Results in Engineering 14 (2022) 100456

Contents lists available at ScienceDirect

Results in Engineering

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Effect of threshold suction on the prediction of the permeability function by using the statistical method

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ARTICLE INFO

Keywords:
Soil-water characteristic curve
Statistical method
Capillary flow
Permeability function

ABSTRACT

The statistical method is commonly adopted by many researchers for the estimation of the permeability function of unsaturated soil. The statistical method is developed based on the assumption that the pores are randomly connected to each other. Water can only flow through the water phase in those pores. In other words, the statistical method is only applicable when the water phase in soil is in a continuous condition. However, past studies indicated that the water phase in unsaturated soil became discontinuous when the soil suction goes beyond the residual suction. When the water phase is discontinuous, the water in some of the pores may not be able to provide the flow path because water cannot flow into the surrounding pores. Therefore, there will be a threshold suction where the statistical method becomes inapplicable for suctions beyond this threshold value. In this parametric study, the effect of the threshold suction on the estimated permeability function obtained from the statistical method was investigated.

1. Background

The statistical method, which was firstly proposed by Childs and Collis-George [1]; is commonly used for the estimation of the permeability function (or the hydraulic conductivity function, HCF) from the soil-water characteristic curve (SWCC). It is observed that the current statistical models [1–6] adopt the maximum suction of 10^6 kPa. Zhai et al. [7] indicated that major assumption adopted in the statistical method is that the pores are randomly distributed and connected to each other. The water in those connecting pores provides the flow path for the capillary water.

The statistical method was proposed based on the Poiseuille's equation, which considers that water flow in soil is mainly dominated by the liquid flow. Zhai et al. [8] and Zhai et al. [9] indicated that water is attached to soil particle due to the adsorptive force and the attached water moves in soil mainly in the vapour form when the suction is higher than 3100 kPa. In this case, it seems that the maximum suction of 3100 kPa which is proposed for the liquid flow condition should be adopted in the statistical method. On the other hand, Vanapalli et al. [10], and Zhai et al. [11] indicated that the entire soil suction can be divided into three zones, such as boundary effect zone, transition zone and residual zone. Those three zones are divided by air-entry value (AEV) and the residual suction. Soltani et al. [12] proposed a framework for the determination of air-entry value from the fitting parameters of SWCC. When the suction is less than AEV (boundary effect zone), the water phase is in a continuous condition; when the suction is higher than the residual suction (residual zone), the water phase is in a discontinuous condition. It seems that the residual suction should be adopted as the threshold suction in the statistical method because the water phase is discontinuous when the suction goes beyond the residual suction. However, it should be noted the current method [13,14] for determination of the residual suction is empirical and it is difficult to have a theoretical solution of the residual suction.

Based on the above literature review, the current statistical method adopts 10^6 kPa as the maximum suction for the prediction of the HCF. In this paper, the effect of the threshold suction on the predicted HCF is

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https://doi.org/10.1016/j.rineng.2022.100456
Received 15 April 2022; Received in revised form 18 May 2022; Accepted 19 May 2022

Table 1
Fitting parameters and residual suction for the soils.

Soils	Name	a(kPa)	n	m	C_r(kPa)	ψ_r (kPa)
Good drainage soil	A	5	1	1	1500	89.88
	B	5	5	1	1500	11.09
	C	5	0.5	1	1500	216.4
Poor drainage soil	D	500	1	1	1500	2217
	E	500	5	1	1500	972
	F	500	0.5	1	1500	4816

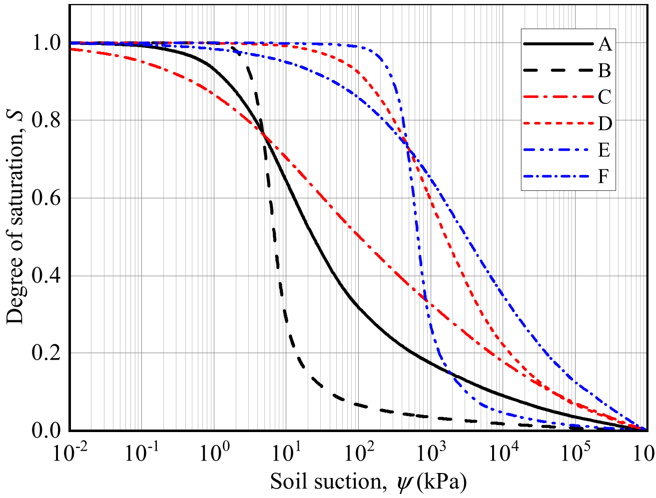

Fig. 1. Six sets of SWCCs as defined by the fitting parameters in Table 1.

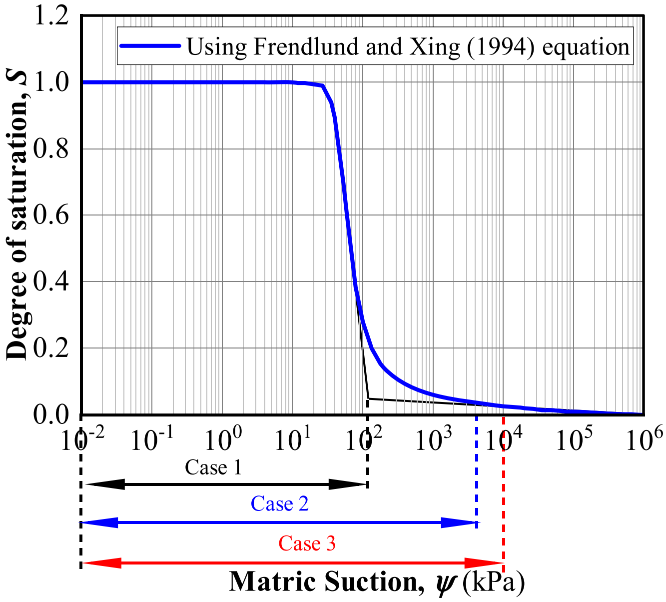

Fig. 2. Illustration of three cases for the estimation of HCF.

investigated by the parametric study considering different maximum suctions in the statistical method.

2. Statistical model for the prediction of the HCF from SWCC

Childs and Collis-George [1] firstly proposed the statistical model for the prediction of the HCF from SWCC as shown in Equation (1).

$$k = M\sum_{\rho=0}^{\rho=R}\sum_{\sigma=0}^{\sigma=R}\sigma^2 f(\rho)\delta r f(\sigma)\delta r \tag{1}$$

where, k is the hydraulic conductivity, M is constant value to match theoretical and experimental curves at a single point; ρ and σ are radii of pores that to be randomly connected; $f(\rho)$ and $f(\sigma)$ are the pore size densities corresponding to radii of ρ and σ, respectively, R is the maximum radius of the pore.

Marshall [2] modified Equation (1) and proposed a summation form as follows:

$$k = \frac{n^2}{N^2}\sum_{i=2}^{N}\left[(i)^2-(i-1)^2\right]r_i^2 = \frac{n^2}{N^2}\left[r_1^2+3r_2^2+5r_3^2+\ldots(2n-1)r_n^2\right]\Big/ 8 \tag{2}$$

where, n is porosity of soil; i is the interval number; N is total numbers of pores taken account; r_i is radius of pore.

Kunze et al. [3] divided the volumetric water content domain evenly for the determination of $(u_a\text{-}u_w)_i$ as follows:

$$k_w(\theta_w)_i = \frac{k_s}{k_{sc}}A_d\sum_{j=i}^{m}\left\{(2j+1-2i)(u_a-u_w)j^{-2}\right\}, i=1,\ \ 2,\ \ \cdots,\ \ \mathrm{m} \tag{3}$$

where.

$k_w(\theta_w)_i$ is the predicted hydraulic conductivity corresponding to the *i*th interval (m/s); *i* is the interval number; *j* is a count from "*i*" to "*m*"; *m* is the total number of intervals; θ_s is the saturated volumetric water content; θ_L is the lowest volumetric water content; k_s is the measured saturated hydraulic conductivity (m/s); k_{sc} is the calculated saturated hydraulic conductivity (m/s); A_d is the adjusting constant.

Mualem [4] and Fredlund et al. [5] showed that Childs and Collis-George [1]'s equation could be expressed in an analytical form as in Equations (4) and (5), respectively.

$$k(\theta_w) = \frac{\displaystyle\int_0^{\theta_w}\frac{(\theta_w-\vartheta)}{\psi^2}d\vartheta}{\displaystyle\int_0^{\theta_s}\frac{(\theta_w-\vartheta)}{\psi^2}d\vartheta} \tag{4}$$

$$k_r(\psi_k) = \frac{\int_{\psi_k}^{\psi_r}\frac{\theta(y)-\theta(\psi_k)}{y^2}\theta'(y)dy}{\int_{\psi_{AEV}}^{\psi_r}\frac{\theta(y)-\theta(\psi_{AEV})}{y^2}\theta'(y)dy} \tag{5}$$

Zhai and Rahardjo [6] proposed to evenly divide the suction domain and proposed the summation form of the statistical method for the estimation of HCF from SWCC as follows:

$$k(\psi_{x,d}) = k(\psi_{ref,d})\frac{\left\{\sum_{i=x,d}^{N}\left[\frac{\left(S(\psi_{x,d})-S(\psi_i)\right)^2-\left(S(\psi_{x,d})-S(\psi_{i+1})\right)^2}{(\psi_i)^2}\right]\right\}}{\left\{\sum_{i=ref,d}^{N}\left[\frac{\left(S(\psi_{ref,d})-S(\psi_i)\right)^2-\left(S(\psi_{ref,d})-S(\psi_{i+1})\right)^2}{(\psi_i)^2}\right]\right\}} \tag{6}$$

where $k(\psi_{x,d})$ is the calculated drying hydraulic conductivity at a given suction of ψ_x.

$k(\psi_{ref,d})$ is the hydraulic conductivity at the reference point, $\psi_{ref,d}$ is the suction corresponding to the reference point, $S(\psi_{ref,d})$ is the degree of saturation corresponding to the reference point, ψ_i is the soil suction in the drying process, $S(\psi_{x,d})$ and $S(\psi_i)$ are the degrees of saturation corresponding to soil suctions of $\psi_{x,d}$ and ψ_i, respectively, and N is the total number of the divided SWCC segments.

It seems that all those equations adopted zero as the minimum radius or 10^6 kPa as the maximum suction which is based on the original Childs and Collis-George [1]'s equation as shown in Equation (1). However, water flows in soil may not be in the liquid form. Therefore, the HCFs are estimated by using the statistical method with different threshold

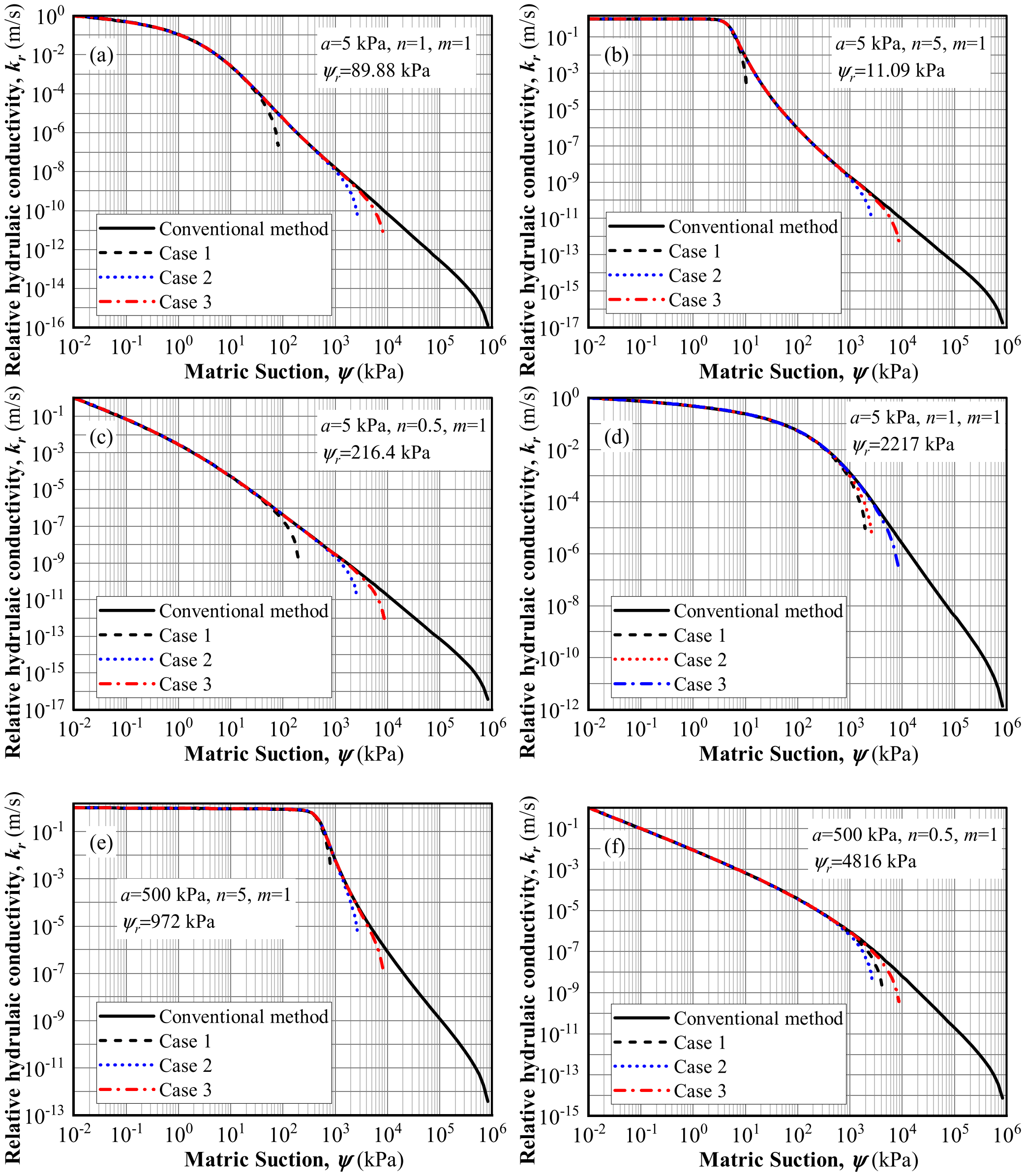

Fig. 3. Illustration of the predicted HCFs from three cases and the conventional method. (a) For Soil A; (b) for Soil B; (c) for Soil (C); (d) for Soil D; (e) for Soil E; (f) for Soil F.

suctions. Consequently, the effect of the threshold suction on the estimated HCF is discussed.

3. Parametric studies on the effect of threshold suction on the estimated HCF

Fredlund and Xing [15]'s equation, as illustrated in Equation (7), is one of most popular equations for the representation of the SWCC and adopted in this study. Rahimi et al. [16] conducted the parametric study on the effects of hydraulic properties on the stability of slopes under rainfall. Rahimi et al. [16] considered two types of soil, namely good drainage soil and poor drainage soil. In this study, both good drainage soil and poor drainage soil were adopted for the parametric study as listed in Table 1. The fitting parameters in Fredlund and Xing [15]'s equation and the residual suctions for the soils are also given in Table 1. The six sets of SWCCs as defined by those fitting parameters are illustrated in Fig. 1.

$$S = \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)}\right] \frac{1}{\left\{\ln\left[e + \left(\frac{k\psi}{a_f}\right)^{n_f}\right]\right\}^{m_f}} \tag{7}$$

where, S is the degree of saturation, a_f, n_f *and* m_f, are fitting parameters, and C_r is an input value which is a rough estimation of the residual suction. Wang et al. [17] illustrated different values of C_r on the effect of performance of Fredlund and Xing [15]'s equation.

From the computation of the hydraulic conductivity using the statistical method with different maximum suctions, it is observed that the estimated results near the maximum suction are forced to approach zero. As a result, if the hydraulic conductivity corresponding to suction ψ_i is estimated, a higher value than ψ_i should be adopted in the estimation. As shown in Equations (5) and (6), the maximum suctions are represented by ψ_r and ψ_N, respectively. If different values of ψ_r and ψ_N are adopted, then the values of the denominator in both equations will be changed, which in turn change the estimated results of the hydraulic conductivity.

In this paper, the HCF for soils with typical SWCCs are estimated using the statistical method by considering three cases such as: Case 1 using the residual suction as determined from the graphical method [14] as the threshold suction; Case 2 using 3100 kPa as the threshold suction; Case 3 using 10^4 kPa as the threshold suction, as shown in Fig. 2. The estimated HCFs from those three cases were compared with the conventional method which adopts the maximum suction of 10^6 kPa. Subsequently, the effects of the suction domain on the estimated HCF are discussed based on the analyzed results.

The predicted HCFs from those three cases are compared with those predicted from the conventional method with the maximum suction of 10^6 kPa and illustrated in Fig. 3.

As shown in Fig. 3, the same relative hydraulic conductivity can be obtained in the low suction from those three cases and the conventional method for all the soils adopted in this analysis. It is observed that within the range of [1/3 times of the threshold suction to the threshold suction], the calculated HCF are underestimated. In other words, the predicted results with the threshold suction of 3100 kPa may underestimate the HCF within the range of [1000 kPa, 3100 kPa]. Based on the theory of hydroscopic water from Plaster [18]; the capillary water flow can work within the suction up to 3100 kPa. Both the theories on the film flow from Zhai et al. [14] and the vapour flow from Zhai et al. [9]; indicated that water will flow through the adsorbed water film or the vapour form in the high suction range.

4. Conclusions

The statistical methods for the prediction of the HCF from SWCC are reviewed and it is observed that all methods adopted a maximum suction of 10^6 kPa. Recent studies indicated that the capillary water flow may not be significant in the high suction range and it is reasonable to set a lower value than 10^6 kPa for the maximum suction in the statistical method. By comparison of the predicted HCF from three cases and the conventional statistical method, it is observed that the maximum suction does not affect the estimated HCF in the low suction range. The predicted results within the suction range of [1/3 threshold suction to the threshold suction] can be significantly affected by the values of the threshold suction adopted for the prediction.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to acknowledge the financial supports received from the National Natural Science Foundation of China (No. 52078128), China Huaneng Group Co. Ltd. (No. HNKJ19-H17).

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