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Multiphase flow in a porous medium using Lattice Boltzmann Method and grid verification

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Related Publications

- Ashirbekov, A., Kabdenova, B., Monaco, E., & Rojas-Solórzano, L. R. (2021). Equation of State's Crossover Enhancement of Pseudopotential Lattice Boltzmann Modeling of CO2 Flow in Homogeneous Porous Media. *Fluids*, 6(12), 434.
- In Progress
  - Numerical Study of the Effect of Viscosity Ratio on the CO2 Injection Through a Homogeneous Aquifer Using a Crossover-EoS Pseudopotential Lattice Boltzmann Model

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Outline

1. Motivation
2. Aims and objectives
3. Lattice Boltzmann Method, its principles and governing equations
4. Method for grid verification
5. Validation of formulation
6. Domain verification; Results of multiphase analysis
7. Conclusion

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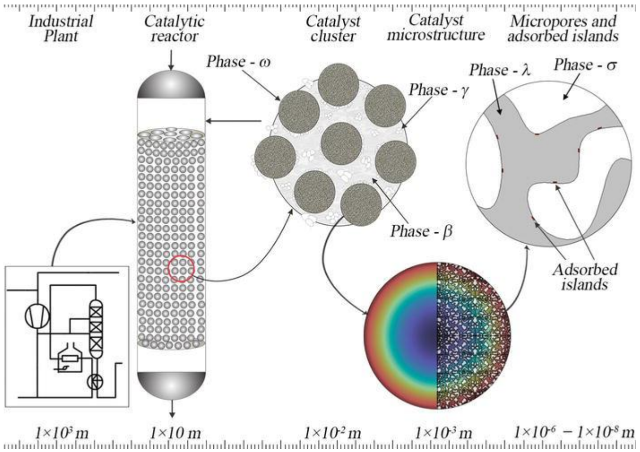
Motivation

- Fluid flows with multiscale and multiphase phenomena
- Energy systems: power plants, fuel cells, generators, turbines
- Medicine: blood flow
- Geoscience: CO2 sequestration, oil recovery



Industrial plant as an example of multiscale process (Cordero et al., 2018)

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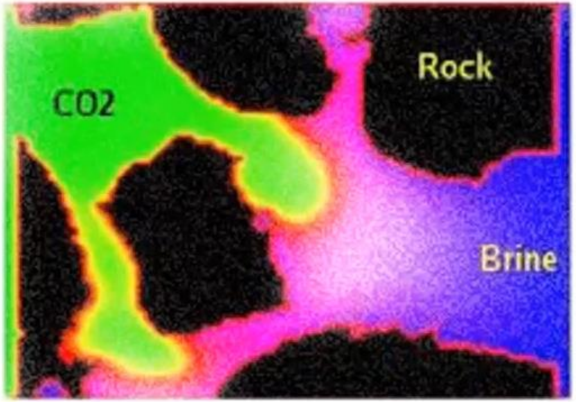
Motivation: CO_2 sequestration

- Long-term storage of excess carbon dioxide captured from the atmosphere
- One of the key strategies for reducing CO2 emission rates
- The global emission of CO2 alone can rise by 6.41 billion tonnes, 18% of 2021, by 2030 by estimations of EIA



Injection of CO_2 into the porous medium of water saturated oil

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Motivation: CO_2 sequestration modeling

- CO_2 sequestration involves modeling of porous medium, which is small scale and multiphase
- Is analyzed using different approaches and methods
- Lattice Boltzmann method was tried, however only with color-fluid model, and without discretization verification
- Hypothesis: pseudopotential LBM modeling, verified with domain size analysis, can give useful insight into mechanics of CO_2 sequestration

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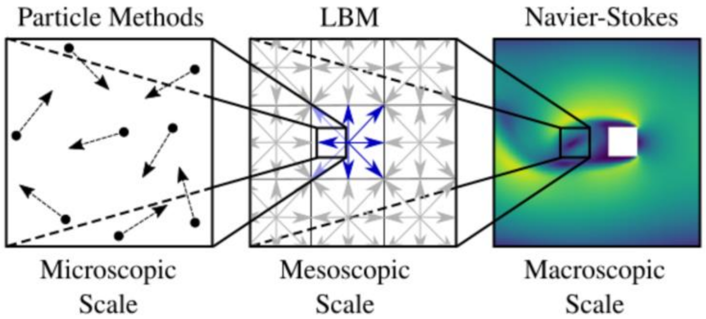
Aims and objectives

- Main goal: model the immiscible displacement flow in a porous medium and apply grid verification

1. Introducing the multi-component LBM formulation
2. To develop validation static case to confirm formulation stability
3. To develop and perform domain size verification procedure

Lattice Boltzmann Method

- Finite volume, the volume of fluid, and level-set methods - most used models, traditional FVM CFD
- Those methods are macroscopic, with assumption of fluid as a continuum
- Lattice Boltzmann Method (LBM) – mesoscale method
- Assumes fluid as a collection of particles



Lattice Boltzmann within scale framework (Bravo, 2019)

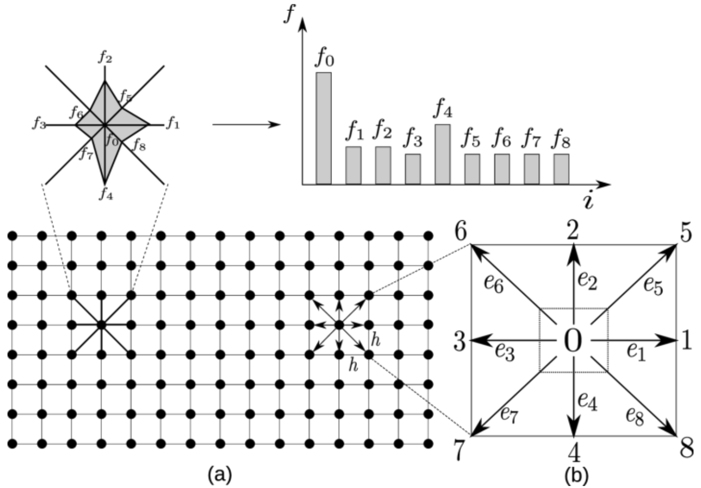
LBM Principles

- Particle interactions but focus on macroscopic behavior

- Fluid is treated as a collection of discrete particles on a uniform grid

- Based on microscopic models and kinetic theory

- Deals with interactions of particles



LBM lattice, probability function and directions, 2D case
("Lattice Boltzmann Method", 2021)

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Governing equations

$$f_i^{\,j}(\mathbf{x}+\mathbf{e_i}\Delta t, t+\Delta t) - f_i^{\,j}(\mathbf{x},t) = -\frac{1}{\tau^j}\left(f_i^{\,j}(\mathbf{x},t) - f_i^{\,j,eq}(\mathbf{x},t)\right)$$

streaming (LHS) and collision (RHS) steps of distribution function, derived from Boltzmann equation

$$f_i^{\,j,eq}(\mathbf{x},t) = \omega_i \rho^j \left[1 + \frac{\mathbf{e_i}\cdot\mathbf{u}^j}{c_s^2} + \frac{\left(\mathbf{e_i}\cdot\mathbf{u}^j\right)^2}{2c_s^4} - \frac{\left(\mathbf{u}^j\right)^2}{2c_s^2}\right]$$

Maxwell-Boltzmann equilibrium

$$\rho^j \mathbf{u}^j = \sum_i f_i^{\,j}\,\mathbf{e_i}$$

momentum – relation to physical density

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Governing equations (cont.)

$$\mathbf{F}_{int}{}^{j}(\mathbf{x},t) = -G(\mathbf{x},\acute{\mathbf{x}})\,\rho^{j}(\mathbf{x},t)\sum_{i}\omega_{i}\psi^{j}(\mathbf{x}+\mathbf{e_{i}}\Delta t,t)\,\mathbf{e_{i}}$$ interaction between components

$$\mathbf{F}_{wet}{}^{j}(\mathbf{x},t) = -g_{wall}{}^{j}\rho^{j}(\mathbf{x},t)\sum_{i}\omega_{i}s(\mathbf{x}+\mathbf{e_{i}})\mathbf{e_{i}}$$ interaction with obstacles

$$\boldsymbol{u}_{eq}{}^{j} = \mathbf{u}^{j} + \frac{\tau^{j}}{\rho^{j}}(\mathbf{F}_{int}+\mathbf{F}_{wet})^{j}\Delta t$$ forces effect transferring to distribution function

$$\psi^{j}(\mathbf{x},t) = \rho_{0} - exp(-\rho(\mathbf{x},t)/\rho_{0})$$ pseudopotential

$$v = c_{s}^{2}\sum_{j}\chi^{j}(\tau^{j}-0.5)$$ kinetic viscosity

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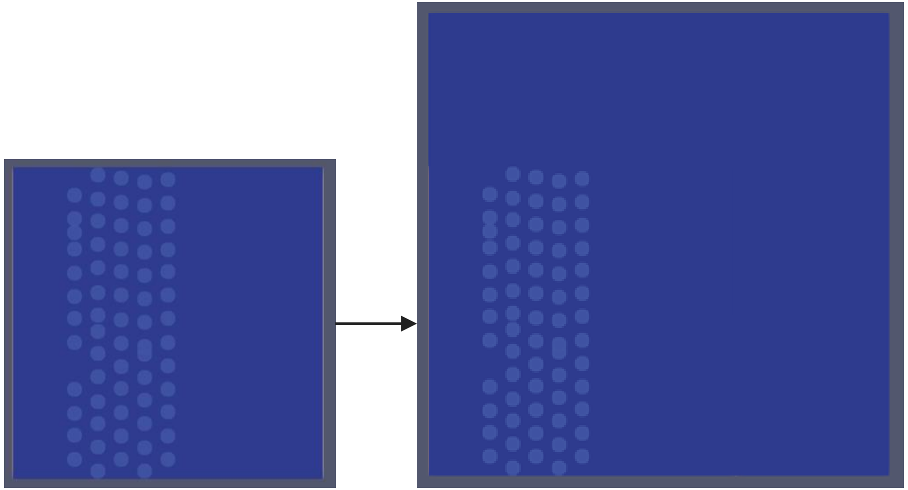
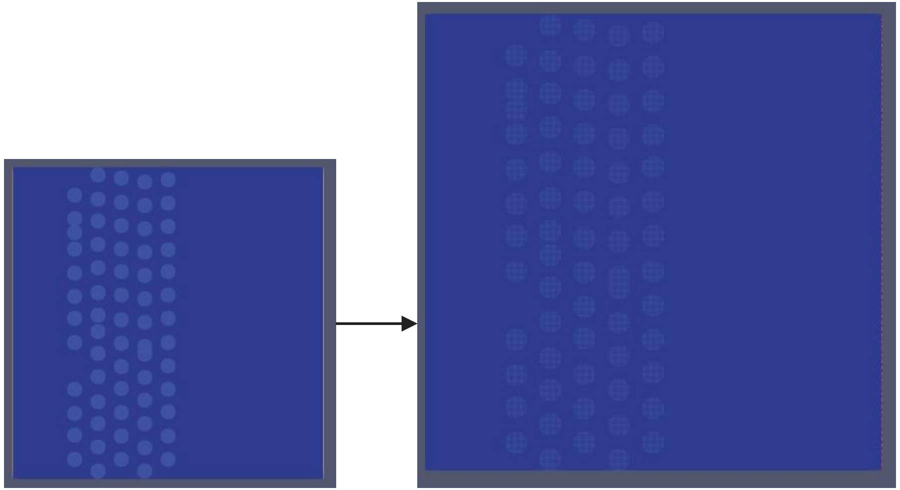
Grid verification

- Lack of studies to perform the proper domain verification
- In 3D typically done by increasing number of directions (D3Q15 → D3Q27)
- Not commonly applied in 2D, and is not trivial with lack of software capabilities

Grid verification – issues



Simple increase in domain size



Coordinate scaling

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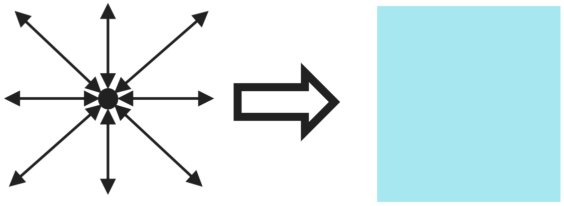
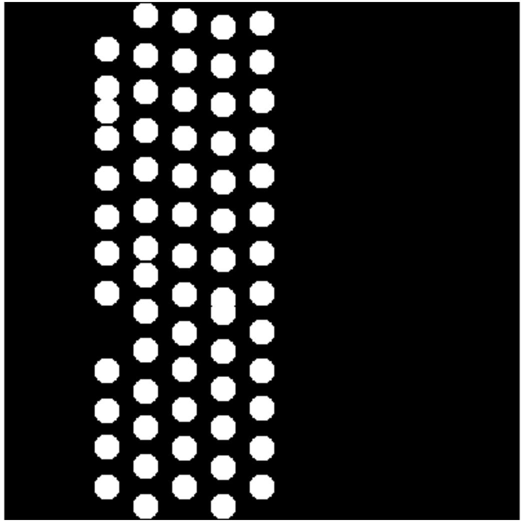
Grid verification – proposed solution



Particle $(\mathbf{x},\ i, v_x,\ v_y, v_z)$

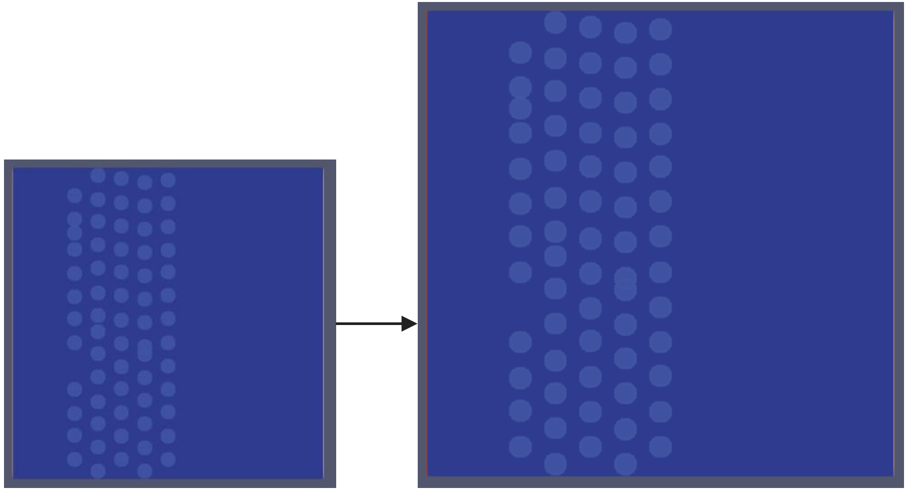
Pixel $(\mathbf{x}, \text{opacity}, R,\ G, B)$

- Conversion of the domain into pixelated image, each lattice point is one pixel
- Color and opacity may be used to decode state of lattice point (e.g. components, initial velocities)
- Perform scaling using image scaling techniques



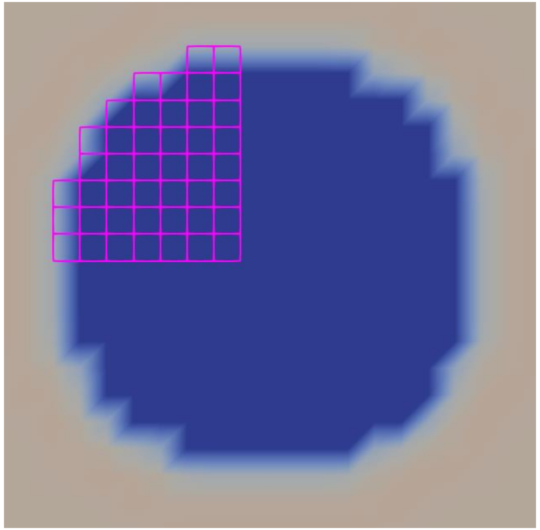
Domain converted to image

Grid verification – proposed solution

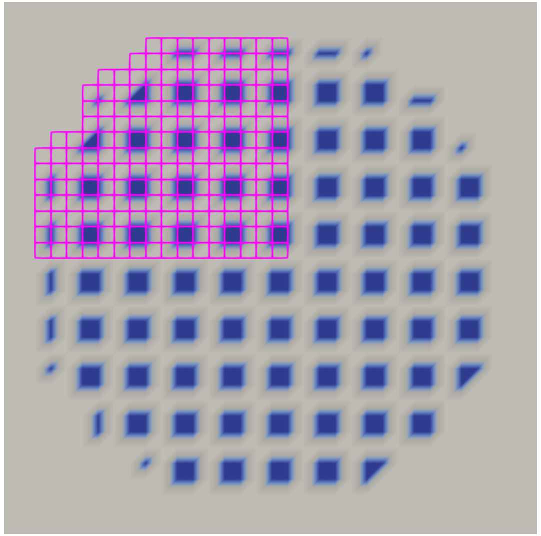


Scaling using image processing, nearest neighbor scaling

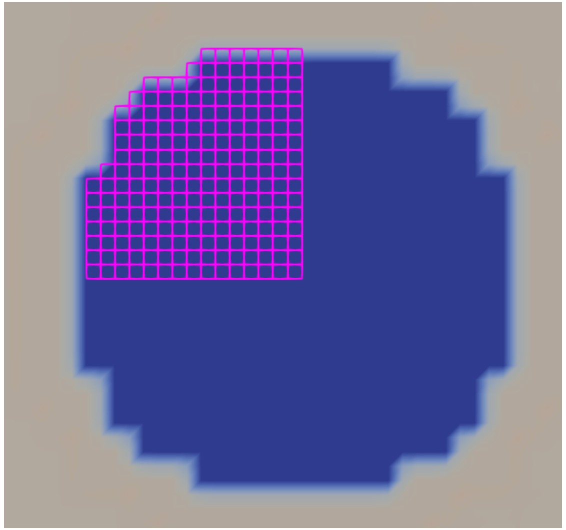
Grid verification



Original



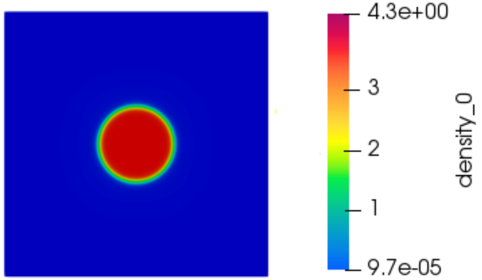
Coordinate scaling



Proposed solution

Water in red, CO_2 is in blue

Validation – droplet test

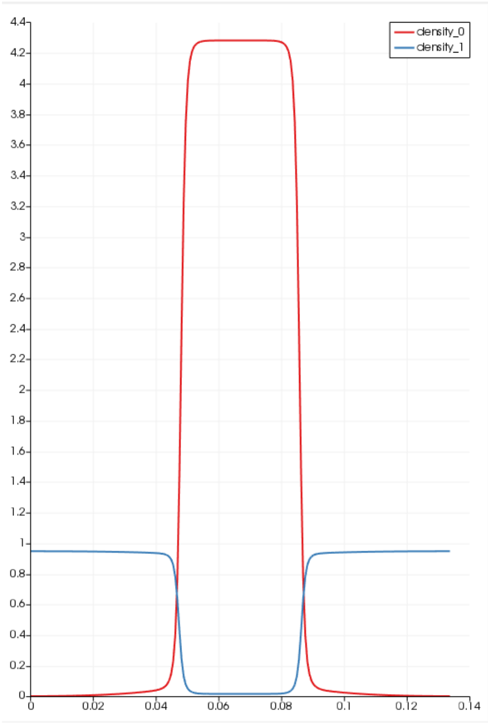


201×201 lattice unites

$g = 1$

$\rho_{water} = 4.3, \rho_{CO2} = 1$

$\tau_{water} = \tau_{CO2} = 1$

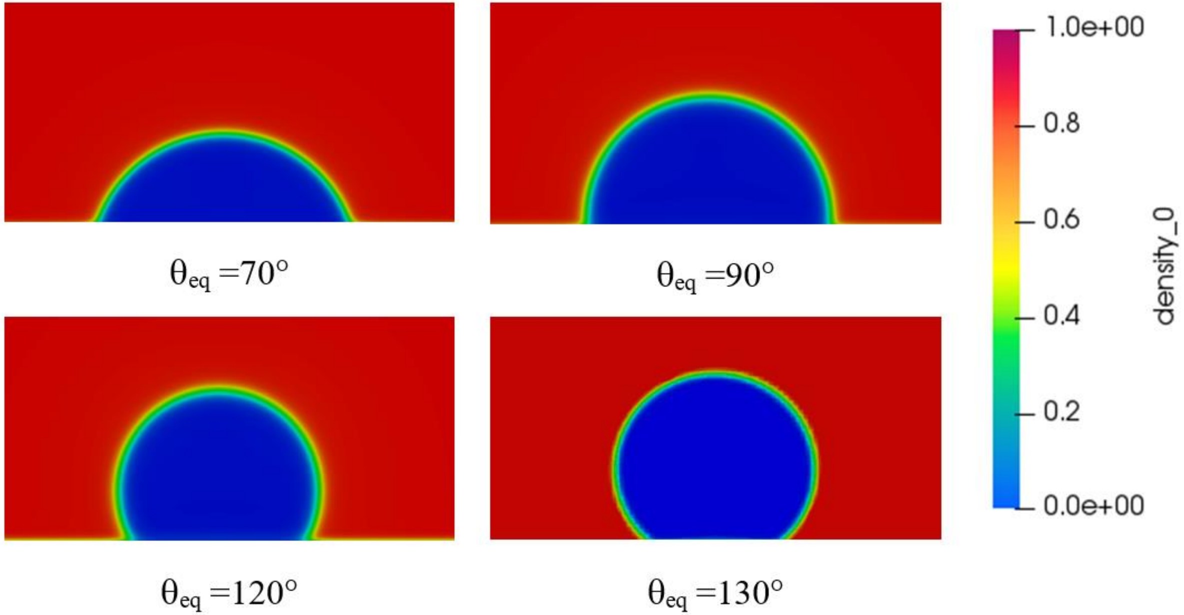


Density profile (in LU) at the cross-section of the domain taken at the horizontal

Water in red, CO_2 is in blue

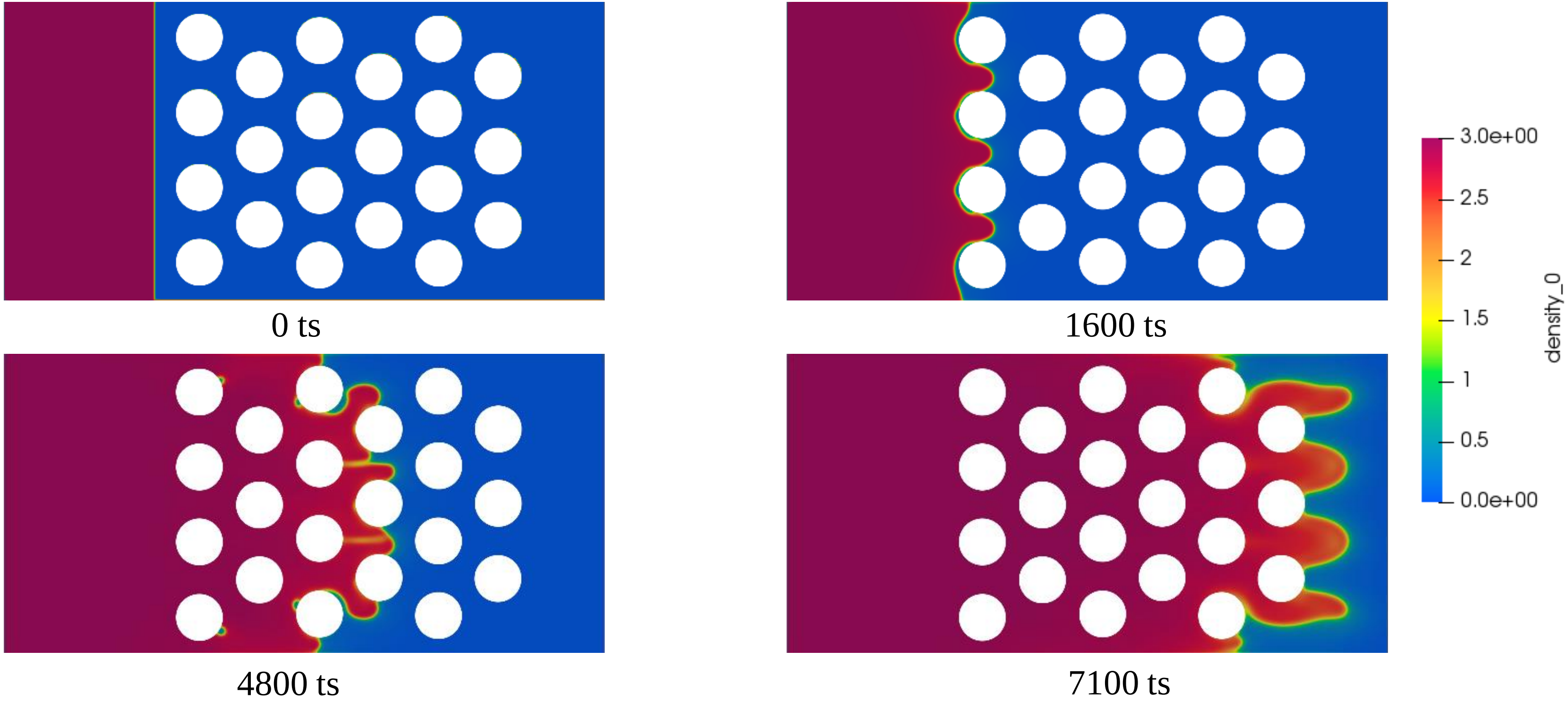
Validation – contact angle, wettability

θ_{eq}	$g_{CO_2,wall}$	$g_{H_2O,wall}$
70°	0.2	-0.2
90°	0	0
120°	-0.2	0.2
130°	-0.3	0.3

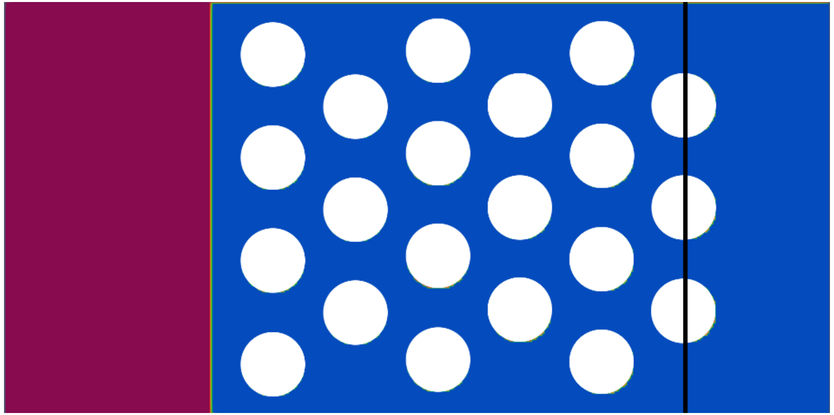


Porous medium model

Water in blue, CO_2 in red, $\theta_{eq} = 70°$
viscosity ratio of 1

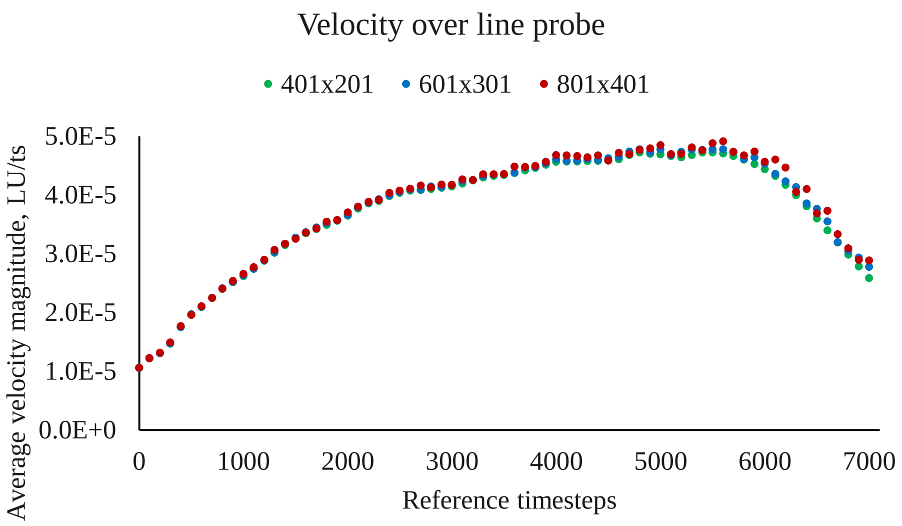


Domain size verification



Line probe location (shown in black) in the grid-independence analysis of CO_2 penetration LBM model

Domain size verification



Average velocity magnitude along the probe line versus timesteps

Grid size, LU^2	CO_2 flux, ×10^-7 LU/ts	Relative error (%)
401×201	3.590	-
601×301	3.651	1.7%
801×401	3.679	0.76%

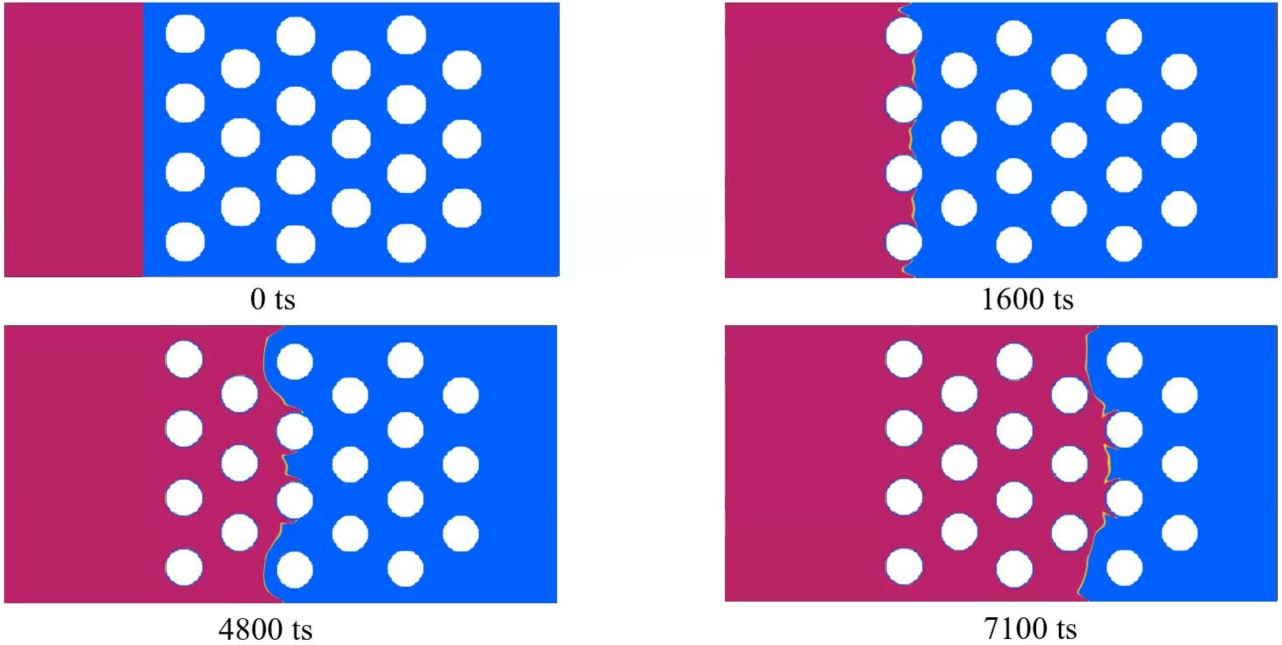
Time-space average CO_2 flux over a probe line integrated over 7100 timesteps

CO_2 sequestration conditions

Temperature (K)	308	318	328	338
Water viscosity (Pa · s)	7.4×10^{-4}	6.1×10^{-4}	5.2×10^{-4}	4.4×10^{-4}
CO_2 viscosity (Pa · s)	7.2×10^{-5}	6.2×10^{-5}	5.0×10^{-5}	4.2×10^{-5}
Water density (kg/m^3)	994	990	986	980
CO_2 density (kg/m^3)	815	735	645	535
Interfacial tension (mN/m)	36.0	34.5	33.4	32.7
log(Ca)	−3.40	−3.53	−3.73	−3.87
log(M)	−1.01	−0.99	−1.01	−1.02

LBM lattice, probability function and directions,
2D case
(Gooya et al., 2019)

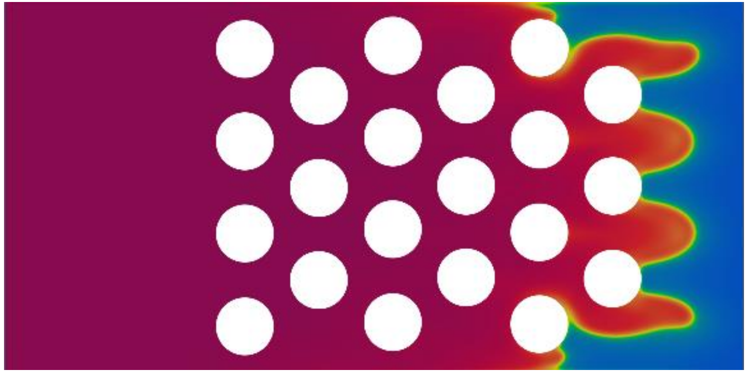
Porous medium models, viscosity ratio of 8.427



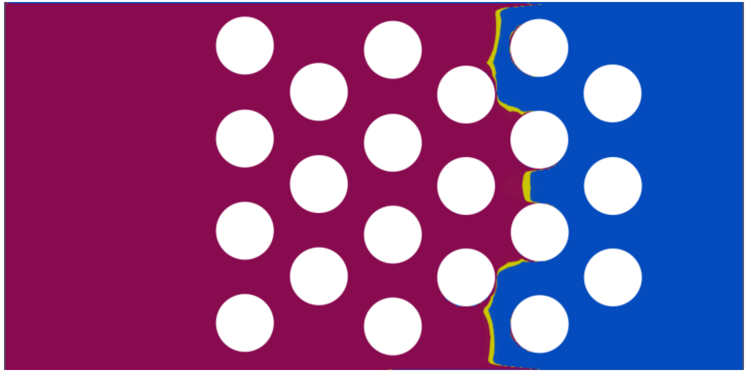
Water in blue, CO_2 in red

Water in blue, CO_2 in red

Porous medium models, comparison at same timestep



Viscosity ratio of 1



Viscosity ratio of 8.427

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Conclusion

- Pseudopotential LBM was applied to the problem of modeling CO_2 sequestration, achieving stable models and giving useful insight into parameters. Domain verification was applied and confirmed grid independence of the model.
    - Validation test using channel model
    - Porous medium model at 1:1 viscosity and 8.427:1 viscosity done
    - Grid verification performed

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Future work

- Capture more details of CO_2 and H_2O interaction featuring both density and viscosity ratios
- Adaption of Peng-Robinson and crossover EoS models, which capture more physics of fluids