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A novel two-polynomial criteria for higher-order systems stability boundaries detection and control

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ABSTRACT

There are many methods for identifying the stability of complex dynamic systems. Routh and Hurwitz's criterion is one of the earliest and commonly used analytical tools analysing the stability of dynamic systems. However, it requires tedious and lengthy derivations of all components of the Routh array to solve the stability problem. Therefore, it is not a simple method to define analytically, stability boundaries for the coefficients of the system characteristic equation. The proposed brand-new criterion is an effective alternative technique in identifying stability higher-order linear time-invariant dynamic system that binds the coefficients of the system characteristic polynomial at the stability boundaries by means of an additional single constant k. It defines the necessary and sufficient conditions for the absolute stability of higher-order dynamic systems. It also allows the analysing of the system's precise marginal stability or marginal instability condition when the roots are relocated on imaginary $j\omega$-axis of s-plane. The criterion proposed by the authors, in contrast to Routh criteria, simplifies the identification of maximum and minimum stability limits for any coefficient of the higher-order characteristic equation significantly. The derived in the paper stability boundary formulas for the polynomial coefficients are successfully used for the proportional integral derivative (PID) controller with single or multiple gains selections in closed-loop control systems.

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1. INTRODUCTION

The research on the stability of higher-order systems was initiated by Edward Routh and Adolf Hurwitz long ago, their theory is being used now by control experts while analysing the stability of dynamic systems and added to many books on control engineering [1-4]. It provides an effective tool for identifying stability condition dynamic system and roots of its system polynomial on the $j\omega$-axis of s-plane. Nevertheless, it does not provide an effective method for identifying precise stability limits of higher-order system operation analytically or numerically by mathematically analysing the coefficients of the system characteristic polynomial. Deriving analytical expressions based on the Routh array is a very tedious and lengthy process. It becomes a formidable task for systems with an order higher than four. Besides, for special cases of all zeros in an array raw, the use of standard Routh procedure does not provide a solution to the problem.

Some researchers have managed to solve specific system stability problems by using the Routh-Hurwitz criterion. In paper [5], the authors used the Hermite-Biehler theorem to derive the Routh-Hurwitz criterion and managed to capture the system's unstable root counting. While performing stability analysis, the Routh array may suffer some singularities. One example is when the first element of a row turns out to be zero. The solution to this case was discussed in some papers [5-7] and textbooks [1-4]. Some researchers have used the ϵ-method to solve the stability problem for the special case when there are zero leftmost elements together with an all-zero row in the Routh array [6]. A minor reconstruction of Routh's array is demonstrated in [7] to solve a particular case of leading elements in the array becoming zero. In reconstructed array, locations of a polynomial root are defined by means of considering first-column sign changes, similar to Routh's method, which eliminates the use of the ϵ-approach.

The singularity in the Routh array would occur in case of all elements in a row become zero. In [8], the authors have presented a solution for the roots of a polynomial in the right-half of s-plane and on the $j\omega$-axis for the case when a few row elements in the Routh array become zero. They have used the continued fraction approach to solve the problem. When a system parameter is of the ϵ-order, the advantage of the ϵ-method of the Routh-Hurwitz criterion for the zero rows was elaborated in [9]. In [10], authors have replaced zero row coefficients with the derivative of the polynomial corresponding to the row next to the zero-row to fill the row as an additional procedure and doing that they have managed to identify the polynomial roots located symmetrically on the right and left and on the $j\omega$-axis. [7].

Importantly, the Routh-Hurwitz criterion unable to determine the case of instability for the case of multiple roots on the $j\omega$-axis of the s-plane [2, 4, 11]. Routh array does not provide a solution for the number of multiple $j\omega$-axis roots unless solving it with the auxiliary polynomial. However, even the application of auxiliary procedure does not show sign change in the first column of Routh's array for some unstable systems that have repeated multiple roots on $j\omega$-axis and no roots of the system polynomials in the right half s-plane [11]. In [10], the authors are managed to count the number of roots on $j\omega$-axis that are complex polynomials. The authors in [12] have investigated possible relation between the multiplicity of $j\omega$-axis poles and the zero rows numbers in the Routh array. The main outcome was a proof that the existence of multiple zero rows in the Routh array is a source of instability of the system despite sign change in the first column. In paper [13], authors have aimed at the modelling of cyclic physical phenomenon and investigated harmonic oscillations of systems at the borders of stability regions. Stability boundary oscillations are used in many science and engineering applications [13]. The authors in [14, 15] conducted boundary locus analysis to achieve a stable control system design. The authors identified stability regions of controller coefficients based on a solution of characteristic equation in s domain ($s=j\omega$). In the research paper [13], the authors have identified the harmonic oscillation boundary of systems by matching the roots of the characteristic polynomial with amplitude-angle ($M - \theta$) plane and representing roots of the polynomial as $\lambda = Me^{j\theta}$.

Another common method of n-th order systems stability studies is related to analysing numerical eigenvalues of n state equations [16, 17]. However, it does not simplify the solution of the problem for the n-th order system, the dimensions of a matrix of eigenvalues and matrix A, i.e. (λI-A), are of the same n-th order. Therefore, the level of complexity of stability problem solution is the same as to look into the roots of the original n-th order system characteristic polynomial. In other words, it requires calculation numerically the roots λ of n-th order polynomial to verify the stability of a given system. Therefore the analytical solution of the problem is not possible. The new theory of stability was initially introduced in [18] and successfully used to identify the boundary conditions analytically for up to sixth order systems. The Laplace transform of polynomial equation is introduced, and the manipulation of signals and systems in terms of stability in the Laplace domain explained [19]. Another work presents simple tools to quickly determine whether a given system is stable, and to determine the value range of coefficients [20]. Global asymptotic stability of the equilibrium point of a delayed system given by a higher-order delayed differential equation of retarded type with several time-varying delays is exaplined [21]. A higher-order shear deformation theory is used to determine the stability of elastic plates in [22]. Stability boundaries and lateral posture control is disceribed in [23].

The literature review has shown that so far there is no any systematic and exact solution for stability problem of linear higher-order dynamic systems that can identify exact stability boundaries of system behaviour through the coefficients of its polynomial equation and doing that is able thoroughly to analyse and differentiate marginal stability or instability of systems at the boundary regions of stability. The importance of such theory could also contribute to closed-loop controllers design and selection of gains for the controller of dynamic systems. The closed-loop controller gains are part of the system characteristic polynomial coefficients and, therefore, stability limits of the coefficients can be used, in turn, to identify stability limits for the gains. The method described in this paper aims to solve these problems. Besides, it can precisely define the number and types of conjugate $j\omega$-axis roots on the s-plane while the dynamic system is at the stability boundary region and their influence on marginal stability or instability for some special cases of zero coefficients.

In the current paper in section 2 presents a completely modified and simpler approach for identifying stability of higher-order time-invariant linear dynamic systems with only two polynomials as an alternative to the renowned Routh-Hurwitz criterion and any other method. The discovered criterion and algorithms for system stability are new and have never been published in relation to the stability control of dynamic systems. The algorithms in this paper have been developed intuitively based on certain systematic relations of the coefficients of system characteristic polynomial at the boundaries of stability. The algorithms are successfully applied to various types of higher-order dynamic systems as well as polynomials of some selected engineering applications with closed-loop controllers. The presented algorithms are essential tools to identify marginal stability or instability of the systems for the case of multiple roots of the polynomials on *jω*-axis of the *s*-plane. Section 3 demonstrates the use of the developed theory for defining stability limits for single and multiple gains of closed-loop controllers for various engineering systems with higher-order dynamic models. This method is successfully tested on the model of a hard disk drive with a single-gain lead compensator [24] and the model of a two-inertia system with multiple gain controller design [25].

2. RESEARCH METHOD

2.1. General stability criteria

In general, the characteristic polynomial for the higher-order dynamic system can be presented as follows:

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2}\, s^{n-2} + \dots + a_1 s + a_0 \ = 0 \tag{1}$$

One of the conditions of possible stability is that all the coefficients of the polynomial must be positive real numbers [23]. However, positive values of the coefficients alone do not provide the stability of the system. The current paper presents stability criteria of the higher-order systems with all positive values coefficients as well as when some coefficients have zero values, which leads to special cases of marginal stability or instability. The general necessary stability criteria for any *n*-order dynamic system (where $n \geq 3$) can be solely expressed by the set of two nonlinear (2) or (3) with the introduction of an additional unknown variable k that couples both equations together. If the system order *n* is an odd number, then two equations are presented, as follows:

$$\begin{aligned} a_n &= \quad (a_{n-2} - (a_{n-4} - \dots - (a_3 - a_1 k)k) \dots)k \\ a_{n-1} &= (a_{n-3} - (a_{n-5} - \dots - (a_2 - a_0 k)k) \dots)k \end{aligned} \tag{2}$$

If the highest order of the system *n* is an *even* number, then two equations are presented differently, as follows:

$$\begin{aligned} a_n &= \quad (a_{n-2} - (a_{n-4} - \dots - (a_2 - a_0 k)k) \dots)k \\ a_{n-1} &= (a_{n-3} - (a_{n-5} - \dots - (a_3 - a_1 k)k) \dots)k \end{aligned} \tag{3}$$

It can be seen from (2) and (3) that unknown parameter *k* must be a real positive number to ensure that coefficients a_n and a_{n-1} are positive real numbers, which is an obvious stability condition for the system. The fundamental law of marginal or boundary stability of any dynamic system with order n ≥3 is stated as follows: "if (2) or (3) are satisfied and there exists a solution of these equations with at least one common *k* as a positive real root, then all the coefficients in (1) are having stability boundary values and the system under consideration is in the state of marginal or boundary stability condition". At this stage, some of the roots of characteristic polynomial (1) form conjugate pairs and strictly located on the imaginary *jω*-axis of the *s*-plane. Therefore, (2) or (3) represent the necessary and sufficient criteria to define accurately stability boundary value for all the coefficient of systems with characteristic polynomial order n ≥3, provided algebraic (2), or (3) have at least one common positive real solution for *k*. In other words, if conditions (2) or (3) satisfy, then the dynamic system is in the state of marginal stability or instability, i.e., it is precisely in between the stable and unstable zones of behaviour. The boundary values for the coefficients of the *n*-th order system (1) can be obtained by mathematically excluding unknown *k* from both (2) or (3). The newly developed (2) or (3) have no analogy to any stability criteria shown so far in the literature. The relationship between the coefficients of the characteristic polynomial at the state of system stability boundary regions has been discovered intuitively. Still, it can be verified by any other method that describes stability boundary conditions for a dynamic system.

3. RESULTS AND ANALYSIS

3.1. Stability range for the closed-loop control systems

In (4), *R(s)* is the input signal, *Y(s)* is the output signal, *H(s)* is the feedback signal, *G(s)* is the plant model (system under observation), and *K(s)* is the controller model. In (2) or (3) can be successfully applied to identify stability ranges for the gains of the closed-loop control system (4). The s-domain transfer function for the closed-loop control system can be expressed as follows:

$$\frac{Y(s)}{R(s)} = \frac{G(s)K(s)}{1+G(s)K(s)H(s)} \quad (4)$$

3.1.1. Case of single gain controller design

The stability analysis of a system with a single gain controller can be demonstrated on the model of a hard disk drive with the lead compensator. The plant model of the hard disk drive system can be expressed as follows [19]:

$$G(s) = A/B \quad (5)$$

$$A = n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0,$$

$$B = d_{10} s^{10} + d_9 s^9 + d_8 s^8 + \cdots + d_4 s^4 + d_3 s^3 + d_2 s^2,$$

where:

$$n_4 = 1.197 \cdot 10^{26},\ n_3 = 2.12 \cdot 10^{29}, n_2 = 5.826 \cdot 10^{34},$$

$$n_1 = 4.366 \cdot 10^{37},\ n_0 = 6.189 \cdot 10^{42},\ d_{10} = 1,\ d_9 = 5336,$$

$$d_8 = 4.124 \cdot 10^{9},\ d_7 = 1.302 \cdot 10^{13},\ d_6 = 4.216 \cdot 10^{18},$$

$$d_5 = 6.72 \cdot 10^{21},\ d_4 = 1.198 \cdot 10^{27},\ d_3 = 7.496 \cdot 10^{29},$$

$$d_2 = 9.668 \cdot 10^{34}.$$

The lead compensator with a proportional gain k_p can be presented as follows:

$$K(s) = k_p(4s+2)/(s+2) \quad (6)$$

Substituting (5), (6) into (4) and assuming $H(s) = 1$, yields the following close-loop system characteristic polynomial of 11th order dynamic system, where:

$$a_{11} = d_{10}, a_{10} = d_9 + 2d_{10},\ a_9 = d_8 + 2d_9,$$

$$a_8 = d_7 + 2d_8, a_7 = d_6 + 12d_7, a_6 = d_5 + 2d_6,$$

$$a_5 = d_4 + 2d_5 + 4k_p n_4, a_4 = d_3 + 2d_4 + 2k_p(2n_3 + n_4),$$

$$a_3 = d_2 + 2d_3 + 2k_p(2n_2 + n_3),$$

$$a_2 = 2d_2 + 2k_p(2n_1 + n_2), a_1 = 2k_p(2n_0 + n_1),$$

$$a_0 = 2k_p n_0.$$

For the eleventh (odd) order characteristic polynomial, two stability boundary polynomials can be presented as (2). By substituting all the coefficients into (2) and dividing one by another, the proportional gain k_p can be excluded from the resulting single algebraic 6th order stability boundary equation with variable k as follows:

$$p_6 k^6 + p_5 k^5 + p_4 k^4 + p_3 k^3 + p_2 k^2 + p_1 k + p_0, \quad (7)$$

where coefficients are functions of only given constant parameters.

The solution of (7) yields four real and two complex k roots. In accordance with rules of stability, only real roots of (7) could be considered for the marginal stability of the closed-loop system. Four real roots are $0.4912*10^{-6}$, $0.0139*10^{-6}$, $0.0077*10^{-6}$, $0.0006*10^{-6}$. Value of $k_p$at the state of marginal stability can be calculated from (32) and presented as follows:

k_p= C/D, where (8)

$$C = dd_{11} - dd_9 k + dd_7 k^2 - dd_5 k^3 + dd_3 k^4 - dd_1 k^5,$$

$D = nn_5 k^3 - nn_3 k^4 + nn_1 k^5$.

Substituting four real roots of (7) into (8) yield three positives and one negative values of k_p. Negative value leads to instability of the system because of the coefficient a_0 of the system is directly proportional to k_p, i.e. $a_0 = 2k_p n_0$, and cannot be negative. As a result, the minimum stability limit for the k_p is zero, i.e. $k_{min} = 0$. The remaining three calculated positive value for k_p are 0.0079, 0.2119, 0.1726. Solving for the roots of eleventh order characteristic polynomial for these three values of k_p yields a pair of roots located on the imaginary axis of s-plane $\pm j0.1427 * 10^4$, $\pm j0.8488 * 10^4$, $\pm j1.1411 * 10^4$, respectively. The analysis of all solutions shows that only one gain value k_{pmax} =0.0079 corresponds to the marginal stability condition of the closed-loop system, where all the roots located at the left half of the s-plane.

3.1.2. Case of multiple gain controller design

The advantage of applying (2) and (3) for stability analysis of higher-order closed-loop dynamic systems can be demonstrated for the case of applying multiple gain controllers to the system. The criteria (2) and (3) were tested on the example of the model of a two-inertia system with a proportional-differential (PD) controller. The plant model of such a two-inertia system can be expressed as follows [20]:

$$G(s) = n_0/(\, d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0), \text{ where:} \tag{9}$$

$n_0 = 0.0625$, $d_4 = 1$, $d_3 = 2$, $d_2 = 1.5$, $d_1 = 0.5$,

$d_0 = 0.0625$.

Substituting (9), $K(s) = k_p + sk_d$ into (4) and assuming $H(s) = 1$ yields the following fourth-order characteristic polynomial of the closed-loop system:

$$d_4 s^4 + d_3 s^3 + d_2 s^2 + (d_1 + n_0 k_d)s + (d_0 + n_0 k_p) = 0 \tag{10}$$

The two stability boundary polynomials (3) for the characteristic polynomial (10.45) can be presented as follows:

$$d_4 = kd_2 - k^2(d_0 + n_0 k_p) \tag{11}$$

$$d_3 = k(d_1 + n_0 k_d) \tag{12}$$

By dividing (11) by (12), the following expression for k_d can be derived:

$$k_d = [d_2 d_3 - d_3(d_0 - n_0 k_p)k - d_1 d_4]/n_0 d_4 \tag{13}$$

Substituting (13) into (12) yields the following quadratic equation:

$$(d_0 d_3 + d_3 n_0 k_p)k^2 - d_2 d_3 k + d_3 d_4 = 0 \tag{14}$$

The solution of (14) can be presented as follows:

$$k = [d_2 \pm \sqrt{d_2^2 - 4d_4(d_0 + n_0 k_p)}]/2(d_0 + n_0 k_p) \tag{15}$$

The stability boundary is achieved when the expression under the square root is equal zero, and the solution of (15) yields a single positive answer for k (stability rule). As a result, at the stability boundary condition for the system the expression for a maximum limit of k_p can be derived from (15) as follows:

$$k_p^{max} = (d_2^2 - 4d_0 d_4)/(4d_4 n_0) \tag{16}$$

The minimum limit of k_p can be obtained from the condition that for a stable system, all the coefficients of characteristic polynomial (10) must be positive. Therefore, the coefficient $d_0 + n_0 k_p$ must have a positive value and the minimum value for k_p can be calculated as follows:

$$k_p^{min} = -d_0/n_0 \tag{17}$$

To provide absolute stability of the closed-loop system, the following condition for k_p must be provided:

$$k_p^{min} < k_p < \ k_p^{max} \tag{18}$$

For any value of k_p within limits (18), two values for ss are be calculated from (15) and subsequently two corresponding limit values for k_d can be calculated from (13). An additional condition for the system stability is that the minimum limit for k_d must be more than one calculated from the corresponding coefficient of the system characteristic polynomial, i.e.

$$k_d^{min} > -d_1/n_0. \tag{19}$$

Using all the stability conditions (13), (15) to (19), the following graph of a function $k_d = f(k_p)$ for the boundary values can be obtained, as shown in Figure 1. For all the boundary values of the system gains, the solution of the characteristic polynomial (10) yields one pair of conjugate roots at the imaginary axis of s-plane, i.e., the system is at the condition of marginal stability.

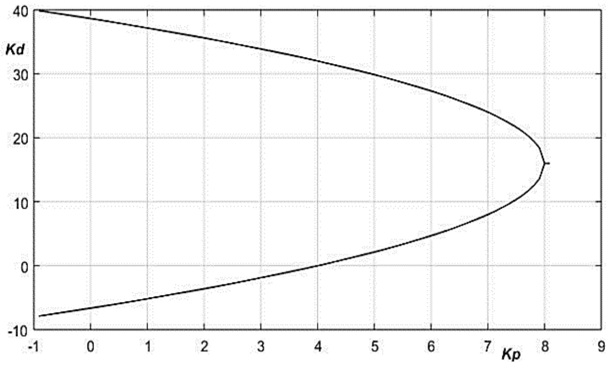

Figure 1. Stability boundary curves for $k_d = f(k_p)$

Figure 2 shows the region of absolute stability of the system that lies in between the upper and lower lines of the graph. The highest range of stability is at $k_p^{min} = -1$, where -8 $< k_d < 40$. At k_p^{max} = 8, the stability region is reduced to a single value k_d =16. In case of applying PID controller $K(s) = k_p + sk_d + k_i/s$ to the model of the two-inertia system [19], the following fifth-order characteristic equation can be obtained:

$$d_4s^5 + d_3s^4 + d_2s^3 + (d_1 + n_0k_d)s^2 + (d_0 + n_0k_p)s + +n_0k_i = 0 \tag{20}$$

Two stability boundary polynomials (2) for the characteristic polynomial (20.55) can be presented as follows:

$$d_4 = kd_2 - k^2(d_0 + n_0k_p), \tag{21}$$

$$d_3 = k(d_1 + n_0k_d) - k^2n_0k_i \tag{22}$$

By dividing (21) by (22), the following formula for k_p can be obtained:

$$k_p = (d_2d_3 - d_1d_4 - n_0d_4k_d)/(n_0d_3k) - \tag{23}$$

$$-(d_0d_3 - n_0d_4k_i)/(n_0d_3)$$

Substituting (23) into (21) yields the following quadratic equation:

$$(n_0k_i)k^2 - (d_1 + n_0k_d)k + d_3 = 0 \tag{24}$$

The solution of (24) can be presented as follows:

$$k = [d_1 + n_0k_d \pm \sqrt{(d_1 + n_0k_d)^2 - 4d_3n_0k_i}]/(2n_0k_i) \tag{25}$$

The stability boundary is achieved when the expression of square root in (25) is equal zero, and the solution of (25) yields a single positive answer for k (stability rule). This condition yields the following boundary equation for k_d:

$$(n_0^2)k_d^2 + (2n_0d_1)k_d + d_1^2 - 4n_0d_3k_i = 0 \quad (26)$$

The solution of (26) yields the boundary equation for k_d as follows:

$$k_d = -d_1 \pm 2\sqrt{d_3n_0k_i} \quad (27)$$

A stability boundary is achieved when the expression of square root in (27) is equal to zero, i.e., when $k_i = 0$. Therefore, for the absolute stability of the closed-loop system, the following condition must be satisfied:

$$k_i > 0 \quad (28)$$

For any value $k_i > 0$, formula (27) yields two limiting values for k_d. The additional condition for the system stability is that the minimum limit for k_d must be more than one calculated from the corresponding coefficient of the system characteristic polynomial, i.e.

$$k_d^{min} > -d_1/n_0. \quad (29)$$

By substituting the two limiting values of k_d into (25) and subsequently into (23), the remaining two limiting values for k_p can be obtained. An additional condition for the system stability is that the minimum limit for k_p must be more than one calculated from the corresponding coefficient of the system characteristic polynomial, i.e.

$$k_p^{min} > -d_0/n_0. \quad (30)$$

Using all the stability conditions (23), (25), (27) to (30), the following 3D graph of function $k_p = f(k_d, k_i)$ for the boundary lines of k_p, k_d gains versus a few values of k_i is shown in Figure 2. The absolute stability of the system is confined within the space outlined by the limiting values of three gains. Figure 3 shows the only 2D view of the lines shown in Figure 2. The maximum values for k_p, k_d, k_i gains are defined by the terminal condition when $k_p^{min} = k_p^{max}$ for raising in steps values of k_i (28) and is calculated on MATLAB software. Increasing k_i reduces that stability range of the system, i.e., stability ranges for the other two gains. For all the boundary values of the system gains the solution of the characteristic polynomial (55) yields one pair of conjugate roots at the imaginary axis of s-plane, i.e., the system is at the condition of marginal stability. An exception is for the points where $k_p^{min} = k_p^{max}$. Figure 4 shows a 2D graph of $k_p = f(k_d)$ for a single value k_i=0.

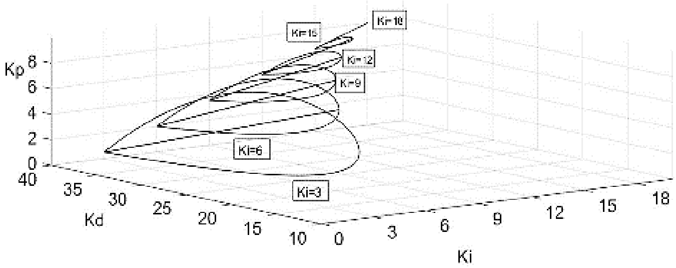

Figure 2. 3D Stability boundary curves for $k_p = f(k_d, k_i)$

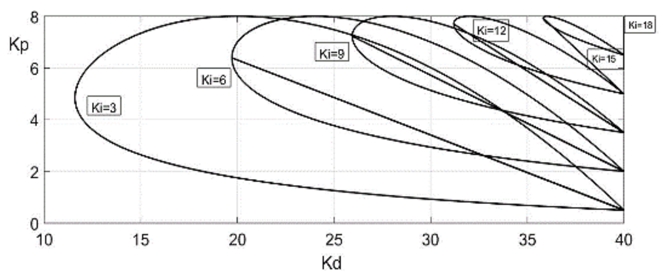

Figure 3. 2D Stability curves for $k_p = f(k_d, k_i)$

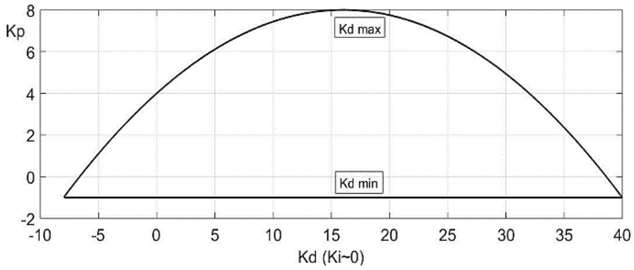

Figure 4. 2D Stability boundary curves for $k_p = f(k_d)$, $k_i = 0$

If $k_i = 0$, then $k_d^{min} = -8 < k_d < 40$ and $k_p^{min} = -1 < k_p < 8$ as shown in Figure 4. At the left intersection of lines ($k_d^{min} = -8$ and $k_p^{min} = -1$), the roots of the closed-loop system are: -1.0000+0.7071i; -1.0000-0.7071i; 0.0000+0.0000i; -0.0000-0.0000i; -0.0000-0.0000i. At the right intersection of lines ($k_d^{max} = 40$ and $k_p^{min} = -1$), the roots of the closed-loop system are: -2+0.0000i; -0.0000+1.2246i; -0.0000-1.2246i; -0.0000+0.0000i; -0.0000+0.0000i. When k_i reaches its maximum value ($k_i^{max} = 18$), the plots on Figure 2 and Figure 3 are converged to a single point and other gains reach their single maximum values, i.e. $k_p^{max} = 8$, $k_d^{max} = 40$. The roots of the system characteristic polynomial at this point are -2.0000+0.0000i; -0.0000+0.8660i; -0.0000 -0.8660i; 0.0000+0.8660i; 0.0000-0.8660i, i.e., the system has double conjugate roots on imaginary axis of s-plane.

4. CONCLUSION

The paper presents an effective and simple tool for the analytical solution of the stability problem of higher-order linear time-invariant dynamic systems. It has a significant advantage compared to the Routh-Hurwitz technique. The proposed universal stability criteria (2) or (3) establish unique relations between the stability boundary values of the system characteristic polynomial coefficients and the newly introduced additional parameter k. It is a new approach, and there are no similarities found to these criteria in the literature. The newly-developed method is a universal one and can be applied to any higher-order dynamic system. The authors of this paper have discovered and established a set of general expressions (2) or (3) that can be applied for derivation of necessary stability criteria for any order linear time-invariant dynamic system. These results are new and have not been published currently in the literature and were obtained for special cases of marginal stability when the same exact set of zero coefficients the system can be either in the state of marginal stability or marginal instability, i.e., the system exhibits a dual behaviour. Section 3 is dedicated to the use of criteria (2) and (3) to provide marginal and absolute stability for the closed-loop control systems with proportional, derivative, and integral gains. The paper discusses in detail the derivation of equations for precise stability boundary values of k_p, k_d, k_i gains based on the two-polynomial criteria (2) and (3). The obtained results of the analytical calculation of precision stability boundary values for a multiple-gain higher-order closed-loop system do not have an analogy currently in the control theory. The results obtained in this paper prove that the developed system stability criteria or algorithm for stability analysis of a higher-order linear dynamic system is a step forward in analysing stability conditions of complex dynamic systems and deriving precise analytical expressions for multiple gains of closed-loop control systems.

REFERENCES

[1] N. S. Nise, "Transient response stability," in *Control Syst. Eng.*, 5th Int. Student ed. Singapore: Wiley, chapter. 6, sec. 2–4, pp. 291–306, 2008.
[2] R. C. Dorf and R. H. Bishop, "The stability of linear feedback systems," in *Modern Control Systems*, 12th ed. Prentice-Hall, chapter. 6, sec. 2, pp. 391–398, 2011.
[3] M. Gopal, "Concept of stability and the Ruth Stability Criterion", in *Control Systems, Principles and Design*, 2nd ed. McGraw Hill, chapter. 5, sec. 4, pp. 314-326, 2003.
[4] B. Kuo and M. F. Golnaraghi, "Stability of linear control systems," in *Automatic Control Systems*, 7th ed. Englewood Cliffs, NJ: Prentice-Hall, chapter 6, sec. 5, pp. 334–343, 1995.
[5] M. Ho, A. Datta, and S. P. Bhattacharyya, "An elementary derivation of the Routh-Hurwitz criterion," *IEEE Trans. Autom. Control*, vol. 43, no. 3, pp. 405-409, March 1998.
[6] M. M. Fahmy and J. O'Reilly, "A note on the Routh-Hurwitz test," *IEEE Trans. Autom. Control*, vol. AC-27, no. 2, pp. 483-485, April 1982.
[7] K. S. Yeung, "Routh-Hurwitz test under vanishing leading array elements," *IEEE Trans. Autom. Control*, vol. AC-28, no. 1, pp. 104-106, January 1983.

[8] K. Khatwani, "On Routh-Hurwitz criterion," *IEEE Transactions on Automatic Control*, vol. 26, no. 2, pp. 583-584, 1981.
[9] S. Pillai, "The ε method of the Routh-Hurwitz criterion," *IEEE Transactions on Automatic Control*, vol. 26, no. 2, pp. 584-584, April 1981.
[10] S. S. Chen and J. S. H. Tsai, "On the singular cases of the complex Routh algorithm for stability tests," *IMA J. Math. Contr. Inform.*, vol. 10, pp. 71–82, March 1993.
[11] R. N. Clark, "The Routh-Hurwitz stability criterion, revisited," *IEEE Control Systems*, vol. 12, no. 3, pp. 119-120, 1992.
[12] M. A. Choghadi and H. A. Talebi, "The routh-hurwitz stability criterion, revisited: the case of multiple poles on imaginary axis*", IEEE Transactions on Automatic Control*, vol. 58, no. 7, pp. 1866-1869, July 2013.
[13] B. B. Alagoz, "On the harmonic oscillation of high-order linear time-invariant systems*", Computer Science, Discrete Mathematics, arXiv*, pp.1-12, 2014.
[14] N. Tan, "Computation of Stabilizing PI and PID controllers for processes with time delay", *ISA Transactions*, vol. 44, no. 2, pp 213-223, April 2005.
[15] T. V. Moghaddam and Abbasi Y. "Tuning a fractional order PD and PID controller with lead compensator for integrating time-delay systems", *Journal of Electrical and Control Engineering*, vol. 2, pp 34-41, 2014.
[16] J. Chen, P. Fu, "Stability analysis of polynomially dependent systems by eigenvalue perturbation", *IEEE Transactions on Automatic Control*, vol. 62, issue 11, pp. 5915-5922, 2017.
[17] L. Wang, B. K. P. Horn, G. Strang, "Eigenvalue and eigenvector analysis of stability for a line of traffic", *Studies in Applied Mathematics Journal*, vol. 138, no. 1, pp. 103-132, September 2016.
[18] N. Mir-Nasiri, M. H. Ali, "A new algorithm to control dynamic stability of higher-order systems", *Proceedings of IEEE International Conference on Control Systems, Computing and Engineering*, November 2018.
[19] Mark A. Haidekker, "Solving Differential Equations in the Laplace Domain, Linear Feedback Controls," *Elsevier*, pp. 27-56, 2013.
[20] Mark A. Haidekker, "Stability Analysis for Linear Systems, Linear Feedback Controls," *Elsevier*, pp. 139-148, 2013.
[21] Takashi Amemiya, "Delay-independent stability of higher-order systems," *International Journal of Control*, vol. 50, no. 1, pp. 139-149, 1989.
[22] J. N. Reddy, N. D. Phan, "Stability and vibration of isotropic, orthotropic and laminated plates according to a higher-order shear deformation theory," *Journal of Sound and Vibration*, vol. 98, no. 2, pp. 157-170, January 1985.
[23] Erwin E. H., *et al*., "Stability boundaries and lateral posture control in parkinson disease, *Motor control*, vol.5, no. 3, pp. 254-269, August 2001.
[24] A. Nath, S. Kaitwanidvilai, "High-performance HDD servo system using GA based fixed structure robust loop Shaping control," *Proceedings of IEEE International Conference on Robotics and Biomimetics (ROBIO)*, pp. 1854-1859, January 2010.
[25] G. Zhang, J. Furusho, "Speed control of two-inertia system by PI/PID control", *IEEE Transactions on Industrial Electronics*, vol. 47, no 3, pp. 603-609, July 2000.

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