PROBABILISTIC DOMAINING OF ALUMINUM AREAS IN IRON DEPOSITS

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ABSTRACT

Geostatistical modeling of Iron grade (Fe) in metalliferous deposits is a rationale stage in further analysis of mine design such as mine planning and mineral processing plant optimization. This procedure is becoming important in the case when Fe is controlled by a co-variable that impacts negatively or positively the mechanical characteristics of the steel production. For instance, AL2O3 in most of the iron deposits in a certain level of concentration is helpful to increase the mechanical properties of the steel and in some other level plays as a gangue material which leads to prolonging the mineral processing procedure. Therefore, its 3D modeling is as significant as spatial modeling of Iron, in which it can give the idea of spatial distribution for aluminous areas in a mineral deposit. Geostatistical simulation is a powerful tool that able the practitioners to come up with the uncertainty quantification and consequently the probabilistic description of those areas. However, independent simulation in such a case that there exists a reasonable correlation between these two variables doesn't guarantee that the generated outputs preserve this correlation. Conditional co-simulation instead can be applied to check whether this intrinsic characteristic is reproduced properly. In this study, turning band co-simulation algorithm is applied to generate the iron and aluminum spatial distribution in a metalliferous deposit located in Brazil. The results then compared to those produced by independent simulation.

KEYWORDS

Iron deposits, Aluminum, Geostatistical modeling, co-simulation, Turning Band simulation

INTRODUCTION

With depletion of appropriate ores with proper grade distribution, for instance, high concentrations of iron with fewer trace elements which is acceptable for steel production (Mukherjee and Whiteman, 1985), new multivariate geostatistical techniques and approaches are used for constructing of block model which is based on complex multi element deposits with grade uncertainty. In blast furnace operations, presence of such the elements in iron deposits can have either negative or positive impact on mechanical property of iron processing and operation of smelter. Despite the fact that blast furnace burden composed of iron ore sinter, which is suitable for blast furnace performance, depletion of ores with high-grade iron leads to high concentration aluminum or other trace elements (Mukherjee and Whiteman, 1985). One of the co-variables that presents in many metalliferous deposits is aluminum oxide (AL2O3). High concentration of this element in sinter can lead to reduction of strength and negatively affects characteristics of prepared sinter (Das et al. 2001; Kumar et al. 1995). Furthermore, heavy of aluminum loading to the blast furnace results in large slag volume in furnace which leads to high consumption of coke (Hino et al. 2003). Experiment done by Okazaki et al. (2003) showed that concentration of aluminum oxide should be less than 1.5% in adhering fines to obtain proper pore structure, which is important in coalescing and reshaping process. Therefore, depending on market demand for quality of iron, existence of trace elements such as aluminum oxide with desired range of iron leads to better performance in furnace operation procedures. Mentioned negative factors can be partly neutralized by the addition of calcium wire (FeCa) or other number of solutions. However, the first solution to be implemented is avoidance of high aluminum concentration in iron deposit (Rosenqvist and Terkel, 1983). Geostatistics offer a range of methods, techniques for estimation, analysis and mapping of multivariate information or elements distributed in a particular region (Wackernagel, 2003; Chile and Delfiner, 2012). Geostatistics was initially originated in mining engineering for mathematical computations of ore deposits in the early 1950s (Sichel, 1952) and still has wide application of improvements in mining engineering for its significant role in determination of useful zones in deposits that can be considered from economical side. Basically, geostatistical simulations propose more reliable evaluation of grade distribution due to the fact that it produces many possible scenarios of each block, while the other deterministic geostatistical approaches such as kriging provides only one unique value for each block (De-Vitry, Vann and Arvidson, 2010). Existence of a good correlation among the elements motivates one to use co-estimation or co-simulation for the purpose of spatial modeling. These multivariate Geostatistical approach are useful for reproducing the intrinsic correlation after modeling. Among others, turning bands co-simulation algorithm can be

applied for such a spatial modeling. In this paper, it is of interest to employ the turning bands co-simulation for two cross-correlated variables (Fe and Al2O3) obtained from a Brazilian iron deposit to define the beneficial probabilistic area of iron with aluminum less than 1.5%. The results are then compared with those resulted from turning bands simulation, in which these two random functions are independently simulated.

METHODS

Turning bands (co)-simulation

Turning bands simulation is an approximate algorithm based on multi-Gaussianity assumption of the underlying random field that first introduced by Matheron (1973) and then extended in some organized program codes (Lantue´joul, 1994; Emery and Lantue´joul, 2006). The principal concept of this algorithm is based on first, drawing plenty of lines with random orientation and second, simulating a one-dimensional Gaussian random field along each line (Lantuéjoul, 1994, 2002). Having the covariance model fitted to the primary declustered normal score variable, the covariance function is derived in one-dimensional random fields. Turning bands simulation provides a non-conditional multi-dimensional random field compatible with the target covariance model, in which the simulated values are practically standard Gaussian (Emery and Lantue´joul, 2006). In order to generate the conditional realizations, the non-conditional simulation so obtained should be progressed through one post-processing kriging step (Chilès and Delfiner, 2012; Emery, 2008; Journel and Huijbregts, 1978). The steps to perform the conditional turning bands simulation are as follow:

1- Exploratory data analysis: this step is needed to detect the possible errors and outliers.
2- Declustering: the scarcity of data in some regions makes the sampling patter irregular and statistical parameters possibly biased. One idea is to account for the weights of each location by cell-declustering technique to correct the pseudo skewness in the global distribution of the dataset (Deutsch and Journel 1998; Goovaerts 1997).
3- Transform the variable to normal score standard: since the turning bands simulation algorithm is based on the multi-Gaussianity assumption of the input data, the variable should be transformed to standard Gaussian with mean 0 and variance 1. The step can be done through Gaussian anamorphosis (Rivoirard, 1994) or quantiles-based approach (Deutsch and Journel, 1998).
4- Variogram analysis: direct experimental variogram is computed over the normal score values and the proper models are fitted by means of either manual or automatic paradigms.
5- Independent simulation: turning bands simulation first, non-conditionally simulates the values in a specified region by the information obtained from spatial continuity analysis in step 4 and then, back transform the realizations to the original space.

In turning bands co-simulation, it is of interest to stochastically simulate the cross-correlated variables (more than two). In this case, the cross-covariance function is needed to construct such a one and multi-dimensional Gaussian random fields in the region. The non-conditional step is the same as tuning band simulation, however, in part of the conditioning mechanism, the co-kriging must be used rather that kriging in order to hold the multivariate characteristics (Carr and Myers, 1985; Myers, 1989; Gutjahr et al., 1997; Emery, 2008). The general workflow is similar to turning bands simulation previously explained except that the items 1, 2 and 3 should be implemented for each variable separately. In variogram analysis, since the co-kriging system is established on the basis of cross-covariance matrix, it is necessary to calculate the experimental cross-variogram along with direct variograms. For n variables, $n(n+1)/2$ experimental variograms should be considered (Journel and Huijbregts, 1978). In order to fit the theoretical direct and cross-variograms models, linear model of coregionalization (LMC) should be fitted to all n(n+1)/2 experimental variograms as a linear combinations of equivalent structures together with the identical ranges, but different in sills, (Chiles and Delfiner, 2012; Wakernagel, 2003). The most tedious part of this job is to construct the permissible positive semidefinitness conditions in fitting the sill matrices. Once this constraint corroborated, the model can be used in variance-covariance matrix in the co-kriging system required in the conditioning process (Goovaerts, 1994).

RESULTS

3. Presentation of dataset

3.1. Case study

The study area, Carajás Mine, is located in Parauapebas municipality, state of Para in the Northern Brazil. The case study provided by Vale, is based on data set of iron deposit which includes iron

(Fe) and trace elements as aluminum oxide (Al2O3), manganese (Mn) and phosphorus (P). Geological and lithostructural setting of the Carajás deposits refers to Grao-Para Group's metasidementary and metavolcanic rocks which has two formations; Parauapebas Formation's volcanic rocks (Meireles et al., 1984) and Carajás Formation's ironstones (Beisiegel et al., 1973).

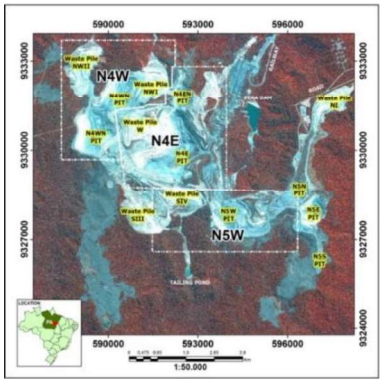


Figure 1 - Top view of mine study area in Para state, Brazil (Paradella et al., 2015).

3.2. Exploratory Data analysis

The dataset consists of 1380 samples obtained from boreholes campaign. Initial step for all mineral resource estimation projects is exploratory analysis of data. First of all, possible outliers should be detected. In this case, they can be removed or replaced by a top-cut value. Second step is identifying duplicated samples and to "mask" them. Declustering as the third step is to assign the weights to the sample points in order to make the global distribution representative (David, 1977 and Deutsch, 1989). To do so, no outlier and duplicated samples detected in the dataset. Declustering has been done in a dimension of $100m \times 400m \times 15m$ and the univariate statistical parameters have been calculated (Table 1).

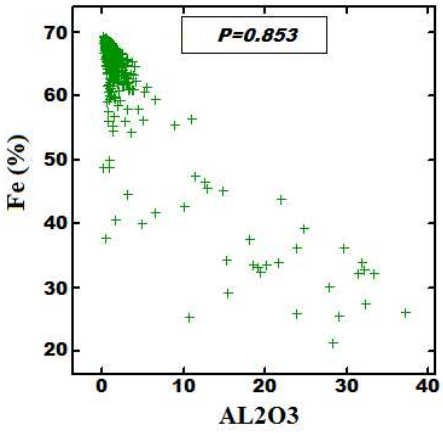


Figure 2 - Correlation between the declustered Fe and Al2O3.

In the case of multivariate analysis, the correlation coefficient parameter is a good measure of dependency. As it can be seen from figure 2, the correlation between declustered original Fe and Al2O3 is 0.853 which encourages one to use co-simulation rather than independent simulation. Beside of that, the heterotopic characteristic of sampling points implies that the co-simulation methodology turns out much satisfying results. Heterotopic data means that some of the variables share some locations of samples in the data set (Wakernagel, 2013).

3.2 Transformation of the Variables into Normal Score Standard

In geostatistical simulation methods, it is required to map the data to Gaussian space. Transformation of the variables to normal score standards is necessary in order to get Gaussian distribution, in which the mean and variance are 0 and 1, respectively (Deutsch and Hournel, 1998). This can be implemented by Gaussian anamorphosis through Hermite polynomial expansion (Rivoirard, 1994). In figure 3,

transformation to normal score for Iron (Fe) and Aluminum oxide (Al2O3) is shown. Table 1 shows the statistical descriptions of the declustered and transformed values of Fe and Al2O3.

Table 1 – Statistical parameters of Fe and Al2O3 before and after transformation

	Fe declustered	*Fe transformed*	*Al2O3 declustered*	*Al2O3 transformed*
Number of samples	996	996	615	615
Minimum	4.80	-3.04	0.10	-2.92
Maximum	69.17	3.04	37.20	2.92
Mean	52.78	0.00	2.06	0.00
Standard Deviation	20.76	1.00	4.79	1.00

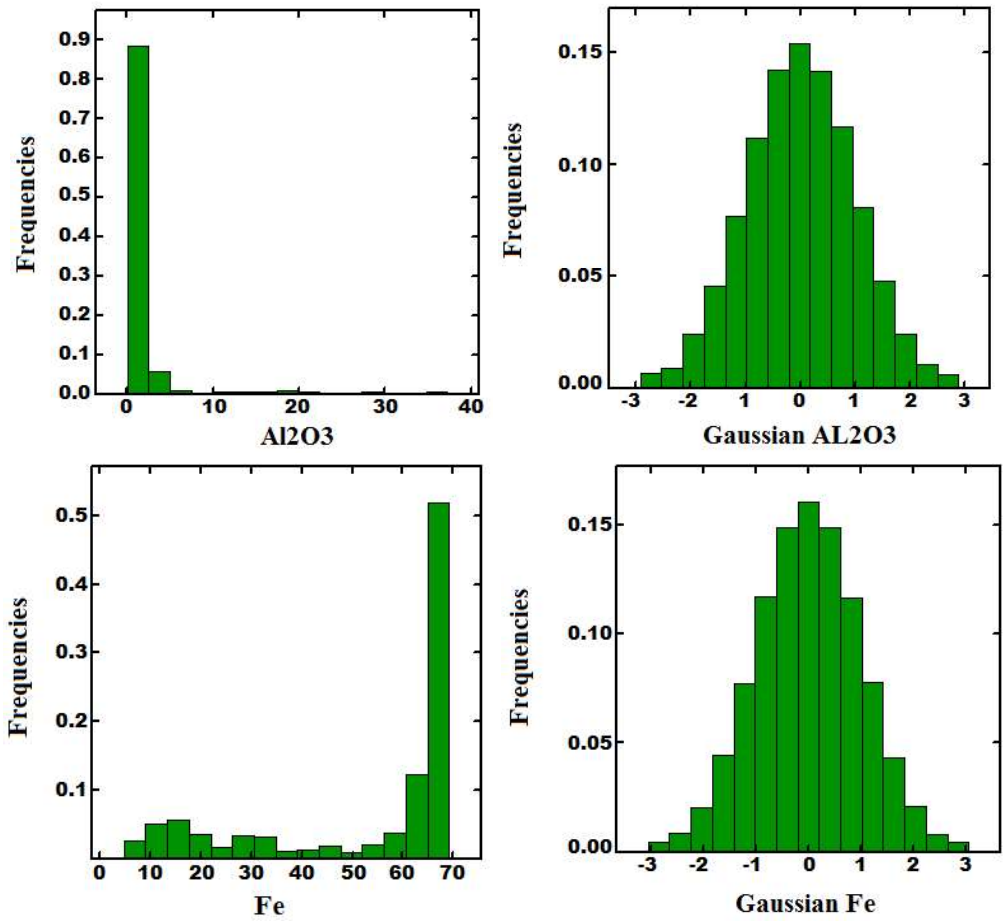


Figure 3 - Original declustered (left) and Normal Score Transformed (right) histograms of Al2O3 and Fe.

3.3 Examination of the multivariate and bivariate Gaussianity

The presence of an interesting positive correlation coefficient among the variables and its univariate transformation (Fig 3) to Gaussian random field does not ensure that the multivariate distributions are also Gaussian (Leuangthong and Deutsch 2003) (a critical assumption for implementing TBCOSIM). One important specification is to examine the multivariate Gaussianity by checking the homoscedasticity and linearity among the transformed cross-correlated variables (Johnson and Wichern, 1998). As an example, Figure (4) illustrates the scatterplot between two underlying elements (Fe and Al2O3) and one can see that the bivariate character is somehow in agreement with homoscedasticity and linearity definitions at small lags. However, the recognition of bivariate normality is somehow demanding in large lags (Emery, 2005).

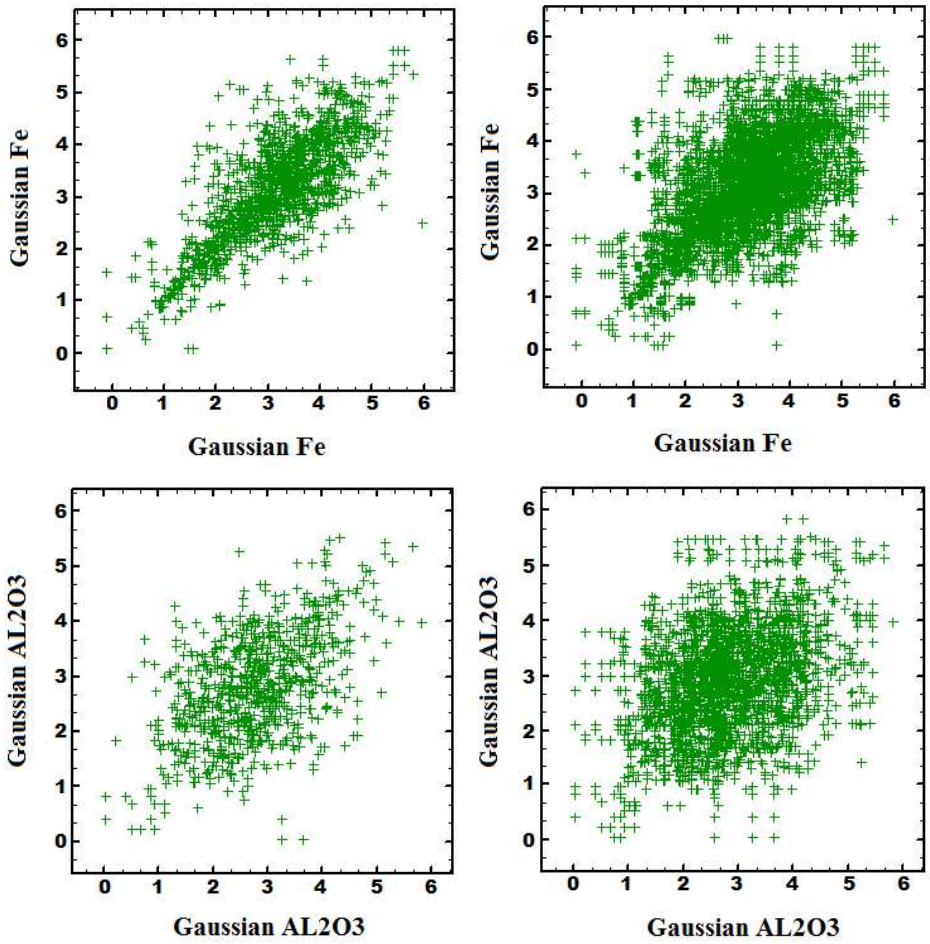
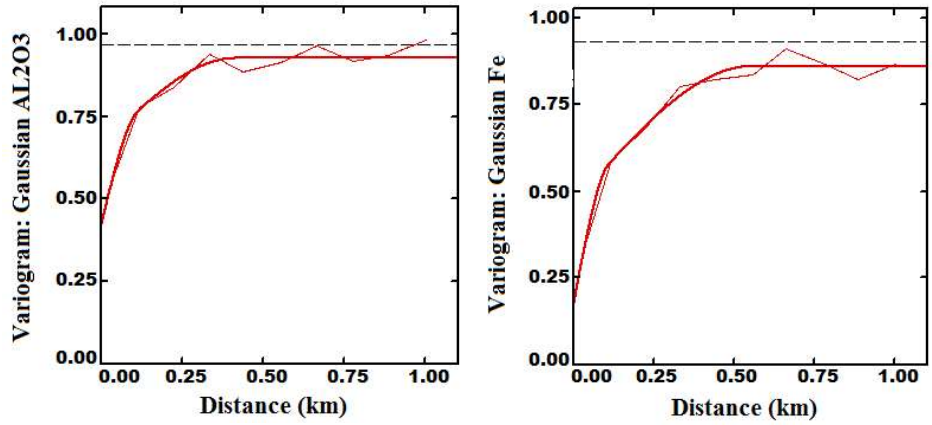


Figure 4 - Bivariate Gaussianity examination at lag (30 m) (left and lag (100 m) (right).

3.4 Spatial Continuity

Direct variograms were calculated for independent Turning Bands Simulation and cross-variograms were computed for Turning Bands Co-Simulation over normal scored aluminum oxide (Al2O3) and iron (Fe). Variogram fitting was done semi-automatically based on linear model of coregionalization (Journel, A. G., & Huijbregts, C. J. (1978)). For the sake of simplicity, the isotropy is considered for both variables. Therefore, the omni-directional variogram taken into account for whole direct and cross-variograms. Figure 5 shows the fitted direct and cross-variogram for Al2O3 and Fe. The equation for the cross-variogram is shown in equation 1:



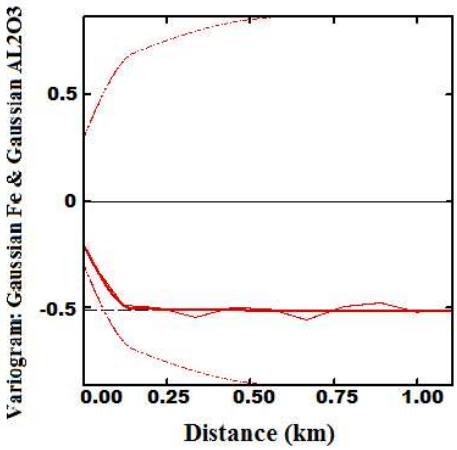


Figure 5 - Direct (upper) and cross-variogram (lower) for the Al2O3 and Fe.

$$\text{Spatial continuity: } \begin{pmatrix} \gamma_{Fe}(h) & \gamma_{Fe-Al2O3}(h) \\ \gamma_{Fe-Al2O3}(h) & \gamma_{Al2O3}(h) \end{pmatrix} = \quad (1)$$
$$\begin{pmatrix} 0.4402 & -0.2129 \\ -0.2129 & 0.2149 \end{pmatrix} Nugget +$$
$$\begin{pmatrix} 0.3041 & -0.2906 \\ -0.2906 & 0.2811 \end{pmatrix} Spherical(149.53m, 149.53m) +$$
$$\begin{pmatrix} 0.1929 & -0.01285 \\ -0.01285 & 0.368 \end{pmatrix} Spherical(598.62m, 598.62m)$$

3.5 Turning Bands Simulation and Co-Simulation

All simulations were conducted on a grid with 10 m * 10 m * 10 m dimension. Type of neighborhood for conditioning process by kriging and co-kriging is moving with maximum distance of 800 m, larger than the maximum range in variography. The number of realization is 100 giving more reliability to conduct the methods with more confidence. Figure 6 shows the E-type maps reproduced by taking average of all realization within each block obtained from co-simulation and independent simulation after back-transformation to original space.

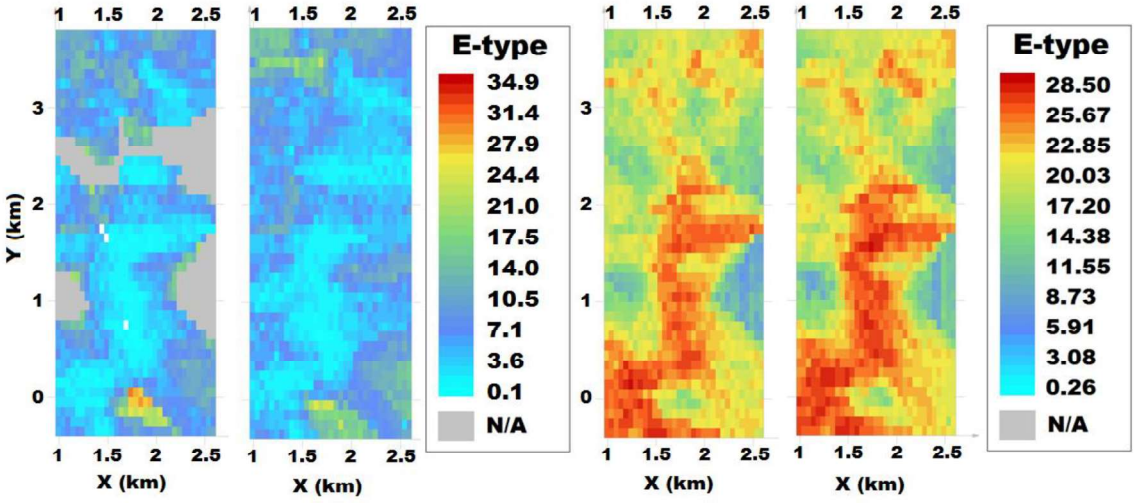


Figure 6 - E-type map of TBCOSIM and TBSIM of Al2O3 (left) and Fe (right). (Elevation 805 m)

3.6 Validation

This section focuses on the comparison between correlation coefficient calculated over the 100 realizations of both independent and co-simulation methods. Correlation coefficient that restitutes by turning bands simulation is 0.105 which is very different from original correlation (0.853, figure 2), while

turning bands co-simulation is 0.498 which is 5 times closer to original correlation coefficient. In case of turning bands simulation, the small correlation coefficient based on realizations can be explained in virtue of the fact that independent simulation do not take into consideration the intrinsic correlation between co-variables in multi-element deposits (Madani and Ortiz, 2017).

Table 2 - Correlation coefficient between Al2O3 and Fe through 100 realizations.

	TBCOSIM	*TBSIM*	*Original dataset*
Correlation Coefficient	0.4918	0.105	0.853

3.7 Probabilistic illustration of Al2O3

As it was mentioned earlier, in order to obtain proper pore structure of iron for processing, the concentration of aluminum should be less than 1.5% beneficial in mineral processing system for coalescing and reshaping (Okazaki et al. 2003). Therefore, it is of interest to identify the possible target areas with small amount of aluminum. Since these areas are not deterministic, the geostatistical simulation methodology provides this opportunity to probabilistically detect those regions. To do so, the output of tuning bands co-simulation (100 realizations) is taken into account for computing such a probability. Probabilistic illustration of Al2O3 below 1.5 % is shown in Figure 7.

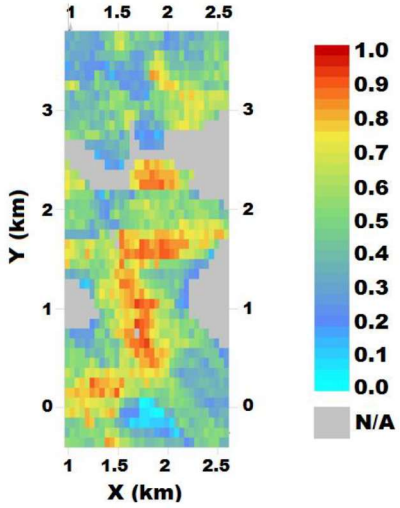


Figure 7 - Probabilistic illustration of Al2O3 below 1.5%. (Elevation 805 m)

DISCUSSION

The depletion of metalliferous deposits with high content of iron leads mining industry to mine metalliferous deposits with higher concentration of other trace elements which sometimes makes the processing less efficient or makes the quality of iron lower. However, geostatistical methods such as estimation and simulation can be applied to identify the blocks with interest of range for trace elements. For instance, trace element, aluminum can lead to high viscosity of slag which can cause unfavorable effect on furnace processes where metals are smelted.

Comparing to estimation (kriging), simulation methods generate more reliable results of spatial grade distribution as it can reproduce multiple scenarios of a deposit, whereas estimation only turns out one scenario of a deposit (De-Vitry, Vann and Arvidson, 2010). Reproduced spatial variability of the variable is another interesting characteristics of geostatistical simulation, which is not guarantee in the case in traditional interpolation such as kriging. The simulation can also be employed in determining the uncertainty in the recoverable resource above or below cut-off grades, Net Present Value (NPV) calculation, and cash flows of a project, geometry of the optimal open pit and identification of useful blocks.

CONCLUSIONS

Geostastical estimation and simulation methods are used for analysis of univariate and multivariate data in space. However, depending on the data distributed, estimation methods are not sufficient to provide with reliable results for constructing the block model. One of the main reasons is that estimation (kriging) methods are biased as they produce only unique realization for analysis, while geostatistical simulation methods provide with multiple realizations giving more reliable evaluation of grade distribution. Another issue occur when independent simulation and co-simulation methods are compared. First of all, independent simulation do not consider the intrinsic correlation between co-variables in the given data, while co-simulation takes into account interdependency between co-variables. Consequently, the results of correlation coefficient computed for independent simulation is much lower than co-simulation. Secondly, as it can be shown in figure 6, the E-type map of TBCOSIM shows higher grades at the bottom area of the map, while E-type of TBSIM shows with less grades. So, it can be stated that TBCOSIM is more reliable than TBSIM. Geostatistical simulation methods can produce the probabilistic determination of areas of interest with specific cut-off grade as in an example in figure 7. Because of mentioned different results for both methods, it is encouraged to use co-simulation in multi-element deposits with good intrinsic correlation among the co-variables. However, other factorization methods such as Projection Pursuit Multivariate Transform (PPMT) or minimum/maximum autocorrelation factors (MAF) can also be applied for better reproduction of global and spatial correlation.

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