Detecting the event of a single photon loss on quantum signals

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We design a scheme for detecting a single photon loss from multi-modal quantum signals transmitted via a fiber or in free space. This consists of a special type of unitary coding transformation, the parity controlled-squeezing, applied prior to the transmission on the signal composed by information and ancilla modes. At the receiver, the inverse unitary transformation is applied -decoding, and the ancilla mode is measured via photon detection. The outcome reveals whether a photon loss has occurred. Distortion of the information part of the signal caused by an ancilla photon loss can be corrected via unitary transformation while loss of a photon from the information part of the signal can be detected with the probability exponentially close to unity but cannot be corrected. In contrast to the schemes of decoherence free subspaces and quantum error correction protocols, this method allows one to make use in principle of entire Hilbert space dimensionality. We discuss possible ways of synthesizing the required encoding-decoding transformation.

I. INTRODUCTION

Photonic quantum information processing [1, 2] is a fast advancing domain with break-through promises in the area of communication security and computation. The main hindrance in the implementation of photonic schemes is the irreversible process of photon losses which may occur during propagation of a signal in free-space, fibers or integrated photonic circuits. As a consequence many works have been dedicated to this subject suggesting various elegant solutions. One class of schemes has been deployed aiming to the realization of noiseless amplification of quantum signals [3–8] in analogy to classical amplifiers. The most celebrated protection scheme against photon losses for distributing entangled pairs of photons over long distances is the one of quantum repeaters [9–11]. Finally, one main class of protection schemes extends the ideas of quantum error-correcting codes for qubits [12] to photonic modes [13–22].

In this work we propose an error-detection scheme for photonic signals and we focus on the single-photon losses that usually occur with a much larger probability compared the higher order events [14, 15, 23]. The task of error-detection is simpler than the one of error-correction and as consequence the restrictions on the encoding subspace of the signal, present in every quantum error-correcting algorithm, can be ultimately lifted. Concerning the resources required for the scheme's implementation. We assume as an extra resource a controllable quantum ensemble of atoms that acts as a mediator to the fields. The latter is of similar implementation challenge as the practical requirements in error-correcting codes [24].

Detection of accidental irreversible changes is well developed for classical signals [25] being one of the main problems of the coding theory. This task can be accomplished by encoding - adding to the main signal a logarithmically small fraction of complimentary information, so-called checksums, that allows one to primarily detect the eventual distortion of the transmitted information by decoding -verification that the check sums did not change during the propagation. In this work, we extend this idea to quantum optical signals and propose a way to detect an uncontrolled error of a single photon-loss from a multi-modal photonic information signal by performing measurements on a complimentary ancilla signal containing a number of photons comparable to the number of photons in the information system. This implies entangling quantum states of the signal and the ancilla at the encoding step and applying the inverse operation on the decoding step. After the inverse operation, due to entanglement, errors on the ancilla may reversibly modify the information signal, and hence, one has to compliment the decoding scheme by unitary corrections of such modifications. In contrast, a photon-loss error on the information part is a non-unitary action that can only be detected but not corrected.

The suggested scheme shown in Fig. 1 includes the phases of encoding, propagation, decoding and photon detection on the ancilla mode. Initially, the quantum state vector of multi-mode electromagnetic field $|\Psi\rangle = |\Psi\rangle_{\mathcal{I}} \otimes |\Psi\rangle_{\mathcal{A}}$ is a direct product of a state of K distinguishable information modes

$$|\Psi\rangle_{\mathcal{I}} = \sum_{n_1=0} \cdots \sum_{n_i=0} \cdots \sum_{n_K=0} \psi_{n_1,\ldots,n_i\ldots,n_K}$$
$$\times |n_1\rangle \ldots |n_i\rangle \ldots |n_K\rangle, \quad (1)$$

with amplitudes $\psi_{n_1,\ldots,n_i\ldots,n_K}$ carrying the quantum information, and the vacuum state

$$|\Psi\rangle_{\mathcal{A}} = \sum_{m=0} \widetilde{\psi}_m |m\rangle \quad (2)$$

(with $\widetilde{\psi}_0 = 1, \widetilde{\psi}_{m>0} = 0$) of a single ancilla mode. The
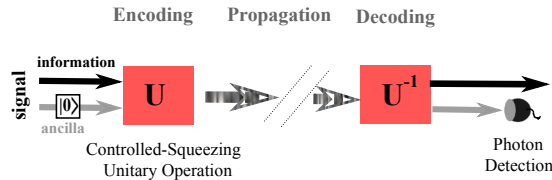
FIG. 1: A schematic representation of the four phases of the detection scheme. *a)* Encoding where the signal's information modes become entangled with the ancilla mode. *b)* Propagation of the 'dressed' signal via the medium. At this phase the incident of a photon loss may occur. *c)* Decoding. *d)* Photon counting on the ancilla part of the signal.

integers n_i and m are the eigenvalues of the photon number operator $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ for the i-th information mode and $\hat{m} = \hat{b}^\dagger \hat{b}$ for the ancilla mode. Each of $K+1$ modes in Eqs.(1)-(2) may correspond to transverse spatial, angular momentum or polarization mode transporting photons to a final destination either through a fiber or in free space. The joint quantum state $|\Psi\rangle$ is subject to the coding $\hat{U}$ and decoding $\hat{U}^{-1}$ transformations prior and after the propagation, respectively. The key idea of the scheme lies on the encoding/decoding actions which are performed with the help of a photon number *parity controlled-squeezing operation*, - an entangling unitary operation producing squeezing of the ancilla mode vacuum state in function of the photon number parity of the information signal modes.

The paper is organized as follows. In Section II we describe the model of single-photon losses and the encoding transformation $\hat{U}$. In Section III we discuss ways for synthesizing $\hat{U}$ gate with the help of the quantum control techniques. We conclude by discussing the results obtained.

II. THE DETECTION SCHEME

The general description of the amplitude damping in quantum channels [14, 15] is based on the theory of open quantum systems and employes the Kraus operators formalism. However this description drastically simplifies [17] under the assumption of a single photon loss and pure states input, where one can remain within the Hilbert space formalism describing a photon loss in mode i by simple action of the annihilation operator $\hat{a}_i$ on the signal state $|\Psi\rangle$, or even a photon loss from a superposition of modes given by the collective photon loss operator

$$\hat{A} + \hat{B} = \sum_{i=1}^{K} \alpha_i \hat{a}_i + \alpha_{K+1} \hat{b} \qquad (3)$$

with $\sum_{i=1}^{K+1} |\alpha_i|^2 = 1$. Parts $\hat{A}$ and $\hat{B}$ accounts for the information and ancilla losses, respectively. Many different

collective photon loss operators may coexist in a system. In the most natural model where every photon has an independent probability ($|\alpha_i|^2 = 1$) of getting lost during propagation, the probability of loosing photons from the signal scales linearly with number of photons in the signal. As a consequence, the total number of photons is restricted by the assumption of single-photon loss errors. For the encoded quantum state $|\Psi\rangle_\mathcal{I}$ of the information system Eq.(1) one estimates the total number of photons on the information part of the signal Eq.(1) as

$$\langle n \rangle_\mathcal{I} = \sum_{n_1=0} \cdots \sum_{n_i=0} \cdots \sum_{n_K=0} (n_1 + \ldots + n_i + \ldots + n_K)$$
$$\times |\psi_{n_1,\ldots,n_i\ldots,n_K}|^2 \qquad (4)$$

and thus we consider only signals with $\langle n \rangle_\mathcal{I}$ low, carrying a finite amount of information. Coding does not change the probability of the photon loss from the information system. In contrast, due to the squeezing, it may and does change the number of photons in the ancilla mode and thus augments the probability of the photon loss. However, as shown below, this augmentation can be done relatively small and thus tolerable. The exact upper limit on the average number of photons depends on the characteristics of the media of propagation which dictates the probability of a single-photon loss error event and has to be found for each particular setting. This task remains beyond the present consideration.

To encode the combined quantum system of $(K+1)$ modes we chose an entangling unitary transformation $\hat{U} = e^{-i\hat{S}(\hat{a}_i^\dagger, \hat{a}_i, \hat{b}^\dagger, \hat{b})}$. The entangling action $\hat{S}$ should be taken such that an error is detectable but also such as that the average number of photons $\langle n \rangle$ on the combined signal remain of the same order of magnitude as $\langle n \rangle_\mathcal{I}$. Among the different possibilities studied, we have identified an action

$$\hat{S} = \Gamma \hat{\Pi} \otimes \left(\hat{b}^\dagger \hat{b}^\dagger + \hat{b}\hat{b} \right), \qquad (5)$$

that fits these requirements. Here $\hat{\Pi} = (-1)^{\sum_{i=1}^{K} \hat{a}_i^\dagger \hat{a}_i}$ is the 'collective' photon number parity operator for the modes of the information system and Γ a real parameter controlling the amount of the ancilla squeezing. We denote the unitary operation corresponding to the action Eq.(5) by $\hat{U}_{PCS}$, and henceforth call it a *parity controlled-squeezing* operation, since this produces squeezing of the ancilla mode controlled by the total parity of the information signal. After the propagation, at the final destination, the entangled signal is decoded by applying $\hat{U}_{PCS}^{-1} = e^{i\hat{S}(\hat{a}_i^\dagger, \hat{a}_i, \hat{b}^\dagger, \hat{b})}$, and the state of the ancilla is subject to a photon number detection.

The additional ancilla mode unavoidably increases the number of possible photon loss channels, since apart from the losses given by the operator $\hat{A}$, one should now take into account the ancilla losses given by the operator $\hat{B}$. The average number of photons

$$\langle n \rangle = \langle n \rangle_\mathcal{I} + \sinh^2 (2\Gamma). \qquad (6)$$

in the combined propagating signal $|\Psi\rangle = \widehat{U}_{PCS} |\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}}$ increases by an average number of photons $\sinh^2(2\Gamma) \ll \langle n \rangle_{\mathcal{I}}$ in the squeezed ancilla mode.

Consider now three possible outcomes of the process: (i) no losses, (ii) a photon has been lost from an ancilla mode, and (iii) a photon is lost from the information part. In case (i), the combined signal is restored intact after the decoding, as a direct product $|\Psi\rangle = |\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}}$. As consequence no photons are detected on the ancilla mode and the signal state being unentangled is not affected by the measurement. In case (ii), the resulting state $|\Psi'\rangle = \widehat{U}_{PCS}^{-1} \widehat{B} \widehat{U}_{PCS} |\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}}$ reads

$$\begin{aligned}|\Psi'\rangle = {} & \cosh\left(2\Gamma\widehat{\Pi}\right) |\Psi\rangle_{\mathcal{I}} \otimes \widehat{b} |0\rangle_{\mathcal{A}} \\ & - \sinh\left(2\Gamma\widehat{\Pi}\right) |\Psi\rangle_{\mathcal{I}} \otimes \widehat{b}^\dagger |0\rangle_{\mathcal{A}}\end{aligned} \qquad (7)$$

where the first term $\sim \widehat{b} |0\rangle$ vanishes and therefore the ancilla mode turns to be is in the first excited state. The measurement on ancilla mode counts one photon and projects the signal state to

$$|\Psi'\rangle_{\mathcal{I}} = \Delta \sinh\left(2\Gamma\widehat{\Pi}\right) |\Psi\rangle_{\mathcal{I}} \ , \qquad (8)$$

with Δ a normalization factor associated with the measurement induced state reduction. By noting that $\sinh(x)$ is an odd function while $\widehat{\Pi}^2 = \widehat{1}$ is the identity, one immediately finds that $|\Psi'\rangle_{\mathcal{I}} = \widehat{\Pi} |\Psi\rangle_{\mathcal{I}}$. This implies that the states with an even total number of photons information mode remain intact, while those having an odd total number of photons experience a π phase shift. This transformation of the information system is reversible, one simply needs to apply the unitary operation $\widehat{\Pi}_i = e^{i\pi\widehat{a}_i^\dagger \widehat{a}_i}$ on every mode of the information system.

In case (iii), the total state after decoding

$$|\Psi'\rangle = \sum_{i=1}^{K} \alpha_i e^{i2\Gamma\widehat{\Pi} \otimes \left(\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b}\right)} |0\rangle_{\mathcal{A}} \widehat{a}_i |\Psi\rangle_{\mathcal{I}} \qquad (9)$$

implies that the photon loss results in squeezing for the ancilla mode after decoding phase, such that the distribution of the even photon number states in this mode reads

$$P_{\{m\}} = \sum_{i=1}^{K} |\alpha_i|^2 \frac{m_i! \sqrt{1 - \tanh^2 4\Gamma} \tanh^{m_i} 4\Gamma}{(m_i/2)! \, (m_i/2)! 2^{m_i}}. \qquad (10)$$

Deriving Eq.(10) we have employed the well-known photon distributions for squeezed states. For such a distribution Eq.(10), the typical numbers of photons in the single ancilla mode is large, being of the order of $\langle m_i \rangle \sim \exp(4\Gamma)$. This implies that once a photon has been lost from a mode i of the information system, the vacuum state of the ancilla mode gets strongly squeezed and a large even number of photons can be detected as

a consequence, considering a Γ parameter of moderate order.

The scheme has been designed under the assumption that only single-photon loss can take place on the propagating combined signal. To conform with this assumption we need to restrict ourselves to information signals with a low number of photons, Eq.(4), and furthermore take care that the encoding operation does not introduce many additional, see Eq.(6). In Fig. 2 the fast increasing function $\sinh^2(2\Gamma)$ describing the mean number of additional photons is plotted against the squeezing parameter Γ. Restricting the number of photons by choosing a moderate strength parameter Γ does not necessarily contradict the requirements for a reliable detection of an error on the signal. In Fig. 2 the probability of detecting no photons $P_{\{0\}}$, Eq.(10), in the case of a single photon loss on the information part of the signal is plotted as a function of Γ. One may observe that there is a parameter window in the vicinity of $\Gamma = 0.85$ where the wanted requirements are satisfied without contradiction, i.e., $P_{\{0\}} < 0.1$ and mean number of additional photons remains below 10.
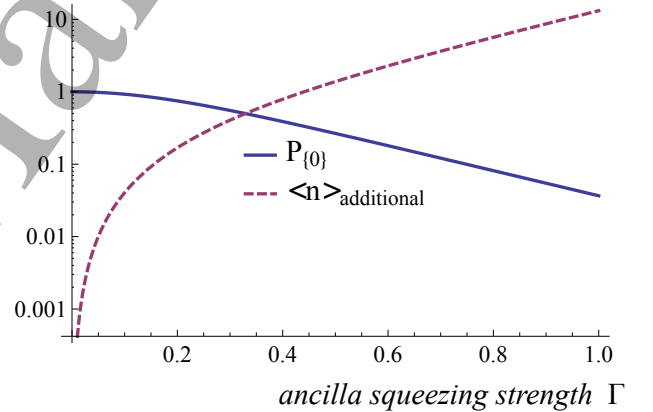
FIG. 2: The probability of no photon detection on the ancilla mode, $P_{\{0\}}$, in the case of a single photon loss on the information part of the signal during propagation. This should be kept as close to zero as possible so that the cases "no loss" and "loss" on the information part of the signal are well distinguished by the scheme. The mean number of additional photons $\sinh^2(2\Gamma)$ on the propagating signal which results from the encoding action $\widehat{U}_{PCS}$. This should be kept as low as possible in order to avoid simulation of further photon losses. The two constrains re-conciliate in the region $0.8 < \Gamma < 1$ where $P_{\{0\}} < 0.1$ and mean number of additional photons remains below 10

III. CONSTRUCTING THE CONTROLLED-SQUEEZING OPERATIONS

We now address the question: What kind of the physical interaction can result in the controlled-squeezing coding action Eq.(5)? Let us consider the simplest case of just one information mode such that the required coding transformation is given by the aforesaid parity controlled-squeezing gate

$$\widehat{G}_{PCS} = e^{i\Gamma \cos(\pi \widehat{a}^\dagger \widehat{a}) \otimes (\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b})}, \tag{11}$$

noticing that $(-1)^{\widehat{a}^\dagger \widehat{a}} = \cos(\pi \widehat{a}^\dagger \widehat{a})$. We show how one can synthesize such a gate with the help of an intermediate system with $SU(2)$ symmetry, such as a collection of non-interacting two-level systems which can be described in terms of three Pauli operators $\widehat{\sigma}_x, \widehat{\sigma}_y$, and $\widehat{\sigma}_z$ entering the polarization, dispersion, and population inversion components of the Bloch vector, respectively.

Let $\widehat{V}_{i,tl} = \mu \widehat{a}^\dagger \widehat{a} \widehat{\sigma}_z$ be the interaction Hamiltonian between the intermediate system and the information system. Physically this corresponds to the Stark shift in the two-level systems induced by the electric field of the information photons. Let also the Hamiltonian $\widehat{V}_{a,tl} = \kappa \left(\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b}\right) \widehat{\sigma}_x$ describe the interaction of the intermediate system with the ancilla, implying that the total polarization of the two-level systems parametrically pumps the ancilla photon mode. As a possible physical realization of such a Hamiltonian, one may think about parametric pumping of the ancilla oscillator by a classical field at the double frequency, at the condition, where the direct pumping is forbidden by symmetry, but gets aloud in the presence of polarization induced by the ensemble of two-level systems. We assume that the coupling energy μ can be done both positive and negative by a proper choice of the intermediate system parameters. We also assume the intermediate system initially in the eigenstate $|1\rangle_{\mathcal{TL}}$ of the operator $\widehat{\sigma}_x$ with eigenvalue 1, such that initially, the entire system is in the quantum state $|\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}} \otimes |1\rangle_{\mathcal{TL}}$.

In order to synthesize the required coding transformation, we first apply the interaction $\widehat{V}_{i,tl}$ for a time interval $\pi/2\mu$, we then apply the interaction $\widehat{V}_{a,tl}$ during a time interval Γ/κ, and finally apply the interaction $-\widehat{V}_{i,tl}$ during a time interval $\pi/2\mu$, thus bringing the entire compound system to the quantum state

$$\begin{aligned}
|\Psi\rangle_{\mathcal{C}} &= e^{i\frac{\pi}{2}\widehat{a}^\dagger \widehat{a} \otimes \widehat{I} \otimes \widehat{\sigma}_z} e^{-i\Gamma \widehat{I} \otimes (\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b}) \otimes \widehat{\sigma}_x} e^{-i\frac{\pi}{2}\widehat{a}^\dagger \widehat{a} \otimes \widehat{I} \otimes \widehat{\sigma}_z} \\
&\quad \times |\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}} \otimes |1\rangle_{\mathcal{TL}} \\
&= \exp[-i\Gamma \left(\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b}\right) \otimes e^{i\frac{\pi}{2}\widehat{a}^\dagger \widehat{a} \otimes \widehat{\sigma}_z} \\
&\quad \times \widehat{\sigma}_x e^{-i\frac{\pi}{2}\widehat{a}^\dagger \widehat{a} \otimes \widehat{\sigma}_z}] |\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}} \otimes |1\rangle_{\mathcal{TL}}. \tag{12}
\end{aligned}$$

We now make use of the fact that the operator $e^{i\frac{\pi}{2}\widehat{a}^\dagger \widehat{a} \otimes \widehat{\sigma}_z} \left(\widehat{I} \otimes \widehat{\sigma}_x\right) e^{-i\frac{\pi}{2}\widehat{a}^\dagger \widehat{a} \otimes \widehat{\sigma}_z} = \cos(\pi \widehat{a}^\dagger \widehat{a}) \otimes \widehat{\sigma}_x +$

$i \sin(\pi \widehat{a}^\dagger \widehat{a}) \otimes \widehat{\sigma}_y$ while the operator $\sin(\pi \widehat{a}^\dagger \widehat{a})$ vanishing for all number states of the information system and their linear combinations thus yielding

$$|\Psi\rangle_{\mathcal{C}} = |1\rangle_{\mathcal{TL}} \otimes e^{-i\Gamma \cos(\pi \widehat{a}^\dagger \widehat{a}) \otimes (\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b})} |\Psi\rangle_{\mathcal{I}} \otimes |0\rangle_{\mathcal{A}}. \tag{13}$$

This means that the intermediate system is not anymore entangled with the information system and ancilla compound, while the latter experience the required coding transformation. By setting the intermediate system's initial state to the eigenstate $|1\rangle_{\mathcal{TL}}$ of the operator $\widehat{\sigma}_x$ corresponding to the eigenvalue -1, one obtains the decoding transformation

$$|\Psi\rangle_{\mathcal{C}} = |-1\rangle_{\mathcal{TL}} \otimes e^{i\Gamma \cos(\pi \widehat{a}^\dagger \widehat{a}) \otimes (\widehat{b}^\dagger \widehat{b}^\dagger + \widehat{b}\widehat{b})} |\Psi\rangle_{\mathcal{I}+\mathcal{A}}, \tag{14}$$

while the intermediate system remains disentangled from the rest.

Thus we have shown how to synthesize the parity controlled squeezing and it's inverse for a single information mode. The generalization on the multimode case is straightforward – sequential application of the same procedure to each information mode yields the required encoding Eq.(9).

IV. DISCUSSION

We have proposed a scheme for detecting the event of a single photon loss on photonic signals composed by discrete modes propagating via a quantum channel. It invokes an ancilla mode which 'dress' the signal during the propagation in such a way that: *a)* a signal which did not loose any photons remains intact, *b)* a photon loss from the information system can be identified by photon detection on the ancilla modes, and *c)* a loss of a photon on the ancilla is also detectable and creates an effect on the information state that can be corrected. This seems to be a workable alternative to well established quantum error-correcting algorithms for bosonic modes, imposing (in contrast to the latter) no restrictions on the encoding space of the signal and requiring a single ancillary mode. On the other hand, the suggestion of this current work seems to be incompatible with quantum photonic computing processing being only helpful for heralded quantum communication.

The main tool of the approach is the conditional squeezing of the ancilla modes, which in the regime of strong squeezing and in the situation where one of the information carrying photons is lost, this results in a strong signal indication, i.e., the presence of an even number of photons on the ancilla modes. In contrast, the loss of an ancilla photon, after decoding, yields the presence of a single photon on that mode. In the latter case, the resulting state of the information system also gets distorted, but this distortion is reversible and can be eliminated by proper unitary correction. One can synthesize the required encoding/decoding transformations with the help of quantum control techniques. We have proposed a

way for the implementation of the transformation which offers opportunities for complete preservation of the signal, synthesized by a quantum control protocol that utilizes as a resource an ensemble of atoms. One can also note, that by logarithmically increasing the number of the ancilla modes one can construct (in the spirit of its classical counterpart) a more elaborate detection scheme that would not only detect the photon loss but would also precisely point the information mode which has lost the photon, if such information is required. Finally it is important to mention that the proposed detection scheme fails to distinguish different events when the assumption of single photon events is not met. The open question remains whether a coupling Hamiltonian can be devised, possibly in the presence of additional ancillary modes, so that one arrives to an efficient detection scheme in the regular situation where more than one photons can be lost during propagation.

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