




Research Article

Estimation of Minimum Torsional Reinforcement of Reinforced Concrete and Steel Fiber-Reinforced Concrete Members

Hyunjin Ju ¹, Sun-Jin Han,² Dichuan Zhang,³ Jong Kim ⁴, Wei Wu,⁵ and Kang Su Kim ⁶

¹Post-Doctoral Research Fellow, Department of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, Feistmantelstrasse 4, 1180 Vienna, Austria

²Ph.D. Candidate, Department of Architectural Engineering, University of Seoul, 163 Seoulsiripdaero, Dongdaemun-gu, Seoul 02504, Republic of Korea

³Assistant Professor, Department of Civil and Environmental Engineering, School of Engineering, Nazarbayev University, 53 Kabanbay Batyr Ave., Astana 010000, Kazakhstan

⁴Professor, Department of Civil and Environmental Engineering, School of Engineering, Nazarbayev University, 53 Kabanbay Batyr Ave., Astana 010000, Kazakhstan

⁵Professor, Department of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, Feistmantelstrasse 4, 1180 Vienna, Austria

⁶Professor, Department of Architectural Engineering, University of Seoul, 163 Seoulsiripdaero, Dongdaemun-gu, Seoul 02504, Republic of Korea

Correspondence should be addressed to Kang Su Kim; kangkim@uos.ac.kr

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The current code specifies a minimum torsional reinforcement ratio to prevent possible brittle failure after torsional cracking in concrete members. However, since there are many researches, in which even the concrete members with the minimum torsional reinforcement fail to secure sufficient reserved strength after torsional cracking, continuous research needs to be carried out. Accordingly, in the authors' previous research, a minimum torsional reinforcement ratio was proposed based on the reserved strength concept and was extended to the steel fiber-reinforced concrete members in order to suggest the minimum fiber factor as the minimum torsional reinforcement ratio. In the present study, a pure torsion test was carried out on reinforced concrete and steel fiber-reinforced concrete members after a brief introduction on the above, and the proposed model was verified based on the test results. The test results of six torsional specimens were compared with those of the proposed model, and it was found that the proposed model provides a reasonable evaluation on the torsional failure mode of the specimen according to the reserved strength ratio.

1. Introduction

With the development of construction materials and concrete engineering, concrete members are becoming increasingly irregular and slender. Therefore, the importance of torsional design is increasing, although it was not considered important in the design of concrete structures [1, 2]. As in the case of flexural and shear design of concrete members, the minimum torsional reinforcement should be determined by the torsion design to prevent brittle failure of the members after the occurrence of torsional cracking. However, based on the minimum torsional reinforcement ratio presented in the

current structural standards [3–6], the reserved strength of members subjected to torsion is often evaluated to be on the unsafe side [7]. In addition, the current structural standards fail to consider the correlation between transverse and longitudinal reinforcement or fail to offer the minimum longitudinal reinforcement for torsion in most cases.

In a previous research [8], authors proposed a minimum torsional reinforcement ratio to ensure sufficient reserved strength exceeding the torsional cracking strength (T_{cr}) by introducing the reserved strength factor (λ). Both the longitudinal and transverse reinforcement ratios were considered in the proposed method. In addition, the

authors proposed a minimum torsional reinforcement requirement not only for reinforced concrete (RC) members but also for steel fiber-reinforced concrete (SFRC) members using the minimum fiber factor (F_{\min}) and suggested an integrated approach for both RC and SFRC members. In the present study, the proposed minimum torsional reinforcement ratio of the RC and SFRC members was introduced briefly, and the proposed model was verified through experimental research on a total of six torsional specimens.

2. Minimum Torsional Reinforcement

2.1. Reinforced Concrete (RC) Members. In the previous research [8], the minimum amount of torsional reinforcement was derived with the reserved strength concept, in which the ultimate torsional strength (T_n) should be larger than certain strength related to cracking strength (λT_{cr}). The torsional cracking strength (T_{cr}) of the reinforced concrete member was provided based on the existing experimental results [9, 10]. In addition, for the torsional strength (T_n), the ACI318 code [3] was used. The torsional strength can be expressed with either transverse or longitudinal reinforcement from the equilibrium condition of forces in the space truss model [8], and with the relation between T_n and λT_{cr} , the minimum amount of torsional reinforcement in transverse and longitudinal directions was derived. If the reinforcement amount is expressed by reinforcement ratio and the ratios in two directions (ρ_t and ρ_l) are added together, the total minimum torsional reinforcement ratio ($\rho_{\text{tot,min}}$) is derived as follows:

$$\rho_{\text{tot,min}} = 0.34 \sqrt{f'_c} \frac{A_{cp}}{A_0} \frac{p_h}{p_{cp}} \left(\frac{f_{yt} \cot \theta + f_{yl} \tan \theta}{f_{yt} f_{yl}} \right), \quad (1)$$

where f'_c is the compressive strength of concrete (MPa), A_{cp} is the area enclosed by the outer perimeter in the concrete section, A_0 is the cross-sectional area closed by shear flow, p_h is the centerline perimeter of the outermost closed transverse stirrup, p_{cp} is the outer perimeter length of the concrete section, f_{yt} is the yield stress of transverse torsional reinforcement, and f_{yl} is the yield strength of longitudinal torsional reinforcement. The reserved strength factor (λ) was determined to secure a reserved strength greater than 35% based on the results of existing experiments [8]. Also, in equation (1), $\tan \theta$ can be calculated by $\sqrt{\rho_t f_{yt} / (\rho_l f_{yl})}$ [11], which is limited to $0.3 \sim 3$ [8]. If either one of the longitudinal and transverse reinforcement requirements is determined in design, the minimum torsional reinforcement requirement in the other direction can be determined using equation (1) and $\tan \theta$.

2.2. Steel Fiber-Reinforced Concrete Members. In recent years, studies have been actively conducted to incorporate steel fibers to replace the complicated reinforcement details of RC members or the minimum reinforcement requirement [1, 7, 12–20]. In the previous research of the authors [8], equation (1) was extended to suggest the minimum amount of torsional reinforcement, which is suitable for SFRC members to which the minimum fiber factor (F_{\min}) is introduced.

According to the thin-walled tube theory [21], because steel fibers near the center of the cross section of the SFRC member contribute little to torsional moment resistance, the steel fibers only within the effective thickness (t_d) were assumed to be effective to resist torsional moment [22]. With the effective thickness, the effective volume fraction of the steel fibers was also derived [8] and the steel fibers resisting torsion within the effective thickness were simply assumed to be evenly distributed in the transverse and longitudinal directions. These can be regarded as the equivalent reinforcement ratios of the steel fibers that resist in the transverse and longitudinal directions, that is, ρ_t^f and ρ_l^f .

Since steel fibers exhibit a very high tensile strength, the maximum stress is determined by the bond stress between the fiber and the concrete [19]. Therefore, the maximum stress of the steel fiber ($\sigma_{\text{max}}^{\text{sf}}$) was calculated by the bond strength between the steel fiber and the surrounding concrete as follows [8]:

$$\sigma_{\text{max}}^{\text{sf}} = 0.41 \tau_{\text{uf}} \frac{F}{V_f}, \quad (2)$$

where τ_{uf} is the ultimate bond stress, which is $2.5 f_{ct}$ [23], f_{ct} is the tensile strength of concrete ($0.33 \sqrt{f'_c}$) [24], and V_f is the fiber volume fraction. In addition, F represents the fiber factor considering the geometry and volume fraction of the steel fibers and can be calculated as $l_f V_f \alpha_f / d_f$, where l_f and d_f are the length and diameter of a fiber, respectively, and α_f is the bond factor according to the type of steel fiber [25].

The previous research [8] found that, in the case of SFRC members with reinforcing bars, the sum of the torsional reinforcing bar and steel fibers should meet the minimum torsional reinforcement ratio represented in equation (1). In other words, if the equivalent reinforcement ratio based on the sum of reinforcement ratios for reinforcing bars ($\rho_{\text{tot}}^{\text{rebar}} = \rho_t + \rho_l$) and steel fiber-reinforcement ratio ($\rho_{\text{tot}}^{\text{sf}} = \rho_t^f + \rho_l^f$) is greater than equation (1), a sufficient reserved strength can be secured against the torsional load. Thus, the relationship $\rho_{\text{tot}}^{\text{rebar}}$, $\rho_{\text{tot}}^{\text{sf}}$, and $\rho_{\text{tot,min}}$ of equation (1) is expressed by the following equation:

$$\rho_{\text{tot}}^{\text{rebar}} + \rho_{\text{tot}}^{\text{sf}} \frac{\sigma_{\text{max}}^{\text{sf}}}{(f_{yt} + f_{yl})/2} \geq \rho_{\text{tot,min}}. \quad (3)$$

Therefore, in the form of the required minimum fiber factor, F_{\min} is derived as follows:

$$F_{\min} = 1.5 (\rho_{\text{tot,min}} - \rho_{\text{tot}}^{\text{rebar}}) \frac{f_{yt} + f_{yl}}{\xi \sqrt{f'_c}}. \quad (4)$$

In case equation (4) gives a negative value, it means that steel fibers to ensure the reserved strength is not necessary. Additionally, for the SFRC members without reinforcing bars, $\rho_{\text{tot}}^{\text{rebar}} = 0$, the minimum fiber factor (F_{\min}) can be calculated by substituting equation (1) into $\rho_{\text{tot,min}}$ and replacing the yield strengths (f_{yt} and f_{yl}) with the maximum stresses of the steel fibers ($\sigma_{\text{max}}^{\text{sf}}$) of equation (2).

3. Experimental Program

3.1. Details of Test Specimens. In the present study, a pure torsion test was conducted on a total of six test specimens, as shown in Table 1, and the proposed equations for calculating the minimum torsional reinforcement requirements (i.e., equations (1) and (4)) were verified based on the test results. The main variables of the test specimens include compressive strength of concrete (f'_c), longitudinal reinforcement ratio (ρ_l), transverse reinforcement ratio (ρ_t), and incorporation of steel fibers. As shown in Figure 1, the length of each specimen was 3,000 mm, the cross-sectional width and height were 350 mm and 500 mm, respectively, and the net cover thickness was 20 mm. In addition, the measurement section in which torsional failure was induced is the center of the member, and stirrups were placed densely in other sections to prevent failure. As summarized in Table 1, D13, D16, and D19 steel bars were used in the test specimens, and the average yield strengths of each rebar were 489.8 MPa, 467.5 MPa, and 500.4 MPa, respectively. In addition, the transverse reinforcement ratios of the specimens (ρ_t) ranged from 0.34% to 0.91%, the longitudinal reinforcement ratios (ρ_l) from 0.43% to 0.98%, and the total reinforcement ratios (ρ_{tot}) from 0.77% to 1.89%. Also, the $\rho_t f_{yt} / \rho_l f_{yl}$ ratio of all specimens was designed to be less than 1.0, considering the details of reinforcing bars placed in typical structures.

Of the six test specimens, steel fibers were mixed in the MTF25-0.77 and MTF25-N specimens. The MTF25-0.77 specimen has the same cross-sectional details as those of the MT30-0.77 specimen, which is an RC member. The MTF25-N specimen is a member in which only longitudinal reinforcement is placed without transverse reinforcement. The steel fiber volume fraction (V_f) of the SFRC specimen was 2.0%, and a hook-shaped steel fiber with a diameter (d_f) of 0.5 mm, a length (l_f) of 30 mm, and a tensile strength of 1,200 MPa was used in the test specimen.

Table 2 shows the mix proportion of the concrete in the test specimens. Portland cement type I and coarse aggregates with a maximum size of 25 mm were used. The concrete compressive strengths (f'_c) of the MT30 series, MT40 series, and MTF25 series were 29.3, 40.3, and 24.0 MPa, respectively. The MTF25 series specimens were originally designed with the same mix proportion as the MT30 series specimens. However, it was estimated that their compressive strengths were somewhat lower due to the fiber balling phenomenon during concrete placement [26].

3.2. Test Setup and Measurements. Figure 2 shows the details of torsional loading and displacement measurements. As shown in Figures 2(a) and 2(c), a frame was fixed at a point 300 mm away from the right end of the test specimen. As shown in Figures 2(a), and 2(b) at a point 300 mm away from the left end, a 600 mm long torsion arm was installed to introduce a torsional moment (T). Moreover, as shown in Figure 2(c), rollers were installed in the upper and lower parts at the right end to release the longitudinal restraints, while an arc bearing was placed at the lower part of the left end of the test specimen so that torsional rotation can occur, as shown in Figure 2(b). The load was applied using a 500 kN capacity actuator in

displacement control, and tests were performed until the load decreased to less than 80% of the maximum strength.

As shown in Figure 1, nine strain gauges were installed in longitudinal reinforcement. In all specimens except for the MTF25-N specimen, a total of 12 strain gauges were attached to the transverse reinforcement, six on the side and six at the bottom of transverse reinforcement. Additionally, as shown in Figure 2(d), two LVDTs were installed on the front and back sides of the test specimen, respectively, at a location 800 mm away to the left and the right from the center of the test specimen.

4. Test Results and Discussion

4.1. Torsional Behavior of Test Specimens. Figure 3 shows the crack patterns and failure modes of the test specimens. All the specimens underwent torsional failure within the planned test sections. In addition, since the test specimens have a relatively small amount of torsional reinforcement, longitudinal and transverse torsional reinforcements yielded after the occurrence of cracks, as shown in Figures 4–6. The MT30-0.77 specimen with the smallest amount of torsional reinforcement exhibited a decrease in load immediately after the torsional cracking, as shown in Figure 4. The MT30-1.32 specimen with longitudinal reinforcement more than twice that of the MT30-0.77 specimen showed a critical torsional crack angle of approximately 45° at the final failure, and more cracks occurred compared to the MT30-0.77 specimen. However, while the MT30-1.32 specimen showed typical torsional failure, the member suffered premature failure as the load was rapidly applied due to the malfunction of the actuator at the point of torsional cracking, as shown in Figure 4.

As shown in Figure 5, the MT40-1.32 specimen showed a decrease in load as the critical torsional crack propagated rapidly after the torsional cracking strength. Ultimately, it failed to ensure the reserved strength and showed a similar behavior to that of the MT30-0.77 specimen. This is because the amount of torsional reinforcement placed in the specimen was not sufficient to ensure the reserved strength, and consequently, the concrete struts failed, as shown in Figure 3(c). In the case of the MT40-1.89 specimen with the largest amount of torsional reinforcement, multiple torsional cracks occurred in the test section, as shown in Figure 3(d). One critical crack was gradually propagated with the load increasing and caused the member failure. In addition, as shown in Figure 5, the MT40-1.89 specimen exhibited the highest reserved strength among all specimens and showed very ductile behavior up to the ultimate strength even after torsional cracking.

The MTF series specimens are members in which steel fibers are incorporated. The MTF25-0.77 specimen has the same reinforcement details as the MT30-0.77 specimen. As shown in Figure 3(e), the MTF25-0.77 specimen showed a tighter and denser distribution of cracks compared to the MT30-0.77. Moreover, it was observed that the crack opening was well controlled by the bridging effect of the fibers even after the occurrence of critical cracks. However, the damage was concentrated on the upper surface of the concrete at near the failure, as shown in the right side of Figure 3(e), and accordingly, significant strength improvement did not occur

TABLE 1: Details and properties of specimens.

Specimen names	f'_c (MPa)	Longitudinal reinforcement		Transverse reinforcement			Steel fiber reinforcement			
		Steel bars (ρ_l)	f_{yl} (MPa)	Steel bars (ρ_t)	f_{yt} (MPa)	S (mm)	V_f (%)	l_f (mm)	d_f (mm)	Fiber shape
MT30-0.77	29.3	6-D13 (0.43%)	489.8	D10 (0.34%)	467.5	180				Without fibers
MT30-1.32	29.3	6-D19 (0.98%)	500.4	D10 (0.34%)	467.5	180				Without fibers
MT40-1.32	40.3	6-D19 (0.98%)	500.4	D10 (0.34%)	467.5	180				Without fibers
MT40-1.89	40.3	6-D19 (0.98%)	489.8	D13 (0.91%)	489.8	120				Without fibers
MTF25-0.77	24.0	6-D13 (0.43%)	489.8	D10 (0.34%)	467.5	180	2.0	30	0.5	Hooked
MTF25-N	24.0	6-D13 (0.43%)	489.8	Without transverse steel			2.0	30	0.5	Hooked

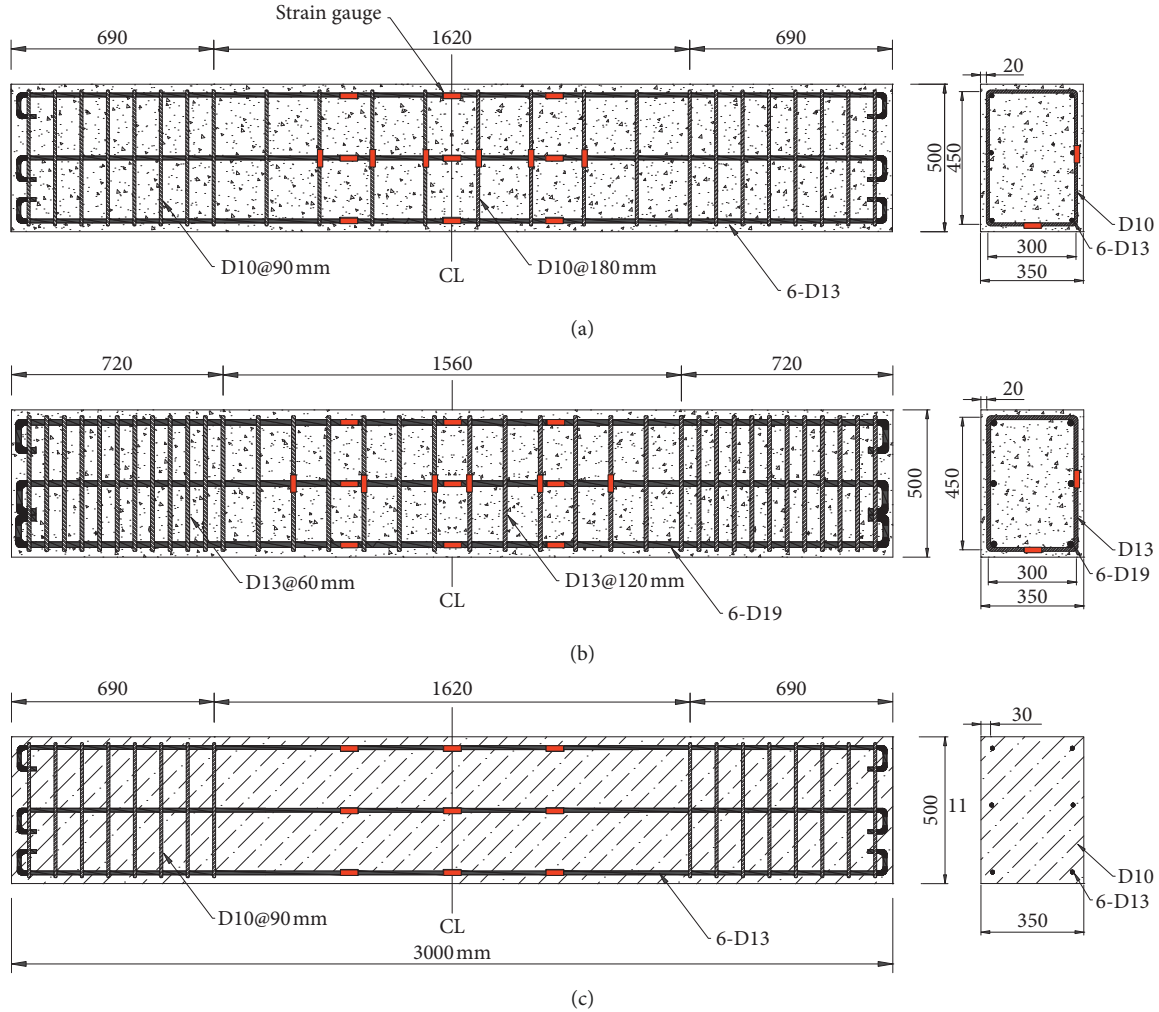


FIGURE 1: Details of specimens: (a) MT30-0.77; (b) MT40-1.89; (c) MTF25-N.

TABLE 2: Mix proportion.

Specimen series	W/C (%)	S/a (%)	Weight per m ³					
			W	C	S	G	AE	SF
MT30 series	47	42	158.78	339.72	754.20	1042.24	0.15	—
MT40 series	40	42	158.78	395.10	712.11	1038.54	0.17	—
MTF25 series	47	42	158.78	339.72	754.20	1042.24	0.15	157.2

W/C: water-cement ratio; S/a: sand percent of total aggregate by weight; W: water; C: cement; S: sand; G: coarse aggregate; AE: air-entraining agent; SF: steel fiber.

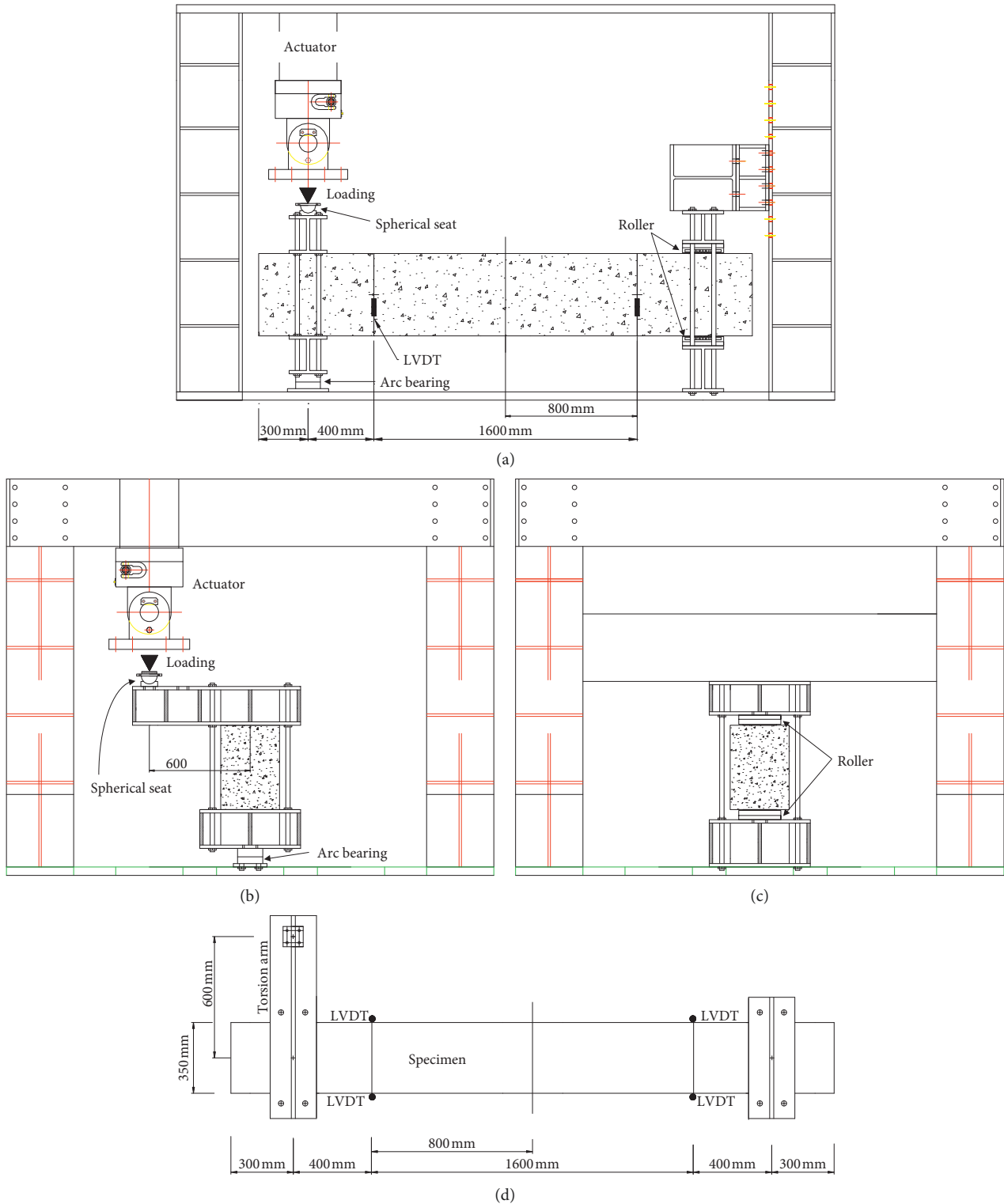


FIGURE 2: Test setup and location of LVDT: (a) front view; (b) left-end view (free to rotation); (c) right-end view (fixed against rotation); (d) top view of specimen and location of LVDT.

when compared to the MTF30-0.77 specimen, as shown in Figure 6. This result appears to be due to the fiber balling phenomenon in the MTF25-0.77 specimen, which was confirmed by comparing fractions collected from the upper part of the MTF25-0.77 specimen shown in Figure 7(a) and those of

the MTF25-N specimen shown in Figure 7(b). It is expected that if the members were well manufactured so that the fibers can be evenly distributed, further reserved strength could be secured after torsional cracking even in the MTF25-0.77 specimen. Meanwhile, no transverse reinforcement was

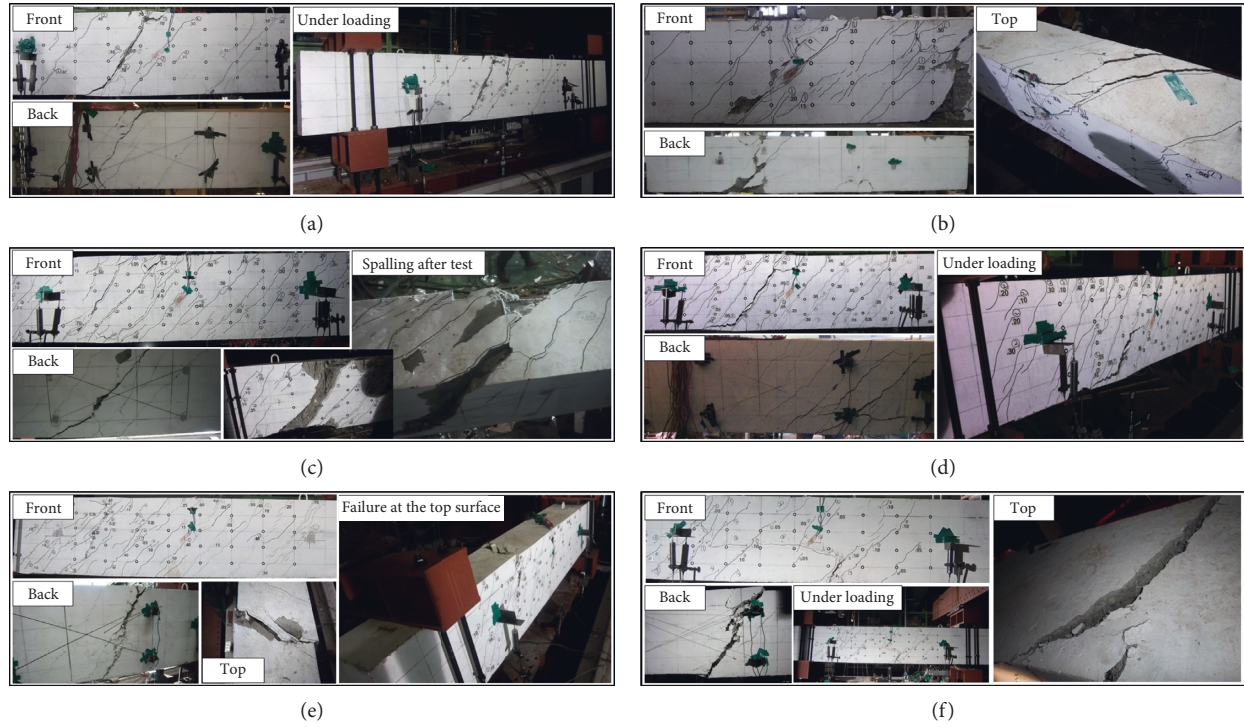


FIGURE 3: Crack pattern and failure of specimens: (a) MT30-0.77; (b) MT30-1.32; (c) MT40-1.32; (d) MT40-1.89; (e) MTF25-0.77; (f) MTF25-N.

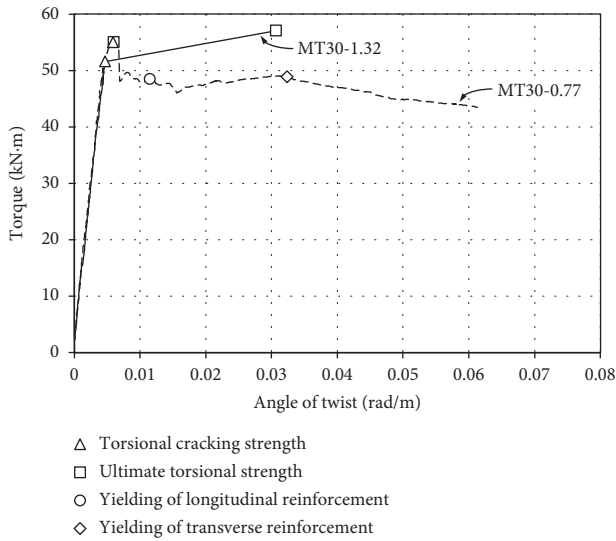


FIGURE 4: Torsional moment-twist of MT30 series.

placed; however, longitudinal reinforcement and 2% steel fibers were incorporated in the MTF25-N specimen. As shown in Figure 6, the MTF-25N specimen showed a rapid decrease in load after torsional cracking and reached failure as the width of the critical torsional crack was greatly expanded, as shown in Figure 3(f). Consequently, it failed to ensure the torsional reserved strength.

4.2. Evaluation of Reserved Strengths of Test Specimens. Table 3 summarizes the results of torsional tests performed in this study. The torsional cracking strengths of the MT40

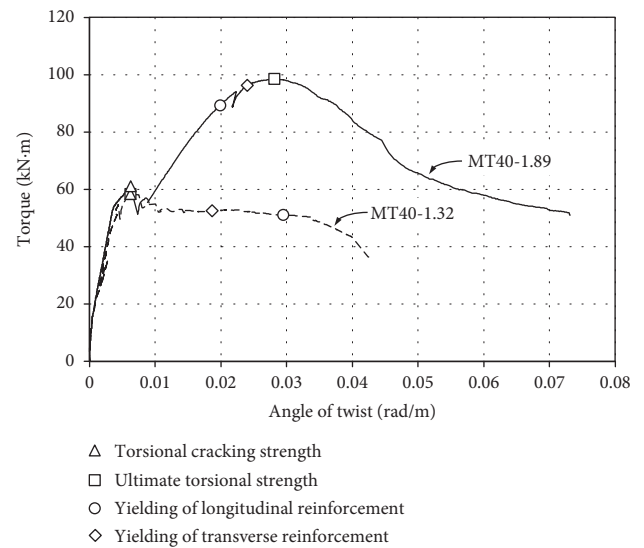


FIGURE 5: Torsional moment-twist of MT40 series.

series with a compressive strength of 40.3 MPa were greater than those of the MT30 series with a compressive strength of 29.3 MPa. The MT40-1.89 specimen with the largest amount of torsional reinforcement showed the highest torsional strength of 98.4 kN-m, whereas the MTF25-N specimen without transverse reinforcement showed the lowest torsional strength of 54.0 kN-m.

The MT30-0.77 specimen failed to exhibit the reserved strength after torsional cracking strength due to a low reinforcement ratio. In contrast, the MT30-1.32 specimen with

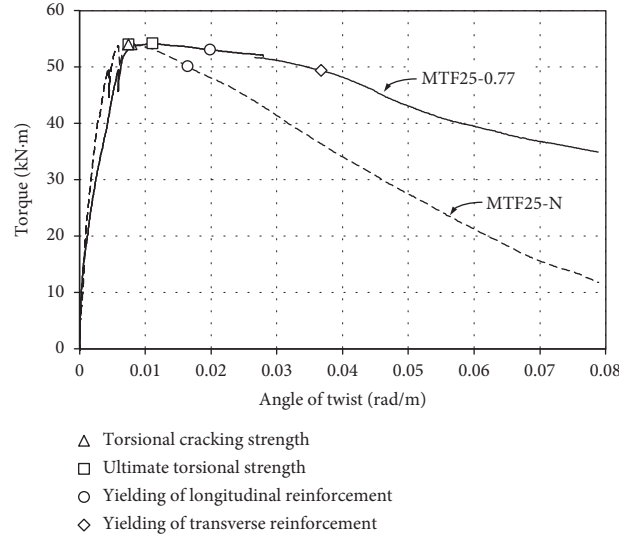


FIGURE 6: Torsional moment-twist of MTF25 series.



FIGURE 7: Fractions of SFRC members: (a) MTF25-0.77; (b) MTF25-N.

TABLE 3: Test results and validation of the proposed model.

Specimen names	At cracking		At ultimate		At yielding of steel				Reserved strength ratio T_u/T_{cr}	Ductility index θ_u/θ_{cr}
	θ_{cr} (rad/m)	T_{cr} (kN·m)	θ_u (rad/m)	T_u (kN·m)	Transverse θ_{yt} (rad/m)	Longitudinal T_{yt} (kN·m)	θ_{yl} (rad/m)	T_{yl} (kN·m)		
MT30-0.77	0.0060	55.00	0.0060	55.00	0.0324	48.83	0.0116	48.49	1.00	1.00
MT30-1.32	0.0048	51.55	0.0307	57.04	0.0307	57.04	0.0307	57.04	1.11	6.46
MT40-1.32	0.0063	58.29	0.0063	58.29	0.0187	52.51	0.0295	51.14	1.00	1.00
MT40-1.89	0.0063	60.97	0.0282	98.40	0.0240	96.19	0.0200	89.23	1.61	4.48
MTF25-0.77	0.0080	54.00	0.0111	54.15	0.0367	49.39	0.0198	53.04	1.00	1.39
MTF25-N	0.0075	53.97	0.0075	53.97	-	-	0.0164	50.08	1.00	1.00

more longitudinal reinforcement showed an increase in strength after cracking; however, the increase was not significant (i.e., $\lambda = 1.11$). Even the MT40-1.32 specimen with the same cross-sectional details as the MT30-1.32 specimen failed to exhibit the reserved strength. This suggests that since the torsional cracking strength is high in the member with a relatively high concrete compressive

strength, a larger amount of torsional reinforcement is required to ensure the reserved strength. The MT40-1.89 specimen with 167% greater transverse reinforcement compared with the MT40-1.32 specimen exhibited sufficient reserved strength after torsional cracking, as shown in Figure 5. The reserved strength ratio ($\lambda = T_n/T_{cr}$) was estimated to be 1.61, which is greater than the target reserved

strength ratio of 1.35 proposed in the previous research [8]. On the contrary, the MTF series specimens (MTF25-0.77 and MTF25-N), which are SFRC members, showed no reserved strength. This is because the steel fiber volume fraction ratios (V_f) of 2.17% and 8.45% are required to satisfy the minimum fiber factor (F_{min}) presented in equation (4), as shown in Table 4, whereas V_f of these specimens was 2%.

Table 3 shows the ratio of the twist angle per length at the maximum torsional strength (θ_u) to that at the torsional cracking strength (θ_{cr}) as the ductility index. The MT30-1.32 specimen exhibited the highest ductility index of 6.46 despite a low reserved strength ratio. In addition, the ductility index of the MT40-1.89 specimen with the highest reserved strength was 4.48, indicating a significantly high ductility index compared to other specimens except for the MT30-1.32 specimen. Meanwhile, the MTF25-0.77 specimen failed to ensure the reserved strength but showed a ductility index of 1.39. In other words, it can be confirmed that the deformation capacity (i.e., ductility) of the torsional member is not significantly correlated with the reserved strength (T_n/T_{cr}). Since the torsional member should be at least designed to have the sufficient resistance capacity of the member against loads, the most important thing in the member design is to satisfy the design strength. The deformation capacity will be the next consideration. Therefore, as described above, the securing of the deformation capacities (i.e., ductility) of torsional members cannot guarantee that the strength of the member can be ensured. Thus, it is reasonable to specify the minimum torsional reinforcement ratio based on the reserved strength rather than the deformation capacity for the safe design of torsional members.

Figures 8(a) and 8(b) show the minimum torsional reinforcement ratio ($\rho_{tot,min}$) with respect to $\rho_t f_{yt}/\rho_l f_{yl}$ and f'_c , respectively. Since the minimum torsional reinforcement ratio specified in the ACI318 code [3] does not consider the ratio of $\rho_t f_{yt}/\rho_l f_{yl}$, it is shown only in Figure 8(b), and not in Figure 8(a). In addition, the reinforcement ratios in six test specimens are shown in the graphs of Figure 8. The specimens which failed to secure the reserved strengths (i.e., $\lambda \leq 1.35$) from the test results were indicated by triangles (\triangle) and those ensuring the sufficient reserved strengths (i.e., $\lambda > 1.35$) by circles (\circ). As shown in Figure 8 and Table 3, only the MT40-1.89 specimen ensured a reserved strength greater than 35%. It can be seen that the failure modes predicted by the proposed model are consistent with the test results, except for the MT30-1.32 specimen. However, it should be noted that the MT30-1.32 specimen failed prematurely as the torsional moment was applied rapidly to the member due to the malfunction of the actuator. On the contrary, Figure 8(b) indicates that all the specimens have the torsional reinforcement greater than the minimum amount specified in ACI318-14. Nevertheless, all the specimens did not have enough reserved strengths except for MT40-1.89, and as shown in Table 3, the MT30-0.77 and MT40-1.32 specimens showed abrupt failures right after torsional cracking. This suggests that the minimum

torsional reinforcement ratio specified in ACI318-14 does not ensure a proper margin of safety in design.

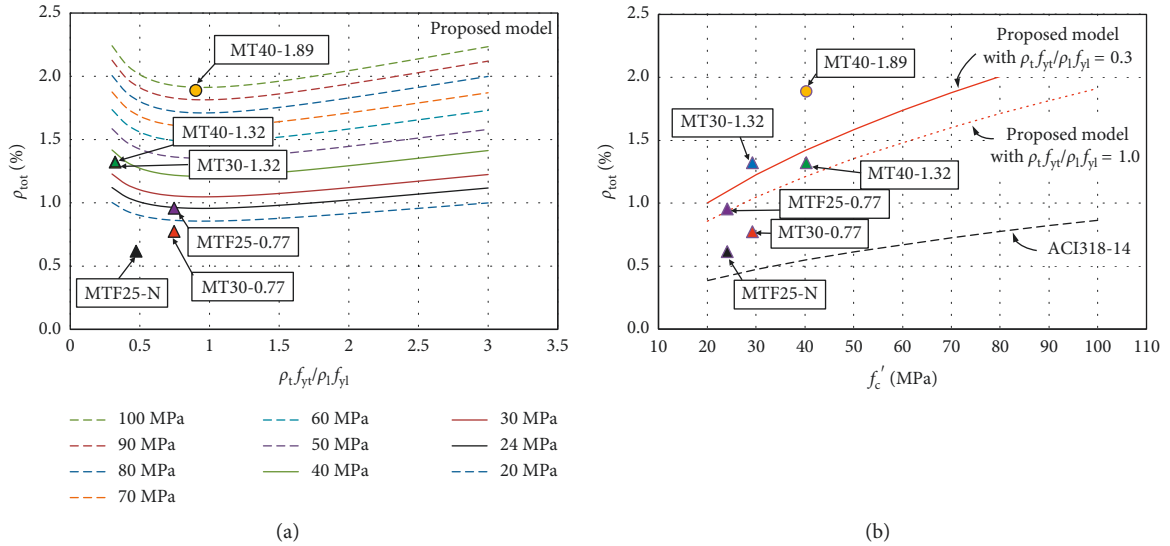
4.3. Required Minimum Amount of Steel Fibers for Ensuring Proper Reserved Strength. As described above, the remaining specimens, except for the MT40-1.89 specimen, failed to ensure the reserved strength. Therefore, additional reinforcement is required for the specimens to achieve a target reserved strength higher than 35% (i.e., $\lambda > 1.35$). In addition, when the member is reinforced with steel fibers, the required minimum amount of steel fibers (F_{min}) should be incorporated as represented in equation (4). Table 4 summarizes the minimum amount of steel fiber reinforcement (F_{min}) required for each specimen. In the case of the MT40-1.89 specimen, which showed sufficient reserved strength ratio in the test, and the MT30-1.32 specimen, which underwent premature failure due to the actuator malfunction, the required minimum fiber factor (F_{min}) was calculated to be zero. In addition, the minimum fiber factor (F_{min}) required for the MTF25-N specimen without transverse reinforcement was the largest (5.07), whereas the required fiber factor (F_{min}) for the MT40-1.32 specimen with a relatively higher torsional reinforcement ratio (ρ_{tot}^{rebar}) was estimated to be as small as 0.31. The minimum fiber factor (F_{min}) of the MTF25-0.77 specimen was smaller than that of the MT30-0.77 specimen with the same torsional reinforcement ratio (ρ_{tot}^{rebar}). This is because the concrete compressive strength (f'_c) of the MTF25-0.77 specimen was lower than that of the MT30-0.77 specimen and, therefore, resulted in a rather small estimated minimum torsional reinforcement ratio ($\rho_{tot,min}$) represented in equation (1). When the same fibers are used (i.e., $l_f = 30$ mm, $d_f = 0.5$ mm, and $\rho_f = 1.0$) as those incorporated in the specimens fabricated in this study, the steel fiber volume fraction (V_f) required for each specimen to ensure a reserved strength of 35% or more is estimated, as shown in Table 4. As mentioned in Section 4.2, the volume fraction ratio of steel fibers required for the MTF25-0.77 specimen was estimated to be 2.17%; however, 2.0% of steel fibers were incorporated in the test specimen, which failed to satisfy the amount of fiber reinforcement necessary to ensure a reserved strength of 35%. Even if this is taken into consideration, the test results show the reserved strength to be very low due to the fiber balling phenomenon as mentioned above. In the case of the MTF25-N specimen without transverse reinforcement, the required fiber volume fraction was estimated to be as high as 8.45%. However, since it is in fact impossible to incorporate such a large amount of steel fibers, it is difficult to replace torsional reinforcement with only the steel fibers in this case [12, 27]. Therefore, the design for torsion should be carried out using an appropriate amount of longitudinal and transverse reinforcement for such members.

5. Conclusions

In the present research, an experimental study on the total of six torsional members was carried out to verify the equation

TABLE 4: Application of the proposed minimum fiber factor to SFRC.

Specimen names	Total torsional reinforcement, ρ_{tot} (%)	$\rho_{\text{tot, min}} - \rho_{\text{tot}}^{\text{rebar}}$ (%)	Effective volume fraction, ξ	Required minimum fiber factor, F_{min}	Designed minimum fiber factor, F_{min}	Steel fiber used in design			
						V_f (%)	l_f (mm)	d_f (mm)	ρ_f
MT30-0.77	0.774	0.27	0.38	1.85	1.85	3.08	30	0.5	1.0
MT30-1.32	1.322	-0.15	0.38	-1.06	0.00	0.00	30	0.5	1.0
MT40-1.32	1.322	0.05	0.38	0.31	0.31	0.51	30	0.5	1.0
MT40-1.89	1.887	-0.71	0.38	-4.35	0.00	0.00	30	0.5	1.0
MTF25-0.77	0.774	0.17	0.38	1.30	1.30	2.17	30	0.5	1.0
MTF25-N	0.434	0.66	0.38	5.07	5.07	8.45	30	0.5	1.0

FIGURE 8: Estimation of reserved strength for torsional members: (a) total reinforcement ratio versus $\rho_t f_{yt} / \rho_l f_{yl}$; (b) total reinforcement ratio versus f'_c .

to calculate the minimum amount of torsional reinforcement for RC and SFRC members, which was proposed in the authors' previous research. The main test variables included the transverse and longitudinal torsional reinforcement ratios, the compressive strength of concrete, and the incorporation of steel fibers. The following conclusions were drawn from the test results and the verification process of the proposed model.

- (1) As the total torsional reinforcement ratio ($\rho_{\text{tot}}^{\text{rebar}}$) increased, more smeared torsional cracks occurred. In addition, it was found that the additional incorporation of steel fibers in the specimen with the same reinforcement ratio made it possible to improve crack control by bridging effect of the steel fibers.
- (2) If the concrete compressive strength (f'_c) is high in a member with the same reinforcement details, the

cracking torsional moment (T_{cr}) is relatively large. Therefore, a greater amount of torsional reinforcement is needed to ensure the reserved strength.

- (3) The minimum torsional reinforcement ratio ($\rho_{\text{tot, min}}$) proposed for ensuring a reserved strength greater than 35% is calculated considering the transverse and longitudinal reinforcement ratios (ρ_t and ρ_l), steel fiber volume fraction (V_f), and concrete compressive strength (f'_c). It has also been proposed in the form of a minimum fiber factor (F_{min}) to facilitate application to SFRC members.
- (4) It was possible to estimate the failure modes of the RC and SFRC torsional specimens accurately by examining whether the proposed minimum reinforcement requirement is satisfied. However, in the case of SFRC members, the reinforcing effects of steel fibers need to be evaluated more conservatively due to the low workability, fiber balling phenomenon, etc.

Data Availability

The experimental data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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