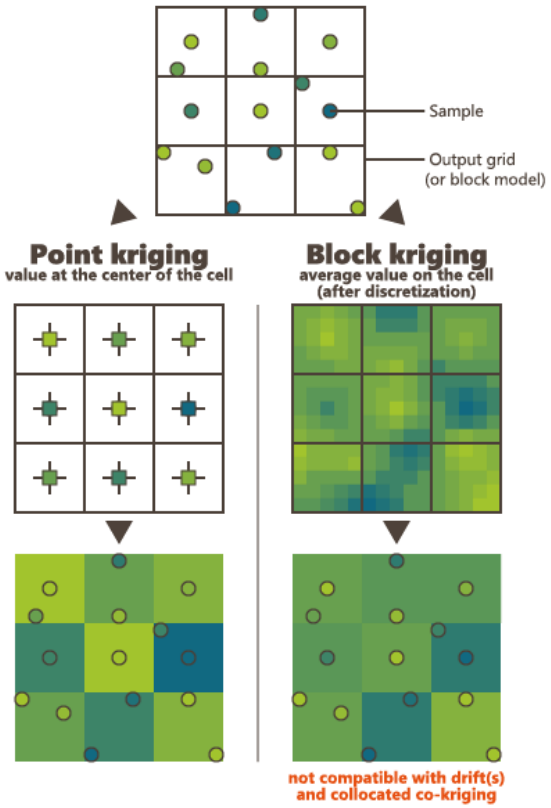

Figure 10. Point and block kriging difference (ISATIS.NEO)

As it can be seen from Figure 11, point kriging sets values to the center of the cell while block kriging divides the cell to the discretization number and sets different values for each division.

4. RESULTS

In the end, as a result of simulation 100 realizations were obtained for SGS and 1 estimation result for SK. The Q-Q plots below demonstrate the 3 realizations from SGS and result of SK in relation to the original dataset. As Figure 11 illustrates, simple kriging gives more deviated results from the original dataset than the SGS (can be seen from the Figures 12-14). 3 random realizations from SGS in figures 12 to 14 demonstrate higher coincidence with the original dataset. In a Q-Q plot, the quantiles of the sample data are plotted against the quantiles of the theoretical distribution. If the points fall approximately along a straight line, it suggests that the sample data comes from the specified distribution. Any deviations from the straight line indicate departures from the specified distribution.

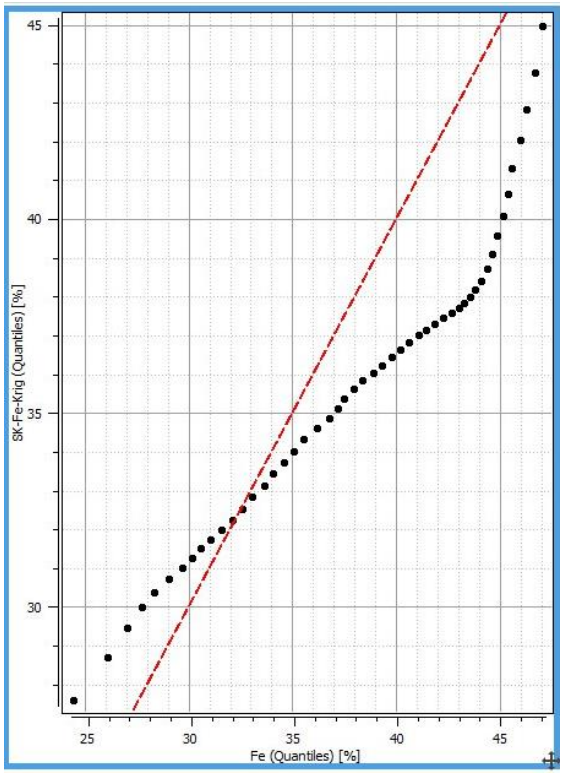

Figure 11. Q-Q plot for SK and Original data

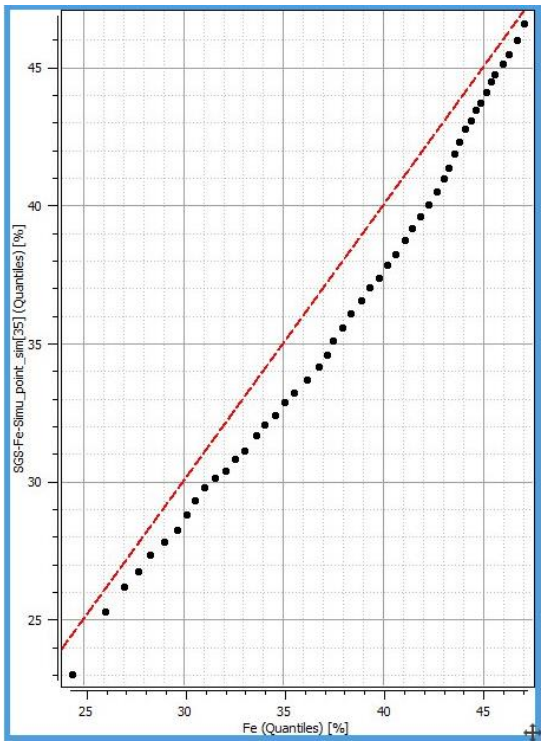

Figure 8. Q-Q plot for SGS realization No. 35

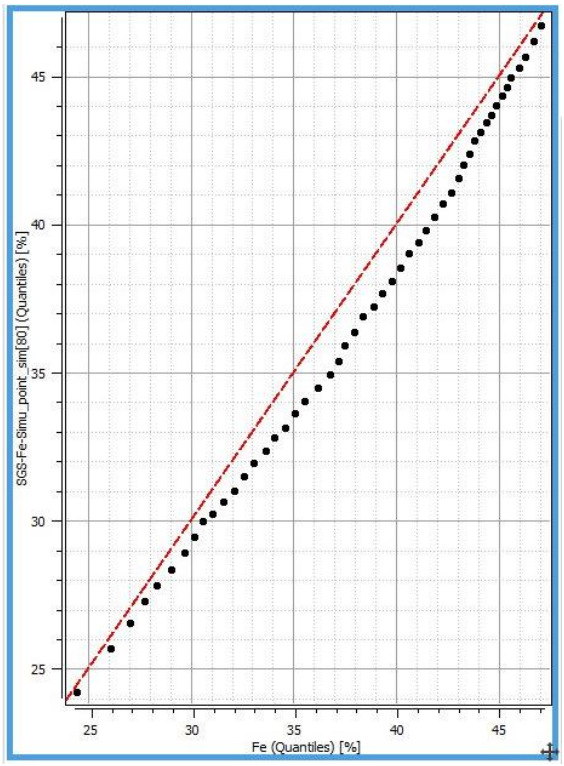

Figure 9. Q-Q plot for SGS realization No.80

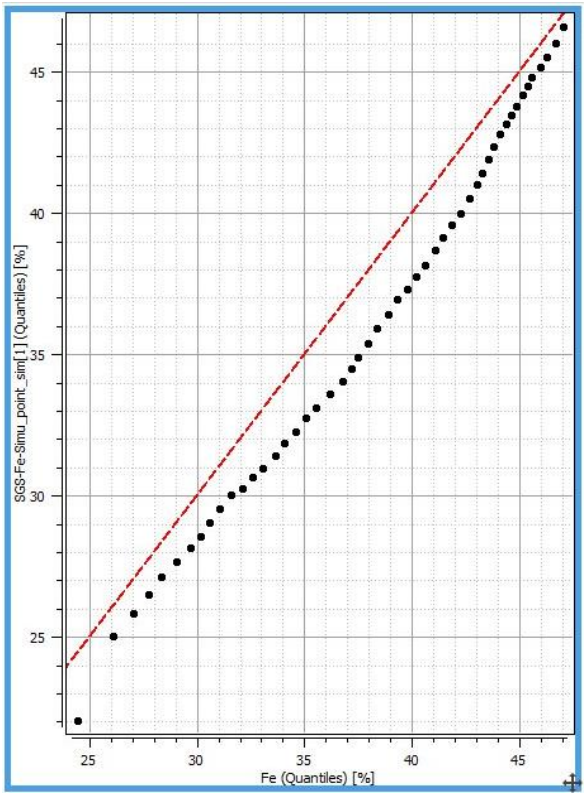

Figure 10. Q-Q plot for SGS realization No.1

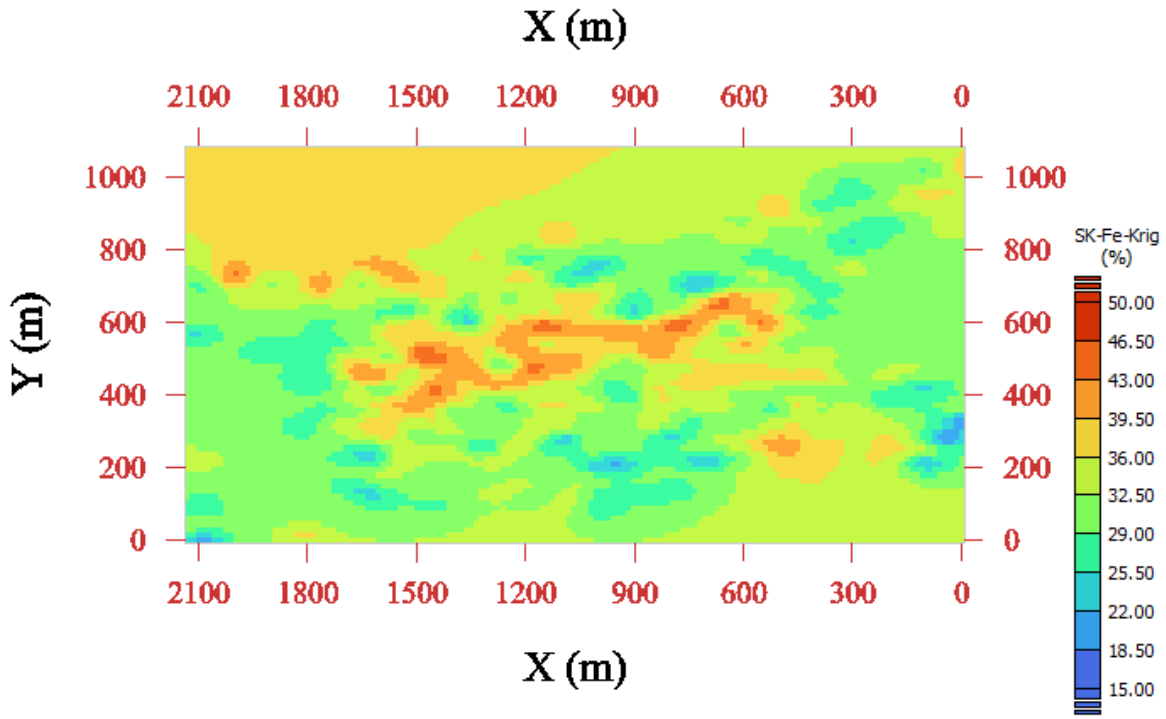

Figure 11. SK E-type map: distribution of iron grade

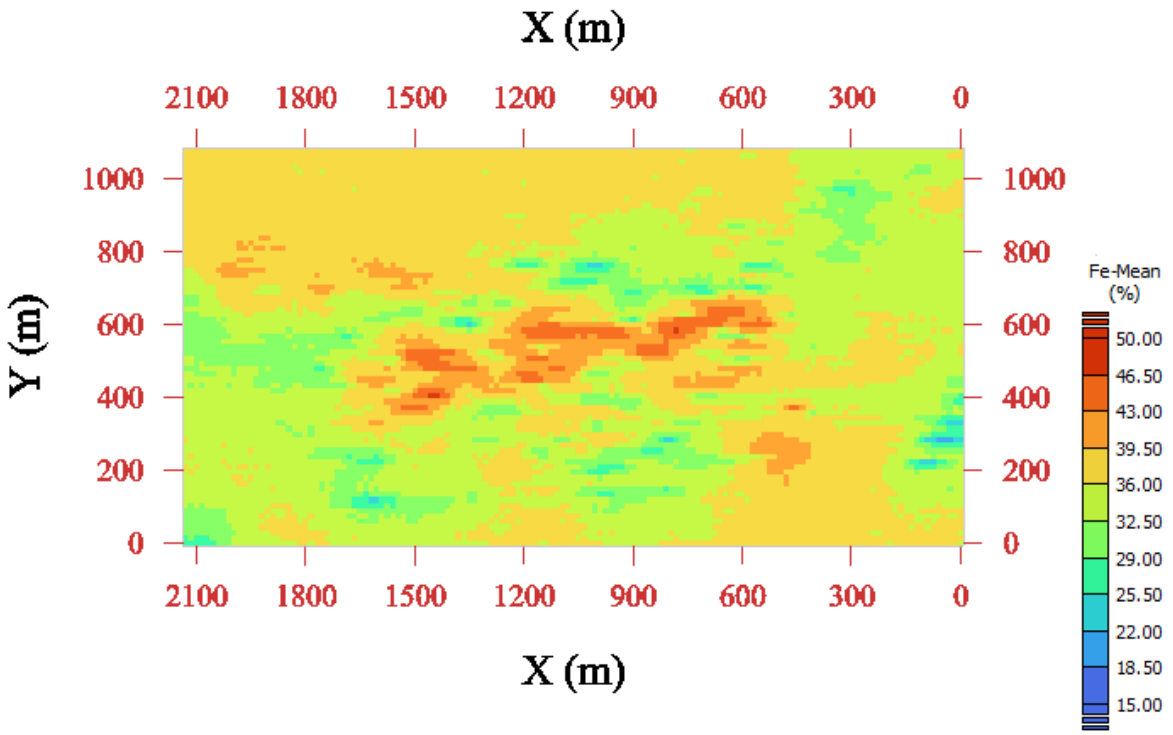

Figure 12. SGS E-type map: distribution of iron grade

The SGS map shows more heterogeneity and detail due to the nature of the simulation, while the SK map appears smoother and less variable due to the influence of the mean of the data. The choice between SGS and SK would depend on the specific objectives of the study, such as whether detailed local variability or a smoothed global estimate is desired.

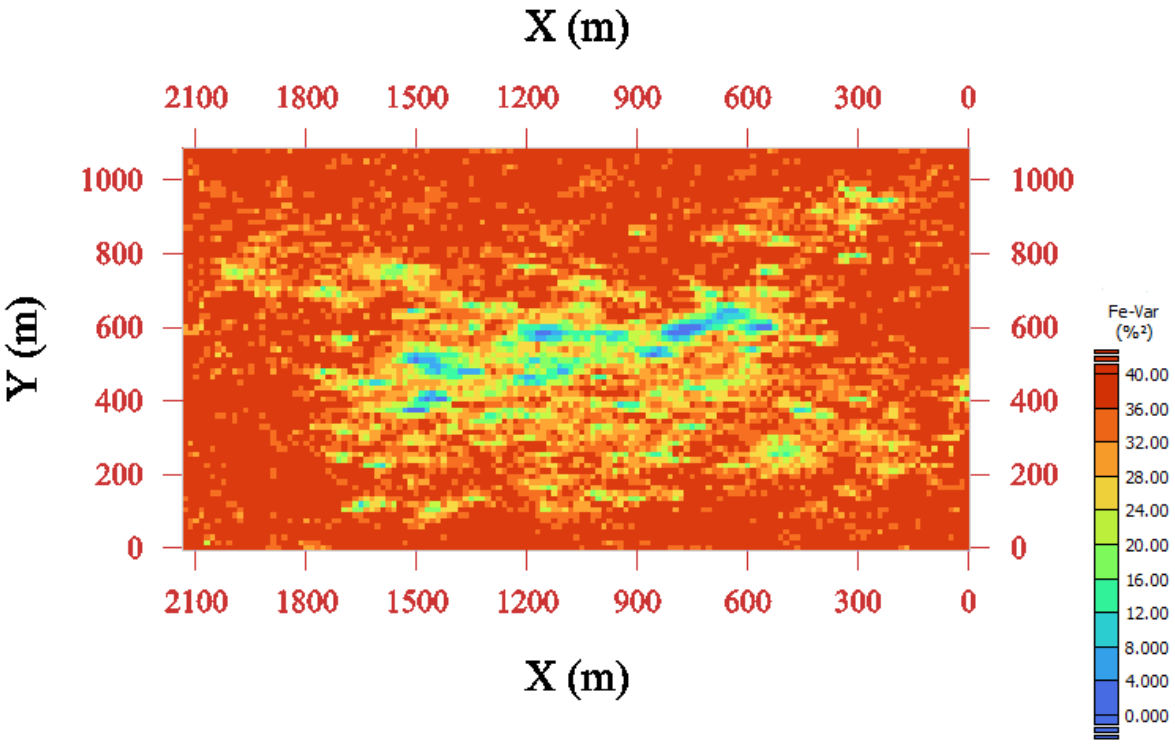

Figure 13. Variance map of Sequential Gaussian Simulation result.

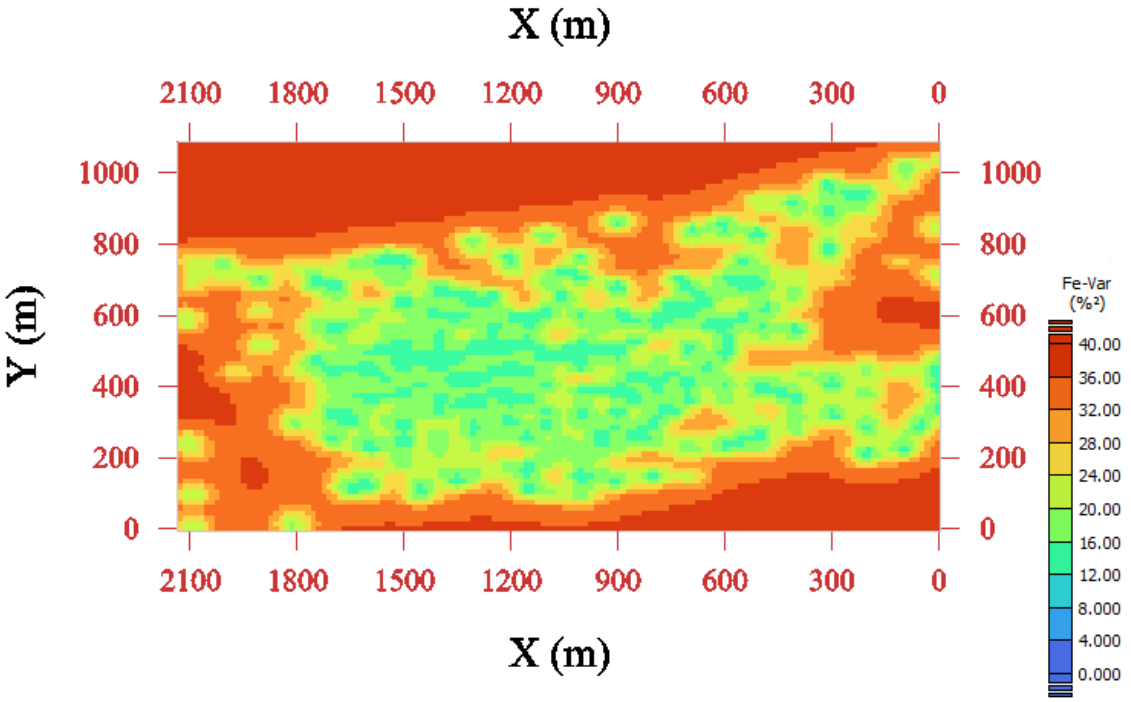

Figure 14. Variance map of Simple Kriging result.

The SGS variance map shows a greater degree of localized uncertainty, which might be due to the simulation capturing more detailed variations in the data. In contrast, the SK variance map indicates a smoother, more homogeneous distribution of uncertainty, which may reflect the influence of the global mean used in the kriging process and potentially less sensitivity to local variations compared to SGS.

Source/method	Mean	Variance
Original dataset	37.71	44.39
Simple Kriging	35.7	17.62
Sequential Gaussian simulation	36.44	44.29

Table 1. Mean and Variance values for 3 cases: original, SK and SGS.

Table 1 compares mean and variance values for 3 cases: original, SK and SGS. Values for SGS were obtained by combining all of the 100 realizations and getting mean from them. As it can be seen from Figure 16, SGS estimates higher grades of iron all over the deposit, thus metal quantity and benefit according to SGS might be higher.

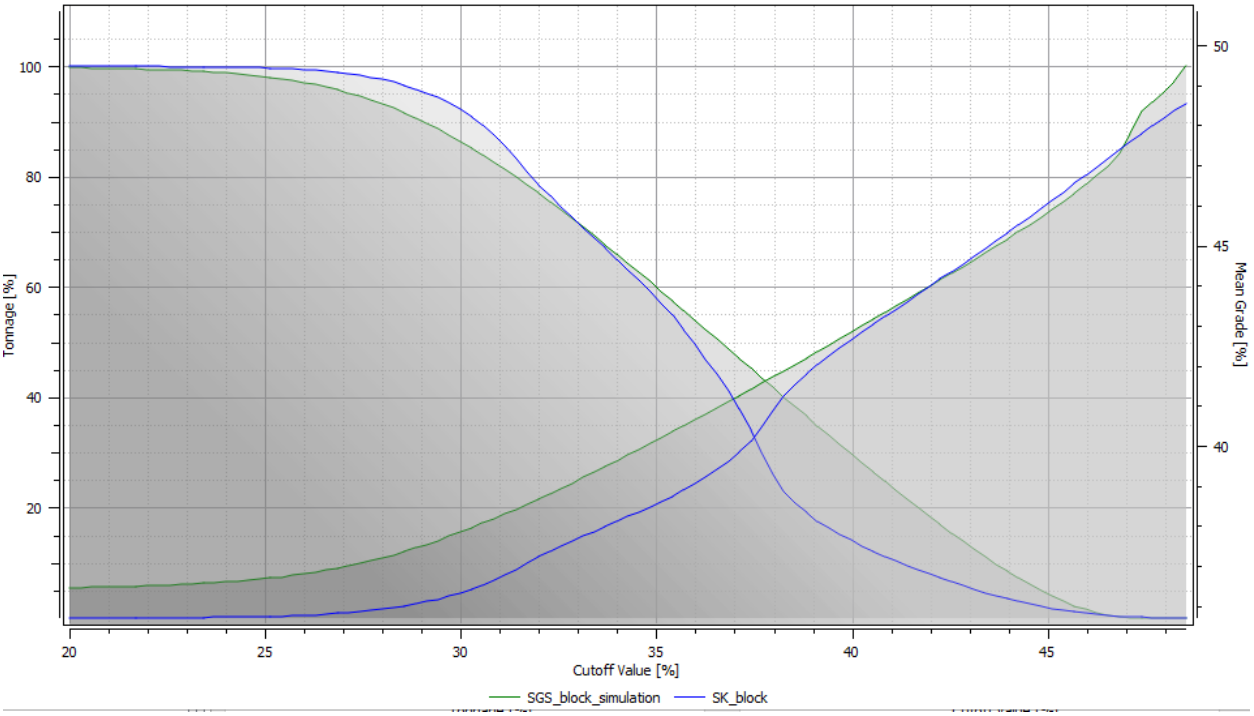

Figure 15. Grade/tonnage graph.

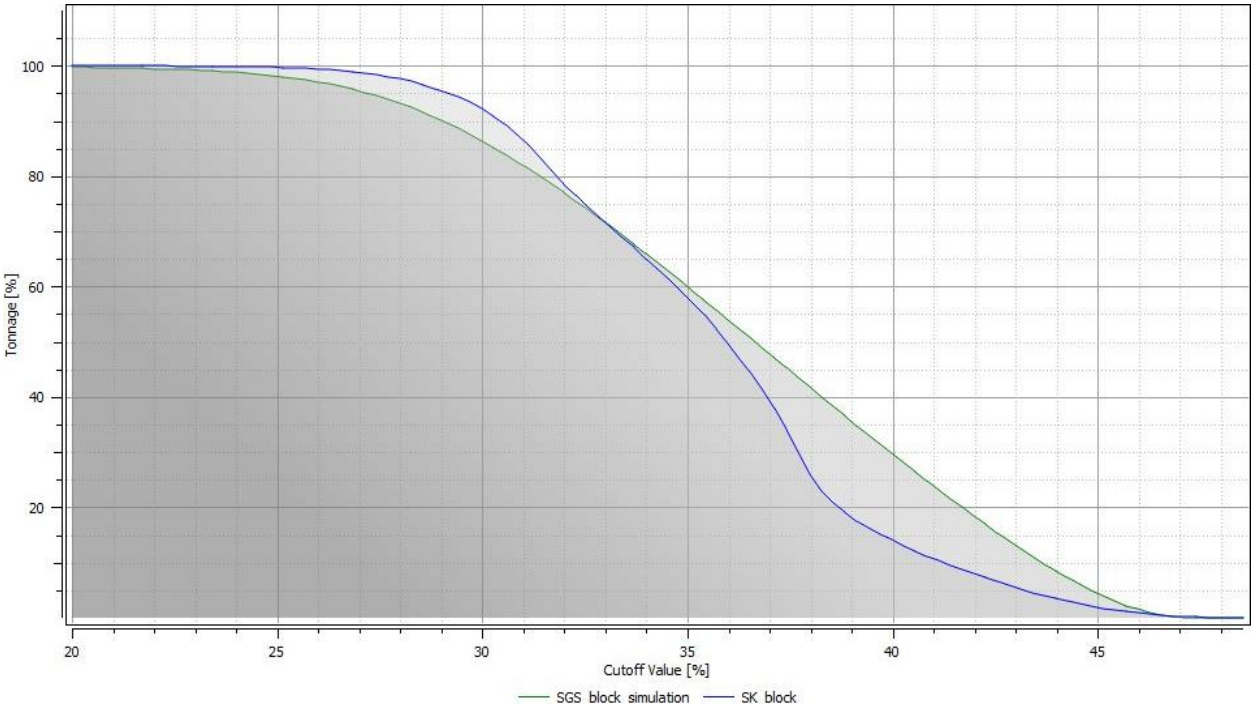

Figure 16. Tonnage over cutoff value graph for SGS and SK.

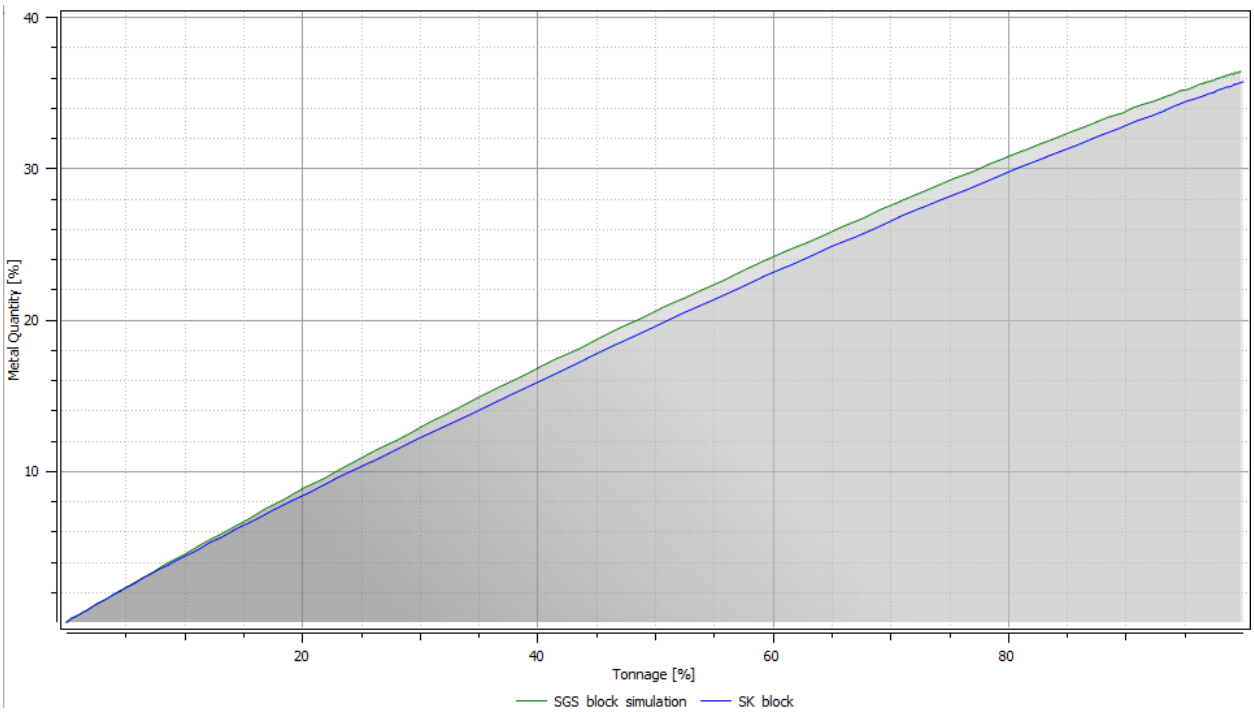

Figure 17. Metal quantity/tonnage value graph.

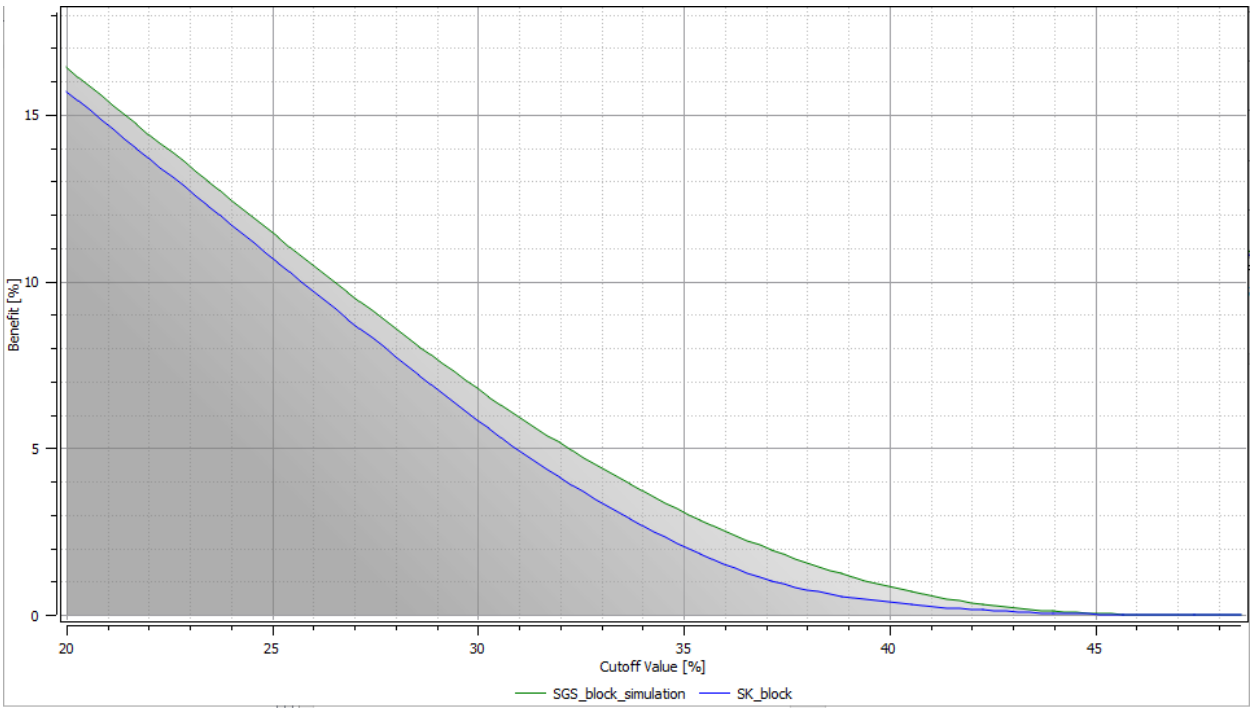

Figure 18. Benefit/cutoff grade graph.

From the tonnage/cutoff grade graph one can conclude that lower cutoff value at SK might have higher tonnage while SGS demonstrates higher tonnage at higher cutoff values in comparison to SK. The differences between SGS and SK are subtle in these graphs. SGS seems to predict a slightly higher tonnage and grade, which might suggest it is identifying more pockets of higher-grade material compared to SK. However, when it comes to the economic benefit and metal quantity, the two methods provide similar estimations, as indicated by the overlapping lines in the corresponding graphs with slightly higher benefit coming from SGS block simulation. Since no economic constraints were implemented in simulations, these values are given approximately by ISATIS.NEO software itself. These results would inform a mining operation on how to balance the cutoff grade for maximizing profit while considering the amount of metal that can be extracted.

5. DISCUSSION

Comparative performance of Sequential Gaussian Simulation and Simple Kriging in estimating iron deposit provided insights into correlation of estimated values and original data, as well as possible outcome from estimated reserves. The utilization of SGS and SK techniques in geostatistical estimation is of paramount importance in accurately characterizing mineral deposits, such as iron, which often exhibit spatial variability. Both methods aim to interpolate values at unsampled locations based on the available data, but they differ in their underlying assumptions and methodologies.

Analysis reveals that the Q-Q plots generated from the Sequential Gaussian Simulation exhibit a higher correlation with the original data compared to those derived from Simple Kriging. This observation suggests that SGS may offer advantages in capturing the complex spatial patterns and variability present in iron deposits.

One possible explanation for the superior performance of SGS lies in its ability to account for spatial trends and non-stationarity more effectively through the simulation of multiple realizations. By honoring the spatial continuity and geological features observed in the data, SGS produces simulated values that closely resemble the distributional characteristics of the original dataset.

In contrast, Simple Kriging relies on a stationary variogram model and assumes constant spatial relationships across the entire study area. While SK is computationally efficient and straightforward to implement, its rigid assumptions may lead to oversimplified representations of the underlying geological structures, particularly in cases of complex spatial patterns or non-linear trends.

Mean value maps of two estimation systems show different distribution of grades. E-map of SK shows

6. CONCLUSION AND RECOMMENDATIONS

In conclusion, the comparative analysis of Sequential Gaussian Simulation and Simple Kriging in estimating a given iron deposit underscores the importance of selecting appropriate geostatistical methods tailored to the specific characteristics of the deposit and research objectives. While both techniques have their merits, the results from Q-Q plots suggest that SGS holds promise in providing more accurate and reliable estimations for a given iron mineralization.

Grade control and its proper estimation is an essential part of mining which would affect feasibility analysis of the mine site. Overestimation would lead to potential profit loss or even would not generate any profit at all, while underestimation would even not indicate profit at feasibility analysis stage and potential mine site would not even open. Since the final objective of every mining project is to get as much profit as possible, high attention is needed to resource estimation. There are some early closure examples of mine sites due to poor resource estimation.

Using a case study in the assessment of mineral resources, this paper compares the effectiveness of SGS and SK. It is essential to acknowledge the limitations and potential biases associated with both SGS and SK techniques. The choice between these methods should be guided by factors such as the spatial characteristics of the deposit, the level of uncertainty tolerance, and computational resources available. Additionally, further research and sensitivity analysis may be warranted to validate the findings and explore alternative geostatistical approaches.

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