AN EXPERIMENTAL ANALYSIS OF UNIDIMENSIONALITY OF OPINIONS AND THE EFFECT OF HOMOPHILY

BY

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Abstract

The goal of the present paper is to experimentally test the unidimensionality of opinions and the effect of homophily on unidimensionality in terms of speed of convergence. The phenomenon of unidimensionality of opinions implies the convergence of opinions in a multidimensional set of issues into one dimension under the assumption that subjects weight the opinions of other subjects symmetrically. But individuals may assign different weights to the opinions of others. One of the reasons is the homophilous relation between the subjects when they interact more with those who share similar characteristics. In the experiment, two types of group are tested: subjects in the control groups exchange with the signals without no information about their groupmates, subjects in the treatment groups exchange with the signals knowing the gender of their groupmates. As a result of the experiment, the opinions on two independent issues of subjects become closer to each other and made an alignment. This result is consistent with the theoretical prediction. Regarding the homophily, subjects who observed characteristics of their groupmates tended to converge in a higher degree than subjects who didn’t observe any characteristics.
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1 Introduction

There is a phenomenon of unidimensionality of opinions. It describes how individuals’ opinions over multiple issues may lie on one dimension after sufficient communication. For example in politics, we used to distinguish people as leftist or rightist in a range of political issues such as immigration policy, tax policy, healthcare system, etc.

DeMarzo et al. (2003) proposed the model where unidimensionality of opinions can be a result of the communication under persuasion bias. The term persuasion bias has been used by DeMarzo (2003) to refer to the process where individuals fail to make an adjustment when they receive the information. In other words, individual treats the new information and the repeated one similarly by assigning equal weight to the opinions of individuals with whom he connected in all period of communication. In the fixed communication network, due to the persuasion bias, the information of individuals shared with each other starts to repeat. Therefore, the convergence of opinions occurs. Before the full convergence of opinions, when the difference of opinions between individuals still exists the unidimensional spectrum emerges. This unidimensional spectrum is the relative positions of individuals in the fixed network determined by the difference of opinions which are the same across all issues. The process of unidimensionality of opinions was experimentally tested by Louis et al (2016). Their findings were consistent with the theory of DeMarzo (2003). However, what happens if we assume that individuals weight other’s information asymmetrically?

Golub and Jackson, (2012), and Melguizo (2018) presented a model where during the opinion formation, individual assigns different weights to the information of other individuals depending on the characteristics of them. This phenomenon is called homophily. Homophily defined by Lazarsfeld and Merton (1954) to refer to the tendency to form a friendship with those who alike in some characteristics. Ac-
According to the study of Golub and Jackson homophily slows down the convergence of opinions. Melguizo (2018) concluded that the more attributes individuals share the more segregated the interactions become which then will cause a disagreement between the groups.

The aim of the present paper is to experimentally test to what extent the homophily affects the formation of unidimensionality of opinions between individuals in a social network.

2 Literature Review

My research is connected to three stands of literature on opinion formation: herding, social learning set ups, and homophily.

2.1 Herding

The field of opinion formation is one of the crucial fields not only in the sphere of behavioural economics but also in psychology and sociology. How our opinion forms and what might effect on its formation are the explanations of the decision-making process.

Our decisions are often influenced by the decision of other people. Before deciding what to do we look what other people do in the same situation and do the same. This phenomenon is called herding behaviour. The model of herding behaviour was presented by Banerjee (1992). They argued that people do what others do by ignoring their own information.

Surowiecki (2004) provided numerous examples and studies of herding and aggregation of information in order to illustrate the idea of wisdom-of-crowd. The
wisdom of crowd implies that people as a group tend to make more accurate decisions rather than an individual.

2.2 Social learning

The opinion formation in a group or social learning was studied in two set ups. The first group of researches made studies based on Bayesian learning, while the other group of research studied the opinion formation using boundedly rational learning.

**Bayesian learning** Acemoglu et al. (2011) based on Banerjee (1992) study developed the model of Bayesian learning by analyzing the sequential learning over a general social network. The main specification of a model was that the subjects act infrequently and have limited control over the timing of their actions.

**Boundedly rational learning** The other group of research studied the model on the idea of boundedly rational learning. DeMarzo et al. (2003) following DeGroot (1974) analyzed the dynamics of opinion formation under the persuasion bias. The term persuasion bias itself is considered as the process of opinion formation when a subject fails to properly adjust his opinion toward repeated information. They proposed several implications of persuasion bias. One of them is a social influence, the other one is unidimensional opinions.

**Experimental studies on persuasion bias** The experimental studies to test the persuasion bias and social influence were conducted by Corazzini et al. (2010) and later by Brandts et al. (2015).

Corazzini et al. (2010) tested the persuasion bias under two treatments: balanced
and unbalanced network structure. Their experimental results were consistent with the theoretical model proposed by DeMarzo. In their experiment participants failed to adjust their opinions when they received similar information. Besides, their study was in line with the hypothesis that the structure of the network determines the social influence of participants and their opinion converges toward the participants who were better connected. They also highlighted that the most influential ones were those who listened to more people than the other.

The experiment conducted by Brandts et al. (2015) also tested the dynamics of opinion formation in the social network. The distinction between their experiment and the experiment of Corazzini et al. (2015) was the different social network structure. They also retested the network of Corazzini et al. (2015). The results of Brandts et al. (2015) were consistent with the theoretical model of persuasion bias. However, their predictions over the influential participants diverge with the former experiment.

All these experimental studies’ results show that the persuasion bias model is a suitable benchmark to study the opinion formation in the network. They focused on the opinions of individuals in one issue. Thus, studies testing the second part of the model related to the unidimensional opinion are still lacking.

There is only one research done to test the phenomenon of unidimensional opinion by this time to my knowledge. Louis et al. (2016) claim that their study is the first one, which permits communication of opinions over more than one issue. In their experiment, subjects in a group of five communicate 10 rounds over two issues. They used three treatments, two of them were with fixed network structure, which was based on a model of DeMarzo (2003), and the third treatment permitted the communication with different subjects in each round. They extended the persuasion bias model and showed that unidimensionality of opinion exists even when there is no fixed network. Following the model of DeMarzo (2003), they also assumed that
individuals weight other’s information symmetrically.

2.3 Homophily

The persistence of disagreement between individuals also was explained under the phenomenon of homophily. The seminal work on homophily was presented by Lazarsfeld and Merton (1954). The main idea of homophily is that individuals with similar attributes tend to listen to each other more than those who don’t have that attribute.

Melguizo (2018) extends the model of DeMarzo, by allowing homophily. Individuals may assign different weight to the information of other individuals. The difference in weight might be due to the difference in characteristics they possess. It might be demographic characteristics or any other attributes. The author stated that individuals tend to put more attention to those individuals who have similar characteristics as he/she has. Under this assumption, the segregation happens among individuals which then will cause a disagreement of opinions.

Golub and Jackson (2012) analysed the speed of learning in the social network under the homophily. They concluded that when homophily develops the convergence of opinions slows down. The experiment’s result carried out by Grimm and Mengel (2014) was consistent with the logic of Golub and Jackson (2012). They tested the convergence of opinion on one issue using different types of listening structures and imposing homophily exogenously.

In this paper, we are going to analyze how the homophily affects the unidimensionality in terms of the speed of convergence.

In order to compare the shape of formation of unidimensionality under the
homophily the experimental set up of the present paper will have two groups. The control group would be the experiment where a group of subjects will communicate on two independent issues in a fixed listening structure without any information about their groupmates. The control group is based on the experiment of Louis et al. (2016). The network structure will be the same. In the treatment group, subjects will also communicate in a similar fixed listening structure on two independent issues but they will know some the demographic characteristics. Thus, the present paper contributes to the existing literature in the field of learning social networks.

The remainder of the paper is organized as follows: Section II describes the theory of unidimensionality and derives the predictions, Section III presents the design of the experiment, experimental results are reported in Section IV; Section V concludes the paper.

3 Background Theory

3.1 Unidimensionality

In my research I am going to compare how two groups in a fixed network structure will come to the unidimensional spectrum of opinions under different conditions. Thus, first I will consider a theoretical model of DeMarzo et al.(2003) for control group where subjects presumed to assign equal weight to those whom they listen and update an information by a weighted average of pieces of information they received before.

According to the model, there is a finite set of agents \( \mathcal{N} = \{1, 2, ..., N\} \) who communicate their opinions over multiple issues. Agent \( i \in \mathcal{N} \) has initial belief \( x_{ik}^0 \) on an issue \( k \) and assigns a weight \( \pi_{ij}^0 \) to an agent \( j \). If assigned weight is \( \pi_{ij}^0 = 0 \) then it means that agent \( i \) ignores agent \( j \). The listening set of \( i \) would be denoted
by \( L(i) \subseteq N \). \( L(i) \) consists of all agents who \( i \) listens to, including himself.

The communication happens over periods \( t \in \{1, 2, \ldots\} \). In \( t = 1 \) agent \( i \) communicates with agents in \( L(i) \) and receives their beliefs on issue \( k, x_{hk}^0 \), for all \( h \in L(i) \). He then updates his belief \( x_{ik}^1 \). In the updating procedure, we assume that agents take into account beliefs they received in the previous period only, and don’t make any difference between beliefs they receive, weighting each of them \( 1/n \) where \( n \) is the number of agents in \( L(i) \) in each period. It can be represented as \( x_t^{t+1} = T x_t \), where \( T \) is a listening matrix composed of the weights of agents assigned to each other. As \( T \) doesn’t change over time agents eventually will share similar beliefs about an issue. The necessary periods of a time that is required for full convergence under the model is \( N^2 \), where \( N \) is the total number of agents who participates in a communication. But in this paper we are interested in the process that happens before the full convergence of opinions.

The differences of opinions that persists before the full convergence determines the relative positions of agents in a given issue. This relative positions of agents are determined by the listening structure they are placed in and don’t have any relation to the initial beliefs they had. The listening matrix \( T \) which reflects the structure of the fixed network can be represented as transition matrix of finite aperiodic irreducible Markov chain. According to the properties of Markov chain, \( T \) provides us with the positions of agents in a long run disagreement by the row and column eigenvectors which corresponds to the second largest eigenvalue \( V^r_2 \) and \( V^c_2 \) of \( T \) respectively. The example of it is given in Prediction section.

As it was stated that the relative positions of agents are characterized by the listening structure, the positions of an agent will be same in any kind of issue under similar listening structure regardless the initial opinion of an agent. It is called as unidimensionality of opinions. In order to test the unidimensionality agents in a control group will exchange their opinions over two issues simultaneously under
similar network structure.

3.2 Homophily

Regarding the subjects in a treatment group in my experiment, they will also communicate with each other over two issues but they will be provided with a characteristic of subjects with whom they communicate. The difference in characteristics among subjects may cause a segregation. In this case, subjects may put different weight to whom he listens to according to their characteristic taking into account the difference in opinions. Thus, in order to determine the weights one subject assigns to the other one, first we should consider how to calculate the differences between opinions across characteristics.

According to the model of Melguizo (2018), individuals endogenously become more homophilous when the characteristics they share with others are salient. The salience of characteristic is determined by the difference of opinions among individuals who posses it or lack it. As a result, if the difference of opinions is small between two subjects they assign higher weight to each other.

Following the model, suppose that $I = \{1, 2, \ldots, n\}$ is the set of characteristics. This characteristics are binary. Each individual may have all of them or some or none of them.

For instance, if $I = \{1, 2, 3\}$, we have 8 different types of individuals: $A$: {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}, {0}. {1, 3} type individual has characteristics 1 and 3, and lacks characteristic 2. $n_i$ denotes number of types that posses the characteristics $i$. For example, in case of those 8 types, $n_{[1]} = 4$, $n_{[−1]} = 4$. The opinions of an individual at time $t$, taking into account the type will be denoted as
Then the average opinion across types possessing characteristic $i$ is

$$\bar{x}^i = \frac{1}{n_i} \sum_{A : i \in A} x^i_A.$$  

Thus, if there are two individuals: $\{1, 3\}$ type and $\{2, 3\}$ type, they share characteristic 3, and are dissimilar with characteristic 2 and 1. The average opinion across two of these individuals possessing characteristic 3 will be $\bar{x}[3] = \frac{1}{2}(x^i_{\{1,3\}} + x^i_{\{2,3\}}).$ Respectively, $\bar{x}[1] = \frac{1}{4}(x^i_{\{1,3\}}), \bar{x}[2] = \frac{1}{4}(x^i_{\{2,3\}})$.

The averages lacking characteristics $i$ s.t. $i = \{1, 2, 3\}$ are $\bar{x}[-1] = x^i_{\{2,3\}}$, $\bar{x}[-2] = x^i_{\{1,3\}}$, and $\bar{x}[-3] = 0$.

This gives us the difference between average opinions across characteristic $i$: $\Delta^i = |\bar{x}^i - \bar{x}[-i]|$. This difference determines the salience of characteristics. This salience will be reason for an individual to put more weight to the opinion of one individual than to the another one. The weight $\{1, 3\}$ type individual assigns to $\{2, 3\}$ type depends on the sum of the values of the shared characteristics between two of them. This value will be denoted as $\lambda^i_3$. It depends on the difference in average opinions possessing or lacking the characteristic $i$: $\lambda^i_3 = \frac{\Delta^i}{\sum_j \Delta^j}$ for all $j$. If $\Delta^i$ is large, the value of this characteristic $\lambda^i_3$ will be low. Therefore if one of the connected with each other individuals lacks characteristic $i$, he will have less weight than those connected individuals who have characteristic $i$. In the case, of our two individuals, $\{1, 3\}$, and $\{2, 3\}$, they share characteristic 3. Therefore they assign to each other $\lambda^i_3 = \frac{\Delta^i}{\Delta^1 + \Delta^2 + \Delta^3}$. If the opinions of $\{1, 3\}$, and $\{2, 3\}$ are almost similar, the $\Delta^i$ will be small, hence $\lambda^i_3$ will be high.
As a result, individual may put different opinion to whom he listens different weight each period of time depending on the difference of opinions.

According to the theory of Golub and Jackson (2016), these characteristics makes the converge of opinions slow.

4 The Experimental Design

In this section, the design of the experiment is described. The experiment included two treatments. Both of the treatments used the same network structure which formed a group of 5 subjects given below and differed in information subjects receive about the other subjects whom they linked with (neighbours). Agents in a control group (CG) observed only the opinions of their neighbours while agents in a treatment group (TG) also observed a characteristic of their neighbours in the form of their gender.

During the experiment both TG and CG played games that replicate the experiment of Louis et al (2018). The model of the main game can be represented the following way: there are two urns in a game: Urn 1 and Urn 2. Each urn consists of 100,000 balls. Urn 1 is filled with red and yellow balls. Urn 2 consists of blue and green balls. The number of balls of each color is random in each urn. The aim of the game is to predict the true amount of red balls in Urn 1 and the true amount of blue balls in Urn 2. Participants predict them in groups of 5 with a network structure as in Figure 1.

Each group plays this game in 3 phases and each phase continues for 10 stages.

At the beginning of each phase, each player is given 2 distinct private signals that are the randomly chosen samples of 100 balls drawn from Urn 1 and 100 balls from urn 2. Group-mates receives samples from same urns, but samples may differ across
Figure 1: The network structure. The arrows show whom the subject listens to. Each subject listens to himself and the nearest neighbours, except Subject 3. Subject 3 also listens to Subject 5, but Subject 5 can not listen to Subject 3.

group-mates. After receiving private signals each player makes a prediction of the total amount of red balls in Urn 1 and blue balls in Urn 2. This is considered as the first stage.

In the second stage, each player can see the predictions of other players with whom they are connected in the first stage. After observing the predictions, each player makes another prediction of true amounts of red balls and blue balls in Urn 1 and Urn 2 respectively.

The same procedure continues for 10 stages. The urns and hence the true number of balls that players should find remains same for all 10 stages of on phase.

In the second phase, each player stays within the group formed before the first stage. The place of each participant doesn’t change throughout the experiment. They are provided with new samples drawn from the new urns. But they are still asked to find the total number of red balls in Urn 1 and blue balls in Urn 2. The same procedure is in the third phase.

The CGs played the game without any demographic information about each other. Players in TGs observed the gender of group-mates whom they are connected with.

At the end of the phase of this game, each player will see his/her points that were obtained in each stage of the phase. Then, one stage will be randomly selected and the reward will be paid according to the points player got in this stage.
The participant’s points are be determined by the following expression:

For each urn, the participants points are:

\[ 100 - \beta_{\text{stage}} \left( \frac{\text{predicted} - \text{true}}{1000} \right)^2 \text{ points.} \]

\( \beta \) - constant that shows the error factor

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>25</td>
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</tbody>
</table>

Then, the points for each urn are added to give the total amount of points.

If in the selected stage the participant’s predictions were the same with the true amounts of red balls in Urn 1 and blue balls in Urn 2, he/she would get 200 points in total.

200 points = 500 tenge.

The experiment was conducted in Nazarbayev University computer lab by using software package ZTree in February of 2019. It lasted 2 days and had 4 sessions. The first and the fourth sessions were held with CGs, the second and the third ones with TGs. Overall, 60 non-economic major students took a participation in the experiment, 15 in each session. Participants were randomly selected among gender. For each session, 10 female and 10 male students were randomly chosen from the list of students who wanted to participate. This procedure was done in order to avoid the cases when players could guess the identity of group-mates. However, in each session only 15 students came on time, thus the proportion of female and male students were not the same.

The proportions were in the following way:
Each session lasted 1.30 hours including the introduction part and the payment part. The payment included the show-up fee of 500 tenge. The experiment also included two other games, maximum they could earn was 2500 tenge. The maximum amount earned among participants was 1969 tenge, minimum 906 tenge, and the mean was 1427 tenge.

## 5 Predictions

By the experimental design, there is a set of agents $\mathcal{N} = \{1, 2, 3, 4, 5\}$.

The Listening sets (Figure 1) is given in a following way: $L(1) = \{1, 2\}, L(2) = \{1, 2, 3\}, L(3) = \{2, 3, 4, 5\}, L(4) = \{3, 4, 5\}, L(5) = \{4, 5\}$

Agents communicate corresponding to the listening structure over two issues and they want to find the true states of issues. Let’s denote them $\theta_1$ and $\theta_2$. Thus agent $i$ has two initial beliefs $x_{i1}^0, x_{i2}^0$ corresponding to $\theta_1$ and $\theta_2$ which will be given $\sim Uniform(0, 100000)$, and $x_{ik}^0$ are i.i.d. $\sim Bin(100, \frac{\theta_k}{100000})$.

In a control treatment subjects will not know any characteristic of other subjects they are connected with. Thus we can assume that they pay equal attention to others.
Then by assigning $\pi_{ij} = \pi_{ii}$ for all $j \in L(i)$, the listening matrix will be:

$$T = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\
0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
0 & 0 & 0 & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}$$

By the given listening matrix we obtain the eigenvalues denoted by $\alpha = \{0, 1, \frac{1}{4}, -\frac{1}{6}, \frac{5}{6}\}$.

Since the listening matrix $T$ can be presented as finite Markov chain transition matrix, the second largest eigenvalue of Markov chain determines the rate of convergence. Therefore, the second column eigenvector and second row eigenvector show the positions of subjects in a given issue before the full convergence occurs.

The corresponding second column eigenvector is $V^c_2 = (-\frac{5}{2}, -\frac{5}{3}, 0, \frac{2}{3}, 1)'$. It implies that in a long run disagreement, the first agent will be in a one extreme point, while the fifth agent will be in other extreme point. Agent 2 will be swayed to the left, while agent 3 and agent 4 will be closer to agent 5, independent of initial beliefs of agents.

The second row eigenvector $V^r_2 = (1, 1, 0, -1, -1)'$ which indicates that the long run disagreement determined by the initial differences of opinions of agent 1 and 5 (or 2 and 4).

The undimensionality has been analysed by the principal component analysis model in Louis et al(2018). PCA derives factors that accounts for variables’ variance. It is used to reduce the large number of variables so that remained linear combinations of few variables still be able to explain the original data. Therefore PCA transforms variables into uncorrelated principal components. In this paper we have two dimensions, that stand for predictions of subjects in Urn 1 and predictions in Urn 2, 2 eigenvalues and 2 eigenvectors. Thus, there are two principal components.
The first principal component accounts for the direction in which statistical variation is greatest in the data, in other words, it is the eigenvector with the largest eigenvalue. Then the second component define the utmost remained variability, the eigenvector with the second large eigenvalue.

Thus, the proportion of the first component denoted by $\beta(t)$ will be used to analyze the degree of convergence of opinions into one dimension. The value of $\beta(t)$ can vary from 0 to 1. If $\beta(t) = 1$ there will be a line and opinions of all subject will lie along that line.

Assume that the initial opinions of subjects with the structure given in a figure 1 are:

$$x_{(Urn1)}^1' = [33257, 55000, 48000, 46000, 39000]$$

$$x_{(Urn2)}^1' = [79190, 80000, 70900, 78000, 69000]$$

By calculating the succeeding opinions of these subject according to the model of DeMarzo et al(2003), we can derive the following possible positions of subjects across stages (Figure 2). In the fourth stage opinions come closer to each other and the line drawn by the first principal component shows the direction of positions of subjects. The proportion of explained variables $\beta_t$ comes to 1 in the sixth stage, when all opinions lie exactly along the line. In the following stages the distance between opinions shortens and the positions in the final stage coincides with the result of the second column eigenvector. Gender indicator in the figure is included to compare the positions under homophily model, but it doesn’t have any effect in a control group.

**Hypothesis 1.** Opinions of Subjects in both treatment converge to unidimensionality

The change of opinions in TG under the homophily model emerges in the following way:

There are $I = \{F, G\}$ set of characteristics where $F$ stands for an indicator of gender and $G$ stands as indicator of groupmate. The characteristics are binary. Thus, there
Figure 2: The predicted changes of opinions across stages in CG. The points in the graph represent subjects in a network. The opinions of subjects about the total number of red balls in Urn 1 and blue balls in Urn 2 are displayed in a horizontal and vertical axis respectively. The line which goes through the opinions projects indicates the first principal component.
are 4 types of subjects according to the given characteristics: \{F, G\}, \{G\}, \{F\}, \{\emptyset\}. 
\{F, G\} represents a subject who is a female and a groupmate, \{G\} - a male groupmate, \{F\} - a female subject who is not a groupmate, \{\emptyset\} - a male subject who is not a groupmate. The groupmate indicator is included because subjects are constrained by fixed network structure.

Suppose subjects and their initial opinions are given as in Table 1, the we calculate the difference between average opinions across characteristics for each listening sets separately.

<table>
<thead>
<tr>
<th></th>
<th>(x^1_{\text{Urn1}})</th>
<th>(x^1_{\text{Urn2}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{FG} 33257</td>
<td>79190</td>
</tr>
<tr>
<td>2</td>
<td>{G} 55000</td>
<td>80000</td>
</tr>
<tr>
<td>3</td>
<td>{G} 48000</td>
<td>79000</td>
</tr>
<tr>
<td>4</td>
<td>{FG} 46000</td>
<td>78000</td>
</tr>
<tr>
<td>5</td>
<td>{G} 39000</td>
<td>69000</td>
</tr>
</tbody>
</table>

Table 1

For example, \(L(1) = \{1, 2\}\) where Subject 1 and Subject 2 are individuals with types \{FG\} and \{G\} respectively. Because we calculate these factors for each listening set separately, we add index which denotes the listening set. First, let’s calculate for Urn 1.

\[x^1_{1}[F] = \frac{1}{2}x^1_{\{F,G\}} = 33257\]
\[x^1_{1}[-F] = \frac{1}{2}x^1_{\{G\}} = 55000\]
\[\bar{x}^1_{1}[G] = \frac{1}{2}(x^1_{\{F,G\}} + x^1_{\{G\}}) = 44129\]
\[\bar{x}^1_{1}[-G] = 0\]

Hence, \(\Delta^1_{1,\text{Urn1}}[F] = 21743\), \(\Delta^1_{1,\text{Urn1}}[G] = 44129\), then the values of shared characteristics in the first stage are
\[
\lambda_{F,(Urn1)}^1 = \frac{21743}{21743 + 44129} = 0.33,
\]
\[
\lambda_{G,(Urn1)}^1 = \frac{44129}{21743 + 44129} = 0.67.
\]

Therefore, in the weighting matrix of the first stage \( T^1 \) Subject 1 assigns the following weights: \( \lambda_{F,(Urn1)}^1 + \frac{1}{2} \lambda_{G,(Urn1)}^1 \) to herself and \( \frac{1}{2} \lambda_{G,(Urn1)}^1 \) to Subject 2. The coefficient multiplied to the \( \lambda_{t,i} \) is the relative fraction of Subject in the subset of Listening set of \( i \)-similar types.

The weighting matrix \( T_t \) changes over time \( t \). The weighting procedure follows (1, in Appendix). Thus, for the first stage:

\[
T^1_{(Urn1)} = \begin{pmatrix}
0.67 & 0.33 & 0 & 0 & 0 \\
0.24 & 0.38 & 0.38 & 0 & 0 \\
0 & 0.25 & 0.25 & 0.24 & 0.25 \\
0 & 0 & 0.32 & 0.37 & 0.32 \\
0 & 0 & 0 & 0.43 & 0.57
\end{pmatrix},
\]
\[
T^1_{(Urn2)} = \begin{pmatrix}
0.51 & 0.49 & 0 & 0 & 0 \\
0.33 & 0.33 & 0.33 & 0 & 0 \\
0 & 0.25 & 0.25 & 0.24 & 0.25 \\
0 & 0 & 0.32 & 0.37 & 0.32 \\
0 & 0 & 0 & 0.45 & 0.55
\end{pmatrix}
\]

According to (1) the opinions of given subjects changes from \( t = 1 \) to \( t = 10 \) as in the graphs given in the Figure 3.

The time of convergence to unidimensionality is negatively proportioned to the difference of subjects in one listening set by characteristics.

**Hypothesis 2** Subjects in CGs reaches the unidimensionality of opinions faster than subjects in TGs: \( \beta'(CG) > \beta'(TG) \) as \( t \to 10 \). This predictions are also consistent with the result of work of Golub and Jackson (2016).

**Hypothesis 3** Groups in homophily treatment where the difference of subjects are higher converges to unidimensionality in later stages than those groups where subjects are more similar to each other.
Figure 3: The knowledge characteristic of the subjects in a listening set slowed down the time when all subjects come to unidimensional opinions.
Consider the weighting matrices given in the example above. Subject 3 assigns almost similar weights for all subjects that he listens to, despite that Subject 4 has different characteristic than him. At the same time Subject 4 assigns less weights to subjects who have different characteristic than she. The less difference across subjects, the less is homophily, consequently the higher is the speed of convergence to unidimensionality.

Melguizo (2018) also states that the more groups are homophilous the higher is the frequency of disagreement.

6 Results

We start analyzing the results for the CGs. Table 2 shows summary statistics of explained variance of opinions across stages $\beta^t$ which indicates the convergence to the undimensionality for both treatments.

T-test showed no difference between means of $\beta^t$ of three phases for control group. Thus, all opinions across phases were pooled together.
Table 2

<table>
<thead>
<tr>
<th>Stage</th>
<th>CG: $\beta^t$</th>
<th>TG: $\beta^t$</th>
</tr>
</thead>
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<tr>
<td></td>
<td>count</td>
<td>mean</td>
</tr>
<tr>
<td>Stage 1</td>
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</tr>
<tr>
<td>Stage 2</td>
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<td>.8932941</td>
</tr>
<tr>
<td>Stage 3</td>
<td>17</td>
<td>.8491059</td>
</tr>
<tr>
<td>Stage 4</td>
<td>17</td>
<td>.8802059</td>
</tr>
<tr>
<td>Stage 5</td>
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<td>.8460235</td>
</tr>
<tr>
<td>Stage 6</td>
<td>17</td>
<td>.8874294</td>
</tr>
<tr>
<td>Stage 7</td>
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<td>.8326941</td>
</tr>
<tr>
<td>Stage 8</td>
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<td>.8491588</td>
</tr>
<tr>
<td>Stage 9</td>
<td>17</td>
<td>.8633235</td>
</tr>
<tr>
<td>Stage 10</td>
<td>17</td>
<td>.8952882</td>
</tr>
<tr>
<td>Observations</td>
<td>17</td>
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</tr>
</tbody>
</table>

**Finding 1.** The nonparametric trend test fails to reject the Hypothesis 1 that opinions of subjects in both treatments converge to unidimensionality ($p=0.001$). $\beta^t$ steadily increases with some fluctuations by stages. The opinions of subjects become closer by stages, but they don’t lie on one line in majority of cases. Nevertheless, it doesn’t deny the existence of trend of the convergence. This finding is consistent with the results of Louis et al(2018). The mean value of $\beta^t$ in stages 6 to 10 for all groups is 0.88 while the result of Louis et al. (2018) showed 0.87.

Figure 4 shows mean of explained variables of opinions $\beta^t$ across stages. The first panel represents the convergence of opinions for all groups. $\beta^t$ for CGs
Finding 2 Subjects in CGs don’t converge to the unidimensionality of opinions faster than subjects in TGs. The second and the third panel of Figure 4 represents the theoretical convergence simulated by the initial opinions of subjects and their real changes in opinions. Subjects in CGs tend to not change significantly their opinions in until 7th stage. The convergence of opinions of subjects in TGs happen faster and steadily increased with a fluctuation in the 4th stage. There is also a jump down in 9th stage, but it might happen because of the typo made by subjects. Therefore, Hypothesis 2 is rejected.

In order to test hypothesis 3, several types of groups were created in TGs which are presented below, where 1 represents female subject, 0 male subject:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
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<tbody>
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<td>Group 10</td>
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<td></td>
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<tr>
<td>Group 8</td>
<td>11100</td>
<td>Group 11</td>
<td>10010</td>
<td></td>
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<tr>
<td>Group 9</td>
<td>01000</td>
<td>Group 12</td>
<td>00001</td>
<td></td>
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</table>

According to the model of Melguizo(2018), Group 7, 9, and 12 where the difference among subjects are small, the convergence of opinions should be higher than subjects
in Group 11. The frequency of disagreement should be higher for Groups 8 and 10 than for subjects in Group 11 and Group 7, 9, 12.

**Finding 3** Figure 5 shows the mean of theoretical predicted convergence rate $\beta_t$ simulated by the initial opinions of subject in give groups and $\beta_t'$ of their actual changes in opinions. The horizontal axis represents the convergence rate $\beta_t'$.

![Figure 5](image)

$\beta_t'$ across stages of Groups 8 and 10 as it was predicted is lower compared to other groups’ $\beta_t'$. Their mean convergence the final stage is a little higher than in their initial stage. Despite of the predictions, the $\beta_t'$ of Groups 7,9,and 12 are lower than of Group 11. The actual convergence rate of Group 11 is close to the predicted one, which means that subjects in this group indeed updated their opinions and weighted their groupmates consistent with the theoretical model. Thus, Hypothesis 3 is partially rejected. Figure 6 (Appendix) displays detailed convergence rates for all groups separately.

### 7 Conclusion

In this paper, we experimentally tested the emergence of unidimensionality of opinions and compared this phenomenon under the homophily. The unidimensionality of opinions proposed by DeMarzo(2003) represents the alignment of opinions of
subjects into one line across all issues. The results of this paper support this theory and it is also consistent with the work of Louis et al (2018). Subjects’ opinions become closer to each other and made an alignment but didn’t reach full unidimensionality which might be because of the heterogeneity of subjects. Regarding the homophily model, the theory of Melguizo (2018) that when subjects are more homophilous they tend to have high disagreement degree was partially validated. According to the experimental results, subjects who observed characteristics of their groupmates tended to converge in a higher degree than subjects who didn’t observe any characteristics. But this might be the result of the fact that subject who doesn’t know anything about his groupmate put much fewer weights to them. The future research can use different types of networks and a large set of characteristics test the effect of homophily to the convergence of unidimensionality of opinions.
8 Appendix

The interaction weighting matrix for the homophily model for one issue at time $t$ is:

$$T' = \begin{pmatrix}
\sum_i \frac{1}{n[i(1)]} \lambda_{1,i}^t & \sum_i \frac{1}{n[i(12)]} \lambda_{1,i}^t & 0 & 0 & 0 \\
\sum_i \frac{1}{n[i(12)]} \lambda_{2,i}^t & \sum_i \frac{1}{n[i(12)]} \lambda_{2,i}^t & \sum_i \frac{1}{n[i(23)]} \lambda_{2,i}^t & 0 & 0 \\
0 & \sum_i \frac{1}{n[i(23)]} \lambda_{3,i}^t & \sum_i \frac{1}{n[i(23)]} \lambda_{3,i}^t & \sum_i \frac{1}{n[i(35)]} \lambda_{3,i}^t \\
0 & 0 & \sum_i \frac{1}{n[i(34)]} \lambda_{4,i}^t & \sum_i \frac{1}{n[i(34)]} \lambda_{4,i}^t & \sum_i \frac{1}{n[i(45)]} \lambda_{4,i}^t \\
0 & 0 & 0 & \sum_i \frac{1}{n[i(45)]} \lambda_{5,i}^t & \sum_i \frac{1}{n[i(45)]} \lambda_{5,i}^t \\
\end{pmatrix} \tag{1}$$

where $n[i(12)] \equiv i \in ((1 \cap 2) \cup (1c \cap 2c)]$. Each row corresponds to weights that each subject assigns to another subject that is in their Listening Set. Let’s look on a third row of the second column: $\sum_i \frac{1}{n[i(23)]} \lambda_{3,i}^t$. This is the weight that Subject 3 assigns to Subject 2 at time $t$. Suppose, the type of Subject 3 is $\{FG\}$, and the type of Subject 2 is $\{FG\}$, Subject 4 is $\{G\}$, Subject 5 is $\{FG\}$. Then, the weight of Subject 3 to Subject 2 takes the following form: $\frac{1}{4} \lambda_{3,G} + \frac{1}{4} \lambda_{3,F}$.

![Figure 6](image-url)
9 References


