Investigation of stope wall convergence-based behavior for narrow vein orebodies

Bachelor of Science in Mining Engineering

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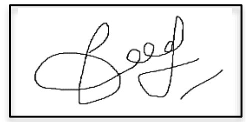
Originality statement

I now declare that this thesis called “Investigation of stope wall convergence-based behavior for narrow vein orebodies” is the output of my work, and all sources of information including figures, tables, on other materials contained in this paper have been acknowledged.

Darkhan Abdir 

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Abstract

Despite the fact that it involves geological complexities like inconsistent vein properties, shear zones, and varied lithological width, narrow-vein mining is an ancient underground mining technique that is currently gaining importance due to the depletion of surface mines. Narrow vein mining targets tabular orebodies known as veins, single or in complex systems that are essential for yielding various minerals like gold and tin. In this thesis, it is suggested that the behavior in deep, hard rock, narrow vein mines is primarily plastic instead of brittle, as the current design tools assume. This is primarily because production demands force stopes to be elongated, leading to stope walls that are significantly longer along the strike than their width. In order to check this hypothesis numerical modeling tool RS2 was used, also, the influence of stress ratio (K=1; K=1.5; K=2; K=2.5), and different stope width (from 1m to 10m) were analyzed. The stope walls deformation and damage increase with mining depth and in-situ stress, as shown by the figures and RS2 simulations. When K=1.5, $\frac{\sigma_1}{\sigma_c} = 0.15$, depth = 65 m, stope wall closure was equal to 0.00210022 m; $\frac{\sigma_1}{\sigma_c} = 0.4$, depth = 750 m, walls converged inside of stope by 0.054073 m; $\frac{\sigma_1}{\sigma_c} = 0.6$, depth = 1500 m, the closure of walls was 0.118056 m. Also, it was concluded that the behavior of stope walls changes with changing stress circumstances; greater depth and K values cause displacement to increase and strength to decrease, respectively, resulting in considerable stress damage. The graph indicates that larger stopes often encounter less wall convergence and it shows that the closure percentage increases as the effective stress ratio (K) increases, underscoring the critical role of stress conditions affecting wall stability and the significance of taking both stope width and stress conditions into account when controlling wall displacement in mines.

Table of Contents

List of Figures

List of Tables

1 Introduction

1.1 Background

Narrow-vein mining is one of the oldest types of underground mining, and nowadays its role in the mining industry is even increasing because of depletion of surface mines (Dominy et al., 1998). A narrow-vein is defined as a tabular orebody in an opening such as a single fracture, or in a complex fracture system (Dominy et al., 1997). Narrow vein deposits are the major source of many products such as gold, tin, uranium, coal, silver, tungsten, etc., have complexities with geology related to mineral veins, shear zones and not continuous behavior in geology, dip and strike, and width of lithology (AMC Consultants, 2023; Maptek, 2021). It is difficult to estimate reserves in narrow-vein mines because of the complicated geology and different ore grades. Efficiency in estimation of narrow vein ore deposit sizes is further limited by operators' challenges with low stope tonnage ratios, considerable wall rock dilution, and constrained tonnages. Furthermore, large-scale automation is not a good fit for narrow-vein mining, especially for in-stope operations (Dominy et al., 1998).

The concept of narrow-vein orebodies is not standardized; instead, it is impacted by regional, national, or author preferences, according to Suorineni (2010) study of the literature descriptions of these orebodies. The range of thickness that is deemed typical for an orebody with thin veins is 2 to 10 meters. According to Suorineni (2010), a narrow-vein orebody geometric properties and the technology used in mining it should both be considered in defining narrow vein. As result of literature review, Suorineni (2010) proposed that an orebody with a width of less than two meters be classified as narrow-vein. This emphasizes the significance of taking into account both geometric and technical considerations when classifying such orebodies in the context of mining. One of the major issues related to narrow-vein mining is finding the balance between dilution and economic aspects of a mine.

1.2. Problem statement

Common narrow-vein mining stope design studies have traditionally relied on visits to existing mines for observations and discussions to adapt best practices (Dominy et al., 1998) in the form of empirical designs. However, these studies lack comprehensive guidelines for planning, selecting, designing, and operating narrow vein mines for continuous optimum net present value (NPV), as it can be seen from narrow vein mine studies in Canada, conducted by CANMET (1999), Lizotte (1993), Larson et al. (1990), and Robertson (1986), where they focused on blasthole mining trials for narrow-vein orebodies. Similar studies outside Canada, in Australia, the Philippines, Austria, Britain, and Sweden, also follow an experiential trial-and-error path in stope design (Suorineni, 2024).

Present narrow vein stope design practices primarily rely on the stability graph (Mathews et al., 1981) and the equivalent linear overbreak/slough (ELOS) method (Clark and Pakalnis, 1997). While the original Mathews stability graph was developed for wide massive orebodies at depths greater than 1000 m and ELOS stability graph was developed from narrow vein mines database, there have been the tendency for mining engineers and researchers to use the graphs interchangeably (Suorineni, 2010). This practice is misuse of the two stope design graphs as it is widely recognized that empirical methods cannot be extrapolated beyond the databases from which they were developed (Franklin, 1993)

The original and modified stability graphs, and the ELOS stability graph both assume structural and stress-induced brittle failures, these approaches as the stope overbreak mechanisms to estimate dilution and ground support. Stope analysis therefore frequently presumes that the rock mass acts elastically. In narrow vein mining, the stability of subvertical tabular excavations controlled by wall closure rather than brittle failures is not well established (Suorineni, 2010). High stresses in confined areas affect excavation and blasting efficiency (An et al. 2018), often hindering material flow and causing overbreak, underbreak, and choking; this was presented in the analysis of Konkola Copper mine made by Ngoma and Mutambo (2020). An et al. (2018) introduced a confinement coefficient in narrow-vein mining blast design defined as the ratio of the blasting crater volume or area formed in an infinite free face to that in a confined face. The blasting confinement coefficient is used to rate the confinement severity. Based on that, a narrow-vein stope blast design chart (Figure 1) was introduced as a guide for preliminary design of narrow vein blasting to optimize production by reducing overbreak (dilution) and underbreak (ore loss).

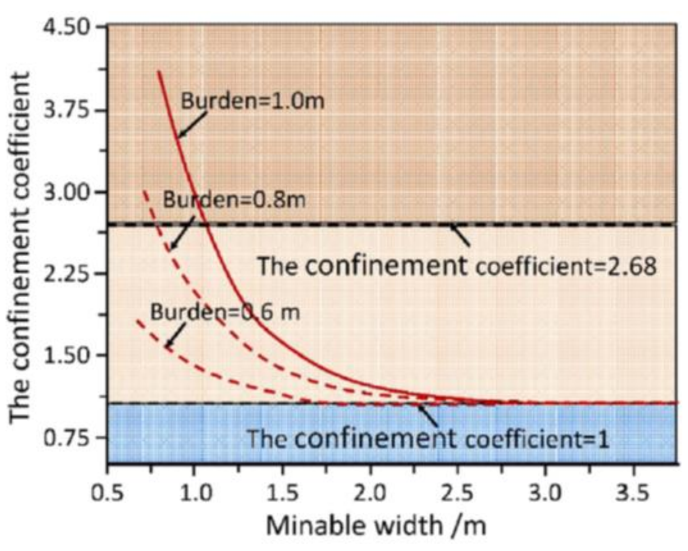


Figure 1. Chart for preliminary selection of burdens for given narrow-vein widths in narrow vein blasting practice (An et al., 2018)

The empirical nature of current practices in the design of narrow-vein stopes underscores the need for a more comprehensive and systematic approach to narrow-vein mining stope design. Empirical methods commonly used for stope design may not be as reliable or applicable at greater depths due to high stress conditions resulting in high confinements. This suggests a need for more sophisticated or adaptable design approaches that can better account for the challenges presented by deeper mining operations (Suorineni, 2024).

1.3 Hypothesis

It is important to highlight that, during the development of the stability graph, the authors (Mathews et al., 1981) explicitly specified its applicability to wide orebodies characterized by long span to short span ratios less than 4:1, termed as two-way span excavations. For excavations with long span to short span ratios exceeding 4:1, they transition into one-way span excavations, where the hydraulic radius becomes insensitive to the long span (Pontow, 2019). In the context of narrow mining stopes, where orebody width is constrained, the research hypothesizes that, for subvertical narrow vein orebodies with dips equal to or greater than 50 degrees, stope lengths have to be extended in order to reach planned tonnage due to the limited stope widths.

The design of current narrow vein mining stopes depends on direct observations and empirical approaches to predict overbreak for ground support and dilution estimates. These methods make the assumption that brittle and structurally controlled failures are the primary causes of overbreak, and stope stability assessments frequently consider the rock mass as elastic material under stress (Dominy et al., 1998; Suorineni, 2012). When the in-situ primary stress is greater than the intact rock uniaxial compressive strength, conditions of high stress are recognized. The work of Hoek et al. (1995) supports this conventional approach by elaborating that a critical damage index, defined as the ratio of the maximum tangential boundary stress to the unconfined compressive strength in the laboratory, surpassing a threshold of approximately 0.4—predicts the onset of brittle failure in underground openings across various rock mass types (Figure 2). Figure 2 shows that for σ_1/σ_c>0.4 the failure is excessive displacement of excavation walls. The wall failure mode is hypothesized to be the dominant failure mode at depth to suggest that stope design under these conditions should be stope wall displacement based rather than stress-induced brittle and structurally controlled failure.

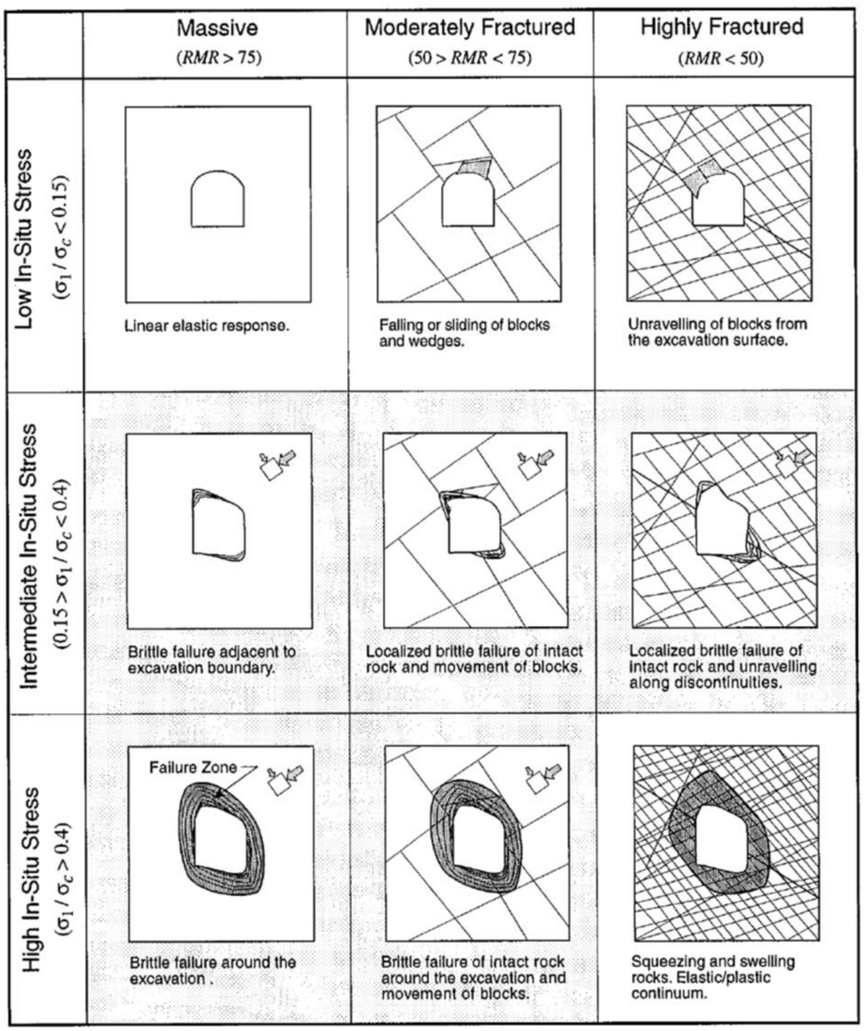


Figure 2. Excavation instability and brittle failure as a function of Rock Mass Rating (RMR) and the ratio of the maximum far-field stress (σ_1) to the unconfined compressive strength (σ_c) modified from Hoek et al. (1995)

Hence, in high-stress conditions, the response of the rock mass may be more ductile, with greater deformation and convergence of the stope walls being the dominant mode of behavior (Suorineni, 2024). This highlights a potential deviation from current narrow-vein stope design approaches where confinement is a well-recognized problem. This implies that extensive hanging wall and footwalls are frequently exposed in one-way span narrow vein stopes where stope lengths are far greater than stope width, and their convergence is influenced by high or low stress conditions, determined by the orebody strike and the orientation of the major far-field principal stress as presented in Figure 3 (Suorineni, 2014). It is hypothesized that since stope walls are much longer along strike than their widths, open stopes in deep narrow vein mines experience inelastic failure as a result of wall displacement and closure.

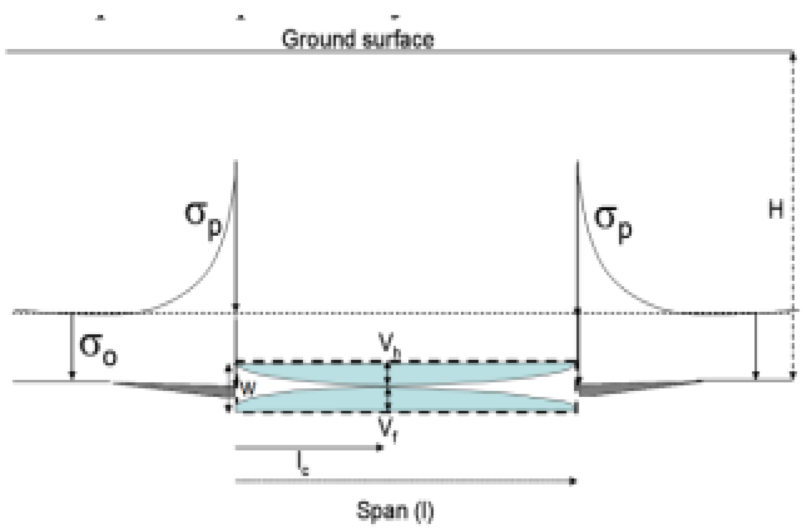


Figure 3. Plan view of stope wall convergence: σ_p - induced stress after excavation; σ_i - far field stress across excavation; V_h and V_f - hanging wall and footwall convergence, respectively (after Ortlepp, 1983, as cited in Suorineni, 2014)

1.4 Objectives

To address the challenges discussed in the ‘Problem statement’ section, this thesis aims to achieve the following objectives:

- Examine the current approaches used to design open stopes for narrow vein mines;
- Use numerical modeling to identify how underground stopes hanging walls/ footwalls deform under different stress conditions;
- Identify factors affecting wall convergence under different stress conditions.

1.5 Significance of the research to the industry

An et al. (2018) stated that narrow-vein mining is a major source for precious metals such as gold, giving example that in China 90% of proven tungsten reserves and half of proven gold reserves, respectively, are contained in narrow vein orebody deposits. Most of the precious metals such as silver, tin, uranium, and especially gold are located in narrow-veins, and narrow vein mining occupies an outstanding position in the world of mining industry (Dominy & Camm, 1998). For this reason, continuous enhancement of narrow-vein mining in Canada is considered a crucial endeavor, as demonstrated by ongoing research efforts outlined in Robertson (1986), Larson et al. (1990), Lizotte (1993), and CANMET (1999) (as cited in Suorineni, 2024).

The selection of mining techniques and the maintenance of operational safety depend heavily on in-stress fields because moderate stress levels encourage the stability of the rock mass, whereas high stress can lead to instability due to inadequate confinement or excessive fracture and failure. Maintaining the stability and safety of mining sites as well as the choice of mining techniques depend heavily on this balance (Bitarafan & Ataei, 2004; Stacey, 1998). The shortcomings of conventional mining system selection techniques are emphasized by Nicholas (1998). These approaches take into consideration in situ vertical stress depending on depth but neglect the impact of larger horizontal stresses. This neglect may have a big influence on the mining systems selection. In particular, the blasting process is severely constrained by the surrounding rocks in circumstances such as longhole blasting of narrow veins with few and confined open faces. This might result in underbreak or overbreak, which could cause unanticipated ore dilution or loss, respectively (Suorineni, 2024).

Incorrect mining systems and inappropriate stope designs have led to catastrophic mine collapses, resulting in tragic loss of lives and significant environmental harm (Yang et al., 2021). A notable instance occurred between 2002 and 2006, with the progressive collapse of the Annensky mine in Eastern Kazakhstan, covering an estimated area of 720,000 m2 by 2006. Makarov (2001) attributed this collapse to the utilization of an unsuitable mining system and stope design. Similar collapses in metalliferous mines have been documented, such as the Quirke Mine in Canada (Suorineni et al., 2011). The catastrophe at Belmoral Mines Ltd. in May 1980 brought to light the grave effects of poor planning, error in mining systems selection, and management in mining operations. The primary cause of the tragedy, which claimed eight miners' lives and put sixteen more at risk, was the improper sizing and installation of a crown pillar between the 200 and 350 levels of the mine during the design phase (Sweeney & Scoble, 2007).

Mathews et al. (1981) acknowledged that the behavior of the rock mass might not exactly follow elastic principles in their development of the stability graph for depths more than 1000 meters (Swan et al., 2005). It is evident that new and more efficient approaches must be developed in light of the difficulties and limitations associated with the techniques used in the design of narrow vein open stope.

The aim of this study is to investigate the behavior of stope walls in narrow vein mines from plastic failure mechanisms, and identify the deformation of these walls under different stress conditions. The results might be useful to create innovative stope design approaches that would completely transform narrow-vein mining and enable the extraction of precious metals in a way that is safer, more profitable, and less harmful to the environment.

1.6 Scope of work

The thesis aims to investigate closure of stope walls in narrow vein mines. The scope of work is divided into two main tasks: comprehensive literature review, and numerical modeling.

The following steps were required in the literature review:

Mining Systems Practices: Examining current methods used in narrow vein mining operations, paying attention to those used in prominent international cases like Canada.

Geotechnical Characteristics: In narrow vein mines, analyzing the mechanical and physical characteristics of the host rocks (hanging walls and footwalls). This will make it easier to comprehend how mine stability and design are influenced by geotechnical behavior.

Focusing on in situ stress states, this study examines the information that is currently available on in situ stress conditions in areas where metalliferous narrow vein orebodies are present.

Numerical modeling consists of the next steps:

Choice of simulation software: The choice of software was based on suitability for the specific analyses required, especially focusing on wall closure based on horizontal and vertical displacements, including the validation of in situ stress states and the study of mine excavation performance.

Hypothesis Testing: The project hypothesizes that open stopes in deep narrow vein mines fail inelastically due to wall displacement and closure, as a consequence of their significantly greater length along strike than width. Numerical modeling will be a key component in testing this hypothesis.

All of these steps will be undertaken to evaluate how stope walls behave under different stress conditions.

2 Literature review

2.1 Introduction to underground stoping

Underground mining refers to a procedure used to extract ore from below the earth's surface. This procedure involves creating openings or rooms within the orebody, which may or may not be backfilled after the ore extraction process is complete. The term "stope" specifically refers to the excavation created by this mining process, essentially the void where the ore is removed from. The process and method of stoping are determined by the orebody characteristics, such as its shape, orientation, and the surrounding rock geotechnical properties, as well as economic factors and safety considerations, and the impact of the process on the environment. The goal of stoping is to maximize ore recovery while minimizing dilution and maintaining stability within the mine (*SME Mining Engineering Handbook*, 1973). The most common mining system used in

Canadian underground hard rock mines is open stope mining. It is characterized by rather single lift stopes with the capacity varying from 20,000 to 100,000 tonnes. The size of the stopes is determined by the states of the host rock mass which must be fair to very good for the method to be applied. Stope are designed to be stable to minimize dilution. Support in the stopes is mostly provided in the back and hanging walls by cable bolting. When applying this method of mining, the greatest concerns are dilution management (King, 2017; Potvin & Hudyma, 2000).

Optimization of underground stoping planning consists of four main aspects: development planning, stope design considerations, production scheduling, and machinery determination and application (Musingwini, 2016). Designing stopes constitutes a crucial component within mine design, and the layout and feasibility of the entire mine are significantly influenced by factors such as stope size, location, orientation, and extraction sequence. Improving the economic efficiency and profitability of underground mining operations requires optimizing stope design through extensive geotechnical data collection. To guarantee safe and economical ore extraction, this requires assessing the physical and mechanical characteristics of the host rock and orebody (Sotoudeh, 2017, as cited in Pontow, 2019).

2.2 Mining systems practices in narrow vein mining

Selecting mining systems and designing mines depend on factors such as optimizing safety, minimizing dilution, maximizing recovery, improving productivity, and reducing costs. Stope design maybe accomplished by using – physical models, analytical approaches, numerical modeling, and or empirical tools guided by field data (Dominy et al., 1998). The strategy becomes more complex with narrow vein mining, where space constraints demand precise extraction methods to ensure valuable minerals are efficiently recovered without significant dilution or safety risks (Dominy & Camm, 1997). Different mining techniques, each with a unique combination of benefits and disadvantages, are available to handle these issues.

Depending on the way the ore is extracted, open stoping is often divided into two main categories: underhand and overhand stoping. Using these techniques, ore is extracted from either above or below the level. Furthermore, combining the two methods into one mining process is possible (Harraz, 2010). The various stoping systems often used are briefly discussed next.

Shrinkage stoping: is a mining system where the void is filled by blasted ore to support the wall and serve as platform for the next stage of drilling and blasting of an ore slice usually two to three meters thick. It is a technique commonly used to extract narrow vein orebodies due to its flexibility to mine out only zones with high grade leaving behind poor areas and constant access to the ore. Generally, a 1 m width of vein is required for safe mining and profitable operations, but 0.8 m thick orebodies are also acceptable. With this mining system, dilution levels can be limited to 10%. The primary weakness of this system is that more than half of the ore cannot be drawn or mucked until all excavation of the stope is completed. This leads to stagnation in production causing losses in profits, and production time. (Dominy et al., 1997).

Longhole (Sub-Level) open stoping: is a mining system used for mining of wide, homogeneous, and steeply dipping ore deposits where the walls of host rock and ore are competent. This technique can also be applied to narrow-veins and is characterized by its non-entry approach, meaning that the physical entry of workers into the stoping areas being blasted is minimized to enhance safety (Dominy et al., 1997). According to Dominy & Camm (1996) the width of stopes used by this method for narrow vein orebodies is 1.5 m. However, Lizotte (1991) stated that 0.5-m thick orebodies can also be mined if appropriate drilling and blasting is used considering good ground conditions. Sublevel open stoping efficiency comes from its ability to allow for simultaneous drilling, blasting, and mucking operations in different parts of the mine, optimizing ore extraction rates and minimizing dilution. The disadvantages of longhole stoping are limited adaptability to the changes in vein characteristics due to the high dependency on good ground conditions, and drilling accuracy challenges in varying ground conditions (Dominy et al., 1997; Lizotte, 1991).

Underhand stoping: alternatively referred to as horizontal-cut underhand or underbreaking stoping, involves the extraction of a vein starting from the top and progressing onwards (Dictionary of Mining, Mineral, and Related Terms, 2003). This method is good from a mechanical perspective because extraction is not against gravity. However it requires high manpower, and the production rate is lower than in other methods (Dominy et al., 1997).

Cut-and-Fill stoping: The cut and fill stoping method is a well-known and extensively used mining technique all over the world to remove ore from tight veins. With this method, the ore is excavated in horizontal levels, and before extracting the next layer, the previous layer is backfilled. This technique, which requires high ore recovery and accuracy, is especially beneficial in situations with weak orebody walls and is suitable for both narrow and wide orebodies. Cut and fill stoping is well known for its high rate of ore recovery, low dilution, and great degree of control over the mining procedure. Nevertheless, the procedure is criticized for its lower production levels and high back cost. The method has several variants including under cut and fill, overhand cut and fill, post pillar cut and fill, and drift and fill (Dominy et al., 1997; Dominy et al., 1998).

Stull stoping: A method used in hardrock mining called "stull stopping" involves placing wooden supports, or "stulls," either randomly or strategically between the vein's floor and roof. The performance of this method is dependent upon the stability and strength of the floor (footwall) and hanging wall, as well as the stulls. In the past, this method of mining was used at depths of 1077 meters, with widths of stopes as wide as 3.7 meters (SME Mining Engineering Handbook, 1973).

Raise stoping: is a method that uses horizontal holes to extract slices from the orebody from dip upwards. Horizontal drilling is carried out from a raise climber in 10-15-meter-long strike panels of vein width, with good recovery rates of 90-95% and minimal dilution of roughly 5% for veins 0.9-1.5 meters wide. This approach is best suited to steeply dipped orebodies (>75 degrees) at 15-meter intervals, allowing for effective mining in narrow vein environments. Stope width is required as wide as 2.5 m, however 1 m is also acceptable (Dominy et al., 1997).

The following narrow-vein mines provide insight examples of narrow vein mining practices and puts into context the significance this research given the challenges that come with operating narrow vein mines.

Gwynfynydd mine operates in North Wales, where gold is located in a 3-6 m quartz vein deposit. The width of the block development is 2.5 m, and sub levels range from 1 to 2.5 m, and the target stoping width is 1.5 m which can vary depending on the geology (Dominy et al., 1998).

Cononish mine is gold mine located in Scotland, where stopes are planned to consist of 10,000 tonnes, with upper limit of lift of 50 m, and strike length of 35-40 m. Stope width is designed as 1.88 m with 2.2x2.7 m breast benches (Dominy et al., 1998).

Curraghinalt mine is quartz vein deposit located in Northern Ireland, where orebody lies dipping between 50 and 80 degrees, with thickness of 0.5-2 m. Stope width is designed as low as 1.25 m, with minimum production rate of 500 tonnes/day (Dominy et al., 1998).

Bendigo Goldfields is a gold deposit located in New Zealand, where quartz veins range between 0.3 m to 2.44 m. Stope widths varied depending on vein thickness and ground conditions. Backfilling was extensively used for support, with empirical approaches based on stope width and depth guiding design (Hutton & Ulrich, 1875).

Kirkland Lake Gold is a gold deposit located in Ontario, Canada. A lower limit for mining excavation is 2.13 m, and the maximum cut is 7.6 m, and a minimum mining height is set as 2.74 m. Cut-and-fill mining is utilized, with stope widths adjusted based on ground conditions and ore grade (Macassa Property, Technical report, 2019).

According to the literature review on existing mining systems used for narrow vein orebodies' extraction the stope width ranges from 0.5 to 3 m depending on the geological and geotechnical conditions. This data may be used in the numerical modeling.

2.3 Factors influencing stope design parameters

This thesis discusses several factors affecting stope design, and according to Feng (2017), stope design is primarily driven by economic and safety considerations, with stability and dilution serving as closely linked factors influencing the design process. Figure 4 illustrates the dependencies between parameter and open stope stability and factors influencing the level of dilution:

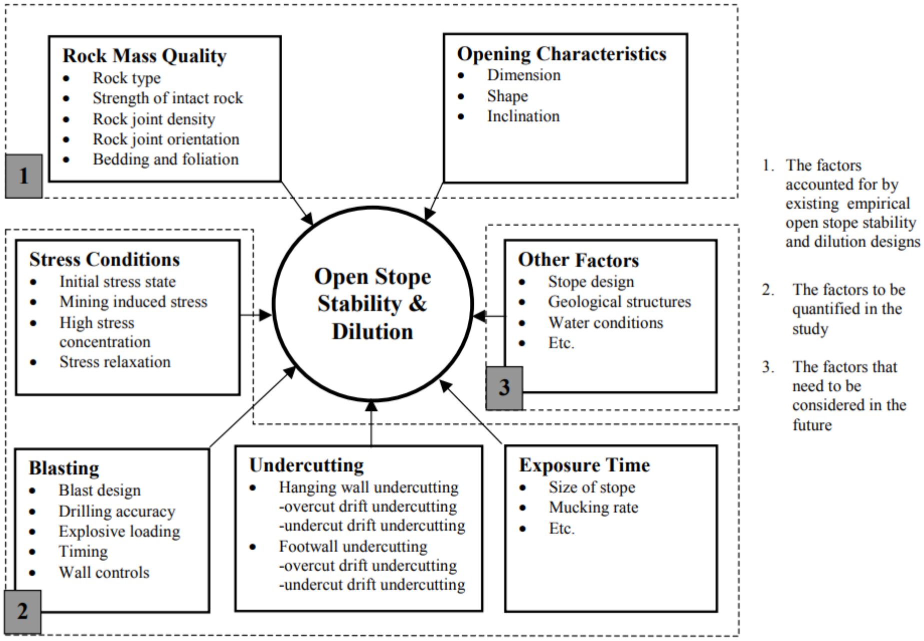


Figure 4. Dependencies between parameters and open stope stability and dilution (Wang, 2004)

2.3.1 Ore grade and orebody size

The stope width, or measurement perpendicular to the vein's dip, is influenced by many factors. The choice of stope width is influenced by the generally thicknesses of narrow vein deposits, which can range from a few centimeters to several meters (0.5 m to 5 m). When combined with complex geology, a greater amount of dimensional and grade inconsistency (nugget effect) frequently makes it difficult to access drilling data and exploit (Dominy et al., 1998).

Since the goal of mining is to maximize profit by mining ore while reducing waste rock, determining the ore grade is essential to stope design. When a mining operation uses the economic block model to identify potential stope locations, a fixed or variable cut-off grade is usually created (Will, 2018). Will (2018) presented a novel step-by-step-based method, where according to the available geological data, the relationship between stope dimensions and optimum cut-off grade is calculated.

Because high-grade veins allow for smaller stopes, limiting dilution with waste rock, ore grade is thought to be the fundamental factor of stope design (Will, 2018). In contrast, wider stopes could be needed for lower-grade resources in order to make them economically viable. The volume of material inside a stope that is acceptable for extraction either remains constant or declines as the cut-off grade increases. This is so because increasing the cut-off grade makes the extraction process more selective and reduces the amount of valuable material. Nonetheless, the stope's physical dimensions—height, minimum, maximum, and width—stay the same regardless of the cut-off grade, therefore, cut-off grade does not affect geometry of the stope, but affects the volume of extracted material, and hence, influences on the cost of mining (King, 2018).

2.3.2 Design of stable stopes

The feasibility, safety, and general success of mining projects are significantly impacted by stope stability, which is a crucial factor in mine design (Wang, 2004). Stope stability is highly dependent on a wide range of parameters, such as rock mass strength, stress conditions, blasting, and mining system.

Stope stability is strongly impacted by depth, with deeper mines undergoing a transition from structurally controlled to stress-induced crushing failure modes. The crucial factor is rockmass strength, upon which stope size and fill stiffness depend. Hence, stope stability difficulties increase with mining depth. This demonstrates how complicated and safety-related deep mining activities become (Wagner, 2019; Wang et al., 2021).

In Suorineni et al. (2011), the significance of the linkages between orebody strike and major far field stress on stope stability is discussed in depth, where it is concluded that inaccuracies in identification of in situ stress direction and complex geometries of orebodies may cause shear loading that is one of the factors influencing rockbursts. In order to evaluate in-situ stress conditions, methods such as numerical modeling, mine openings geometries and, geologic complexity data should be used to validate the in situ stress tensor (Hart, 2003).

The complexities of blasthole stoping in narrow vein mining, as explained by Lizotte (1991) in his thorough analysis of narrow-vein mining, highlight the complex difficulties and restrictions this technique encounters because of the particular geological configurations and operational constraints. In contrast to traditional methods used in larger ore bodies, vertical stress orientations in areas such as the Canadian Shield, where the major principal stress is horizontal, cause substantial wall convergence problems for narrow vein miners. This phenomenon complicates the mining process by increasing the possibility of ore hang-ups and excessive dilution, in addition to endangering the structural integrity of stopes (Lizotte, 1991).

The choice of mining technique also affects stope width, for instance, overhand techniques as in cut-and-fill often use wider stopes than underhand techniques such as underhand stoping (Villescusa, 1998). Artificially supported mining systems, such as cut and fill, rely on passive support from backfill, while shrinkage and VCR stoping system utilize the blasted ore as temporary support for stope walls, which can lead to susceptibility to external dilution and wall damage. Hence, mining operators should pay attention to qualitative and quantitative potentials of mining systems to properly design stopes and manage dilution control (Villaescusa, 1998).

2.3.3 Dilution

According to Pakalnis et al. (1995) “Dilution in mining refers to the contamination of ore with inferior grade ore and/or waste and backfill material”. Dominy et al. (1998) states that there are two primary forms of dilution in mining: "planned" and "unplanned". Planned dilution, which occurs when the orebody is narrower than the minimum mining width, is the process of adding below-cutoff grade material from outside the borders of the orebody intentionally to reserve estimates. On the other hand, unplanned dilution refers to lower-grade material that comes from rock failures, blasting, mixing, or other mining-related events that occur beyond the orebody planned limits of a stope outside the orebody as shown in Figure 5:

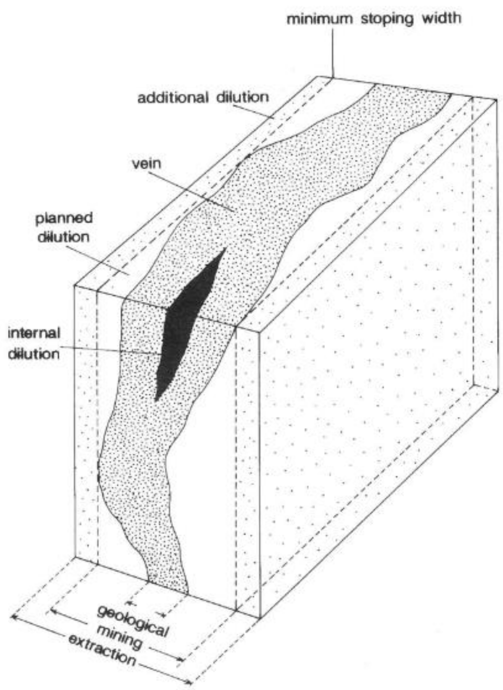


Figure 5. Planned and unplanned dilution illustration (after Dominy et al., 1998)

In Canada, open-stope mining accounts for about half of the ore produced in underground metal mines. This mining system minimizes dilution by leaving limited areas exposed during ore removal. Unmanaged dilution was shown to be a major contributing factor to the closure of several underground mines in a 1988 research (Pakalnis et al., 1995). Additionally, 40% of mines that employed the open-stope approach saw diluting rates that were higher than 20% (Pakalnis et al., 1995). Henning and Mitri (2008) pointed out that nearly 40% of open stoping operations encounter a dilution rate of between 10% and 20% (Jang et al., 2015).

Hefni et al. (2020) reported that the substantial negative economic consequences of unplanned dilution, including the incurred opportunity costs from additional activities such as mucking, haulage, crushing, hoisting, milling, and the processing of waste. Direct costs include those associated with transporting, crushing, lifting, and milling the excess non-ore material as well as the costs of treating and extracting it. Indirect expenses result from equipment damage brought on by ground collapses at open stopes during debris removal (Capes, 2009, as cited in Roux & Stacey, 2017).

According to Mubita (2005), the Konkola Mine in Zambia budgeted $11.30 million in 2002 to address problems brought on by unanticipated dilution. Konkola Mine has a dilution level of 20% or more, and the mine is planning to reduce this value to 10% in the next five years (Ngoma & Mutambo, 2020). Scoble and Moss (1994) reported that in the Thompson mine they studied, minimizing dilution in terms of reducing the spacing of drilling, resulted in an average decrease of 8.5% in dilution, and 6.4% in ore loss, which affected the revenue increase by $98,000. This was five to ten times lower than the cost they would spend without minimizing the dilution.

Certainly, the primary challenges associated with the stoping systems are the unplanned dilution and loss of ore, both of which have a substantial negative impact (Hefni et al., 2020). The most effective method to increase a mine's performance is to minimize ore loss and unplanned dilution.

2.4 Empirical open stope design methods

Empirical design methods for open stopes are broadly categorized into stability graph- design methods and dilution-based design methods. The foundation of these methods lies in the accumulation and analysis of data from various case histories, which inform the development of design criteria and guidelines tailored to specific mining conditions (Wang, 2004).

Present empirical approaches to designing open stopes for narrow vein mining are based on the stability graph technique (Mathews et al., 1981) and the equivalent linear overbreak/slough (ELOS) method (Clark and Pakalnis, 1997). In reality, the Mathews et al. (1981) stability graph is based a database of wide orebodies should not be used for the design of open stopes in narrow-vein orebodies for reasons given earlier. Similarly, because the ELOS stability graph was developed based on a database on narrow-vein mines, it is improper to apply it to the design of stopes in wide orebodies. These strategies are grounded in the understanding of how brittle rock mechanisms lead to overbreak (Suorineni, 2010).

For about three decades, the stability graph method, introduced by Mathews and colleagues in 1981, has been a key tool in designing open stopes. This method saw an increase in its application and reliability when its database grew from 26 case studies across three mines to 175 cases from 34 mines (Suorineni, 2010). The core of the stability graph method is to calculate a dimensionless stability number (N, the original stability number), which assesses the safe dimensions of open stopes by considering factors like in-situ stress, the orientation of key joints, the angle of dip of the orebody and critical joint, and the quality of the rock mass (represented by Q', modified Q – Barton et al. (1974)), and the effect of gravity against a shape factor (S) or hydraulic radius (HR), as detailed in specific equations 1, 2, 3 and illustrated in Figure 6 and

Figure 7. The formula for calculating N' involves Q' and coefficients A, B, and C, which account for the effects of stress, joint orientation, and gravity on stability, respectively, and these factors are identified using the graphs illustrated in Figure 7. Q' itself is calculated by combining the rock quality designation (RQD), joint set number (Jn), joint roughness number (Jr), and joint alteration number (Ja) to quantify the rock mass quality (Suorineni, 2010).

$$N' = Q' * A * B * C \tag{1}$$

$$Q' = \left(\frac{RQD}{J_n}\right)\left(\frac{J_r}{J_a}\right) \tag{2}$$

$$HR = \frac{Area}{Perimeter} \tag{3}$$

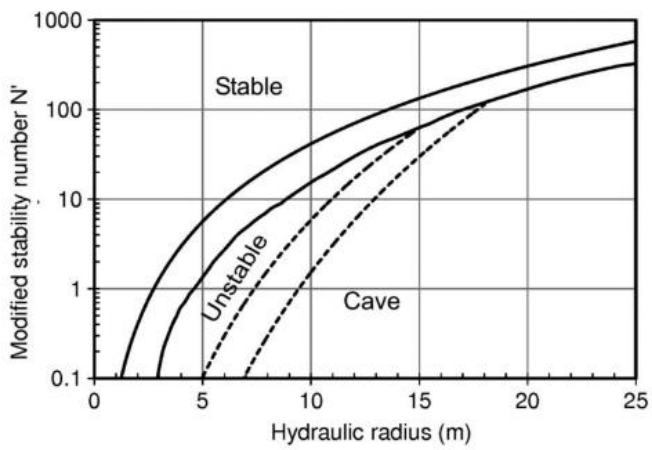


Figure 6. The standard stability graph (Nickson, 1992, as cited in Suorineni, 2010)

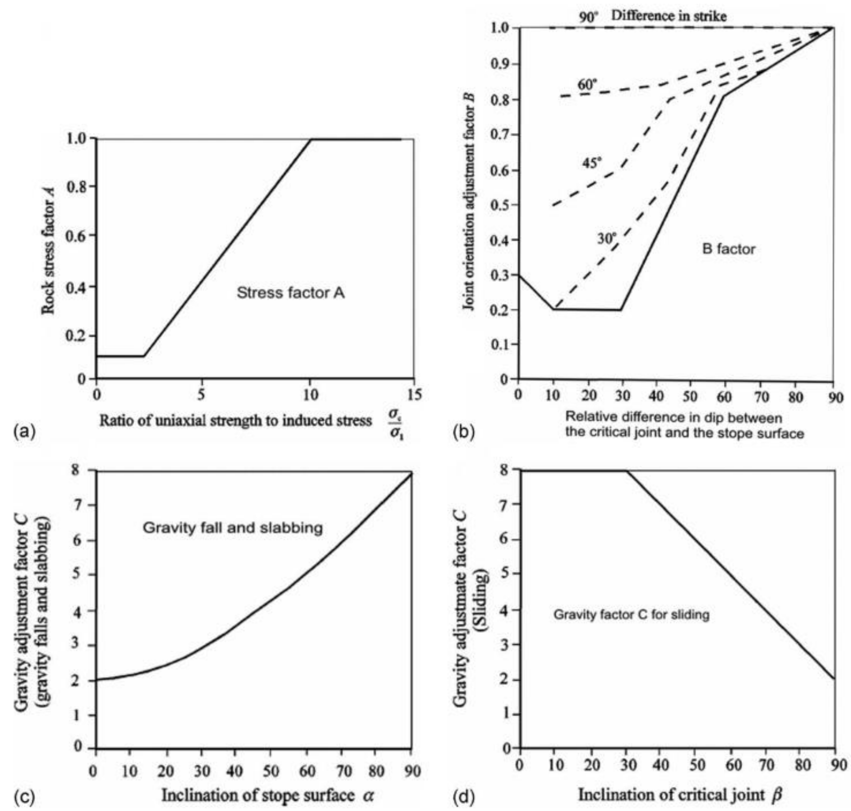


Figure 7. Graphs used in stability graph: a) Stress factor A; b) Joint orientation factor; c) Gravity factor for wedge fall; d) Gravity factor for sliding failure (Nickson, 1992, as cited in Suorineni, 2010)

The stability graph approach uses a graph to classify stope surface stability states into three separate zones: stable, transition (possibly unstable or supportable), and cave (unsupportable). This method is useful for evaluating the stability of stope surfaces. According to Suorineni (2010), this graphical technique enables the case-by-case prediction of the stability requirements of the stope surfaces.

The ELOS stability graph (Clark and Pakalnis, 1997), emerged as a tailored solution for the unique challenges posed by narrow vein ore bodies. Unlike its predecessor, the qualitative stability graph, which primarily focused on massive ore bodies, the ELOS stability graph offers a quantitative approach specifically designed for narrow vein orebodies. The ELOS graph is presented in Figure 8. At its core lies the concept of Equivalent Linear Over-break Slough (ELOS) as presented in equation 4 (Clark and Pakalnis, 1997):

$$ELOS = \frac{Volume\ of\ slough\ from\ stope\ surface}{Stope\ height * Wall\ strike\ length} \tag{4}$$

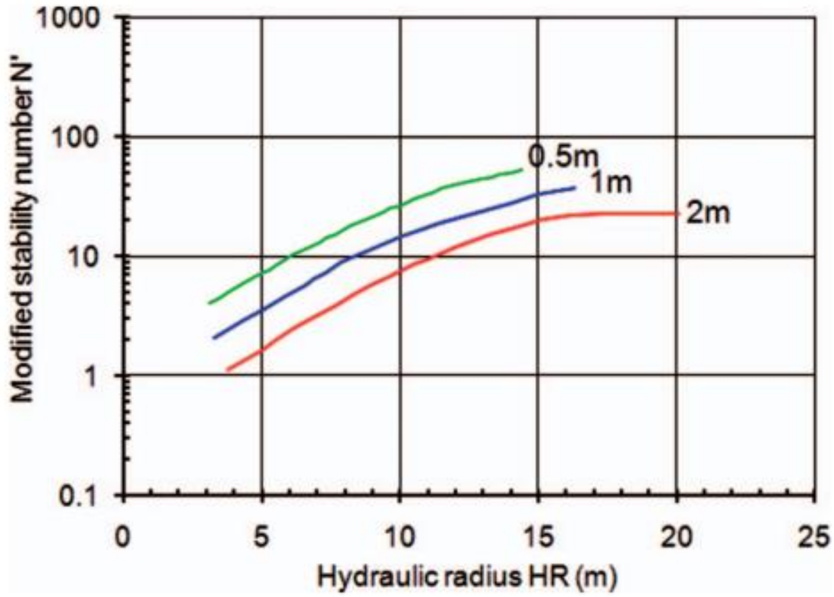


Figure 8. ELOS stability graph (Clark and Pakalnis, 1997)

Papaioanou and Suorineni (2016) developed a dilution-based stability graph, arguing that mining companies prefer to know how much dilution they are encountering to estimate the cost of their operations and profit margins rather than to know qualitative description which is very relative depending what mining operators consider as their acceptable dilution level. The dilution-based stability graph (Figure 9) eliminates this anomaly. Additionally, the dilution-based stability is orebody width independent to eliminate inappropriate use and hence it is referred to as the generalized-dilution-based stability graph (Papaioanou & Suorineni, 2016).

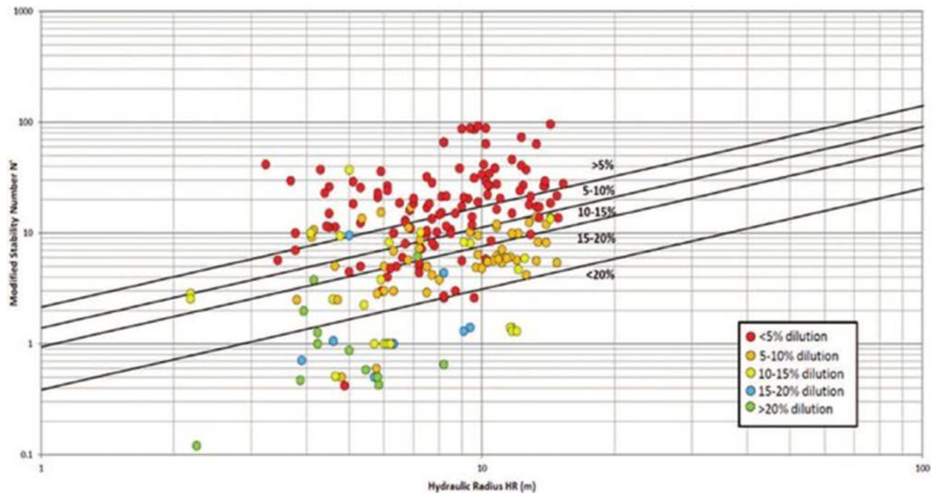


Figure 9. Dilution based stability graph (Papaioanou and Suorineni (2016)

2.4.1 Limitations of empirical design methods

Although helpful for open stope design and indeed in geotechnical engineering, the stability graph approach as an empirical method has drawbacks because of its qualitative zone classifications, such as "stable," "unstable," and "caved," which create uncertainty and is unable to assess dilution directly. Important factors that impact stope stability and dilution are not sufficiently addressed, such as the impact of orebody width on dilution, stress considerations, drilling and blasting procedures, and stope wall undercutting. Moreover, the approach oversimplifies stope geometries, fill surface stability, and fault effects. Its accuracy in forecasting and maintaining mine stability is compromised by its dependence on subjective judgment to define stability zones and its disregard for stope stand-up time, and drilling and blasting accuracy (Suorineni, 2010).

The ELOS method, while offering a more focused attempt at quantifying dilution, also falls short in several areas. It fails to normalize ELOS values by ore body width or to develop stability graphs specific to each ore body class. This lack of differentiation can lead to misleading interpretations of dilution levels, particularly in comparison between wide and narrow ore bodies. For instance, as in Figure 8 an ELOS of 1 meter in a 2-meter-wide ore body signifies a drastically different dilution level compared to the same ELOS in a 10-meter-wide ore body (Suorineni, 2012). Normalizing ELOS by orebody width or developing separate stability graphs tailored to each ore body class is suggested to address this issue effectively (Suorineni, 2010).

Moreover, the method does not adequately address the stability of backfilled stope surfaces, an essential factor in dilution management. The stability graph's omission of this factor suggests the necessity for supplementary methodologies, such as the Mitchell theory, to comprehensively assess backfill stability (Papaioanou & Suorineni, 2016).

Empirical methods neglect factors such as blasting quality and the presence of backfill abutments further limits its applicability in scenarios where backfill stability is crucial for dilution control (Suorineni, 2010). Furthermore, it is difficult to apply empirical approaches—more especially, the stability graph methodology designed for wide orebodies—to narrow vein situations. These techniques frequently fall short of taking into consideration the high confinement and increased vulnerability to blast-induced damages that are typical in narrow vein mining (Suorineni, 2012). The discussion in Suorineni (2010) highlights the critical need for a paradigm change in the direction of more specialized, context-sensitive empirical models that are able to accurately forecast and lessen the unique geotechnical risks associated with narrow vein ore bodies. This change is essential for increasing safety, reducing dilution, and maximizing narrow vein mining operations' overall efficiency.

3 Methodology

Saiang (2023) investigated the stability of narrow vein mine stopes in operational mines where orebodies have widths ranging from 0.7 m to 1.5 m and dip angles between 70º to 85º. The author proposed that high walls of underground mine stopes could behave similarly to open pit slopes where geological structures primarily govern stability. The author noted that in such cases, kinematic analyses can be used to evaluate the stability of various parts of narrow vein open stopes, including the footwall, hanging wall, roof, and floor. As hypothesized in this thesis, and supported by the authors of the Mathews et al (1981) stability graph together with Hoek et al. (1995 – Figure 2) at the depth failure of narrow vein stopes changes from brittle stress-induced and structurally controlled to excessive displacement of the hanging wall and footwall resulting stope closure. Therefore, the methodology employed in this thesis for conducting numerical modeling involves several steps, each aimed at accurately representing the complex interactions of the various factors in narrow vein mining. This thesis will utilize RocScience software RS2 for 2D modeling, which is based on the Finite Element Method (FEM), a highly effective numerical modelling code with extensive applications in geotechnical engineering, to simulate the behavior of open stopes in narrow-vein orebodies at depth.

3.1 RocScience RS2 application

Numerical modeling is an important tool in geomechanics, with which the engineer and scientific researcher can undertake the modeling of complex geo-processes and their interactions, hardly possible to be studied by sole direct observation, physical laboratory experimentations (Cundall and Hart, 1992) and closed form analytical solutions.

Numerical modeling has a wide range of applications in geomechanics, from the study of rock mechanics and fluid flow in porous media to the assessment of the stability of slope, tunneling, foundation construction to the understanding of the complex mine structure. Because of its adaptability and ability to work with intricate geometries and boundary conditions, it is a vital tool for researchers and geotechnical engineers who want to better understand the response of geomechanical systems and increase the sustainability and safety of geological engineering techniques in mining operations (Hammah et al., 2007).

A numerical modelling method called the Finite Element Method (FEM) is used to model and examine complex physical processes. In this method, a continuous system is divided into a set of smaller, discrete components, referred to as finite elements that are connected at nodes. Differential equations are used to characterize the behavior of each element, and the total solution for the system is obtained by adding the solutions of these individual elements. The behavior of the system can be approximated more accurately by decreasing the size of the parts. After the procedure, a set of algebraic equations that can be solved using techniques like Gaussian elimination are produced. This technique has been greatly expedited by modern FEM software, which optimizes the arrangement of these equations to improve usability and solution speed (Singh et al., 2010).

RS2 (Rocscience, 2024) is a software developed by Rocscience company limited, Toronto, Canada, and is a powerful and sophisticated two-dimensional Finite Element Program designed for structural analysis and meant primarily for use in geotechnical applications, mostly in the field of analysis of excavation, slope, tunnel design and mine excavations stability analysis. RS2 uses FEM, for the modeling of mechanical behavior of soils and rocks. The code allows for observing the distribution of stresses, deformation, and failure within geotechnical structures (Rocscience, n.d.).

RS2 is used to model open stopes in narrow vein orebodies. The selection of RS2 is based on the fact that it is a designed software which has remained the most reliable tool up to now, preserving an apparently unattainable level for accuracy in geotechnical modeling (Rocscience, n.d.). It is simple fast and easy to use, and the results are easy to understand. The software is also available in the school with application support by the vendor. More importantly, study hypothesizes that because narrow vein widths are limited getting sufficient tonnage to meet production targets in narrow vein stopes necessitates that stope lengths by largely increased compared to their width and satisfy the criteria for a 2D rather than a 3D analysis. Vlachopoulos and Diederichs (2014) state that in spite of the gradual development of three-dimensional analysis packages utilizing finite element models or finite difference algorithms for stress–strain calculations, two-dimensional (2D) analysis is still used as the primary engineering tool for practical analysis of tunnel behavior and tunnel support performance for design—particularly at the preliminary stage of a project. These authors continue say that the applicability of 2D finite element analysis or analytical convergence confinement solutions to staged support installation depend on the application of an assumed or validated longitudinal displacement profile. Convergence in commonly applied 2D staged models is controlled by boundary displacement or internal pressure relaxation.

3.1 Modelling logic

The expected outcome of this study is going to be the behavior of open stopes in narrow vein mines at depth. Through numerical modeling and analysis, the thesis anticipates gaining a deeper understanding of the mechanisms governing stope failure and deformation at depth and to answer the question: Are current empirical design approaches applicable to the design of open stopes in narrow-vein orebodies at depth? Specifically, the research expects to observe significant wall displacement and closure within the simulated stopes, consistent with the hypothesis mentioned above.

Furthermore, the results will confirm whether the research hypothesis is valid, and identify the dependencies between geomechanical properties, effective stress ratio, stope geometry, and stope wall convergence. Hypothetical scheme of the wall convergence is shown in the Figure 10:

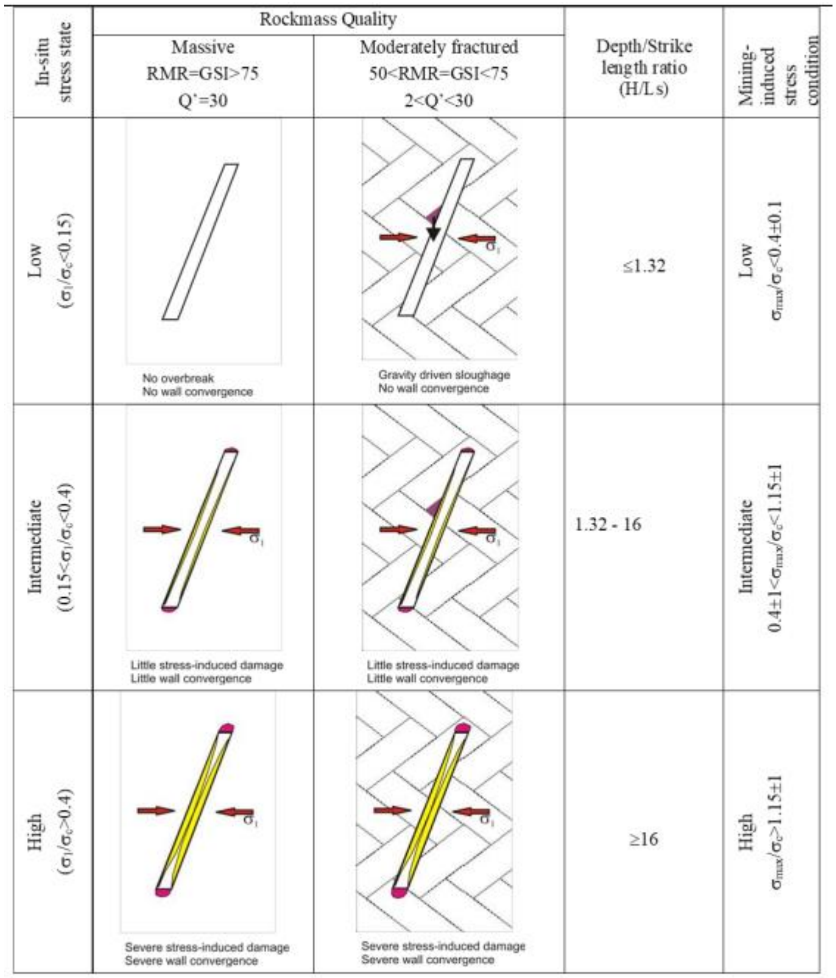


Figure 10. Hypothetical schematic of conditions of narrow vein stope wall convergence (Suorineni, 2024; modified from Hoek et al., 1995)

3.2 Model setup and input parameters

The input data required for numerical modeling will include geological information, hanging wall and footwall properties, stope geometries, and in situ stress states. This information will be collected from various sources, existing narrow vein sites reports, and the literature. Key parameters such as rock strength, rockmass quality, and stress magnitudes will be input into our numerical models to investigate represent the behavior of the open stopes in narrow-veins depending selected spans.

In summary, the input data for our study will encompass a wide range of geological, geotechnical, and observational information, which will be integrated into our numerical models to facilitate a comprehensive analysis of narrow vein open stopes behavior at depth.

By following this approach and methodology, the thesis aims to advance our understanding of stope failure mechanisms in narrow vein mines and contribute to the development of safer and more efficient mining practices. Through rigorous numerical modeling and analysis, we seek to validate our hypothesis and prove the hypothesis of inelastic failure of stope walls. The thesis will focus on the following key parameters for numerical modeling:

- Geomechanical Properties: Modelling will focus on the uniaxial compressive strength (UCS), tensile strength, cohesion, Poisson's ratio, Elastic modulus, and density of both the orebody and host rock. These parameters are crucial for assessing rock mass behavior under stress. The mechanical properties of the orebody and surrounding rock mass significantly impact stope stability and design. Key properties include:
    - **Elastic modulus:** The elastic modulus, also called the modulus of elasticity, is a measurement of how resistant a material or item is to elastic deformation—that is, how quickly it returns to its original shape once an applied force is removed. Its definition is the elastic deformation region's stress-strain curve's slope. A stiffer material is indicated by a greater elastic modulus. The modulus of elasticity (Ei) of rock material is an important feature utilized in the design stage of engineering projects like building dams and tunnels, excavating mines, etc. (Ocak, 2008)
    - **Poisson's ratio:** The Poisson effect, which quantifies a material's propensity to expand perpendicular to the direction of compression and contract transverse to the direction of stretching, is measured by Poisson's ratio. Reflects the rock's tendency to

deform laterally when compressed axially. Understanding this ratio is crucial for accurate stress analysis and predicting potential deformations (Lakes & Wojciechowski, 2008).

- **Cohesion and internal friction angle:** Together define the rock's shear strength. Cohesion represents the initial resistance to shear, while the friction angle defines the angle at which failure occurs. These properties are critical for assessing stope wall stability and designing support systems (Villeneuve et al., 2021).
- **UCS**: The capacity of a material to sustain axial compressive stresses without failing is measured by its uniaxial compressive strength. UCS is an essential input parameter in geomechanics numerical modeling that helps forecast how rock or soil formations will react to different loading scenarios that are experienced in engineering projects. Engineers may make well-informed judgments on the design, stability, and risk mitigation techniques for geotechnical structures and excavations by including UCS values into their models (Gholami & Fakhari, 2017).
- **Unit weight (**γ): Unit weight, which is commonly expressed in kilograms per cubic meter (kg/m3), is the weight of a material per unit volume. It is a measure of the gravitational force applied to a certain volume of material and is affected by moisture content, porosity, and mineral composition. In many geotechnical and geological applications, such as slope stability studies, groundwater flow estimates, and foundation and retaining structure design, unit weight is important (Das & Sobhan, 2016; Coduto et al., 2011).
- **Tensile strength:** Tensile strength in geomechanics is the highest stress a material can bear before breaking or fracturing under tensile (pulling) forces. It gauges a

material's ability to withstand tearing or stretching. When it comes to understanding the behavior of geological materials like rocks and soils, tensile strength is a crucial factor. This is especially true in scenarios where tensional stresses are present, such in faulting, jointing, and fracturing (Fossen, 2016).

- **Stope Dimensions:** Length, width, and height of the stope, considering that narrow vein stopes have limited width.
- **In Situ Stress Conditions:** Direction and magnitude of the principal stresses, with particular attention to effective stress ratio K which is defined as the ratio of horizontal to vertical stress.
- **Failure Criteria:** The Mohr-Coulomb or Hoek-Brown failure criteria can be applied based on the available data.

The model will be created in RS2 based on representative stope geometries identified in the literature and the definition of narrow vein orebodies. Following the guidelines in RS2 modelling, external boundaries, orebody geometries, and stope dimensions were constructed. This helps in establishing a realistic baseline for simulations. The model set up is shown Figure 11. The following different scenarios will be examined:

- Simulating Different Stress Conditions: Simulation of various stress conditions, including varying the direction of the major principal stress relative to the orebody strike will be conducted in the numerical modeling. This can help in understanding the impact of stress on stope walls convergence.

- Analyzing Stope Stability: Utilize the model to assess wall convergence under different conditions, focusing on the impact of stope width and different stress conditions. This involves understanding how the exposed hanging wall and footwalls react to the stress conditions, which is central to the hypothesis of the thesis.

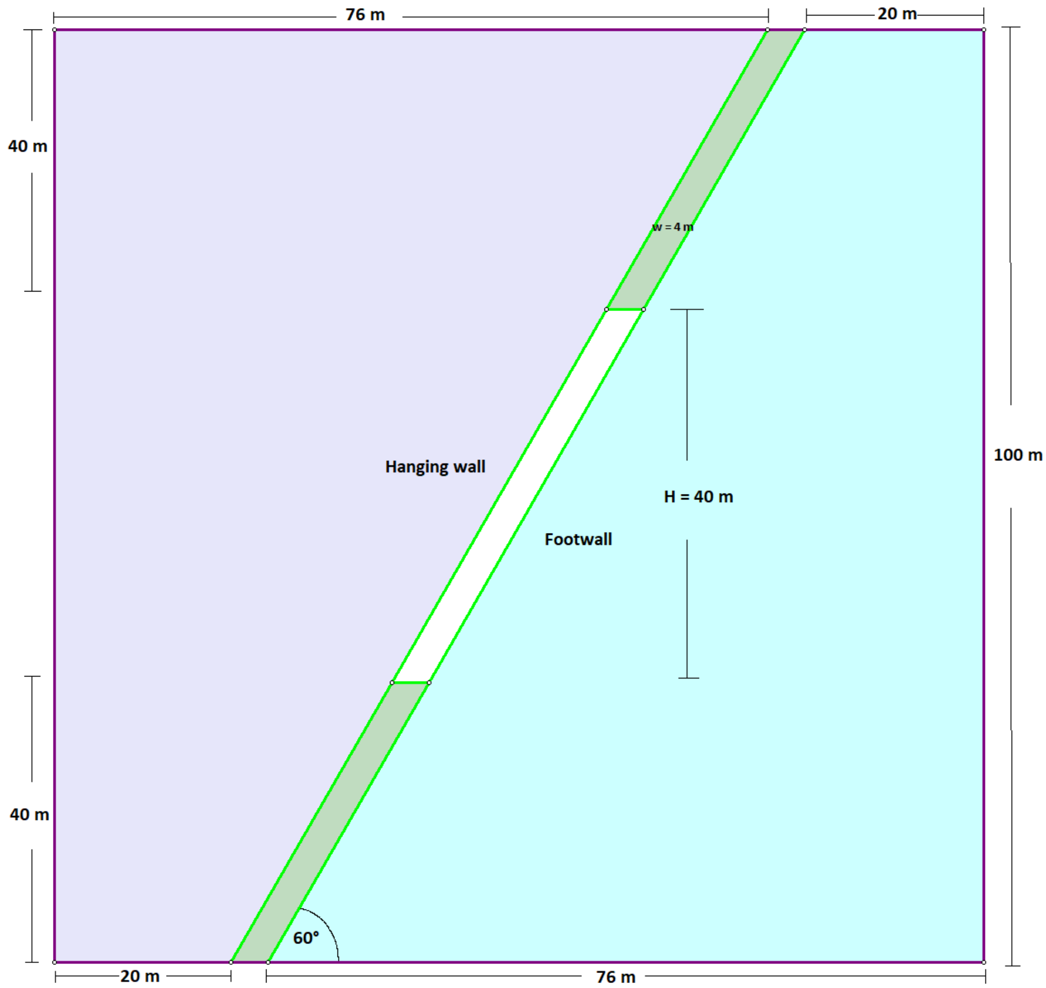


Figure 11. Model in RS2 (Cross section view)

As can be seen from the Figure 11, the orebody is dipping at 60 °, the stope width is 4 m, and the stope height is 40 m. The mechanical properties for hanging wall, footwall, and orebody are set according to Table 1:

Table 1 Mohr Coulomb input parameters (Abdellah et al., 2019)

Parameter	Orebody	Hanging wall	Footwall
Poisson's Ratio	0.26	0.25	0.18
Young's Modulus (GPa)	20	25	40
Unit weight (MN/m3)	0.045	0.028	0.030
UCS (MPa)	90	90	172
Tensile Strength (MPa)	0.31	0.11	1.52
Cohesion (MPa)	10.2	4.8	14.13
Friction angle (°)	43	38	42
Porosity Value	0.04	0.05	0.03

Mine-A taken as a typical narrow vein mine that is situated in the Abitibi region of Quebec within a greenstone belt. It contains approximately 1.1 million ounces of gold in its mineral deposits. The orebody is steeply dipping and consists of pyritic polymetallic veins (containing both gold and copper) with an average thickness of 5 meters. These veins are positioned between a hanging wall made of mafic tuff and a footwall of rhyolite tuff. The orebody is located in a contact zone that ranges from 500 to 1500 meters below the ground. The mining system used is open stoping with stope heights ranging from 20m to 40 meters (Abdellah et al., 2019). This site was chosen as a reference because the orebody is a narrow vein and the rock properties and site conditions are well known in the literature. The model setup for this mine for stope height of 40 m is given in Figure 12.

After material properties were defined, the next step was to create a mesh, which was set to 6 noded triangles with mesh density of 200 nodes on external.

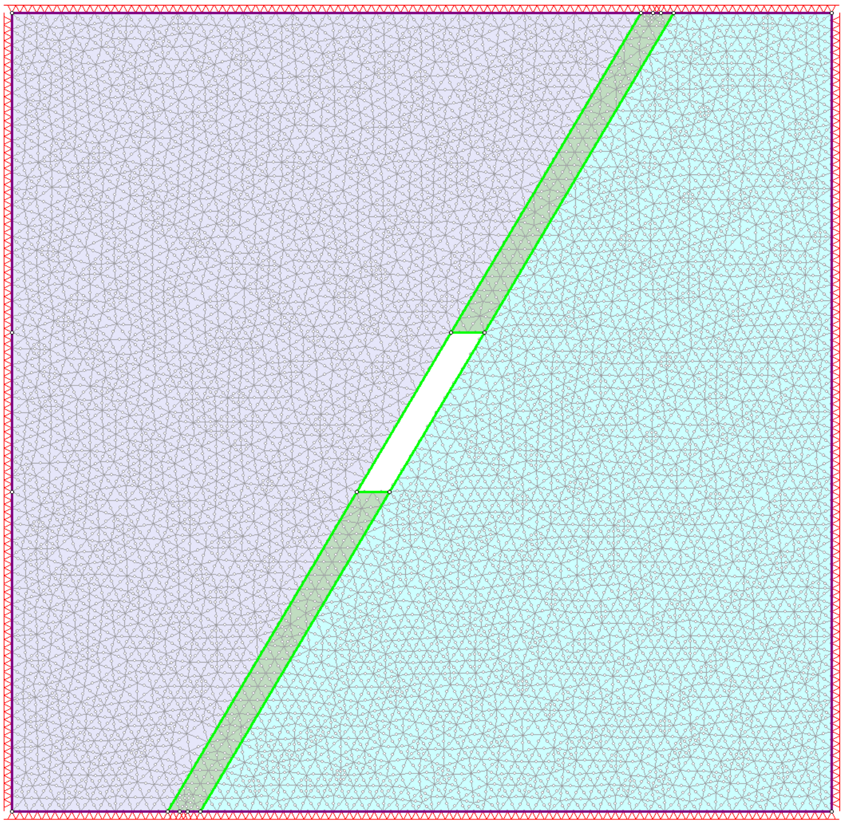


Figure 12. Mesh size in RS2 model

The following different scenarios were identified according to hypothetical scheme:

Case 1: depth = 65 m, $\frac{\sigma_1}{\sigma_c} < 0.15$;

Case 2: depth = 750 m, $0.15 < \frac{\sigma_1}{\sigma_c} < 0.4$;

Case 3: depth = 1500 m, $\frac{\sigma_1}{\sigma_c} > 0.4$

The properties were chosen as presented in Table 1, and effective stress ratio (K) was set to 1.5.

The same model was used to determine the closure of walls under different stress conditions: K=1; K=1.5; K=2; K=2.5.

Then, the width of the stope was varied to analyze how displacement of the hanging wall and footwall differ with the width of opening and stress conditions.

Some analysis used the stope plan view rather than sectional view in order to see how σ_H and σ_h are correlated, and how they affect the deformation of wall closure. The model set up for this case is provided in Figure 13:

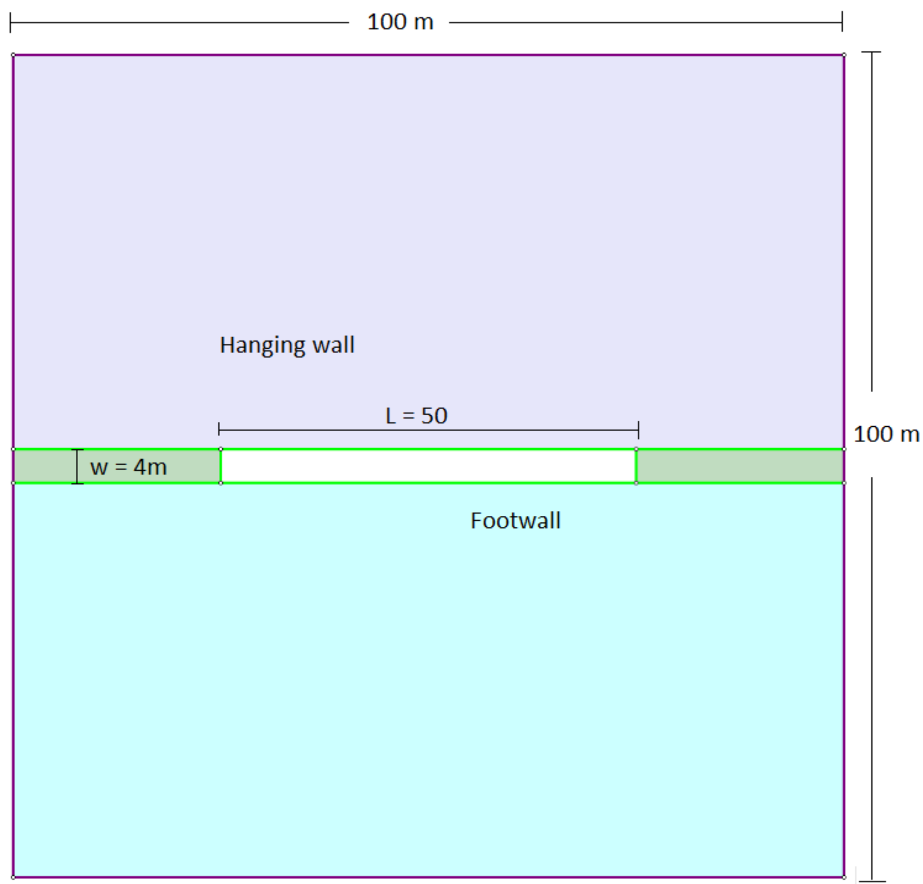


Figure 13. Model in RS2 (plan view)

4 Results and discussion

4.1 Hypothesis check

Case 1: depth = 65 m, $\frac{\sigma_1}{\sigma_c} = 0.14$:

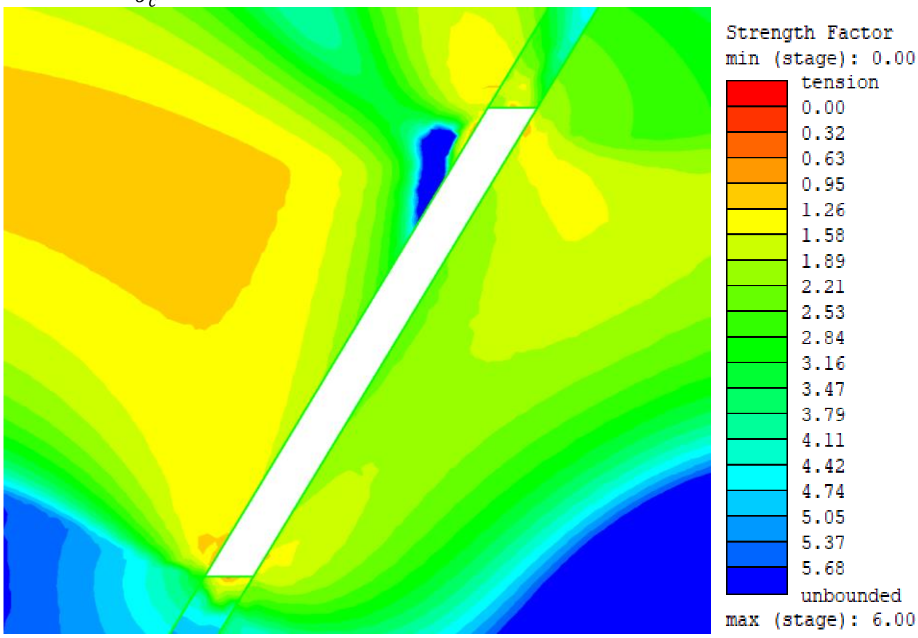


Figure 14. Strength factor (case 1)

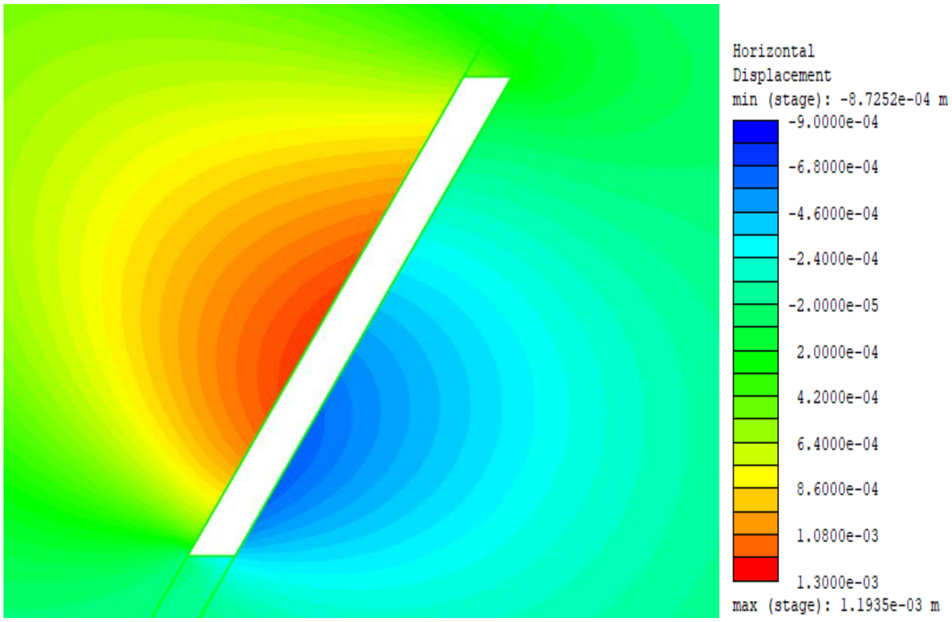


Figure 15. Horizontal displacement (Case 1)

Case 2: depth = 750 m, $\frac{\sigma_1}{\sigma_c} = 0.4$:

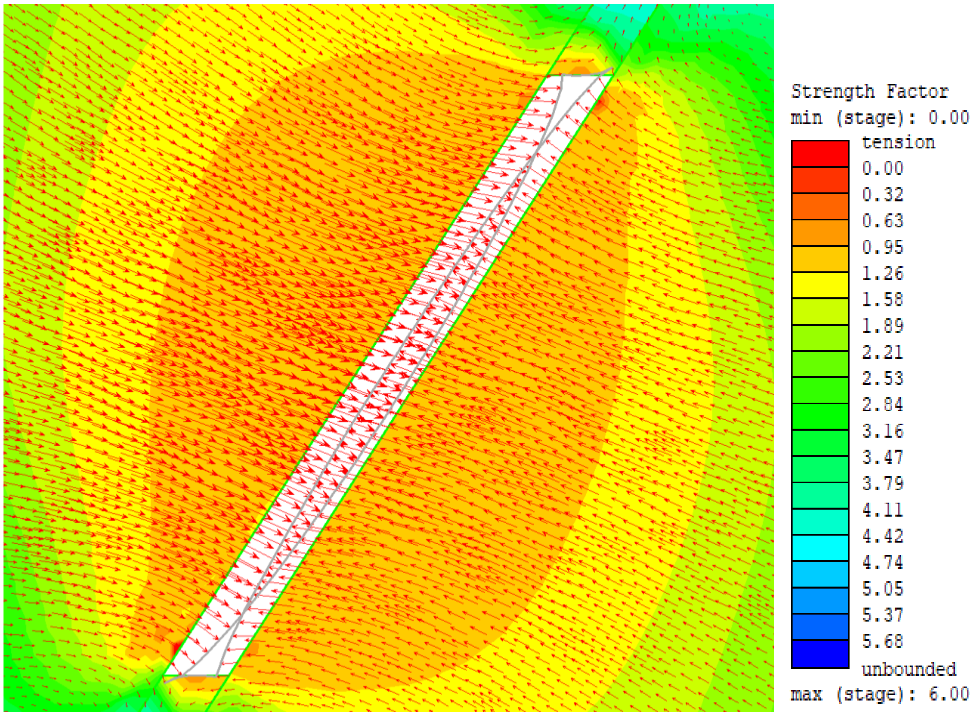


Figure 16. Strength factor (Case 2)

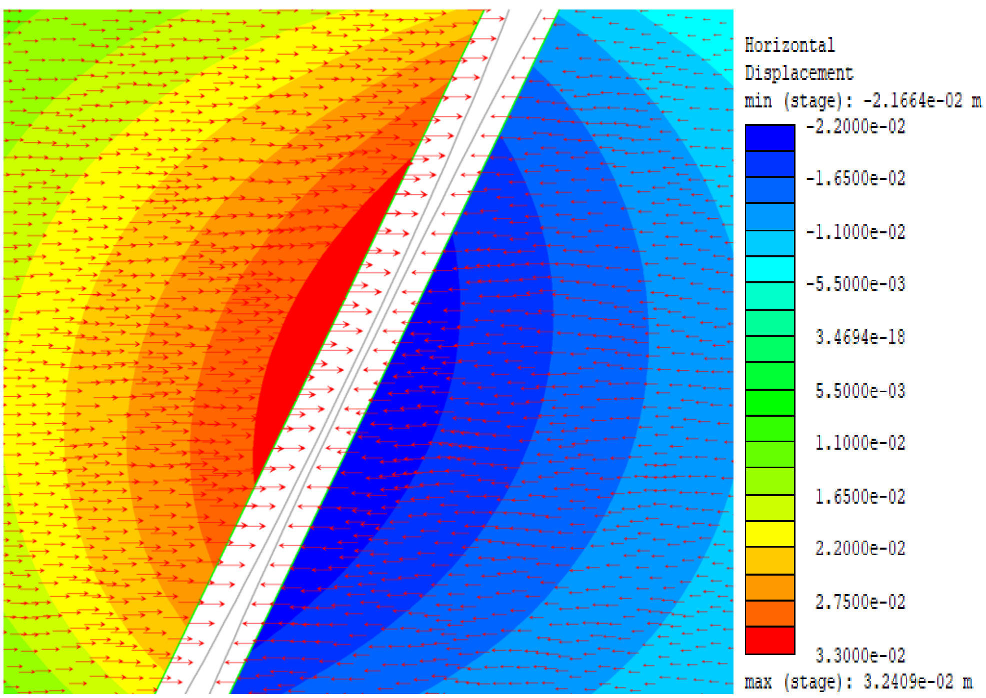


Figure 17. Horizontal displacement (Case 2)

Case 3: depth = 1500 m, $\frac{\sigma_1}{\sigma_c} = 0.6$:

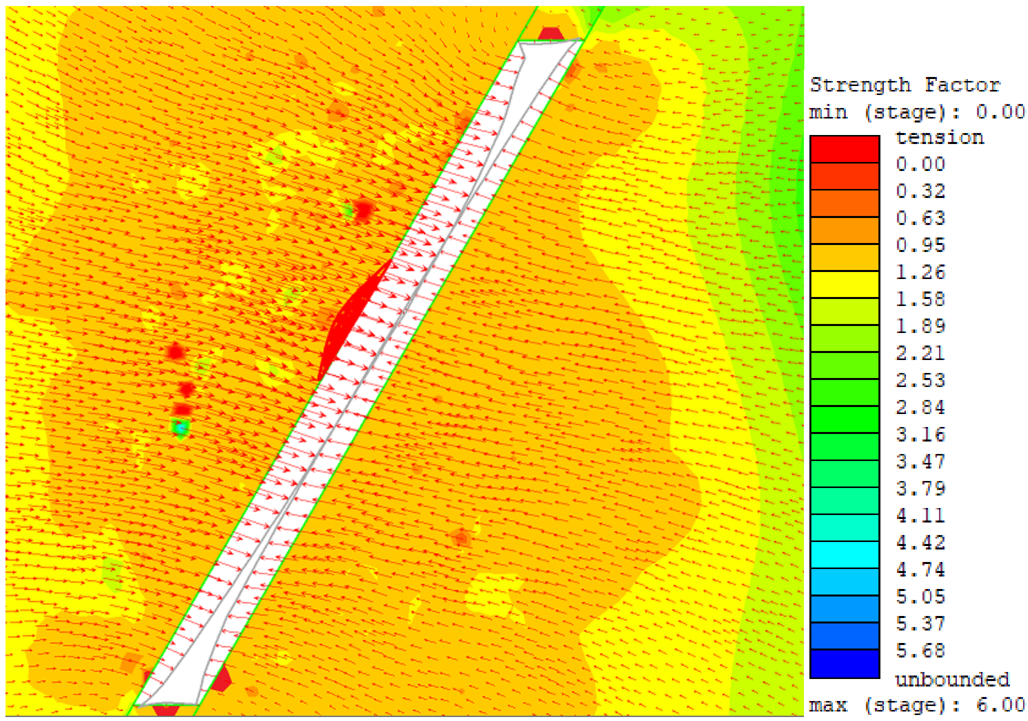


Figure 18. Strength factor (Case 3)

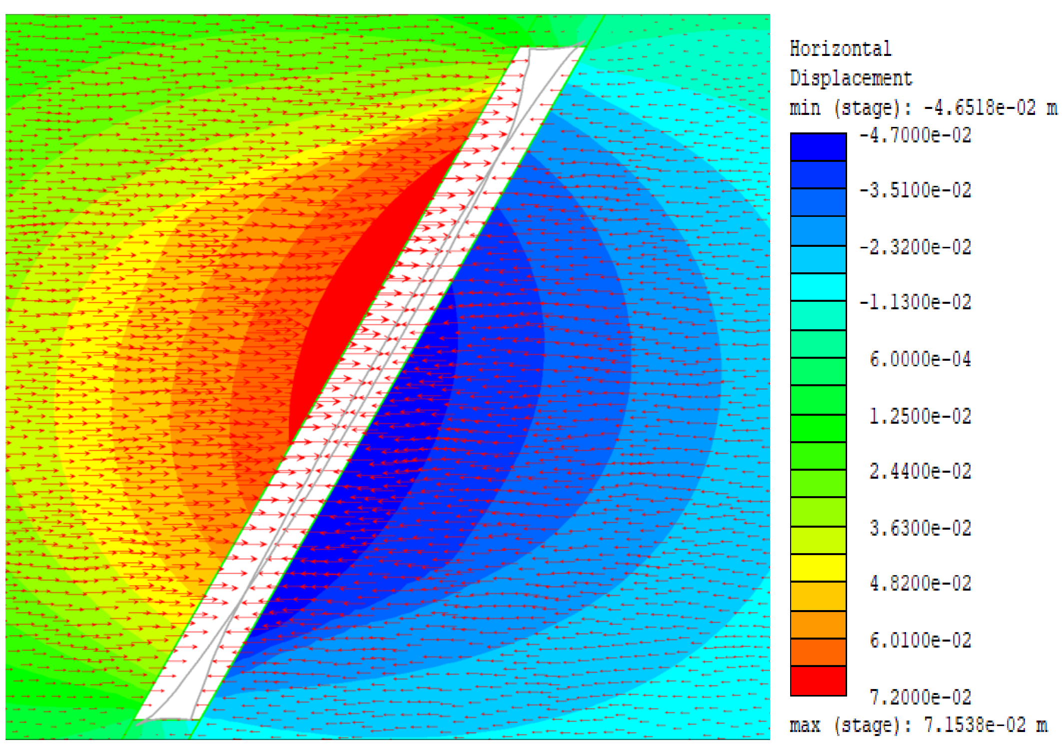


Figure 19. Horizontal displacement (Case 3)

As can be seen from the above figures and summary in Table 2, as depth of mining increases, and in-situ stress state increases, more damage, deformation can be observed. In particular wall closure becomes the dominant mode of failure as depth increases as expected.

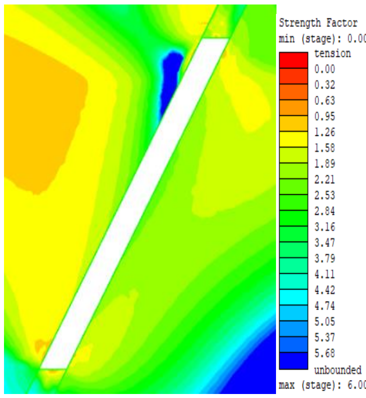
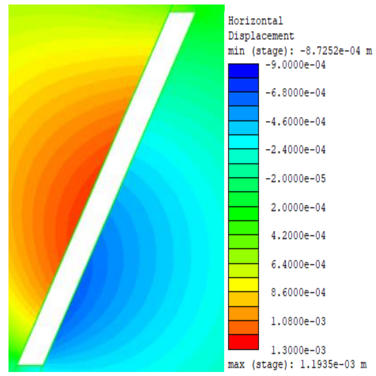
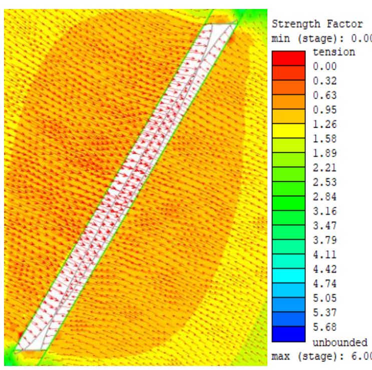
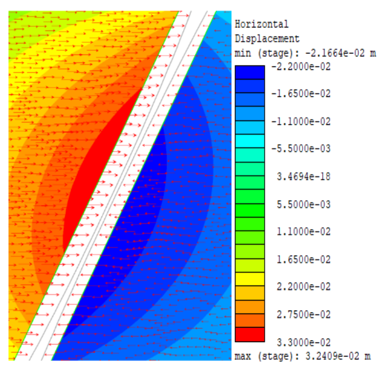
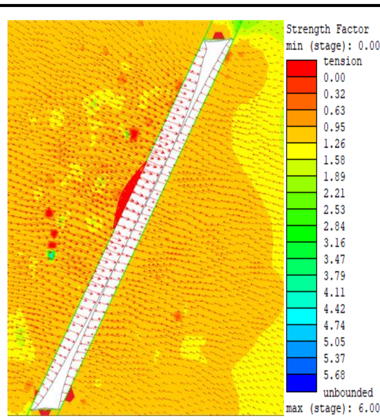
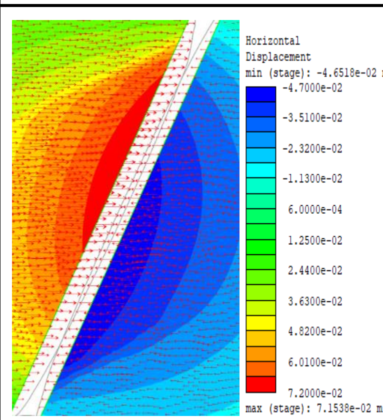
In case 1, depth is shallow, and the ratio of principal stress to UCS of the rock is less than 0.15, which means that the rock's strength is high, we do not observe wall convergence (horizontal displacement observed is 0.001 m in hanging wall, and 0.0008 m in footwall) as shown in Figure 14, and Figure 15 illustrates that around the stope the strength factor is higher than 1, meaning stable rock conditions.

In Case 2, depth is 750 m, and the UCS of rock is higher than in Case 1, but the ratio of the maximum tangential boundary stress to the laboratory unconfined compressive strength is between 0.15 and 0.4, meaning that major principal stress is higher and affects the stope in this case. Here, little stress induced damage (Figure 16 shows strength factor around the stope is between 0.32 and 0.63, and we can observe damage in hanging wall and footwall), and little convergence (Figure 17 shows horizontal displacement: in hanging wall 0.032 m, in footwall - 0.022 m).

Case 3 analyzes the rock under 1500 m, where the ratio of principal stress to UCS is more than 0.4, meaning high stress concentration. Figure 18 shows severe stress induced damages around excavation in both hanging and footwall. Horizontal displacement in the hanging wall is 7.15 cm, in the footwall - 4.65 cm, meaning that walls are converging, and the deformation of walls was significant.

In summary, the results obtained from RS2 simulations and summarized in Table 2 show that stope wall closure becomes a more important consideration than brittle stress-induced and structurally controlled damage in narrow-vein open stope design at depths over 1000 m.

Table 2. Summary of proof of concept

In-situ stress state	Depth /strike length ratio	Strength factor	Wall displacement	Wall closure (cm)
$\frac{\sigma_1}{\sigma_c} = 0.15$	1.3			0.21 No overbreak No convergence
$\frac{\sigma_1}{\sigma_c} = 0.4$	15			5.41 Little stress induced damage Little wall closure
$\frac{\sigma_1}{\sigma_c} = 0.6$	30			11.81 Severe stress induced damage Significant wall convergence

4.2 Stope walls behavior under different stress conditions

The convergence of stope walls and its stability under different stress conditions was analyzed. In RS2 the effective stress ratio: horizontal/vertical in plane, was varied between 1 and 2.5: K=1; K=1.5; K=2; K=2.5, and depth of excavation was varied as: D=1000 m; D=1250 m; D=1500 m; D=1750 m, respectively with K values.

Scenario 1: K=1, Depth=1000 m:

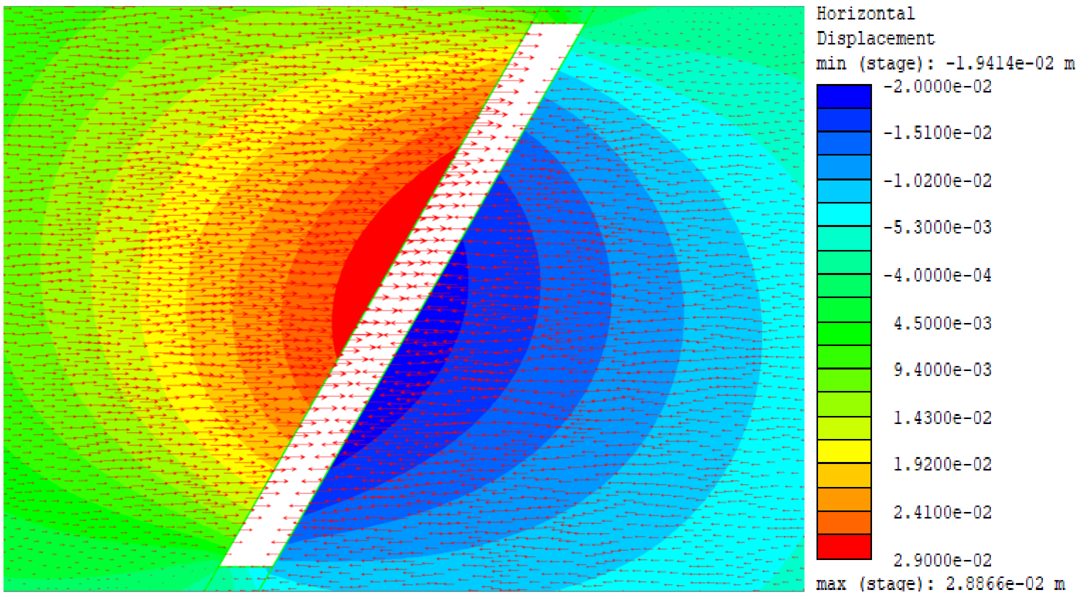


Figure 20. Horizontal displacement of walls (K=1, D=1000m)

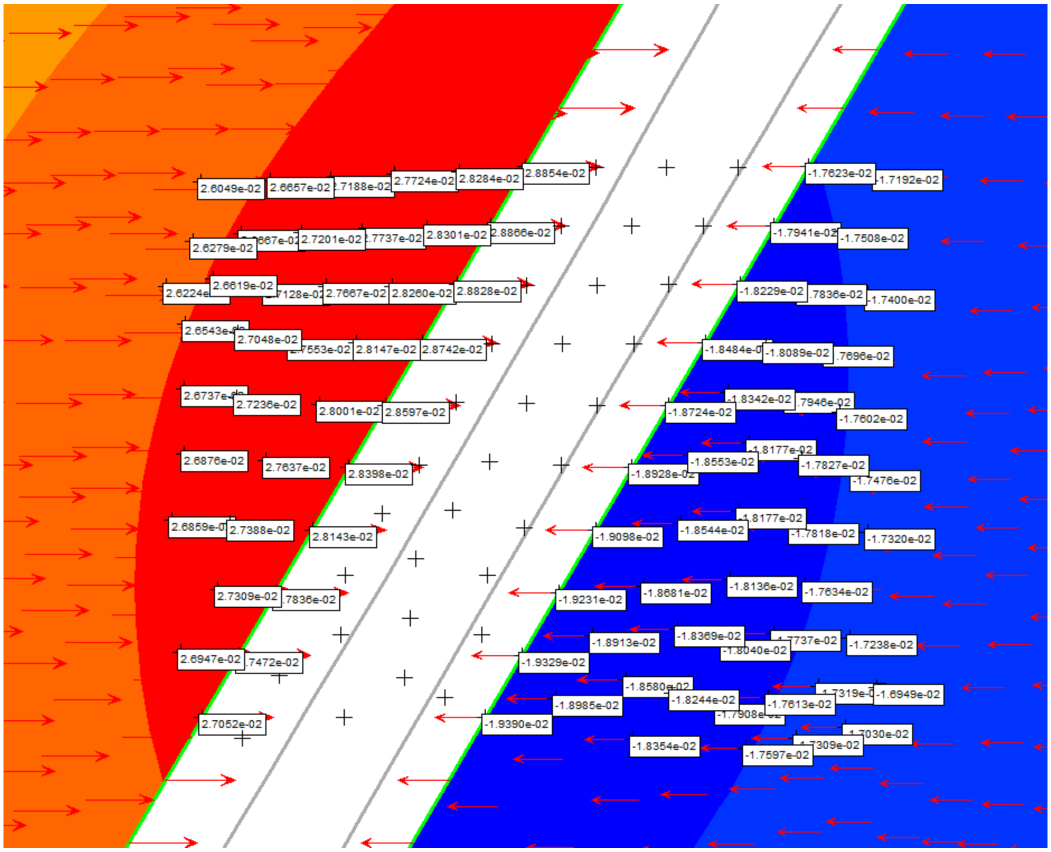


Figure 21. Nodes displacement (K=1, D=1000m)

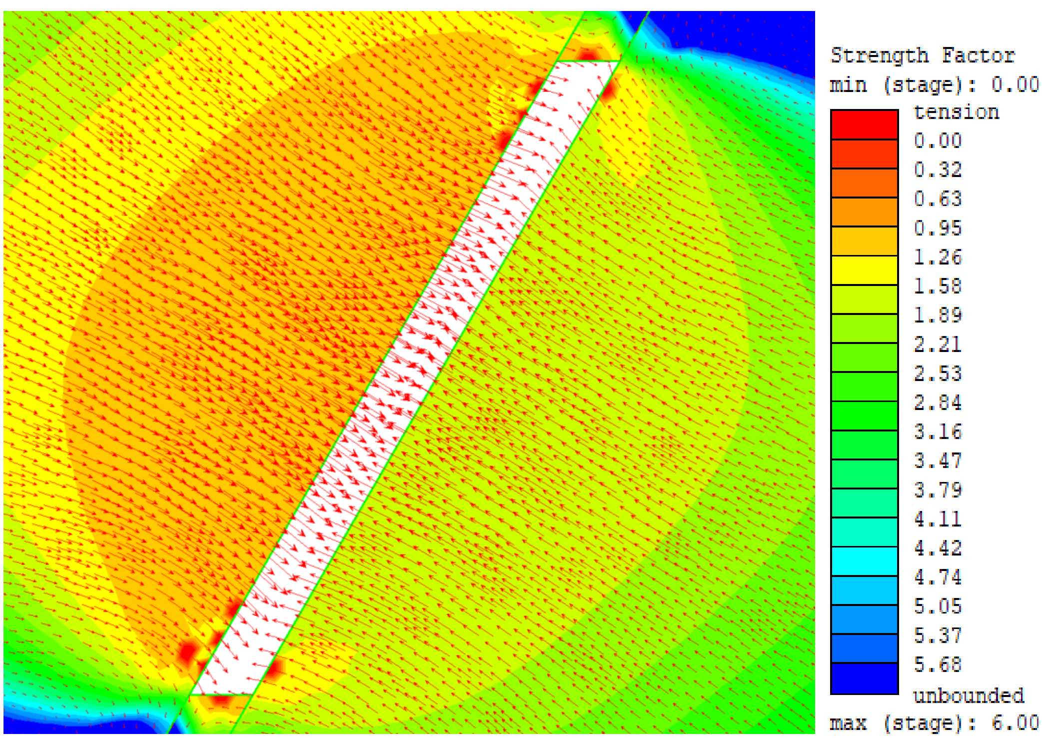


Figure 22. Strength factor (K=1, D=1000m)

Scenario 2: K=1.5, Depth = 1250 m:

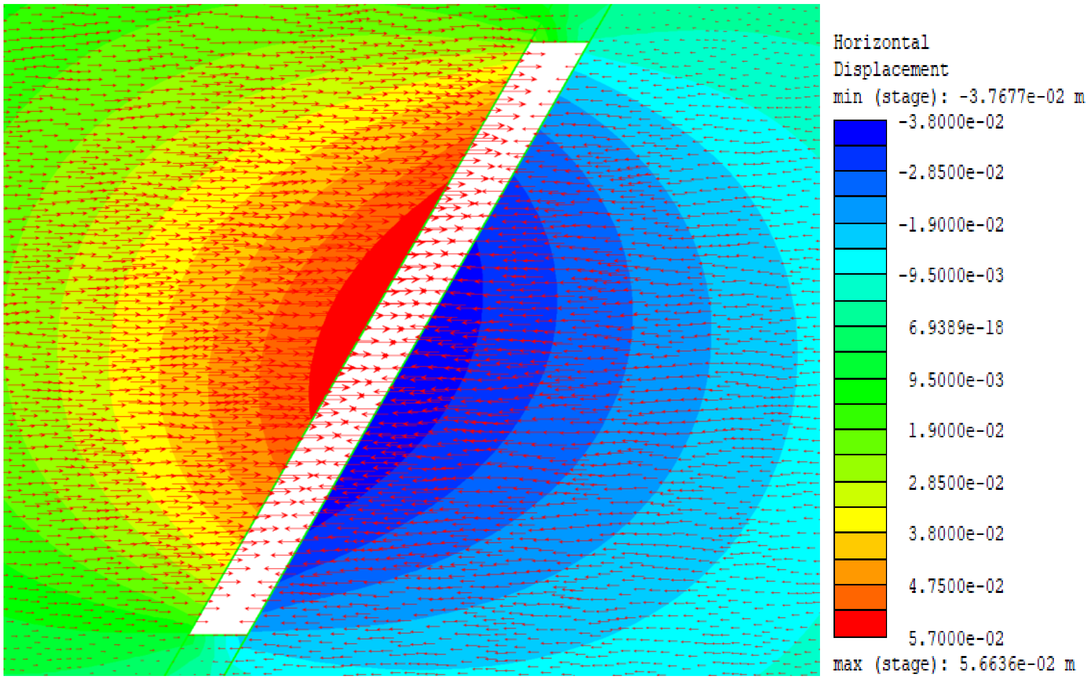


Figure 23. Horizontal displacement of walls (K=1.5, D=1250m)

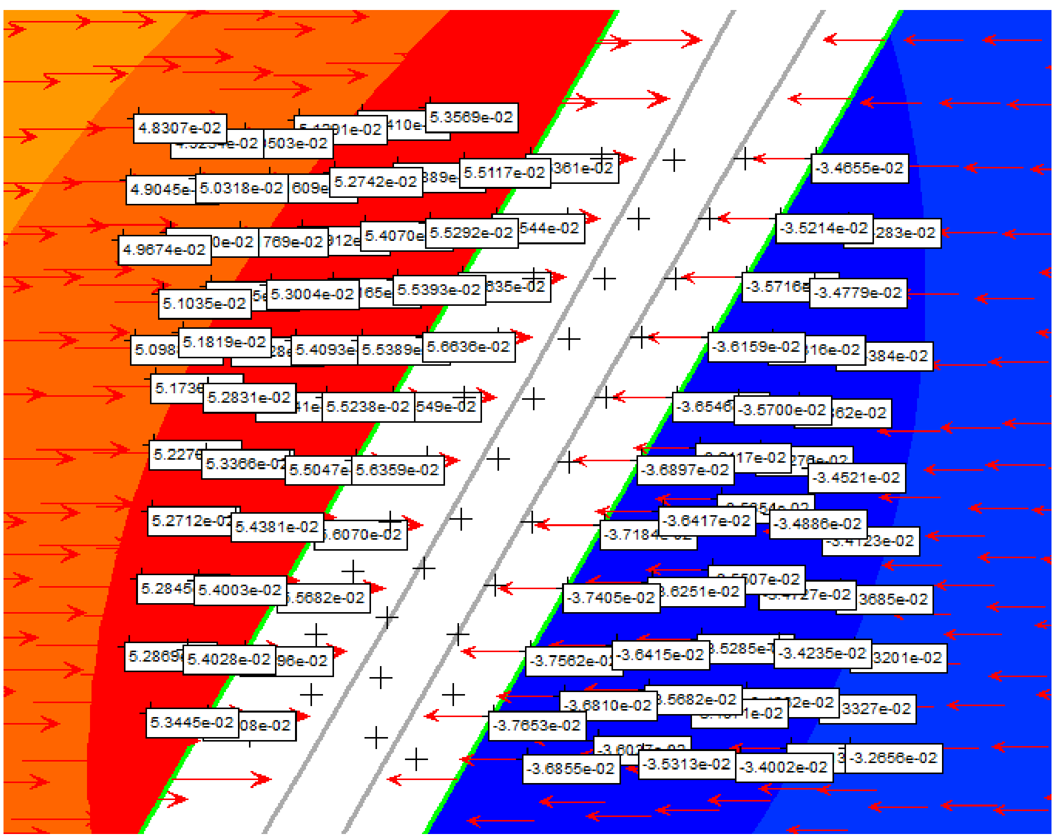


Figure 24. Nodes displacement (K=1.5, D=1250m)

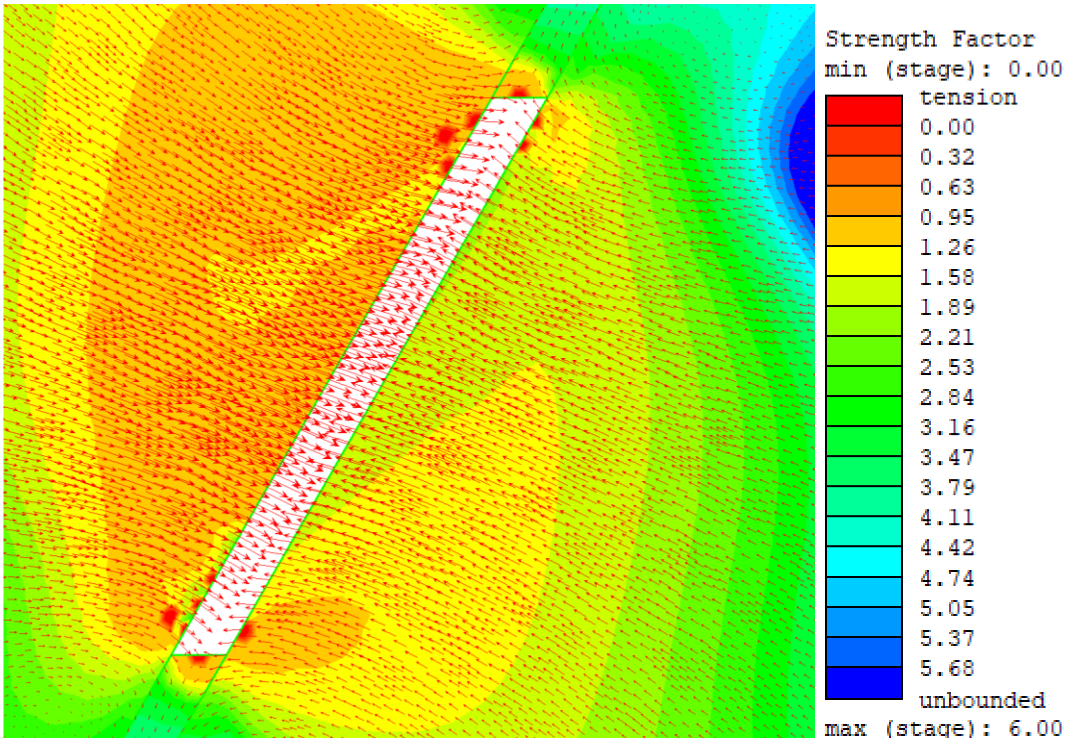


Figure 25. Strength factor (K=1.5, D=1250m)

Scenario 3: K=2, Depth=1500 m:

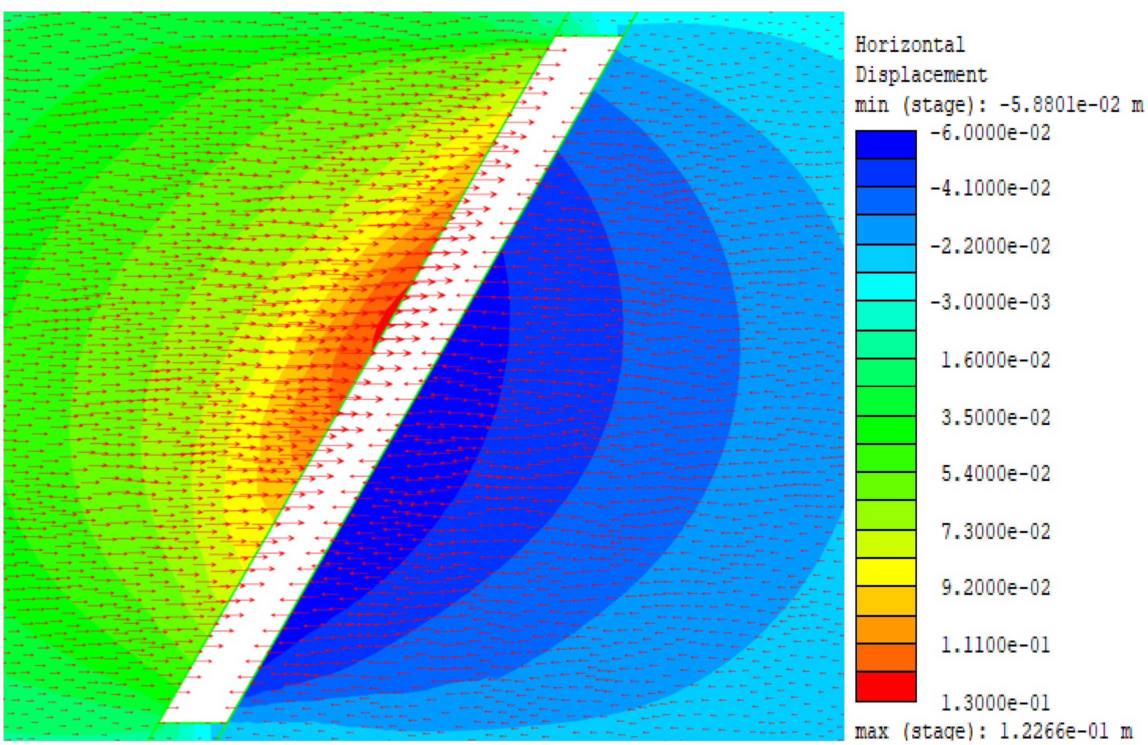


Figure 26. Horizontal displacement of walls (K=2, D=1500m)

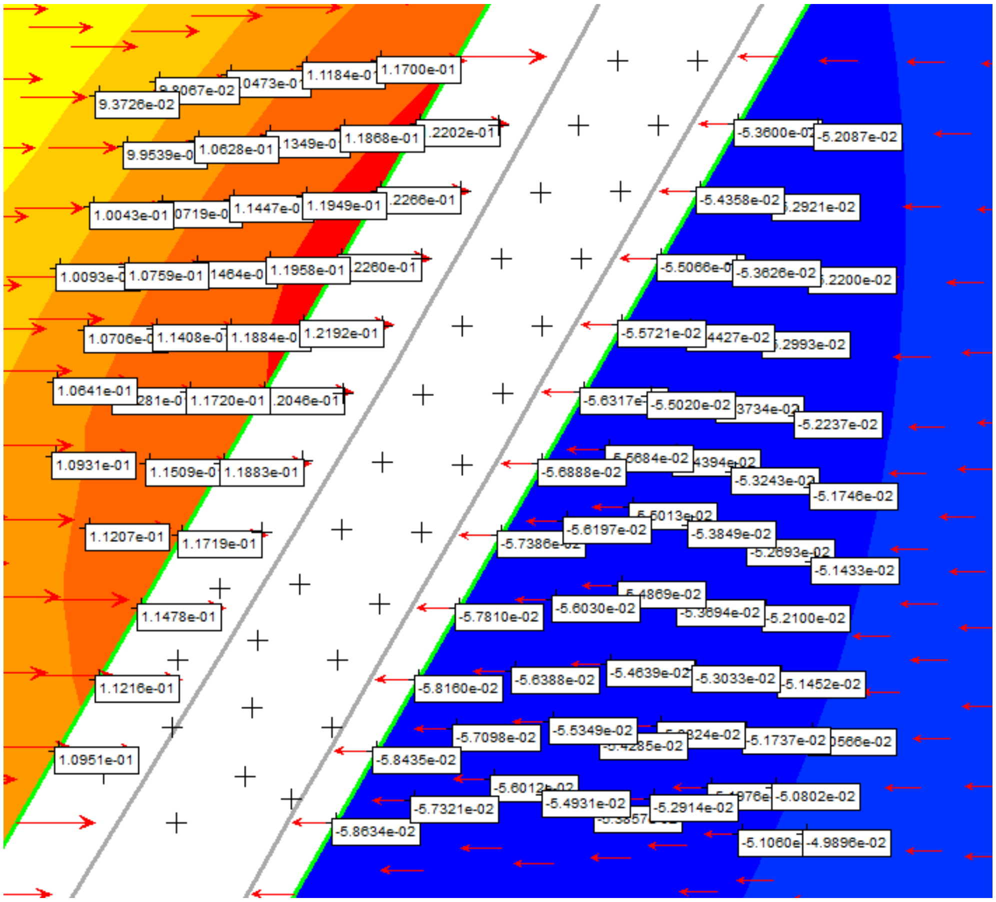


Figure 27. Nodes displacement (K=2, D=1500m)

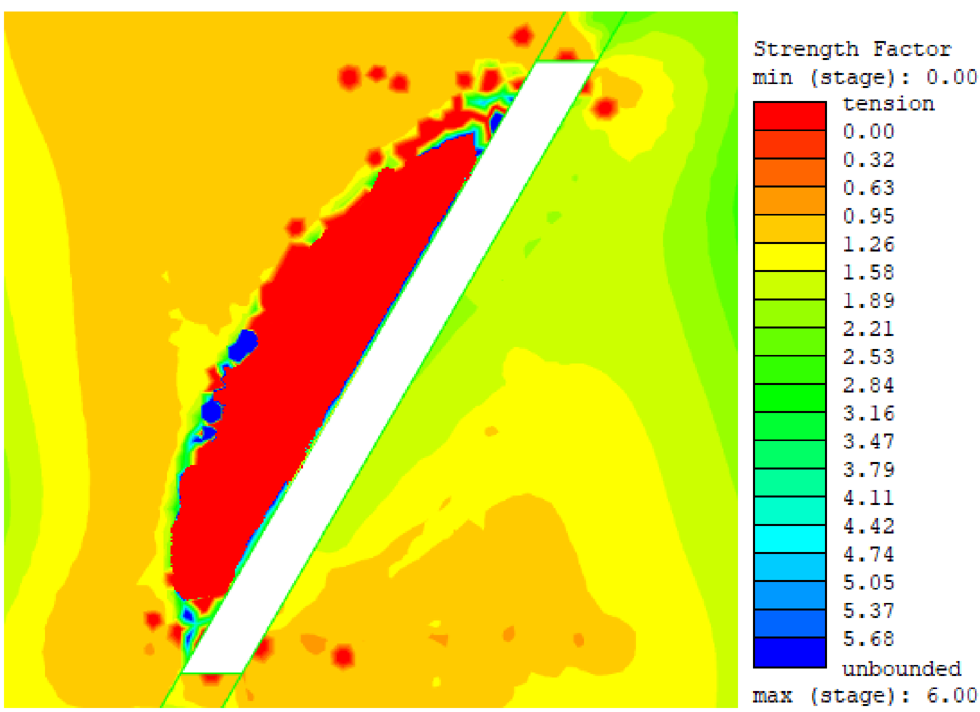


Figure 28. Strength factor (K=2, D=1500m)

Scenario 4: K=2.5, Depth=1750 m:

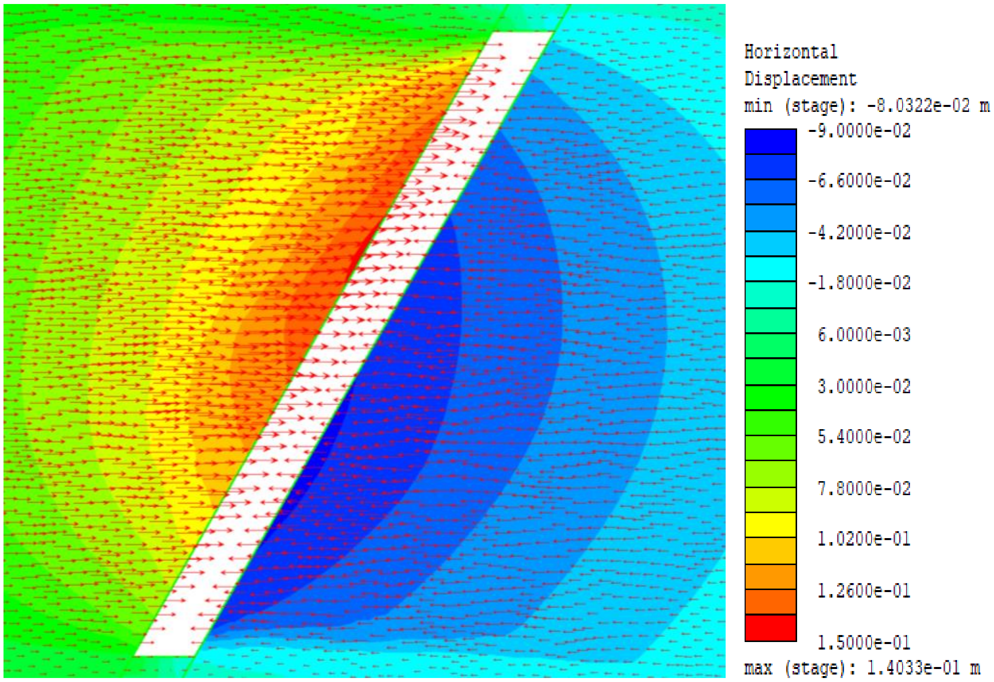


Figure 29. Horizontal displacement of walls (K=2.5, D=1750m)

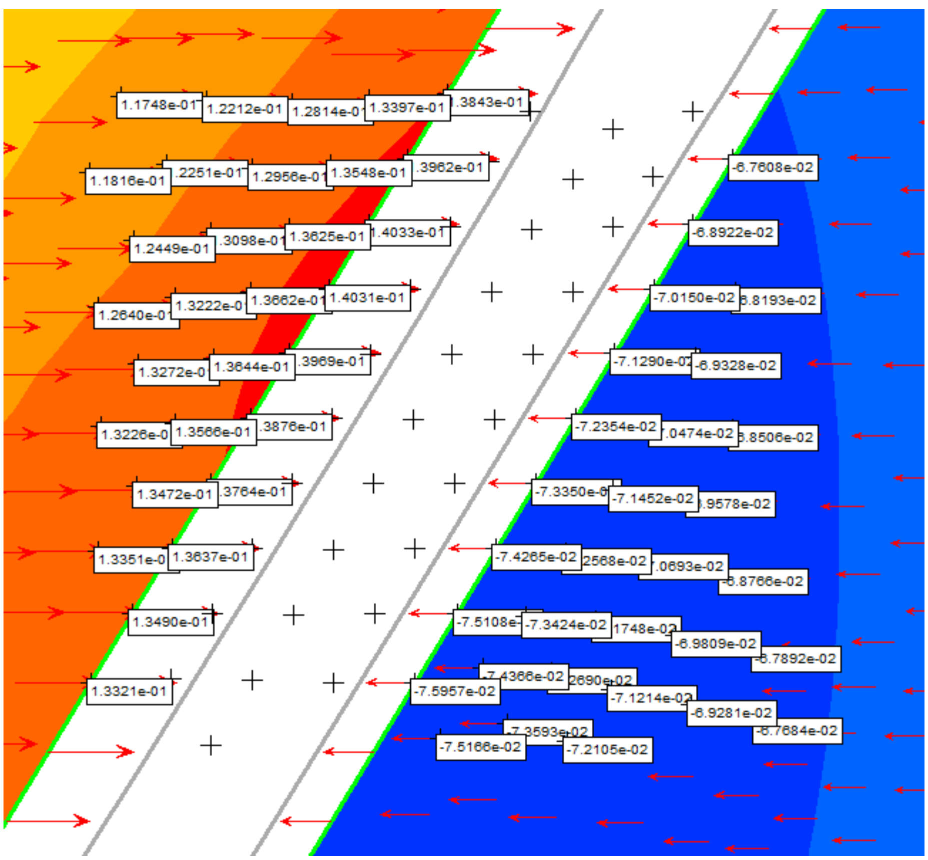


Figure 30. Nodes displacement (K=2.5, D=1750m)

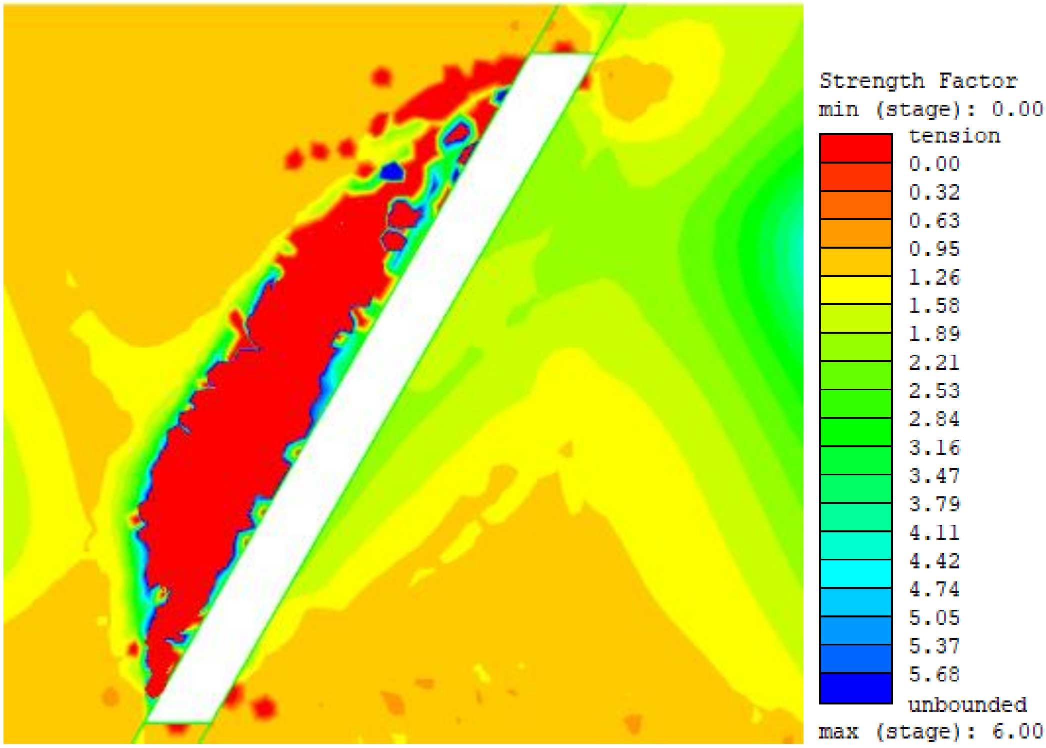


Figure 31. Strength factor (K=2.5, D=1750m)

The analysis presented in Figures 20 to Figure 31 delved into the behavior of stope walls under varying stress conditions, focusing on the horizontal displacement of both hanging wall and footwall, node displacement at the center of the stope, and the surrounding strength factor. This discussion interprets the relationship between the stress ratio (K), mining depth, and the consequential horizontal displacement, alongside how these factors collectively influence stope wall closure.

In the initial scenario (K=1, depth=1000m), a minimal closure of walls (0.004 m, Figure 20) was observed, indicating a stable condition around the stope with insignificant damage. This suggests that at moderate depths and equal hydrostatic stress conditions, stope walls exhibit minimal displacement, and maintain their geometric integrity.

However, as the K value increases to 1.5 with an increased depth of 1250m, both the hanging wall and footwall experience greater displacement (0.056 m and 0.037 m, respectively, Figure 23). The strength factor fluctuating between 0.95 and 1.26 hints at slight damages, particularly at the stope's lower extremities, as shown in Figure 25. This indicates that at moderate increase in horizontal stress relative to vertical stress, stope walls start to deform more significantly, though their overall geometry remains relatively stable.

Under a scenario where the horizontal principal stress is double the vertical stress, the stope wall closure markedly increases to 0.181 m, accompanied by severe stress-induced damages across the hanging wall, back, and floor, as presented in Figure 26 and Figure 28. This condition showcases the pronounced effect of high horizontal stress, leading to substantial deformation and potential structural compromise.

In the most extreme conditions analyzed (K=2.5, depth=1750m), the convergence in the hanging wall is 0.140 m and footwall -0.081 m (Figure 29), which are significant, with a low strength factor indicating a potential for stope collapse. This scenario underscores the detrimental impact of very high horizontal stresses at greater depths, where the integrity of stope walls is critically endangered.

In conclusion, the relationship between K (stress ratio), depth, and horizontal displacement is critical in understanding stope wall deformation. Under varying stress conditions, the stope walls perform differently, with increased K values and depth wall displacements are higher and strength factors reduced and severe stress induced damage occur.

4.3 Stope walls behavior in plan view

In plan view two scenarios were analyzed at depth of 1000 m, when major principal stress is: 1) perpendicular to the stope; 2) parallel to the stope.

1) Major principal stress is perpendicular to the stope:

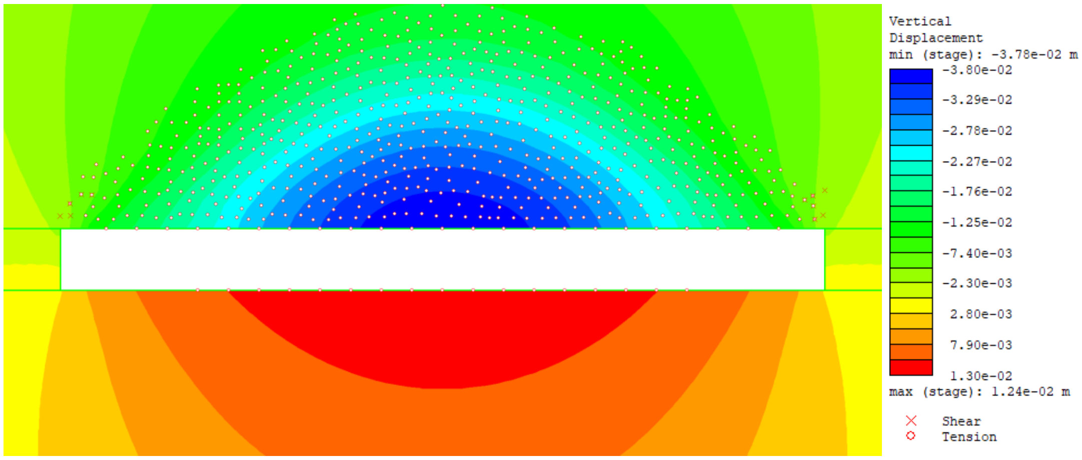


Figure 32. Convergence in stope walls (sigma 1 is perpendicular to stope)

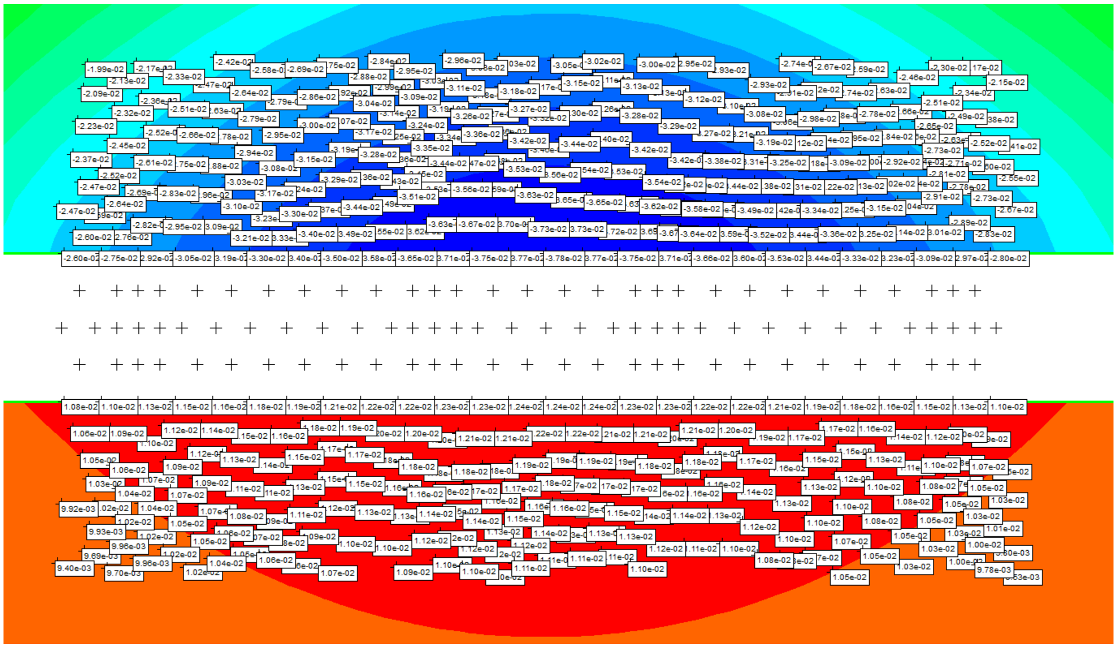


Figure 33. Nodes displacement (sigma 1 is perpendicular to the stope)

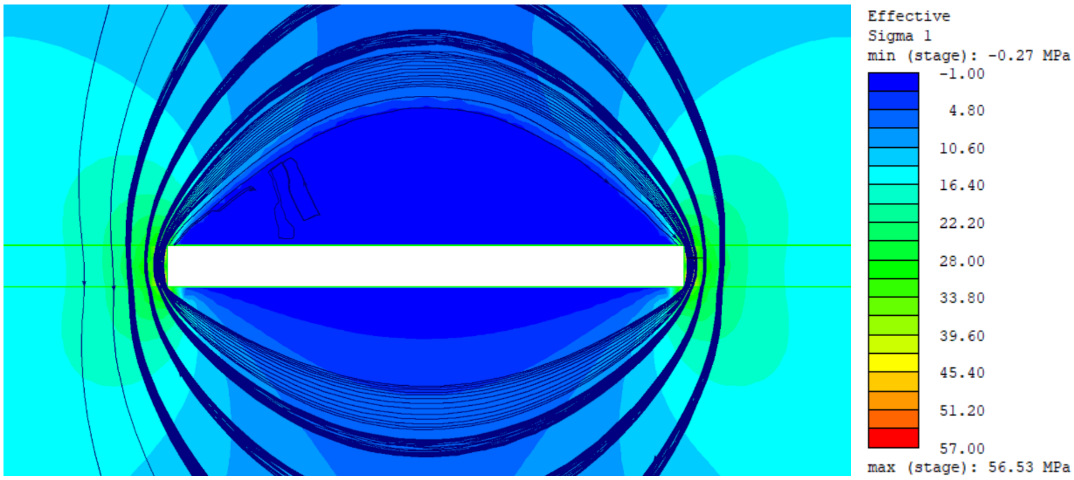


Figure 34. Sigma 1 distribution (sigma 1 is perpendicular to stope)

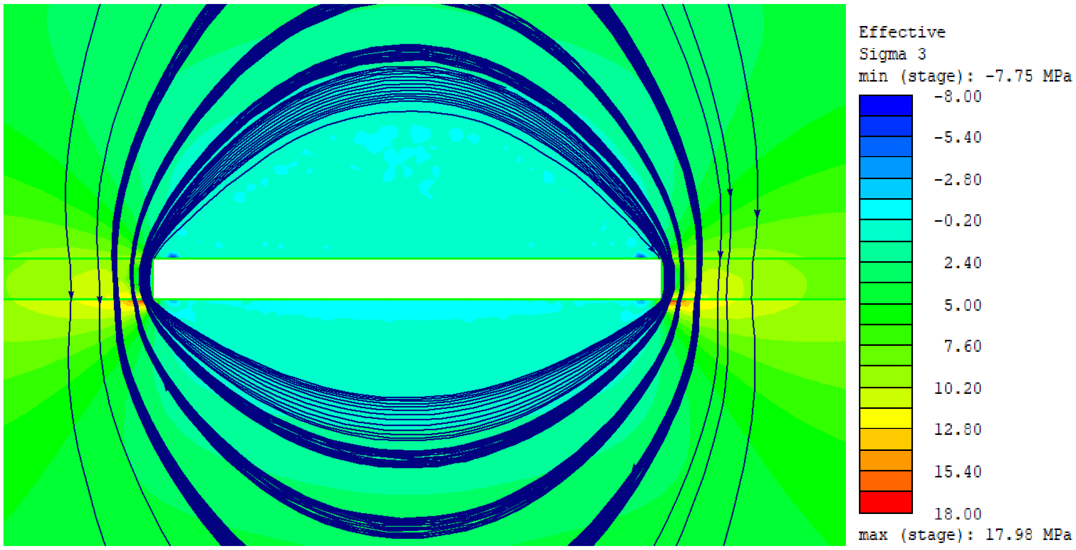


Figure 35. Sigma 3 distribution (sigma 1 is perpendicular to the stope)

2) Major principal stress is parallel to the stope:

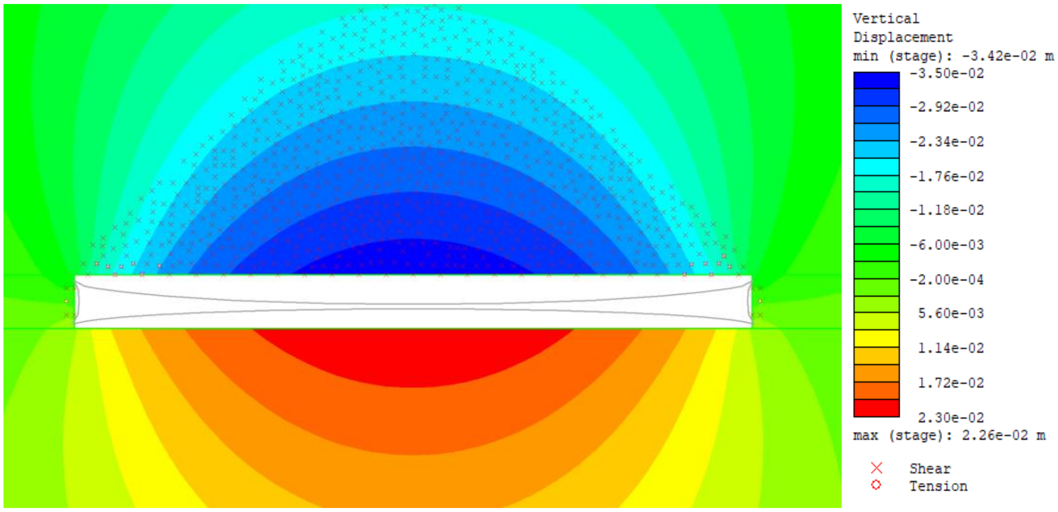


Figure 36. Stope wall convergence (sigma 1 is parallel to stope)

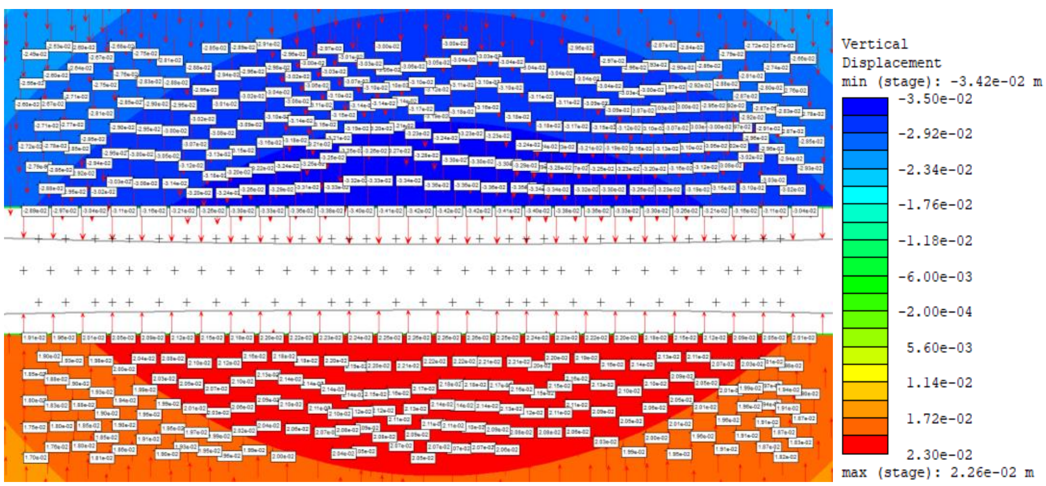


Figure 37. Nodes displacement (sigma 1 is parallel to stope)

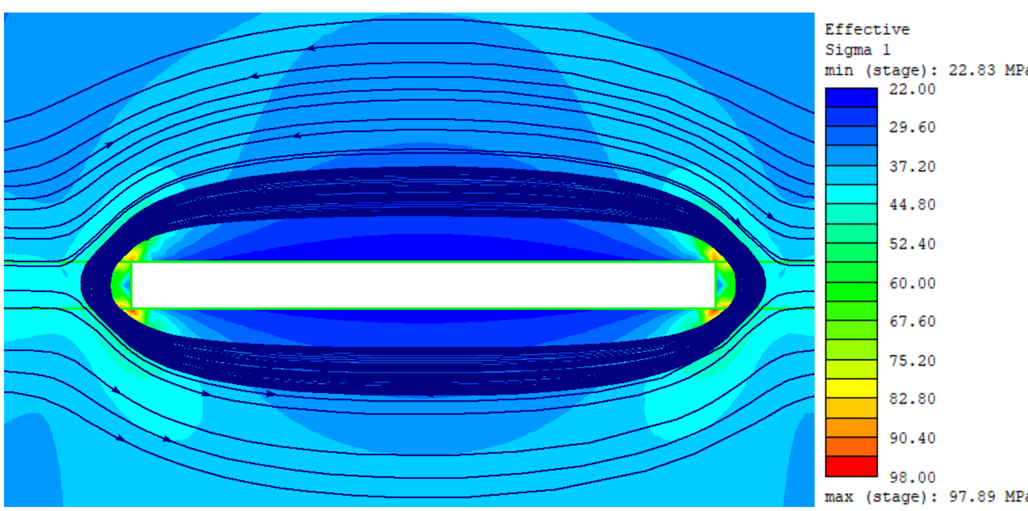


Figure 38. Sigma 1 distribution (sigma 1 is parallel to the stope)

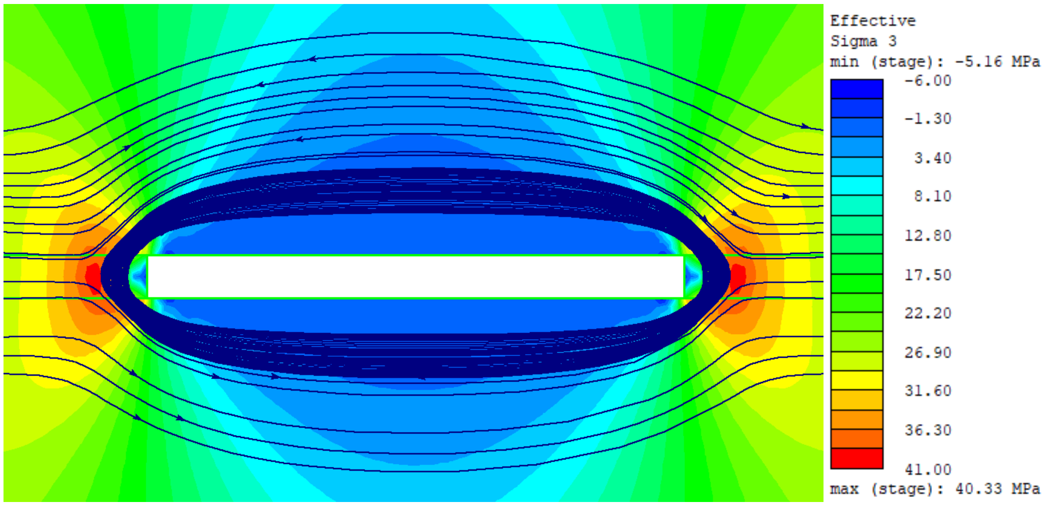


Figure 39. Sigma 3 distribution (sigma 1 is parallel to stope)

When this stress is perpendicular to the stope, it induces tension, in the hanging wall and footwall, leading to the inward movement of the hanging wall and footwall. This scenario, detailed in Figure 32, presents the hanging wall moving into the stope by 3.78 cm and the footwall moving in by 1.24 cm. The stress distribution, as shown in Figure 34 and Figure 35, reveals that while the stope undergoes closure, the hanging and footwalls experience tension, highlighting a differential impact on wall stability.

Conversely, when the major principal stress is parallel to the stope, compressive stress dominates, causing both walls to converge into the stope. Figures 36 and 37 depict this situation with displacements of 2.26 cm in the footwall and 3.42 cm in the hanging wall respectively, emphasizing potential compression stress-induced damage of the walls under these conditions, as illustrated in stress distribution in Figures 38 and 39.

This shift from tension to compression, depending on the in situ maximum principal stress orientation, underlines the complexity of stress management in underground excavations.

4.4 Stope design width (orebody thickness) and closure

In order to observe stope wall convergence, stope width was varied from 1 m to 10 m, and four different situations were analyzed: K=1; K=1.5; K=2; K=2.5. The results obtained from RS2 on horizontal displacement of hanging wall and footwall and percent of closure which are concluded in tables (Appendix A), are shown in graphs presented in Figures 40-47:

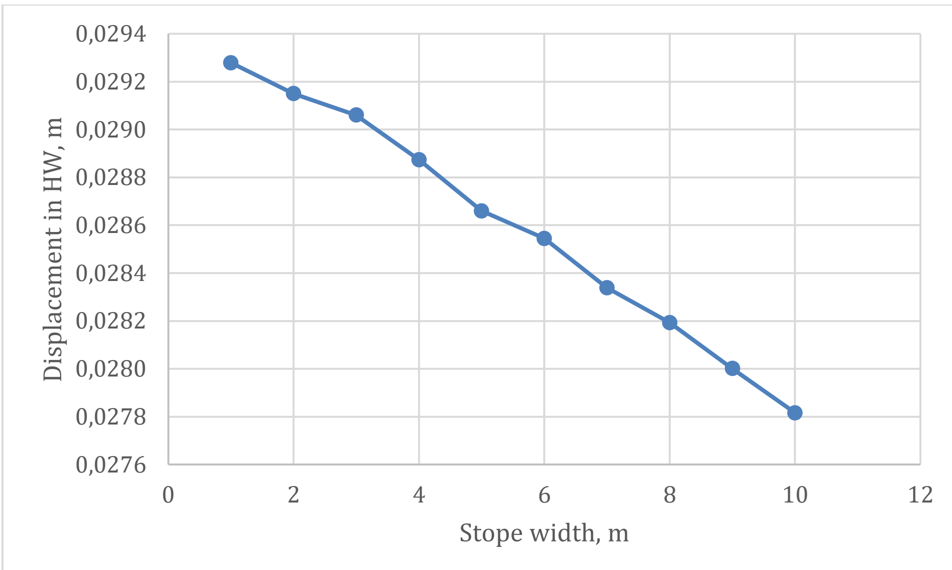


Figure 40. Graph of HW displacement vs stope width, K=1

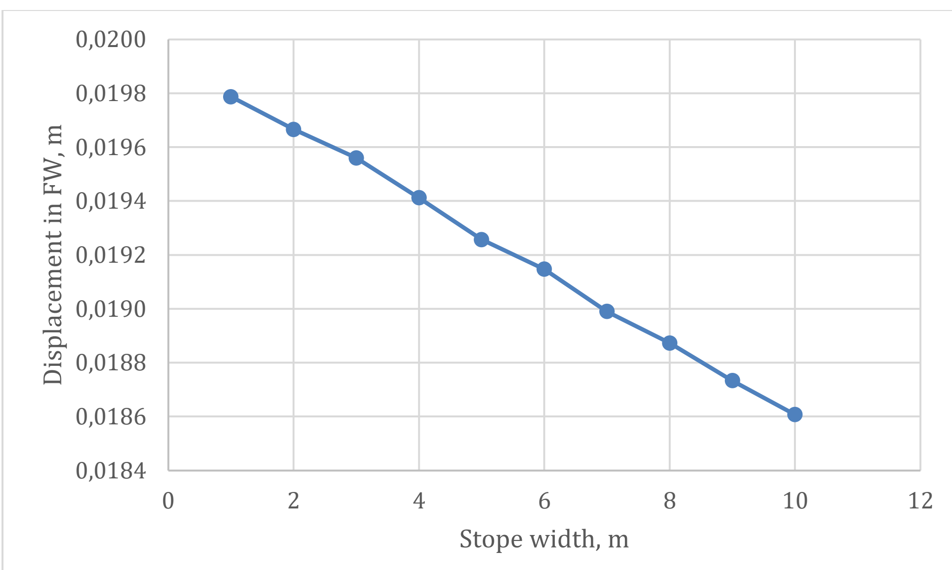


Figure 41. Graph (FW deformation vs stope width, K=1)

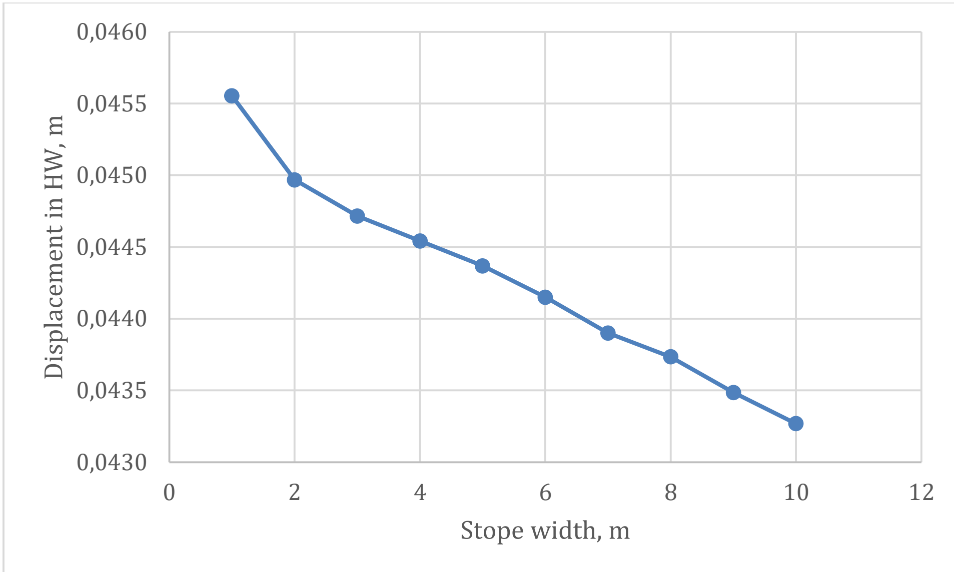


Figure 42. Graph of HW displacement vs stope width, K=1.5

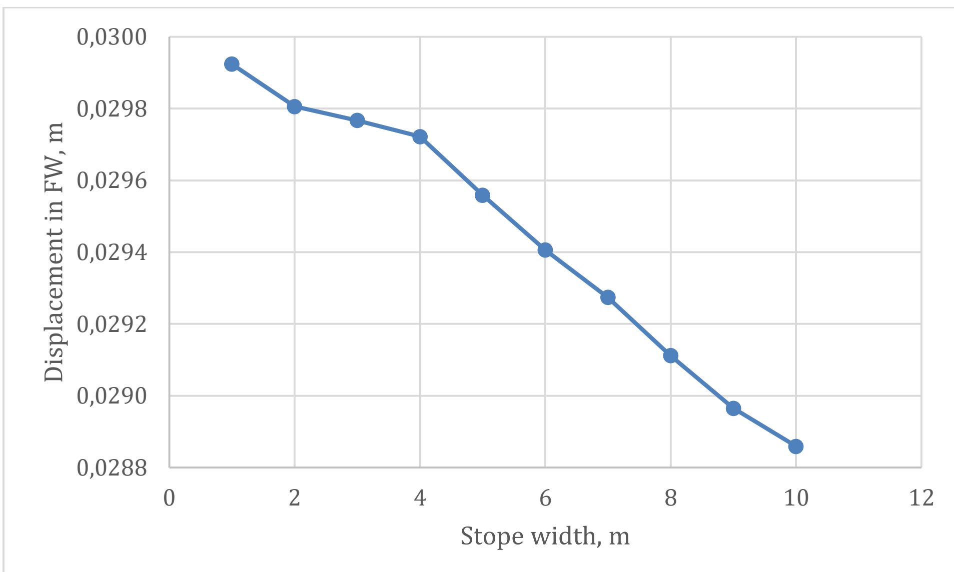


Figure 43. Graph of FW displacement vs stope width, K=1.5

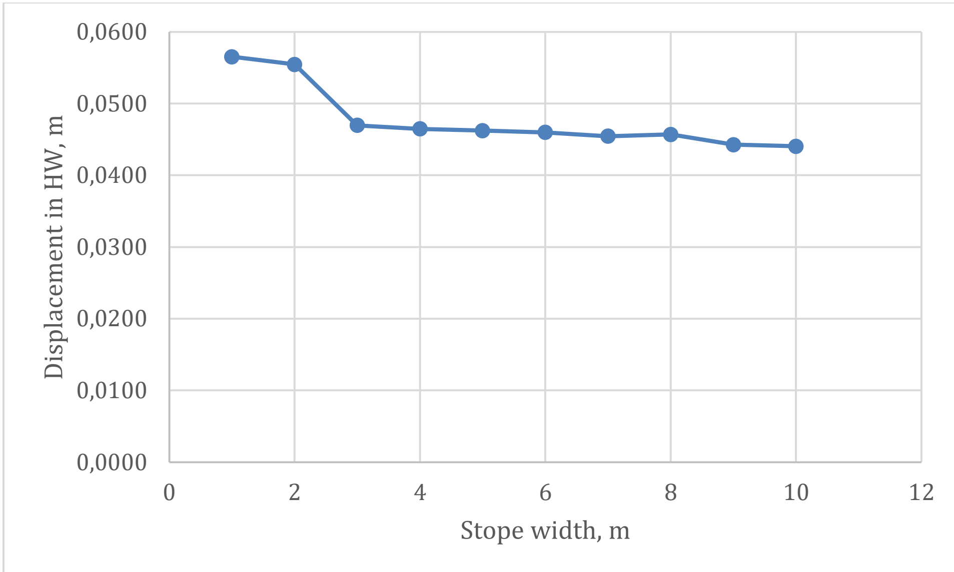


Figure 44. Graph of HW displacement vs stope width, K=2

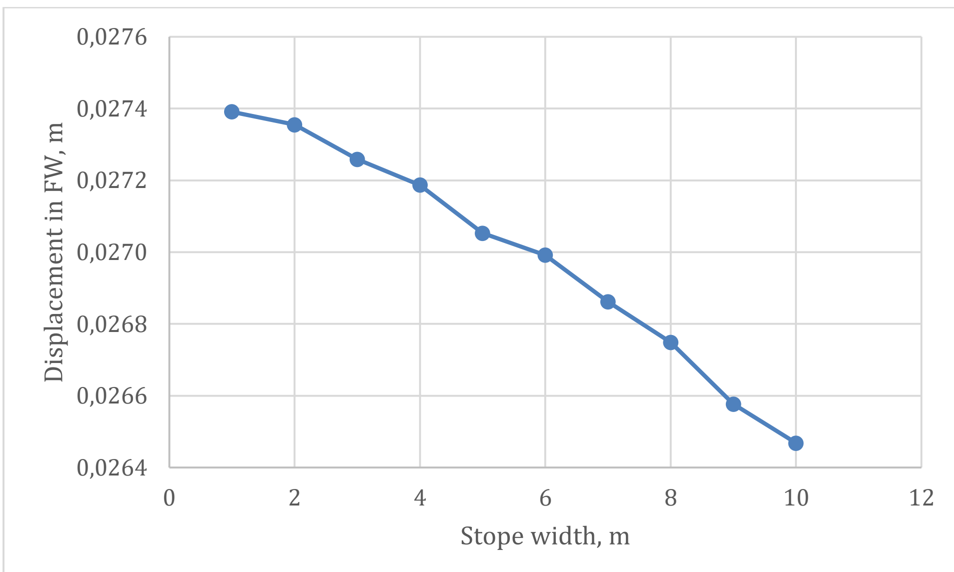


Figure 45. Graph of FW displacement vs stope width, K=2

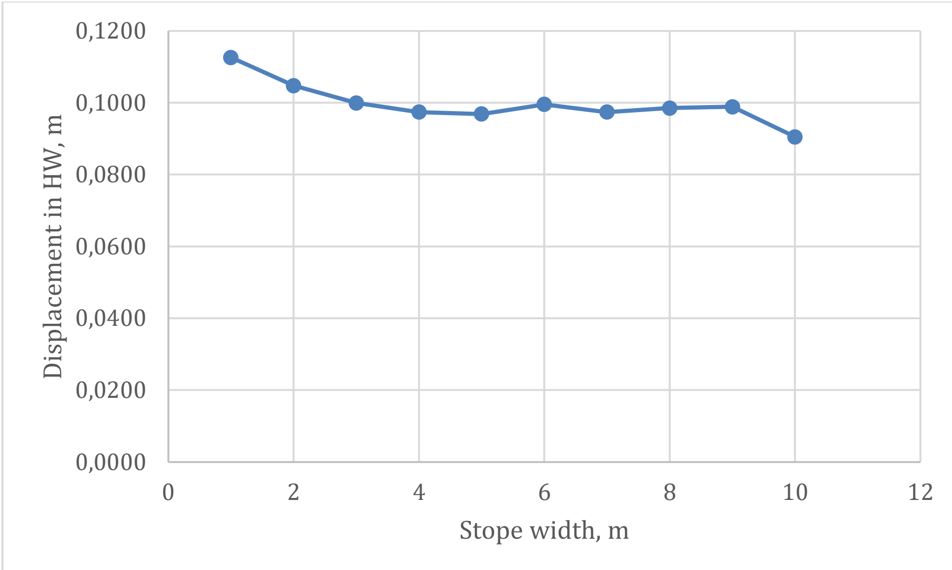


Figure 46. Graph of HW displacement vs stope width, K=2.5

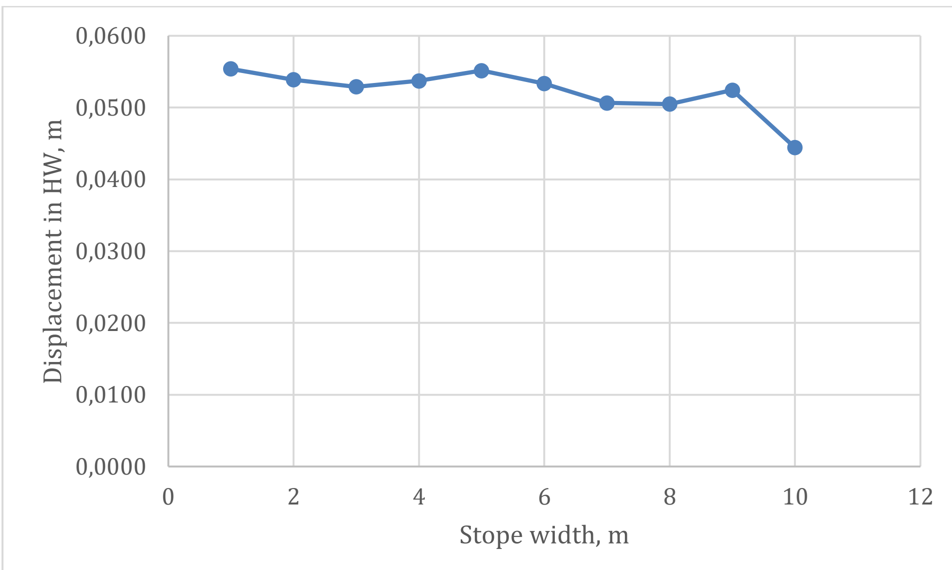


Figure 47. Graph of FW displacement vs stope width, K=2.5

The impact of the stress ratio (K) and the width of stope on stope closure was analyzed. The study's main conclusions include the inverse relationship between stope width and the horizontal displacement of the hanging wall and footwall as well as the direct association between the stope walls' horizontal displacement and the effective stress ratio (K).

There is a reduction in horizontal displacement of the hanging wall and footwall with increasing stope width. This phenomenon may be explained by the fact that wider stopes are less confined and that lateral forces have less of an impact on them. This reduces the tendency for wall closure and increases the stability of the stope structure. In essence, wider stopes mitigate localized stress concentrations that cause severe wall displacement by more uniformly distributing stress over the excavation's width. This observation confirms the issues raised in this thesis on the appropriate use of tools designed for wide stopes (Mathews et al., 1981) for the design of narrow vein stopes and vice versa.

On the other hand, the study shows that the closure of stope walls increases with increasing K ratio. This effect is more noticeable in settings where the horizontal stress is much greater than the vertical stress and is parallel to the orebody. The greater the stope closure at higher K ratios the more important stress orientation and magnitude are in determining how stopes spaces behave, making how we stabilize stopes extremely important.

The results highlight the intricacy of geotechnical reactions in underground mining operations, where the stability and safety of excavations are largely dependent on the geometric features of the stope and the dominant stress regime, and that rocks under high stress conditions may perform in a plastic behavior.

Then, the information from Table 3 - Table 6 were collected and visualized in a graph, particularly focusing on the relationship between stope width and percent of closure:

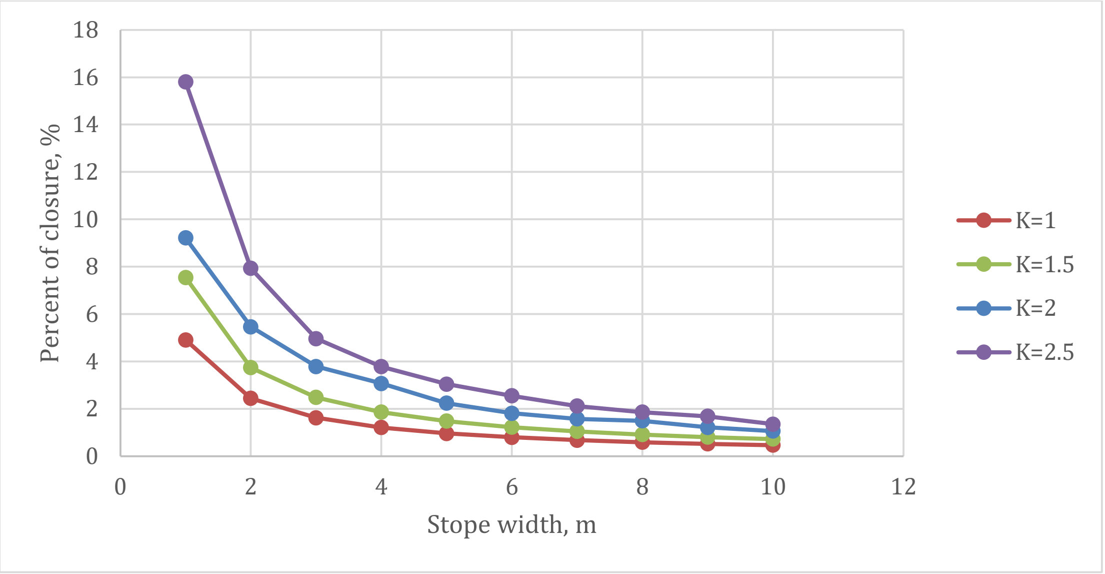


Figure 48. Plot of percent of closure against stope width for different K-ratios

Figure 48 shows a strong correlation between stope width and the percentage of stope wall closure resulting from wall displacement in various narrow vein stopes. The figure clearly shows that for stope widths equal to or greater than 10 m stope wall convergence becomes a less important consideration in stope design and other design tools such as empirical design tools could be employed.

For a given value of K, stope wall closure decreases with increasing stope width. Furthermore, the percentage of closure increases with the effective K-stress ratio, which indicates a higher horizontal stress in relation to a larger vertical stress. This indicates that stress circumstances have a substantial impact on stope wall stability. The higher the K-ratio the steeper the curves are, and hence, the more essential the influence of stope width becomes in minimizing stope wall closure. The graph therefore emphasizes how crucial it is to take into account both the effective stress environment and the stope width when anticipating and controlling wall displacement in narrow vein stopes.

5 Conclusion and Recommendation

The main goal of this thesis was to investigate the stope wall convergence in narrow vein mining. It started with the specific notion that wall displacement and closure are greatly influenced by the orientation of the primary far-field principal stress with respect to the stope, particularly in deep narrow vein mines where the hanging wall and footwall are widely exposed. This research used RS2 for numerical modeling to simulate wall deformations under various stress conditions.

The data provided by the different figures and RS2 simulations clearly shows that damage and deformation of stope walls increase with an increase in mining depth and in-situ stress condition. At shallow depths the wall convergence is low, indicating stable mining conditions. Increased primary stress causes minimal structural alterations at 750 meters below the surface, indicating little damage. For mining safety and efficiency, significant wall deformation caused by large stress concentrations over 1500 meters calls for increased support and monitoring. According to the study's findings, at depths larger than 1000 meters, inelastic failure—which is characterized by severe wall displacement and stress-induced damage—becomes a crucial element in forecasting the stability of rock structures.

The research highlights the importance of stress ratio (K), mining depth, and horizontal displacement in determining the behavior of stope walls under different stress conditions. Minimal wall movement at intermediate depths (1000 m) and in a hydrostatic stress state present stable stope walls. Increased displacement and minor damage show up when the depth rises to 1250 m and the stress ratio reaches 1.5, indicating that walls start to severely deform under higher horizontal load even while overall stability is preserved. Significant wall displacement and severe damage demonstrate the essential impact of large horizontal stresses, endangering structural integrity and signaling a potential for collapse at stress ratios of two and above, with depths approaching 1750 meters. This highlights how crucial stress ratio and depth are for forecasting stope wall behavior and guaranteeing mining safety. This work makes a significant addition by clarifying the effects of two primary principal stress orientations on the movement and stability of stope walls: parallel, which results in shear stress, and perpendicular, which leads to tension.

In underground excavations, the study demonstrates a critical link between the effective stress ratio (K), stope width, and stope wall closure. Broader stopes are shown to have less horizontal displacement and, as a result, improved stability because of less stress concentration. It also highlights the significance of horizontal over vertical stress on structural integrity by showing that a rise in the effective stress ratio results in a greater displacement of the wall. These results highlight how crucial it is to take stress conditions and stope width into account when designing subterranean excavation support systems in order to maximize safety and reduce the chance of wall convergence.

It is recommended to increase the number of simulations up to 15 m wide open stopes, and collect data from existing narrow vein mines to better understand the closure of hanging walls and footwalls. Varying the mechanical properties of the rock, adding geology geometries, faults and joints, would provide additional data to the stope wall convergence-based chart for narrow vein stope design.

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Appendix A

Table 3. Wall closure for K=1

Width	Closure in HW	Closure in FW	Percent closure
1	0.02928	0.01979	4.90660
2	0.02915	0.01967	2.44080
3	0.02906	0.01956	1.62067
4	0.02887	0.01941	1.20715
5	0.02866	0.01926	0.95834
6	0.02855	0.01915	0.79487
7	0.02834	0.01899	0.67613
8	0.02819	0.01887	0.58833
9	0.02800	0.01873	0.51927
10	0.02782	0.01861	0.46423

Table 4. Wall closure for K=1.5

Width	Closure in HW	Closure in FW	Percent closure
1	0.04555	0.02572	7.12780
2	0.04497	0.02971	3.73370
3	0.04472	0.02977	2.48277
4	0.04454	0.02972	1.85660
5	0.04437	0.02956	1.47854
6	0.04415	0.02941	1.22593
7	0.04390	0.02927	1.04534
8	0.04373	0.02911	0.91058
9	0.04349	0.02897	0.80501
10	0.04327	0.02886	0.72128

Table 5. Wall closure for K=2

Width	Closure in HW	Closure in FW	Percent closure
1	0.05652	0.02739	8.39070
2	0.05545	0.02736	4.14045
3	0.04696	0.02726	2.47387
4	0.04648	0.02719	1.84165
5	0.04622	0.02705	1.46552
6	0.04598	0.02699	1.21618
7	0.04545	0.02686	1.03299
8	0.04569	0.02675	0.90554
9	0.04426	0.02658	0.78703
10	0.04405	0.02647	0.70514

Table 6. Wall closure for K=2.5

Width	Closure in HW	Closure in FW	Percent closure
1	0.11258	0.05540	16.79830
2	0.10475	0.05387	7.93120
3	0.09993	0.05289	5.09403
4	0.09739	0.05373	3.77808
5	0.09685	0.05515	3.03996
6	0.09954	0.05335	2.54807
7	0.09741	0.05066	2.11526
8	0.09850	0.05050	1.86248
9	0.09888	0.05244	1.68131
10	0.09050	0.04442	1.34919