Uncertainty quantification of rock quality designation at the Gazestan phosphate deposit

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Several geomechanical variables reflect the characteristics of rock masses and in recent years have been considered as the geometallurgical responses in the mining industry. These variables have a great effect on the energy consumption and in further stages of mineral processing, impacting the throughput of the grinding circuit, metal recovery, and reagent consumption For instance, rock quality designation (RQD) is not only a very popular parameter used to define domains with excellent and poor physical properties, but can also be used to predict relative throughput rates in a SAG milling circuit. Geostatistical modelling of this regionalized variable is significant and nonlinear Geostatistics can be employed for the spatial quantification. In this study, the multi-Gaussian kriging approach is adopted as a nonlinear geostatistical technique for probabilistic domaining of RQD at unsampled locations at a phosphate deposit in Iran. The resulting estimates are checked thoroughly by cross-validation and can be used for defining the probable areas dominated by soft, moderate, and hard rocks applicable for mine design and mineral processing plant optimization

INTRODUCTION

Zoning of subsurface materials based on 3D modelling of quantitative descriptions of rock mass quality is a powerful tool in civil and mining engineering. The geomechanical parameters logged from boreholes are able to provide sufficient information for the quality of rocks and the region of interest to be defined. Rock quality designation (RQD) (Deere et al., 1967) is becoming increasingly used since it gives practitioners, in a simple way, a reasonable idea of the appropriate definition of the corresponding areas. Application fields of spatial RQD characterization in the geosciences include mine design (Shademan et al. 2016; Eivazy et al. 2015), geohazard zonation mapping (Masoud and Khalaf, 2017), tunnelling (Ozturk and Simdi, 2014; Ozturk and Nasuf, 2002), safety design (Oh, 2013), and geometallurgical studies (Burger et al., 2006). Use in the latter application implies that the RQD can be considered as a geometallurgical variable since its modelling has a direct impact on the design and optimization of the grinding circuit and operating parameters for optimum throughput and recovery (SGS, 2015). Moreover, this regionalized variable, in particular, monitors the size distribution of the blasted rock, which dictates the crusher and mill throughput. For instance, Burger et al. (2006) showed that modelling ore domains in a copper-gold mine using RQD zoning unproved the comminution performance, thus benefiting medium- to long-term production scheduling. Dunham, and Vann (2007) also mentioned that this geometallurgical descriptor for rock quality, as an important component in several rock mass quality classification schemes can be used to predict relative throughput rates for SAG milling circuits. Therefore, it is of interest to apply an effective algorithm for modelling the geometallurgical properties, including RQD (Powell, 2013).

Generally speaking, spatial prediction of regionalized variables by a linear geostatistical methodology such as kriging is common practice. The main difficulty with such a robust interpolation methodology is the smoothing effect (Vann, Jackson, and Bertoli, 2003; Chiles and Delfiner, 2012; Sadeghi, Madam, and Carranza, 2015), which affects under- and over-estimation Therefore, such an insufficient model cannot be used for decision-making in optimization of mineral processing plants (Deutsch et al., 20116). In addition to this reasonable criticism, some geomechanical parameters like RQD are non-additive and cannot be modelled by these types of linear interpolation methodologies (Coward et al, 2009). Non-additivity of a variable implies that its mean value does not coincide with the arithmetic average and consequently, 3D mapping of such a variable is problematic. However, in this context there are some applications of different aspects of kriging for spatial visualization of RQD, such as ordinary kriging; Liu et al., 2015; Ozturk and Simdi, 2014), and indirect estimation by integrating the seismic values using indicator kriging (Oh, 2013). Nonlinear geostatistical modelling methods such as geostatistical simulation are strongly recommended as an alternative for spatial characterization of geometallurgical variables since there is no requirement for linear averaging (Carrasco, Chiles, and Segure, 2008; Newton and Graham, 2011; Boisvert et al., 2013). For instance, the RQD can be conditionally simulated by sequential simulation; Cromer, Rautman, and Zelinski, 1996; Assari and Mohammadi, 2017; Assari, Mohammadi, and Ghanbari, 2016; Madani and Asghari, 2013) and turning band simulation (Ellefmo and Eidsvik, 2009). Although simulation approaches turn the joint imcertainty in an efficient avenue, significant computational time and post-processing are needed for managing the realizations (Deutsch, 2016). Other nonlinear geostatistical functions can also be considered, including lognormal kriging, disjunctive kriging, and indicator kriging. Nevertheless, these approaches suffer from a smoothing effect for skewed distribution; Moyeed and Papritz (2002) showed that the estimation results are relatively poor in this latter case.

Multi-Gaussian kriging is another nonlinear geostatistical technique that has been accepted among practitioners due to it having less or no smoothing effect. Two families of multi-Gaussian kriging, called simple and ordinary, have been proposed for recoverable resource assessments and mapping the probabilities in mineral resource evaluation in block modelling of an ore deposit. Simple multi-Gaussian kriging assumes that the mean value is perfectly known through the region, and then restricts its usage only to stationary domains (Emery, 2008; Guibal and Remacre, 1984; Marechal, 1984; Schofield, 1988). To overcome this impediment, Emery (2008) proposed a new approach by substituting the unknown mean by a random variable constant throughout the region in order to meet the true conditional distribution. This method is highly recommended in the case of some domains including trends in the variability of attribute under study, whereas universal kriging is problematic in variogram analysis (Armstrong, 1984; Cressie, 1987).

The first aim of this paper to apply ordinary multi-Gaussian kriging to model the spatial distribution of RQD in a phosphate ore deposit in Iran. Since multi-Gaussian kriging is perfectly developed for uncertainty quantification above a certain threshold, the second aim is to update the current algorithm of multi-Gaussian kriging presented by Emery (2008), so that one can calculate the uncertainty between two consecutive thresholds (applicable for defining the area covered by different rock qualities derived from the RQD table).

METHODS

Multi-Gaussian Kriging

Multi-Gaussian kriging as a nonlinear geostatistical methodology is based on the multivariate normality assumption of its posterior distribution in which the mean and variance are identified based on kriging mediodology (Chiles and Delfiner, 2012). This fundamental theory entails the anterior distribution of the variable under study being Gaussian normal standard with mean and variance equal to 0 and 1, respectively. Simple multi-Gaussian kriging is applicable when the random field is Gaussian and shows stationary characteristics on its first and second order moments (mean and variance), In this study, the ordinary multi-Gaussian kriging is applied to quantification of uncertainty in RQD (subsequent section). So, in order to perform the experiment through multi-Gaussian kriging, the general outlined below should be followed: exploratory data analysis, declustering, normal score transformation of the raw ariable, variogram analysis over the Gaussian values, estimation of the Gaussian variables (simple or ordinary), back-transformation to the original variable, and uncertainty quantification and cross-validation (Figure 1).

1. Exploratory data analysis: the statistical parameters are considered and possible outlying values should be detected.

2. Declustering: in order to provide the representative distribution of the original data, a cell declustering technique can be applied in case of irregular sampling.

3. Normal score transformation: since multi-Gaussian kriging concept is based on the Gaussian distribution, the declustered data needs to be transformed to the normal Gaussian distribution with mean 0 and variance 1.

4. Variogram analysis: an experimental variogram is calculated and an appropriate model should be fitted along the proper anisotropy. This procedure can be implemented through automatic or semi-automatic approaches.

5. Back-trarisformation: the normal score estimated values are back-transformed to original space and the uncertainty can be quantified though some nonlinear functions.

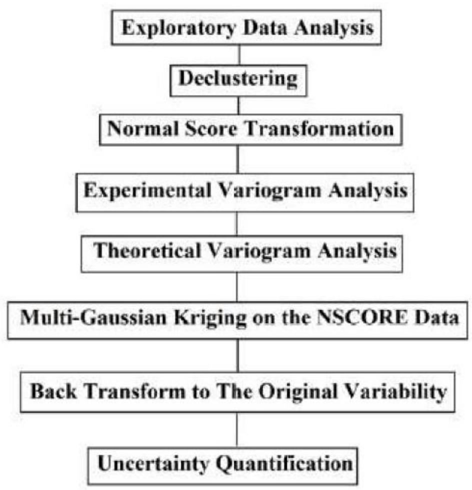


Figure 1. Schematic flow for multi-Gaussian kriging.

One application in this study is to map the probability of finding the estimated RQD between two thresholds that give the related rock mass quality at unsampled locations. In following sections, this idea is presented and discussed by means of a case study.

CASE STUDY: GAZE STAN PHOSPHATE DEPOSIT, IRAN

The Gazestan phosphate deposit is located in Bafg Province in Central Iran (Figure 2). The deposit is hosted in the Rizu Formation, which consists of carbonate sediments, shale, sandstone, and volcanic rocks. Intrusive rocks are present in stocks and dikes, including gabbros, diorite, and diabase with some outcropping within the area, as well as sedimentary and volcanic rocks. Green rocks with acidic components contain phosphate and iron ore mineralization, which in some parts shows facies changes relayed to depth. Alteration in volcanic rocks consists of silicification, feldspathization, chloritization, and formation of mafic minerals such as epidote, tremolite, and actinolite. Apatite mineralization occurs as apatite- magnetite lenses, veins, and nodules, in strongly chloritized pyroxene-bearing mafic diabase. The P2O5 content varies between 3% and 38%, and values of rare earth elements (REEs) in apatite exceed 1.5%. Detailed exploratory studies show that tens of millions of tons of ore with a mean grade P2O5 grade of 10-15 % and about ten (REEs) can be expected in this ore deposit (Jamali, 2008). From a structural perspective, the Gazestan deposit belongs to the central Iran zone and Posht-e-Badam-Bafq Block.

There have been three episodes of faulting and fracturing system in the area, and one of thrusting (Jamali, 2008; Madani and Asghari, 2013).

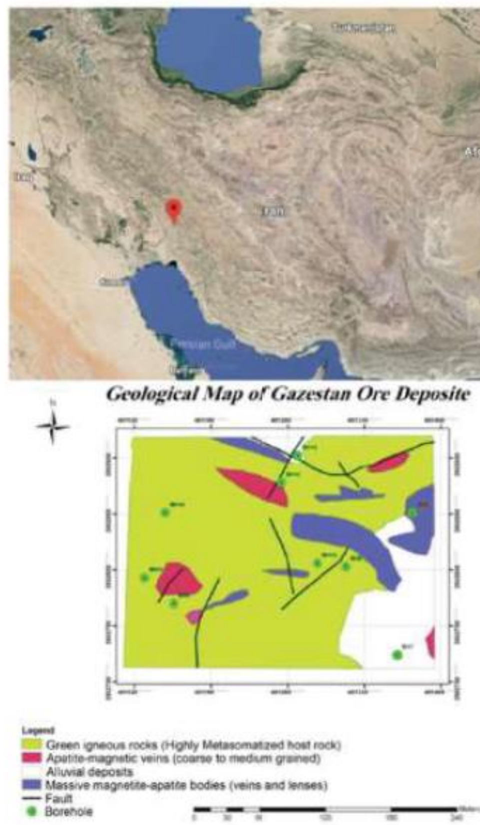


Figure 2. Geological map and location of nine boreholes and the study area.

Using the Gazestan phosphate deposit as a case study, the spatial distribution of RQD was modelled via the ordinary multi-Gaussian kriging algorithm RQD, which was introduced by Deere et al, (1966) as a standalone classification system, is also one of the input variables of the two well-known rock classification systems, the RMR and Q systems. The RQD, which was initially proposed as a measure of the quality of borehole core, is defined as the ratio, expressed as a percentage, of the total length of sound core pieces that are 10 cm or longer to the length of the core run. On the basis of the RQD values, a rock mass can be classified in terms of its quality as very poor, poor, fail, good, arid, very good for RQD values <25%, 25-50%, 51-75%, 76-90%, and 91-100%, respectively.

Exploratory Data Analysis

The data-set consisted of 470 samples from nine boreholes. Due to the paucity of the data, 100 more sample locations were randomly added to the drilling pattern. The purpose of this work is to show the capability of the methodology to quantify uncertainty. To do so by turning band simulation (Emery, 2006), RQD values at those locations are conditionally simulated with respect to the data from the original boreholes (only one realization is applied here). This ensures that the simulated values have the same statistical characteristics as the borehole data and enhance the visualization of RQD variability in the area. Table I shows the statistical analysis of the all 570 samples (original plus synthetic). The RQD varies from 0 to 100%. It is obvious that the rock mass is quite jointed and the quality varies from very poor to excellent.

Table I. statistical parameters of the RQD values in Gazestan deposit.

Parameter	Value
Number of samples	570
Mean	54.031
Variance	1470.947
Minimum	0
Median	54.034
Maximum	100

In order to perform multi-Gaussian kriging, the conditional samples should then be transformed to a Gaussian normal distribution (step. 1). Since the drilling patterns are commonly irregular, a declustering technique is required (Deutsch and Journel, 1998) to provide a representative distribution of the variable of interest in the region Through this technique, the weights are allocated to each location considering the proximity to the surrounding RQD data. A cell dimension of 50 x 50 x 10 m is considered, which corresponds approximately to the dimensions of the drilling grid.

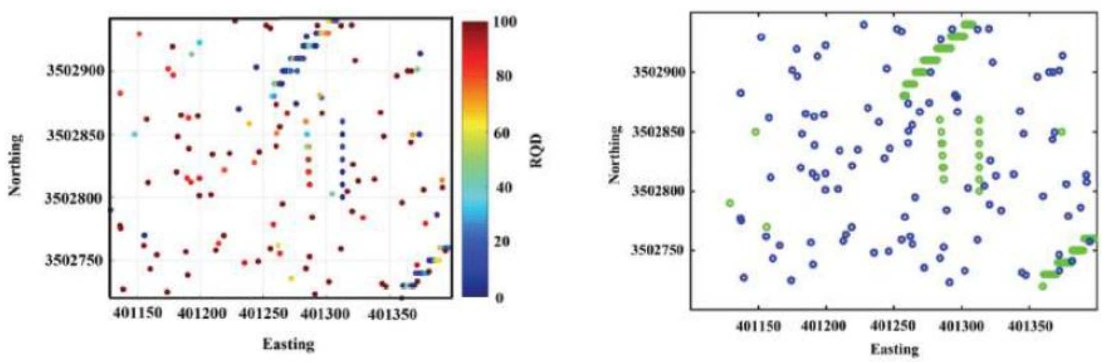


Figure 3. 2D location maps of the available RQD information. Right: distribution of boreholes (green circle: original and blue circle: synthetic); left: RQD values at all the borehole locations.

Step 2 in the multi-Gaussian kriging procedure involves transforming the declustered data to a normal standard distribution N(0,1). The reason why the data should be Gaussian-distributed is that the prediction of uncertainty at unsampled locations is based on conditioning the normal standard data (Chiles and Delfiner, 2012). Therefore, a Gaussian anamorphosis function is used to transform the RQD data into a Gaussian variable with mean 0 and variance 1. In this approach, one constructs the sample cumulative histogram by applying declustering weights. The normal-score-transformed data has a Gaussian distribution with mean approximately 0 and variance 1 (Figure 4).

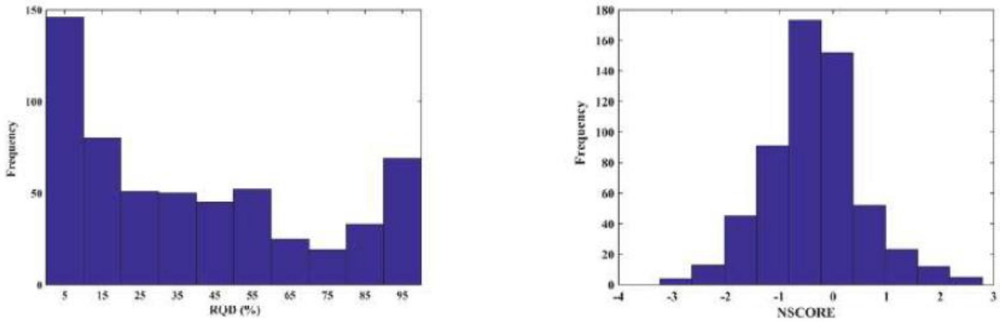


Figure 4. Histogram of original declustered RQD values (left); normal score values (light).

After transformation, it is of interest to check whether the transformed variable follows bivariate normality, since multi-Gaussian kriging is based on the assumption of bivariate normality. This can be implemented by checking the experimental variogram of different order (w) through the transferred Gaussian values. This paradigm is based on the comparison between the usual variogram and the variogram of lower order. The latter corresponds to the madogram, for which the power in the variogram formula is unity. In order to show that the bi-Gaussian assumption between two variables is respected, Emery (2005) verified that plotting the experimental variogram with those different orders against the conventional experimental variogram in log-log coordinates should be almost aligned with the theoretical lines. This theory is employed over the normal score RQD variable in this study, and as can be seen in Figure 5, the points are distributed along the thick solid lines with a reasonable deviation. This graph implies that the normal values are somehow in acceptable agreement with bi-Gaussian characteristics, and one can therefore apply the multi-Gaussian kriging over the underlying data-set.

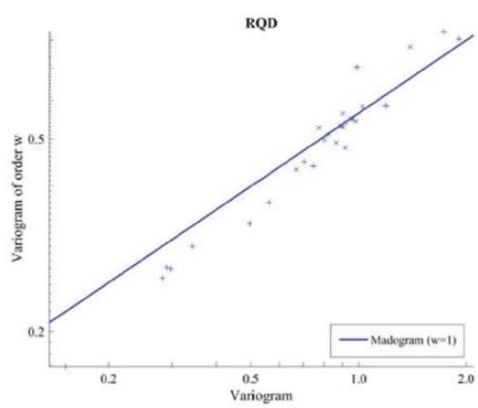


Figure 5. Experimental madograms of the Gaussian variables as a function of their experimental variograms.

Spatial continuity of geological properties, including geomechanical parameters, can be a measure of variation in variance with distance. Experimental variogram y(h) computes the average dissimilarity between data separated by vector h. It is calculated as half the average squared difference between the components of every data pair (Chiles and Delfiner, 2012):
$\gamma(h) = \frac{1}{2N(h)}\sum_{\alpha=1}^{N(h)}[z(u_\alpha) - z(u_\alpha + h)]^2$ is an h-increment of the variable z and N(h) is the number of pairs. Maximum and minimum spatial continuity is a requirement for every geostatistical methodology and should be defined prior to any variogram analysis. After considering the variogram range in different directions, we obtain one isotropy characteristic in the plan view and one perpendicular anisotropy toward the vertical.
After the anisotropy of RQD in the region has been ascertained, variogram analysis is performed on the transformed RQD data to model the related spatial structure. One omnidirectional variogram and one vertical

directional variogram. is calculated and fitted with two spherical models (Figure 6), using two basic nested structures:

$$\gamma(h) = 0.8Sph(70,70,150) + 0.2Sph(150,150,\infty)$$

The plotted models show that the related variogram has geometric anisotropy.

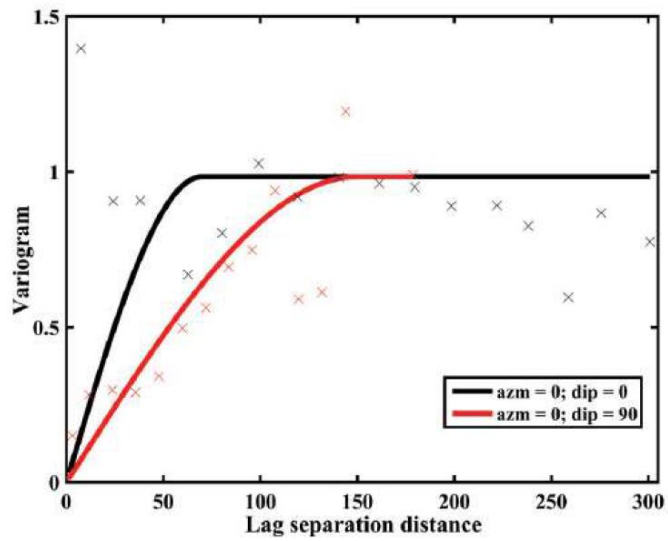


Figure 6. Direct variograms of Gaussian variables (crosses: experimental variogram and solid lines: theoretical variogram modelled to the experimental).

Multi-Gaussian Kriging Results

Multi-Gaussian krigjng is performed on a regular grid with dimension of 5 X 5 X 5 m for prediction of RQD. In this algorithm, ordinary kriging is used since it is independent of the mean and its stationary behaviour (Emery, 2006). The moving neighbourhood is constructed based on the variogram ranges and is set to 200 and 150 for horizontal and vertical dimensions, respectively containing up to 50 original data-points from the surrounding area. The RQD, like just odier geomeclianical parameters, is non-additive (Deutsch and Journel 1998). This means that the averages of these kinds of measurements do not follow the linear averaging (Deutsch, 2013). As a result, when applying kriging or any other estimation methodology, one cannot discretize the block and point support should be considered. In this research the RQD values are estimated in the centre of the block and no discretization is treated in the process of modelling. Figure 7 shows a plan view of the estimated RQD at 80 m elevation after back-transformation to the original distribution as a general consideration.

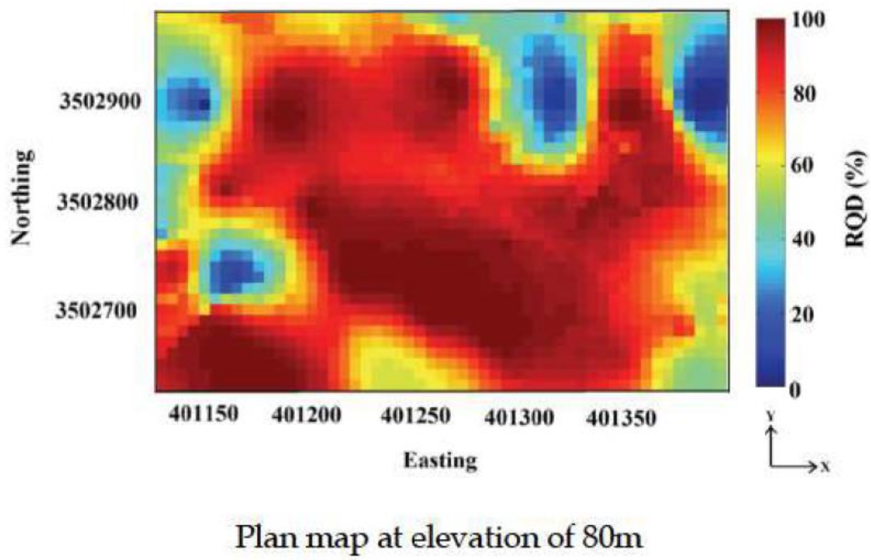
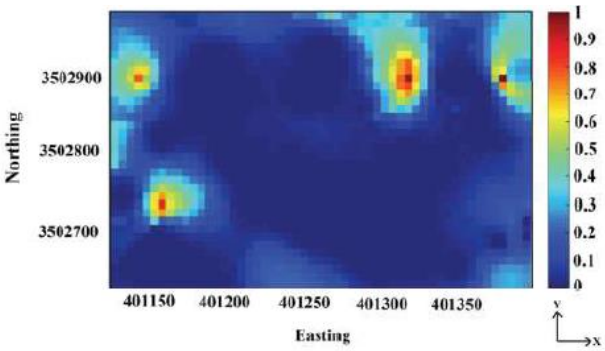


Figure 7. Multi-Gaussian kriging of RQD. High RDQ values are distributed in the middle of the region.

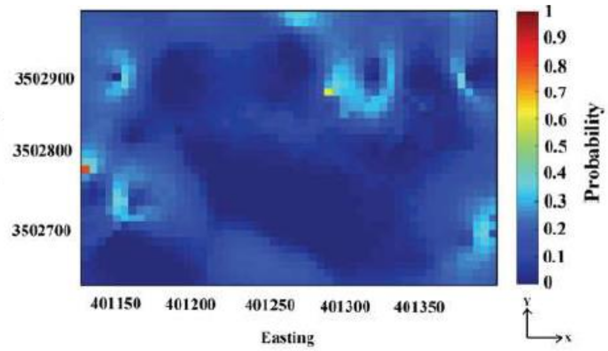
Probabilistic Description for Each Rock Mass Quality Domain

As discussed above, the uncertainty function can be calculated by Equation [5]. In this section we quantify the local uncertainty (probability) of the underlying domains which specify the rock mass quality for that region according to the data obtained from boreholes. These maps are constructed by computing, for each block, the frequency of occurrence of each rock quality domain over 500 realizations drawn from Equation [4] in one section (Figure 8). The maps depict the risk of finding a domain different from others. The sectors with little uncertainty are those in which there is a high probability of a given rock quality domain (coloured red in Figure 8), whereas in those with a very low probability (dark blue in Figure 8) there is little likelihood of finding this domain.

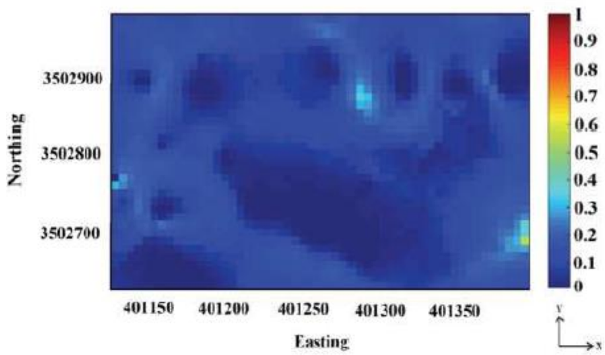
The predictions and the model obtained by ordinary multi-Gaussian kriging, at each data location were validated using a cross-validation procedure (Deutsch and Journel, 1998). The correlation coefficient between the predicted and true RQD is 87.58. The mean relative error is 0.301, which implies that the spatial estimation of data location by cross-validation gives reasonable results and the probabilistic description of RQD variability at unsampled locations can be trusted.



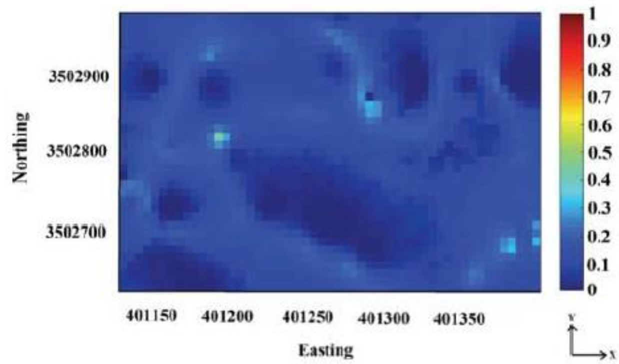
Very poor (plan section)



Poor (3D visualization)



Fair (plan section)



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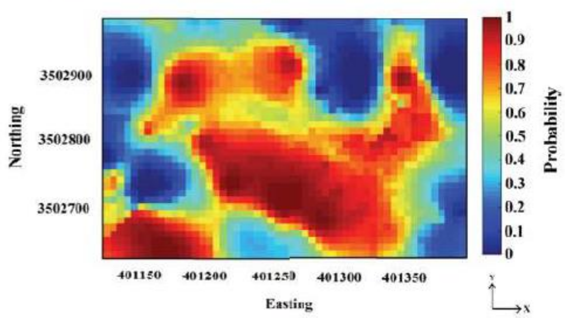

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Figure 8. Probability maps for different rock mass qualities at elevation 80 m.

CONCLUSION

A multi-Gaussian kriging approach for estimating the spatial distribittion of RQD was presented to overcome some limitations of the nonlinear geostatistical techniques. A 2D surface map of RQD was established, allowing a spatial visualization of the RQD classes (very poor, poor, fair, good, and very good) and their associated probabilities, In addition, the associated uncertainty can be validated on the basis of the conditional distributions, and the obtained probability matched the overall results. This technique could be expanded to 3D mapping, and also compared with some other uncertainty quantification techniques in further research.

The current work highlighted several advantages. Theoretical results and the case study demonstrated that this methodology was able to calculate, in a reliable maimer, the joint uncertainty at unsampled locations. The results of this study can serve as basis for future developments in the mapping of geometalluigical variables such as RMR GSI, Q and ore grades. Иге current methodology can assist in the construction of spatial models of the RQD, which are important in mine design and for optimization downstream mineral processing. However, the upscaling issue should be considered when employing the methodology for other geomechanical parameters.

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