SHEAR STRENGTH MODEL OF REINFORCED AND PRESTRESSED CONCRETE BEAMS WITH AND WITHOUT STIRRUP USED IN THE CIS COUNTRIES

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Submitted in fulfillment of the requirements for the degree of Masters of Science in Civil and Environmental Engineering



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December 2018

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DECLARATION

I hereby, declare that this manuscript, entitled "Shear Strength Model of Reinforced and

Prestressed Concrete Beams with and without Stirrup Used in the CIS Countries", is the result of

my own work except for quotations and citations which have been duly acknowledged.

I also declare that, to the best of my knowledge and belief, it has not been previously or

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Abstract

SNiP code is widely used in the so-called post-soviet union or commonwealth of independent states (CIS) countries for the structural designs of reinforced and prestressed concrete structures, and this study aims to present modification factors of the existing shear design equation for the concrete contribution specified in SNiP code. The existing shear strength estimation model specified in SNiP code was initially developed about 90 years ago based on a semi-empirical approach, which means its accuracy and margin of safety level depends significantly on the shear test results available at that time. In this study, a rational modification factor is derived to improve the accuracy of the shear design equation in SNiP code using a current up-to-date large shear database, and the key influencing factors of the shear resistance of beam members, such as the longitudinal reinforcement ratio and presttressing effect, were addressed. To verify the accuracy and safety level of the proposed models, detailed verifications are introduced with comparisons to the other design models used in the international building codes. ACI-DAfStb shear database consisting of a total of 1287 tests results was utilized to verify the modified SNiP shear design code model, and four different shear design code models were also chosen for detailed comparative studies in terms of the margin of safety and accuracy. According to verification results, the SNiP code model with

the new modification factor provided a comparable analytical accuracy and reasonable safety level compared to the current international building code models.

Acknowledgments

First of all, I would like to express my gratitude towards my advisor Professor Deuckhang Lee for the continuous support of my graduate study and research, for his patience, motivation, and provided opportunities. Moreover, I would like to thank the thesis committee, Professors of Civil and Environmental Engineering Department, Professor Jong Kim, for his support and contribution.

In addition, I would like to express special thanks to Professor Karl Heinz Reineck for providing and sharing his shear databases.

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List of Abbreviations & Symbols

 A_p Area of prestressing steel;

 A_{s} Area of longitudinal reinforcement;

Area of shear reinforcement;

 a_g Maximum aggregate size;

a/d Span to depth ratio;

 b_{w} Width of web;

C Horizontal distance from support to inclined crack end;

 c_o Projection of the inclined crack;

COV Coefficient of variance;

d Effective depth of concrete member;

 E_p Elastic modulus of prestressed steel;

 E_s Elastic modulus of steel;

 F_i Point loads subjected to the beam;

 f'_{c} Specified compressive strength of concrete;

 f_{cr} Cracking stress;

 f_{ck} Characteristic concrete compressive strength;

$f_{\it pc}$	Compressive strength of concrete at centroid of section;
f_t	Tensile strength of concrete;
f_{y}	Yield strength of stirrup;
h	Height of beam;
M	Maximum unfactored moment;
M_{cr}	Cracking moment;
M_f	Factored flexural moment;
q_{sw}	The yield strength of shear reinforcements per unit length;
S	Stirrup spacing;
$S_{ m max}$	Maximum allowed stirrup spacing;
S_z	Crack spacing parameter;
S_{ze}	The equivalent crack spacing parameter;
V	Total force at support;
V_c	Shear contribution of concrete;
V_{cal}	Calculated shear strength from design model;
V_{ci}	Shear strength provided by flexure-shear crack;
V_{cw}	Shear strength provided by web-shear crack;

Maximum shear contribution of concrete;

Minimum shear contribution of concrete;

 $V_{c,\max}$

 $V_{c,\min}$

V_f	Factored flexural shear force;
V_p	Vertical component of effective prestress force at section;
V_s	Shear contribution of stirrup;
$V_{\it test}$	Shear strength of the test specimen according to experiment
	results;
Z	Inner lever arm;
$oldsymbol{eta}_{ ho}$	Proposed modification factor;
$\mathcal{E}_{_{\chi}}$	Longitudinal strain;
η	Ratio of horizontal distance from support to inclined crack
	end and projection of the inclined crack;
$ ho_{\scriptscriptstyle w}$	Longitudinal reinforcement ratio;
σ_{cp}	Axial concrete stress at center of gravity center;
φ	Principal compressive stress's the inclination angle;
$arphi_n$	Prestressing effect coefficient;
$arphi_{sw}$	Strength reduction factor of shear reinforcement;

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Chapter 1 – Introduction

1.1 General

Due to the significant influences from the legacy of USSR (Union of Soviet Socialist Republics), Russian SNiP code is widely adopted as a dominant design standard in Eurasia, North Asia and Eastern European countries [1]. Even though these the so-called commonwealth of independent states (CIS) countries occupy the highest portion of the continents in the world, and they also have high construction demands and huge economic potentials in near future, our understanding of the structural design philosophy inherent in SNiP code is very limited due to cultural and language barriers after the Cold War. In addition, since 2015 in Russia, Eurocode is implemented in parallel with existing SNiP code [2], and some Eurasia countries, such as Kazakhstan and Uzbekistan, also currently consider the possibility of introducing new design code models [3]. Thus, at present, these Eurasia countries should bring up capabilities to update and develop their code independently.

The flexural design model specified in the international building codes including ACI318-14, Eurocode2, and SNiP code is almost the same [4-6], however, in spite of the extensive research efforts for the last several decades, the shear resistance mechanisms of a concrete member is still regarded as one of the most challenging topics in the structural engineering research society[8-10]. There is also no clear

consensus regarding the rational shear design method as generally accepted in the flexural design, and consequently different countries possess their own shear design models based on different theoretical and experimental backgrounds [4-7]. In addition, various studies have been still carried out to develop the rational shear design equation for estimating the so-called shear contribution of concrete (i.e., V_c term) in North America, Europe and East Asia [11, 12]. On the other hand, the design equation for estimating the shear contribution of concrete specified in SNiP code was initially developed about 90 years ago in a mostly empirical manner, however, there is no modification for the last several decades [13]. The shear strength model of SNiP code is basically semi-empirical as mentioned, and this means that its analytical accuracy and margin of safety depends significantly on available shear test results. This study aims to propose a rational modification factor for improving the shear design equation specified in SNiP code by utilizing a currently available up-to-date large shear database [11]. In this study, ACI-DAfStb shear databases [5], which contain experimental results for reinforced and prestressed concrete beam members with and without shear reinforcement, were utilized to verify the proposed model. In addition, the accuracy and safety level of the modified SNiP shear strength equation were examined by comparing with the existing international design standards, in which SNiP (Russia and CIS countries),

ACI318-14 (United States), Eurocode2 (European Union), and CSA-A23.3 (Canada).

1.2 Aims & Objectives

This research work aims to present the evaluation of current SNiP code and suggest modification factors to increase its accuracy. It contains several objectives as follows:

- 1. Literature review is required to conduct this research. Shear transfer mechanisms and main concrete parameters, which most influence on shear capacity, should be carefully investigated. Moreover, current SNiP philosophy and derivation process should be clarified.
- Database analysis should be conducted to propose modification factors. In order to do so, the database should be carefully filtered out to fit realistic design approaches.
- 3. Regression analysis of the database for the purposes of the optimization of proposed factors should be performed. During conducting analysis for modification, two main criteria should be met. First is to make SNiP code more accurate and precise keeping main philosophy. Second is to decrease difficulty of a design process. The model must be easy to be utilized in design practice.
- 4. Comparability study should be conducted. The evaluation design codes of

developed countries will help to understand to what extent modification can improve design accuracy.

Chapter 2 – Literature Review

The following section presents a brief information on shear approaches and behaviors for shear design of reinforced and prestressed concrete members. Intense research in shear behavior has observed some significant mechanism of shear resistance in concrete and several fundamental factors such as shear span to depth ratio, concrete strength, and etc., which has a great effect on the capacity of the concrete member. Understanding these concepts plays a crucial role in establishing almost all current shear design models.

2.1 Shear Transfer Mechanisms

Mechanisms of shear transfer in concrete beams still seem to be not fully investigated and discovered due to the difficulty of explaining them by any conventional mechanical theory. There were found many factors that influence to stress distribution after cracking occurs. However, researchers in shear community argued about the level of importance of that basic transfer actions. These actions are following: dowel action of the longitudinal reinforcement, shear in the beam's uncracked compression zone, shear resistance of the shear reinforcement, resistance due to aggregate interlock, and residual tensile stresses across inclined

cracks [14]. Basic shear transfer mechanisms are shown in Fig. 2.1 and described in more detail below.

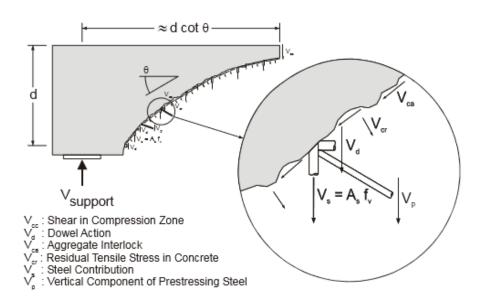


Fig. 2.1. Shear transfer contributing to shear resistance

Dowel action of the longitudinal reinforcement. When crack in concrete member passes across longitudinal bars, additional shear resisting mechanism will occur. This mechanism comes as longitudinal reinforcement will act as dowels. The capacity of load resistance by dowel action is a function of the width of concrete cover under longitudinal reinforcement and capacity of member shear resisting by transverse reinforcement or concrete tensile strength in case of there is no stirrups. An experimental study provided by Watstein et al. [15] showed that dowel action

can resist up to 70 percent of total shear demand during experiencing load equal to half of maximum member capacity.

The uncracked concrete compression zone. The uncracked compression zone plays a huge role in resistance of shear in concrete member. Failure due to shear forces will occur only if inclined shear crack penetrates and passes compression zone. Thus, the capacity of the member to shear resistance is directly proportional to the depth of the concrete compression zone. It means that increasing of compression zone leads to increasing of load-bearing capacity of the member. However, the significance of uncracked concrete compression zone to loading capacity still remains unclear. Researchers such as Reineck [16] states that it contributes only about one-third of all capacity. In the same time Choi et al. [17], Tureyen et al. [18], Park et al. [19] and others believe that compression zone should be considered as only failure mechanism during the calculation of shear resistance of the member.

Shear Reinforcement. Resistance process of shear reinforcement starts after diagonal cracking occurs. The large part of shear is resisted by transverse reinforcement. Typically, during the calculation total shear capacity of a member, it is designed by adding shear reinforcement term to concrete one as in case of 45-

degree truss model, or without consideration concrete term as in case of variable angle truss model. Moreover, shear reinforcement makes member behavior more ductile due to slowing down the increase of inclined cracks. It excludes sudden failure of structure and increases safety level. Thus, many construction codes require a minimum amount of shear reinforcement in all members.

Resistance due to aggregate interlock. A roughness of the surfaces in crack generates interlock action between surfaces in order to prevent sliding. Aggregate interlock is mainly depended on one aggregate size of concrete and the width of developed crack. Resisting capacity of this mechanism increases as increasing aggregate size and/or decreasing crack width. The influence of aggregate interlock to load carrying capacity of member is still controversy. According to Zararis et al. [20], presence of uncracked compression zone above critical inclined crack makes it to be buffer in order to prevent sliding of crack surfaces. Thus, aggregate interlock cannot be considered as a transfer mechanism at all. On the other hand, Reineck [16] stated that interlock is one of the main contributors in resistance of shear by concrete.

Residual tensile stresses across inclined cracks. The ability of concrete to transfer tensile strength through cracks was experimentally proven by Gopalaratnam et al. [21]. It happens as small pieces of concrete still link two sides of the crack. Tensile strength can be transmitted as long as the width of crack exceeds from 0.06 to 0.16

millimeters. Therefore, shear resistance ability due to residual tensile stresses is very significant in the initial stages of crack development. However, Bažant [22] theoretically derived that linking ability of concrete pieces should be ignored during the design process as an amount of carrying capacity is incomparably lower than the capacity of concrete compression zone

2.2 Major factors for shear capacity

Experimental studies of shear showed that there are some significant factors which control member's shear resistance capacity. These parameters are (1) concrete strength, (2) shear span to depth ratio, (3) longitudinal reinforcement ratio, (4) size effect. Nevertheless, a degree of influence of these parameters is still argued by researchers.

Concrete strength. Test results showed that increase of concrete strength leads to increase of shear bearing capacity. Due to the assumption that principal tensile stresses of member causes crack occurrence, it is clear that tensile strength of concrete has a significant influence on resisting capacity of the member. As concrete tensile and compressive strength represent concrete strength, both of them can be used to define the failure mechanism of the structure. However, most of the code models use compressive strength in structure codes due to its simplicity in

derivation. Moreover, experimental results of defining compressive strength are more accurate, while tensile strength results need complex interpretation and they are more scattered.

Independent test results from different authors confirmed the conclusion that the degree of influence of concrete strength to structure member can be altered depending on failure modes. Moody et al. [23] conducted an experiment with 136 beams with both stirrups and without, and stated that shear capacity of structure member increase with increasing concrete strength only until concrete strength reaches 34.5 MPa. Further, it becomes independent of concrete strength. However, tests carried by Kani [24] showed that the shear capacity of rectangular reinforced concrete beams is not affected by concrete strength within range 17.2-34.5 MPa. According to the experimental results of Ferguson et al. [25], the influence rate of concrete strength in high values sharply drops. The reason for this phenomena is a reduction of aggregate interlock shear transfer mechanism. The increase of concrete strength leads to smoother crack interfaces.

Shear span to depth ratio. Property of shear span to depth ratio is inversely proportional to shear capacity was firstly noticed by Talbot, Maurer, and Turneaure. Ferguson et al. [25] illustrated that raise of shear capacity can reach more than two times for the same beam, if the value of span to depth ratio changes from 2.35 to 1.17. Many empirical models consider ratio in the equation in order to

take into account the influence of it. A considerable part of shear strength will be transferred to the support in ratio values less than 2.5, it can be seen a significant increase of strength in members with corresponding ratio values. This phenomenon is mostly recognized as arch action.

Longitudinal reinforcement ratio. According to Kani's [24] test results, 133 rectangular reinforced concrete beams showed that effect of the longitudinal reinforced ratio is significant. Moreover, researchers proved that height of compression zone and crack width are governed by a portion of longitudinal reinforcement. The increase of ratio will lead to expand the height of the compression zone and lessen the crack width. Thus, shear resistance capacity of the member will be increased as well. Due to this effect, many actual construction codes adopt longitudinal reinforcement ratio in their equation.

Size effect. Size effect term means that the bearing capacity of member decreases as the effective depth of it increases. Both experimental results, which were conducted by Kani [26] and Shioya [27], effectively proved that conclusion. Experimental beams with identical concrete strength and longitudinal reinforcement ratio, but different effective depth showed shear force at the failure of the smallest beam was about three times greater than the largest beam. Moreover, calculation by ACI 318-02 for the shear capacity of the largest beam showed two times greater results than actual failure stress in the experiment. To

explain this effect, Reineck [28] and Collins et al. [29] assumed the depth of the beam is directly proportional to the width of crack. Therefore, widening of crack beams results to the reduction of shear transfer mechanism by aggregate interlock, thus increase of beam's depth leads to decrease of shear resisting capacity of the member. One more assumption was made by Bažant [22]. He tried to explain phenomena in terms of fracture energy release. However, the size effect can be seen only in beams without transverse reinforcement. Test results showed that there is no significant effect of increasing depth on shear capacity of the member. It can be explained by fact that the width of crack is primarily governed by stirrups.

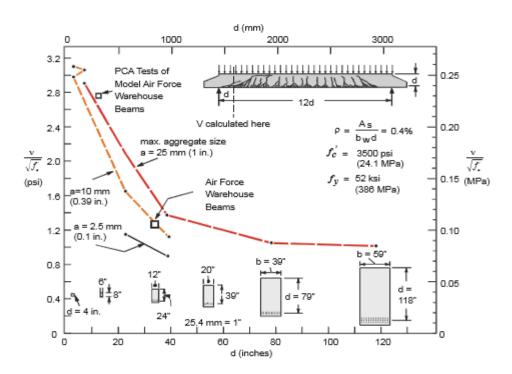


Fig. 2.2. Size effect in shear

2.3 Brief overview of shear strength models

In 1899 Ritter suggested the parallel chord truss model, which describes how shear is used in structural concrete [30]. In the parallel chord truss model, the reinforced concrete suppresses the load, which is similar to how the load is spread in a truss which has the zigzagging load to the support. The path of force is as follows: it flows down by concrete diagonal struts and then raises up to the compression chord with help of transverse tension ties. The bottom chord will have tension and the top chord of the truss will have compression because of equilibrating of the forces. As a result, the equation for calculation of shear capacity was derived for a 45-degree truss. This capacity is equal to the multiplication of strength capacity of individual capacity and the total number of stirrups over the length, d, which is the length divided by stirrups spacing as shown in Eq. (2.1).

$$V_n = \frac{A_v f_y d}{s} \tag{2.1}$$

In the early 1900s in the United States of America, the University of Wisconsin and the University of Illinois conducted a research on the shear capacity of beams [31]. As a result of the research, researchers presented the 45-degree parallel chord truss model. The experiments showed that the prediction of the truss model was less than the shear capacity of beams by around a constant amount. That was the introduction of the idea of a concrete contribution to shear resistance. Initially, the

contribution of concrete was calculated by assuming that concrete resistance stress is equal to a value between 2 and 3 percent of f_c . Therefore, resistance provided by concrete was taken as 2 and 3 percent of f_c multiplied by area. But with the lapse of time that contribution had begun to be connected to the diagonal cracking strength by the reason of its advantageous fit with test data. The U.S. design practices commonly uses assumption, which takes diagonal cracking load as equal to $0.167\sqrt{f_c}$. Empirical data shows that sufficiently correct assessment of shear capacity is insured by "the sum of the diagonal cracking strength plus a shear reinforcement contribution calculated using a 45-degree truss", even though no mechanical reason proposes that the diagonal cracking load is equal to the concrete contribution to shear resistance at ultimate.

In the 1960s, the influence of dowel action and aggregate interlock on shear capacity was heavily studied. Fenwick and Paulay [32] assumed that the contribution of the compressive zone to shear capacity is only 25%. The remainder of shear is being carried by dowel action and aggregate interlock. Zwoyer and Siess [33] researched the stress application in the concrete above flexural cracks of beams without shear reinforcement, suggesting that all shear will be carried in the flexural compression zone. In 1964, Kani [34] presented the "comb" model for assessment of shear capacity. In that model, the teeth of the comb are defined as

the concrete which located between the flexural cracks and backbone of the comb is defined as uncracked concrete above the flexural cracks. From experimental tests, Kani showed a significant influence of span to depth ratio on shear resistance capacity. He observed that beam behavior changes according to the value of span to depth ratio. In short spans, concrete beam behaves as a tied arch to resist shear, while long span concrete beams showed comb-like structure behavior.

Due to the difficulty of shear resistance mechanisms, factors that change these mechanisms, as well as estimation of the shear reinforcement contributions, the shear strength prediction values calculated by models and code provisions' structure are various.

Chapter 3 – SNiP code model

A plane of minimum resistance approach was firstly suggested by Borishanski [12]. He realized that crack propagates through the inclined plane, where shear resistance capacity is minimum. Therefore, the concept idea of the approach is to find an inclined plane with minimum capacity. The advantageous side of the theory is to determine critical section for each beam, not just assume an exact place for any beam as it is assumed in many structure codes. In the proposed theory, two primary hypotheses were assumed: shear failure of concrete and the yielding of vertical reinforcement occur at the same moment, and there is no contribution of the longitudinal tension reinforcement to resist shear. Moreover, it was assumed that total shear is determined by the summation of concrete and transverse reinforcement contribution.

In order to develop an equation for shear resistance provided by concrete, shear tests on 75 RC beams were carried out, and, based on this, a semi-empirical equation for the shear design of an RC beam member was proposed and shown in Eq. (3.1) and Eq. (3.2) [12].

$$V_c = \frac{M_b}{c} \tag{3.1}$$

$$M_b = 1.5\varphi_n f_t b_w d^2 \tag{3.2}$$

The values of the coefficient φ_n are taken equal to:

$$1 + \frac{\sigma_{cp}}{f_c} \text{ for } 0 \le \sigma_{cp} \le 0.25 f_c \tag{3.3b}$$

1.25 for
$$0.25 f_c \le \sigma_{cp} \le 0.75 f_c$$
 (3.3c)

$$5*(1-\frac{\sigma_{cp}}{f_c}) \text{ for } 0.75 f_c \le \sigma_{cp} \le f_c$$
(3.3d)

According to Borishanski, it is obvious that the shear resistance provided by concrete (V_c) depends on the cross section size and the compressive strength of concrete, but the effect of principal compressive stress's (φ) the inclination angle is less clear. To explain the influence of relationship between inclination angle of principal compressive stress (i.e., $\tan \varphi = c/d$) and the shear resistance, the following simple analysis were done by Borishanski, ¹⁸ as shown in Fig. 3.1: "the shear contribution of concrete (V_c) can be estimated to be the sum of the components normal to the principal compressive stress direction, and its direction can be considered in the shear strength model using $\cot \varphi = d/c$. If the direction of the principal compressive stress and critical shear crack is assumed to be coincided, the steeper the inclination of a critical shear crack is, the steeper the direction of the principal compressive stress is. It finally results in the greater

vertical component of the principal compressive stress, and, therefore, the shear contribution of concrete (V_c) also increases".

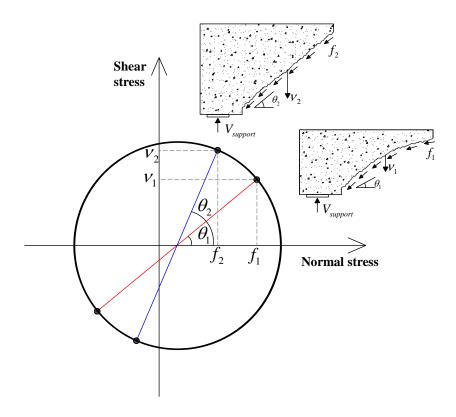


Fig. 3.1 – Effect of principal compressive stress on shear resistance

However, Borishanski claimed that it is very challenging to determine precise contribution of inclined section angle. Therefore, he, by 75 rectangular test beams [13] as shown in Fig. 3.2, concluded that it is directly proportional to shear capacity and added 1.5 coefficient to final equation.

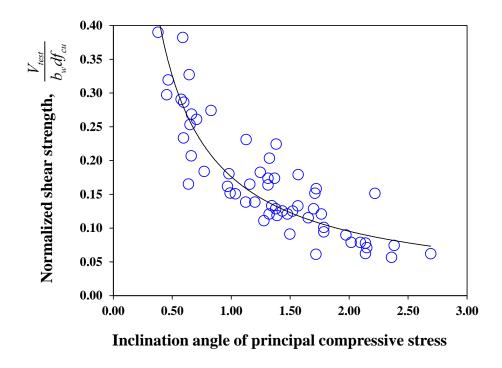


Fig. 3.2 – Distribution of factor K estimated from Borishanski

As it can be seen, SNiP equation uses the tensile strength of concrete. However, most of the practice codes utilize compressive strength. The main reasons to use compressive strength are the difficulty of conducting tensile strength test and results in interpretation problems. In order to avoid these difficulties, SNiP code presents the relationships between the tensile strength of concrete and compressive strength of concrete as shown in Table 2.1. It can be seen that the tensile strength of concrete is ranging from about 15 % and to 5 % of the compressive strength for normal strength concrete and high-strength concrete, respectively.

Table 2.1 Relationship between the compressive and tensile strength of concrete in SNiP

f_{cu} (MPa)	18.5	22	25.5	29	32	36	39.5	43	50	57	64	71
f _t (MPa)	1.55	1.75	1.95	2.10	2.25	2.45	2.60	2.75	3.00	3.30	3.60	3.80

During experimentally study, it was noticed that the shear resistance (V_c) at small and large values of the span to depth ratio (a/d) gets close to a constant values. Therefore, it was decided to express the shear resistance of concrete as follows for small values of a/d:

$$V_{c,\text{max}} = 2.5 f_t b_w d \tag{3.4}$$

For large values of a / d:

$$V_{c,\min} = 0.5 f_t b_w d \tag{3.5}$$

Thus, the maximum and minimum limitations for the shear contribution of concrete was introduced, as follows:

$$V_{c,\min} \le V_c \le V_{c,\max} \tag{3.6}$$

In order to determine contribution of shear reinforcement (V_s) to total shear resistance, it was supposed that all shear reinforcements located in the projection line of the inclined shear crack will contribute resisting force uniformly, as shown

in Fig. 3.3. Moreover, it is assumed that all transverse reinforcements over inclined crack will yield at the moment of shear failure. Hence, the shear resistance provided by transverse reinforcement (V_s) is the multiplication of the yield strength of shear reinforcements per unit length (q_{sw}) and the projection of the inclined crack (c_o) with the safety factor:

$$V_{sw} = \varphi_{sw} q_{sw} c_0 = \varphi_{sw} \frac{A_{v} f_{y}}{s} c_0$$
 (3.7)

where $d \le c_0 \le 2d$

where φ_{sw} is 0.75, which is the strength reduction factor of shear reinforcement. Moreover, experimental studies showed that projection of inclined crack should be limited between d and 2d.

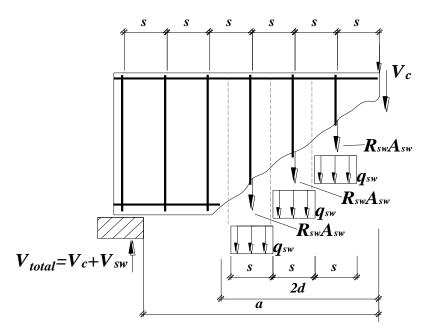


Fig. 3.3 – Shear force equilibrium specified in SNiP code

As it was mentioned above, the total shear resistance strength is equal to sum of the concrete and shear reinforcement contributions, as follows:

$$V_n = V_c + V_{sw} = \frac{M_b}{c} + 0.75q_{sw}c_0 = \frac{1.5\varphi_n f_t b_w d^2}{c} + 0.75q_{sw}c_0$$
 (3.8)

Minimum shear reinforcement. There is a probability of sudden brittle failure of the member with a small amount of shear reinforcement. Shear reinforcement may not be able to resist shear force, which was initially resisted by concrete, after cracking. It typically happens in large span reinforced concrete beams. In order to ensure that shear reinforcement can resist shear, a minimum shear reinforcement value should be established. In large span beams, shear resistance is determined by minimum value:

$$V_{c,\min} = 0.5 f_t b_w d \tag{3.9}$$

The value of shear resistance provided by shear reinforcement, which was taken without reduction factor, in large spans can be found by maximum value of projection of inclined crack ($c_0 = 2d$):

$$V_{sw} = q_{sw}c_0 = 2q_{sw}d ag{3.10}$$

From inequality $V_{sw} \ge V_{c,min}$, it can be found that:

$$q_{\text{sw}} \ge 0.25 f_t b_{\text{w}} \tag{3.11}$$

This condition can be not fulfilled, if the value of M_b will be changed to $M_b = 6q_{sw}d^2$ in Eq. (3.1). By changing this value, Eq. (3.11) will be satisfied.

Except sudden brittle failure, there is probability that crack will be penetrate between two stirrups. In that case, shear will be resisted only by concrete. In order to avoid this situation, limitation for spacing should be established. Maximum spacing ($s_{\rm max}$) should be calculated by assuming that shear can be resisted only by concrete, which again taken without factor, and crack inclination length is equal to $s_{\rm max}$, i.e.:

$$V = \frac{\varphi_n f_t b_w d^2}{c} = \frac{\varphi_n f_t b_w d^2}{s_{\text{max}}}$$
 (3.12)

$$s_{\text{max}} = \frac{\varphi_n f_t b_w d^2}{V} \tag{3.13}$$

Length of projection of inclined crack. As it was written above, the main idea of plane of minimum resistance is to identify inclined plane, which can carry minimum shear force. In other words, minimum value of total force at support should be found. In order to do so, the derivation of equation for total shear at support should be equalized to zero. However, developers of SNiP equation faced with problem of two unknowns in one equation (c and c_0). However, it can be seen

that c is reverse proportional to shear capacity, while c_0 is directly proportional to shear capacity. In order to be able to solve derivative equation, it was assumed that minimum shear will occur, when these two terms are equal. Thus, Boryshanski identified c_0 as equal to c, but not exceeding 2d due to experimental results. Two cases, point load and uniformly distributed load, will be reviewed to clarify derivation process.

Point load. In case of point load, equilibrium force equation can be expressed as follows:

$$V = V_c + V_{sw} + \sum_i F_i = \frac{M_b}{c} + 0.75 q_{sw} c_0 + \sum_i F_i$$
 (3.14)

As it was mentioned before, Eq. (3.14) should be derived in order to find minimum plane of resistance.

$$\frac{dV}{dc} = 0 ag{3.15a}$$

$$\frac{d}{dc}\left(\frac{M_b}{c} + 0.75q_{sw}c + \sum F_i\right) = 0 \tag{3.15b}$$

$$-\frac{M_b}{c^2} + 0.75q_{sw} = 0 ag{3.15c}$$

$$c = \sqrt{\frac{M_b}{0.75q_{sw}}} \tag{3.15d}$$

It should be noted that this value matches minimum of shear capacity, i.e. minimum sum of shear resistance provided by concrete and shear reinforcement $(\frac{d}{dc}(V_c + V_{sw}) = 0)$.

However, c was identified as distance from support to crack location. Crack typically occurs from location of point load. Therefore, the value of c is taken minimum of $\sqrt{\frac{M_b}{0.75q_{sw}}}$ and span length (a). Moreover, test results concluded that

the value of c has to be limited between d and 3d for normal weight concrete. And, c_0 has to be taken as equal to c, but not more than 2d.

Uniformly distributed load. In case of uniformly distributed load, equilibrium force equation can be expressed as follows:

$$V = V_c + V_{sw} + qc = \frac{M_b}{c} + 0.75q_{sw}c_0 + qc$$
 (3.16)

where q is uniformly distributed load subjected to the beam.

Eq. (3.16) should be derived in order to find minimum plane of resistance.

$$\frac{dV}{dc} = \frac{d}{dc} \left(V_c + V_{sw} + qc \right) = \frac{d}{dc} \left(\frac{M_b}{c} + 0.75 q_{sw} c_0 + qc \right) = 0$$
 (3.17)

However, it cannot be just derived assuming $c_0 = c$ as point load case. Because minimum of total shear and minimum of shear capacity do not match as in case of point loading. Therefore, solution of Eq. (3.17) must be divided into several cases in order to two minimums match.

Case 1:
$$c_0 = \sqrt{\frac{M_b}{0.75q_{sw}}} \ge d$$

Solution of equation Eq. (3.17) is $c = \sqrt{\frac{M_b}{q}}$.

Case 2:
$$c_0 = d \ge \sqrt{\frac{M_b}{0.75q_{sw}}}$$

Solution of equation Eq. (3.17) is $c = \sqrt{\frac{M_b}{q}}$.

Case 3: $c_0 = c$

Solution of equation Eq. (3.17) is $c = \sqrt{\frac{M_b}{q + 0.75q_{sw}}}$.

Case 4: $c_0 = 2d$

Solution of equation Eq. (3.17) is $c = \sqrt{\frac{M_b}{q}}$.

It is not practical to consider all four cases during design process. Thus, SNiP simplifies it by finding area application of each case and reducing it to only one condition:

If
$$\frac{q_{sw}}{f_t b_w} > 2.0$$
 take as $c = \sqrt{\frac{M_b}{q}}$, otherwise take as $c = \sqrt{\frac{M_b}{q + 0.75q_{sw}}}$.

In order to summarize design process for uniformly distributed load case, the following flow chart is provided.

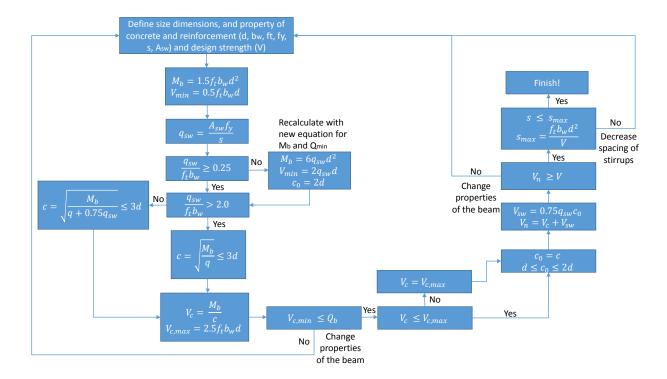


Fig. 3.4. Design process of current SNiP

Chapter 4 – Shear Database

The code provisions for the shear capacity of structural concrete members were primarily derived from test data with respect to the required amount of shear reinforcement and the calculation of maximum shear capacity. Therefore, the detailed verifications using the reliable test sets are of utmost importance for calibrating the design methods. Therefore, the ACI-DAfStb shear database was utilized in this study to verify the shear strength equations specified in SNiP and to check accuracy of empirical equations. Moreover, shear database will be used for derivation of modified SNiP in this paper. This database was collected and developed by Reineck et al.4, and it consists of the shear test results on reinforcement and prestressed concrete simply supported beams with and without transverse reinforcement subjected to both point and uniformly distributed loads.

4.1 Filtration process

Same procedure for filtration processes for both reinforcement and prestressed concrete members were made. To remove some test results with unpractical variables and biased shear test results, the collected shear test results were carefully filtered out based on the following criteria: the test specimens with compressive strength of concrete (f_c') under 12 MPa, the web width of section (b_w) under 70

mm, and the shear span-depth ratio (a/d) smaller than 2.4 were excluded. Filtering samples with small value of compressive strength and web width was done in order to represent real practice. Span depth ratio was taken more than 2.4 to avoid effect of arching action. In addition, to make sure that the specimens failed due to shear but not flexure, only the specimens whose capacity ratios between the shear strength and the shear force at the flexural strength (V_{test}/V_{flex}) are less than or equal to 1.1 were selected, where 10 % margin was introduced to avoid the conservativeness of the nominal flexural strengths determined based on ACI318 code. Only criteria, which was applied in longitudinal reinforcement for reinforcement concrete beams, is presence of reinforcement in the sample. In case of over reinforcement, it would be eliminated by itself, as shear failure cannot be occur in that case.

4.2 Database for reinforcement concrete beams

A total of 954 shear test results were finally selected in this study. Among 954 test data, a number of specimens without shear reinforcement is 784 and number of specimens with shear reinforcement is 170 as it is shown in Fig. 4.1. Moreover, 914 tests were subjected to point loads and 40 specimens were tested under uniformly distributed loads. All filtered data (954 specimens) was used in the comparative study for the evaluation process. It means that the same data was utilized for comparative study of the compressive strength of concrete, the

longitudinal reinforcement ratio, the shear span-to-depth ratio, the effective member depth. Fig. 4.2 shows the distributions of the key test variables involved in the experimental results. Many specimens have the compressive strength of concrete ranged from 12 MPa to 35 MPa, and high strength concrete more than 60 MPa are used in 180 specimens, which are about 61 % and 19 % of a total number of the collected test specimens, respectively. It is noteworthy that the seven tests have compressive strength of more than 100 MPa. It is well known that the effective member depth (d) can significantly affect the shear strengths and behaviors of RC beam members with and without shear reinforcement, and thus this so-called size effect was mainly addressed in the proposed modification factor for the shear design model of SNiP code. Almost 40 % of the specimens have the effective member depth (d) ranged from 250 mm to 300 mm, and the shear database also includes the RC members with the effective depth exceeding 1000 mm, as shown in Fig. 4.2. Longitudinal reinforcement in database is ranged from 0.1% to 6.6%. Moreover, 793 out of 954 samples contain less than 3% longitudinal reinforcement.

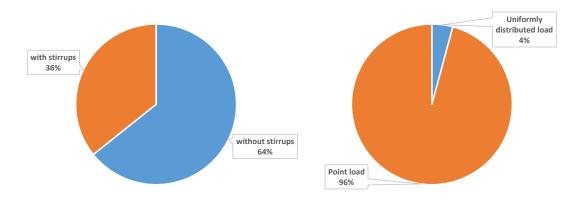


Fig. 4.1. General characteristics of reinforced concrete beams

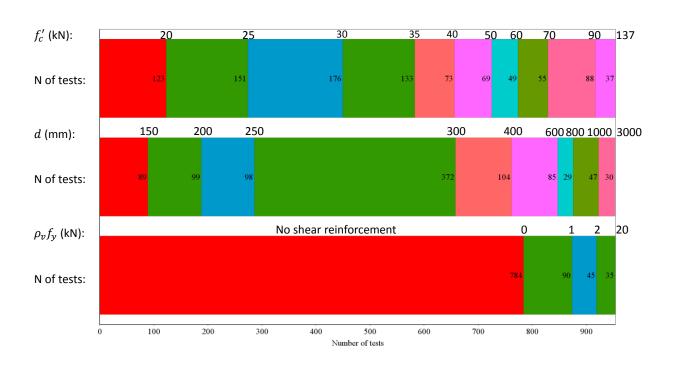


Fig. 4.2. Distribution of parameters of reinforced concrete beams

4.3 Database for prestressed concrete beams

After filtration process, 333 shear test results were chosen. As Fig. 4.3 shows 214 out of 333 specimens do not have transverse reinforcement. From these specimens, 114 tests were used post-tensioned concrete. At the same time, 55 tests out of 119

tests with transverse reinforcement were used post-tensioned concrete. Fig. 4.4 shows the distributions of the key parameters used in the experimental results. Distribution of specimens by the compressive strength of concrete is almost same as in database with reinforcement concrete beams. About 60 % of the specimens have the effective member depth (d) less than 300 mm, and the shear database also includes the prestressed members with the effective depth until 1300 mm. For both with and without shear reinforcement, about half of tests (147 tests: 70 tests without stirrups and 77 tests with stirrups) do not have longitudinal reinforcement in database.

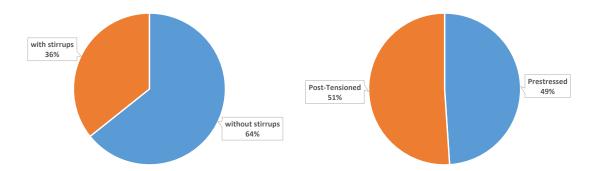


Fig.4.3. General characteristics of prestressed concrete beams

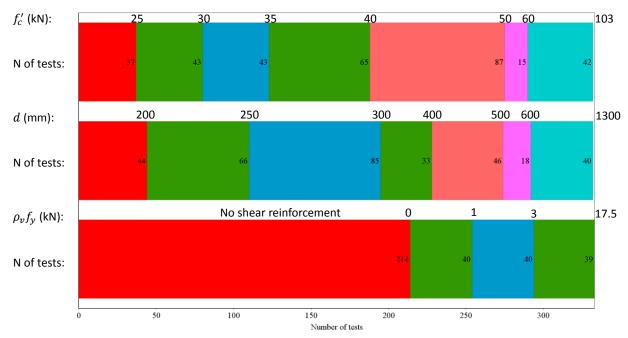


Fig. 4.4. Distribution of parameters of prestressed concrete beams

Chapter 5 – Proposed modification factors

As mentioned in the previous section, the SNiP code model can provide unconservative shear strengths for beam members, and the equation for shear resistance by concrete in the current SNiP code was determined based on the limited number of test results. Moreover, current design process in SNiP code is confusing. Many conditions and no solid expression for inclined crack projection during the design process make it difficult to understand. Furthermore, it relies on designers' ability to find out the minimum plane of resistance. Giving choice for determining minimum plane to the designer can lead to unsafe consequences as there is always the human factor. To overcome such limitations and to more

rationally modify existing equation in the current SNiP code, databases, which was presented in previous sections, were utilized. There are two main strategies of modification. Firstly, up-to-date test results should be used to increase the accuracy of the equation. It means that current empirical equations will be changed by adding some factors, which was not evident or theoretically proven in current SNiP equation derivation moment. Secondly, the new simplified design process will be presented. The important issue to be mentioned is to keep the main philosophy of SNiP.

Modification factors. As it was mentioned before, shear resistance by concrete is fully empirical equation. During the derivation Borishanski assumed that the main parameters affecting on capacity are strength of concrete, size of beam, and inclination angle of the principal compressive stress. However, according to the recent studies, it has been clear observed that the shear strengths of RC beam members are strongly influenced by longitudinal reinforcement ratio (ρ_w). Based on this clear experimental observation, the modification factor β_ρ was chosen as a function of longitudinal reinforcement ratio (ρ_w). The derivation of modification factor (β_ρ) was derived based on the tensile strength of concrete specified in SNiP code. Eq. (5.1) and Eq. (5.2) were derived utilizing the shear test results.

$$V_{test} = \frac{\beta_{\rho} \varphi_n f_t b d^2}{c} \tag{5.1}$$

$$\beta_{\rho} = \frac{V_{test}c}{\varphi_n f_t b d^2} \tag{5.2}$$

Fig. 5.1 shows the distribution of β_{ρ} obtained from Eq. (5.2) utilizing the shear test results, and this distribution trend can be approximated, as follows:

$$\beta_{\rho} = \sqrt[3]{100\rho}$$
 if $\rho > 2.5\%$ take as 1.5 (5.3)

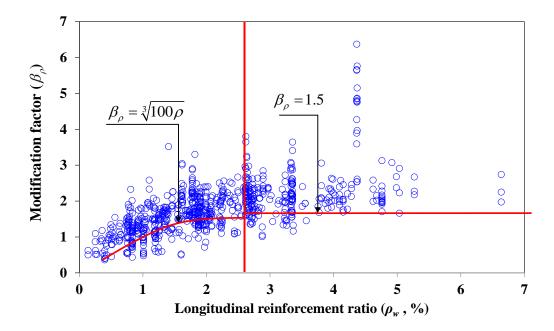


Fig. 5.1. Derivation of β_{ρ} factor

Then, Eq. (3.1) can be finally modified in a simple manner, as follows:

$$V_c = \frac{\beta_\rho \varphi_n f_t b_w d^2}{c} \tag{5.4}$$

It should be mentioned that reinforcement concrete beams without shear reinforcement only was used. Therefore, prestressing factor was taken as unity value. Fig. 5.2 shows influence of modification factor to accuracy of equation. From part (a) of this figure, it can be seen that current SNiP model typically overestimated the shear capacity of lightly-reinforced members whose longitudinal reinforcement ratio are less than 2.0 %, and they also showed the underestimation tendencies as the reinforcement ratio goes up. However, after modification implementation tendency of unconservative results for small ratios was eliminated.

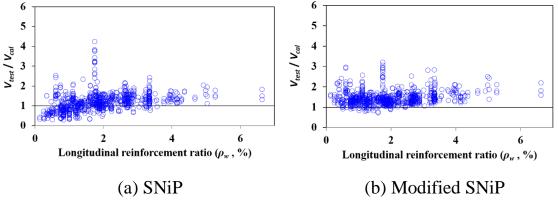


Fig. 5.2. Effect of β_o factor

The second issue is prestressing factor (φ_n). It was also derived fully empirical and not justified by theoretical background. From this factor, it can be seen that maximum effect of prestess can be reached only 25% of total shear capacity. This

underestimation can be explained by absence of well-developed prestressing technologies at that time. This phenomena was shown in Fig. 5.3.

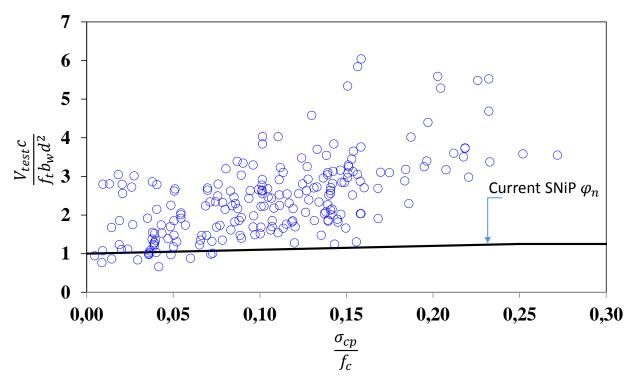


Fig. 5.3. Influence of φ_n for current SNiP

In case of presressed concrete member, concrete will be subjected to a longitudinal compressibe stress, σ_{cp} , by prestressing and shear stress, ν . Final principle tensile stress can be identified by using Mohr's circle as follows:

$$f_1 = \sqrt{v^2 + \left(\frac{\sigma_{cp}}{2}\right)^2} - \frac{\sigma_{cp}}{2} \tag{5.5}$$

At the moment of principle tensile stress will reach cracking stress (f_{cr}), shear stress could be determined from Eq. (5.5) as follows:

$$v_{cr} = f_{cr} \sqrt{1 + \frac{\sigma_{cp}}{f_{cr}}} \tag{5.6}$$

From Eq. (5.6), it can be concluded that the following approximate equation for stress con be used:

$$f = f_t + K\sqrt{\sigma_{cp}} \tag{5.7}$$

Using database for prestressed concrete without shear reinforcement, constant value of K can be found. Analytical analysis showed that constant value should be equal to 1.5. Therefore, final equation for shear resistance provided by concrete can be expressed as:

$$V_c = \frac{\beta_\rho (f_t + 1.5\sqrt{\sigma_{cp}})bd^2}{c}$$
 (5.8)

Fig. 5.4 shows influence of modification factor to accuracy of equation. From part (a) of this figure, it can be seen that current SNiP model typically underestimated the shear capacity. However, after modification implementation tendency of conservative results was eliminated.

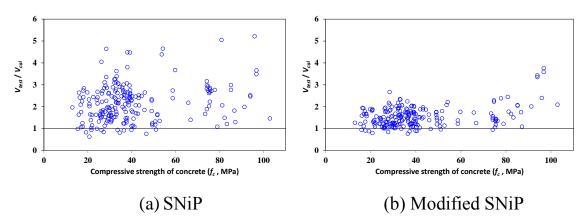


Fig. 5.4. Effect of modified SNiP to prestressed concrete members

Process simplification. The main problem of current SNiP design process regarding to its difficulty is not being able to solve Eq. (3.16) due to two unknowns. Therefore, it requires to consider some cases to eliminate this obstacle. In order to do so, it was decided to assume c_0 as a function of c. By doing this one unknown can be removed. Moreover, the main philosophy of SNiP can be maintained. Thus, η factor was introduced.

$$\eta = c_0 / c \tag{5.9}$$

After analysis by using database, it was concluded that factor cannot be constant value. In order to find influence parameters for η , multiple linear regression was used. Multiple linear regression tries to model the interdependence between two or more explanatory variables and a response variable by adjusting a linear equation to observed data. In regression analysis, significant factors such as effective depth, longitudinal ratio, compressive strength, and span to depth ratio was considered.

Results of analysis showed that influence of span to depth ratio $(\frac{a}{d})$ is the most considerable and could reach up to 90%. Thus, it was decided to take η factor as a function of span to depth ratio. Parametric studies showed that η is reversely proportional to square root of $\frac{a}{d}$ and effect of factor becomes constant in large values of span to depth ratio. The equation for η factor can be expressed as follows:

$$\eta = 0.7 \sqrt{\frac{2.5}{\frac{a}{d}}} \text{ if } \frac{a}{d} > 2.5 \text{ take as } 0.7$$
(5.10)

Moreover, minimum shear resistance provided by concrete shown in Eq. (3.9) should be removed. The reason of this action is minimum equation shows unsafe results and mostly controlled by it. Fig. 5.5 represents comparison of normalized shear strength of minimum SNiP, American code (ACI318), and European Union code (EC2). It can be seen that even minimum value of SNiP equation derives much higher strength results than other two codes.

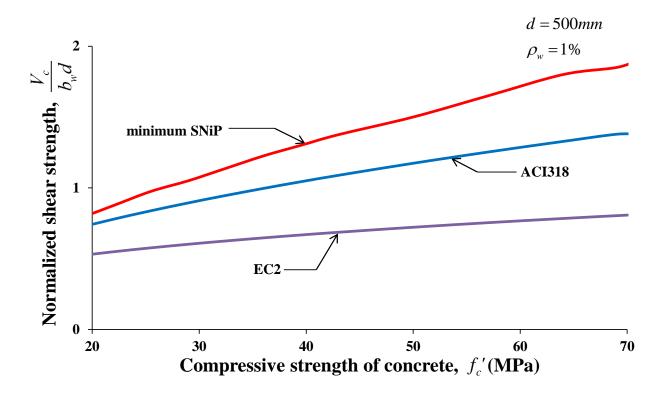


Fig. 5.5. Un-conservativeness in existing SNiP code model's minimum requirement

Additionally, condition at Eq. (3.11) has to be eliminated. According to SNiP, this condition secures from sudden brittle failure of member with small amount of shear reinforcement. SNiP assumed that shear reinforcement cannot be able to arrest crack before it reaches compression zone during sudden failure. However, reinforcement arresting ability is not depended by speed of crack propagation or loading velocity. As maximum spacing is taken into account, crack will face with shear reinforcement during development of crack propagation from flexure crack to compression zone. Therefore, this condition is not theoretically nor practically approved.

Fig. 5.6 shows design steps of modified SNiP code considering all adjustments. It can be seen that design process was simplified. It can be easily understood and there is no unclear conditions for calculating length of inclined crack projection.

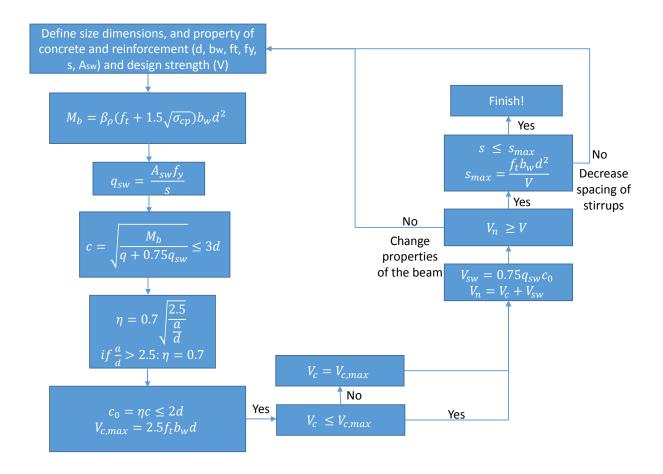


Fig. 5.6. Design process of modified SNiP

Table 5.1 shows mean, standard derivation, and coefficient of variance of the shear strength ratios (V_{test}/V_{cal}) of all test specimens estimated from both current and modified design model, where V_{cal} and V_{test} are calculated shear strength from each design model equation and the shear strength of the test specimen according

to experiment results, respectively. In calculations, all the shear strengths were taken as the nominal strengths without the consideration of the strength reduction factors. From this table, it can be conclude that modification process makes current equation more accurate in both member types, reinforcement and prestressed because COV was decreased in all cases. Moreover, it influences reversely for reinforcement and prestressed concrete. In reinforcement concrete, it became more conservative, hence, making SNiP safer. On the other hand, results for prestressed concrete showed that level of underestimation was gradually decreased. It leads to more economic design consequences.

Table 5.1. Comparison of current and modified SNiP code models

Member type		Reinforced Concrete		Prestressed Concrete	
With or without stirrups		Without stirrups	With stirrups	Without stirrups	With stirrups
Number of tests		784	170	214	119
Current SNiP	Mean	1.20	1.19	2.17	1.47
	Std. Derivation	0.48	0.29	0.84	0.41
	COV	0.40	0.24	0.39	0.28
Modified SNiP	Mean	1.43	1.31	1.55	1. 30
	Std. Derivation	0.43	0.26	0.47	0. 34
	COV	0.30	0.20	0.30	0. 26

Chapter 6 – Shear design models specified

in international building codes

To conduct comparative studies using the filtered shear database, five international design code models were selected, in which SNiP (the CIS region) and its modification, ACI318 (United States), Eurocode2 (European Union), CSA-A23.3 (Canada) are included.

SNiP and modified SNiP are already presented in this paper. Other code standards are described in following sections.

6.1 ACI318-14 - Shear strength model

The current ACI318 shear design provision [5] provides that equation for the shear strength are sum of shear strength resisted by concrete and shear reinforcement.

Calculation for shear concrete resistance is divided into two types. First one is used to calculate shear strength of RC beam member. Second one is utilized in order to estimate shear capacity of prestressed concrete beams.

Shear capacity of RC beam members can be calculated by the simple and detailed equations, but there is no significant difference between two methods in the accuracy of the calculated shear strengths according to Kim [35]. Thus, this study

adopts the simple model presented in ACI318-14 for estimating the shear resistance provided by concrete, which can be expressed, as follows:

$$V_c = 0.166 \sqrt{f_c'} b_w d \tag{6.1}$$

According to ACI318-14, the maximum allowable compressive strength of concrete is restricted up to 69 MPa. It means that the shear stress of RC beams ($v_n = V_c / b_w d$) with the compressive strength of concrete more than 69 MPa should be considered as 1.41 MPa.

Shear strength of prestressed concrete beam provided by concrete also can be calculated by simple and detailed equations. In order to increase accuracy of calculations, detailed approach will be used in this paper.

According to ACI318-14, it considers two types of cracks, web-shear crack and flexure-shear crack. Shear concrete capacity to prevent both type of crack should be calculated and minimum of them should be selected as controlling strength. Shear strength at web-shear and flexure-shear cracks are presented as follows:

$$V_{cw} = (0.29\sqrt{f_c'} + 0.3f_{pc})b_w d + V_p$$
(6.2)

$$V_{ci} = 0.05\sqrt{f_c'}b_w d + \frac{V}{M}M_{cr}$$
 (6.3)

Shear strength provided by transverse reinforcement was derived by 45 degree truss model. Equation is same for both reinforced and prestressed concrete members, and can be expressed as follows:

$$V_s = \frac{A_v f_y d}{s} \tag{6.4}$$

6.2 Eurocode2 (EC2)

Eurocode2 specifies different shear strength models for RC beam members with and without shear reinforcement [6]. For RC beams with transverse reinforcement, the shear strength should be calculated with an assumption that the shear force can be transferred only by transverse reinforcements. It is determined as smaller of:

$$V = \frac{A_{v}}{s} z f_{ywd} \cot \theta \tag{6.5}$$

$$V_{\text{max}} = (\alpha_{cw} v_1 f_{cd} / (\cot \theta + \tan \theta)) b_w z$$
(6.6)

On the other hand, for RC beams without transverse reinforcement, the shear contribution of concrete (V_c) can be calculated, as follows:

$$V_c = (0.12k(100\rho_s f_{ck})^{1/3} + 0.15\sigma_{cp})b_w d$$
(6.7)

where $k=1+\sqrt{200/d} \le 2.0$, $\rho_s=A_s/b_w d \le 0.02$. The shear strength estimated from Eq. (6) shall not be smaller than $(0.035k^{3/2}\sqrt{f_{ck}}+0.15\sigma_{cp})b_w d$.

6.3 CSA-A23.3

The shear design equations in Canadian code (CSA-A23.3) [7] is based on the modified compression field theory (MCFT) [36], and the shear strength of concrete (V_c) can be calculated, as follows:

$$V = V_c + V_s \tag{6.8}$$

where $V_c = \beta \sqrt{f_c'} b_w d_v = \frac{0.4}{1 + 1500 \varepsilon_x} \frac{1300}{1000 + s_{ze}} \sqrt{f_c'} b_w d_v$: concrete contribution

$$V_s = \frac{A_v f_y d_v(\cot \theta + \cot \alpha) \sin \alpha}{s}$$
: reinforcement contribution

In these equations, d_v is the minimum of 0.8h and 0.9d. The longitudinal strain (ε_x) is calculated considering the factored loads including flexural moment and shear force, as follows:

$$\varepsilon_{x} = \frac{M_{f}/d_{v} + V_{f} - \phi_{p}V_{p} - A_{p}f_{po}}{2(E_{s}A_{s} + E_{p}A_{p})}$$
(6.9)

In addition, when ε_x is estimated to be negative value, it should be taken as zero, and this cannot be greater than 0.003. In the calculation of the longitudinal strain (ε_x) using Eq. (6.9), the factored moment and shear force cannot be used for the shear strength evaluations using the shear database, and thus the iterative calculation procedures were used in this study until the assumed flexural moment (M_f) and shear force (V_f) were converged to those calculated from Eq. (6.8). The equivalent crack spacing parameter (s_{ze}) in Eq. (6.8) is the function of the crack spacing parameter (s_z) and the maximum aggregate size (a_g), as follows:

$$s_{ze} = \frac{35s_z}{15 + a_g} \le 0.85s_z \tag{6.10}$$

The angle of the diagonal compression field, θ , is calculated as:

$$\theta = 29 + 7000\varepsilon_{x} \tag{6.11}$$

Chapter 7 – Comparative study

Table 7.1 shows the shear strength ratios (V_{test}/V_{cal}) of all test specimens estimated from each design model, where V_{cal} and V_{test} are calculated shear strength from each design model equation and the shear strength of the test specimen according to experiment results, respectively. All test results were divided into four category: reinforced concrete with and without shear reinforcement and prestressed concrete with and without shear reinforcement. In calculations, all the shear strengths were taken as the nominal strengths without the consideration of the strength reduction factors.

For results of reinforced concrete with and without transverse reinforcement, the CSA-A23.3 code model showed the most accurate prediction results among the existing design codes, in which the average is 1.37 and COV is 0.26 for without stirrups , and 1.33 and COV is 0.18 for with stirrups. The SNiP code model showed relatively poor analytical accuracy in the shear strength estimations on the test specimens for reinforced concrete members. It appeared that the ACI318 code model contain high uncertainty and scattered prediction results because its standard

deviation is the largest. On the other hand, the shear strengths estimated from the modified SNiP code model were agreed well with the test results for reinforced concrete without shear reinforcement, and it showed reasonable accuracy and margin of safety with 1.43 and 0.30 of the average and COV, respectively. In addition, accuracy of modified SNiP and CSA for concrete members with transverse reinforcement is much higher than other codes, as they have the smallest COV. The most conservative results for reinforced members were shown by Eurocode2 with the highest mean values of calculation.

For results of prestressed concrete with and without transverse reinforcement, the average values of all standard codes is higher than reinforced concrete case. It means that codes underestimate effect of prestressing. It can be clear seen by codes such as CSA and SNiP. However, this phenomena is mostly observed in members without shear reinforcement. Generally, accuracy of all codes is almost same. COV is ranged from 0.30 to 0.39 in members without transverse reinforcement, and from 0.26 to 0.37 in members with transverse reinforcement. Same as reinforced concrete case, the Eurocode2 shows the most conservative results.

Table 7.1. Code assessments for reinforced and prestressed concrete members

Member type		Reinforced Concrete		Prestressed Concrete	
With or without stirrups		Without stirrups	With stirrups	Without stirrups	With stirrups
Number of tests		784	170	214	119
Current SNiP	Mean	1.20	1.19	2.17	1.47
	Std. Derivation	0.48	0.29	0.84	0.41
	COV	0.40	0.24	0.39	0.28
ACI318-14	Mean	1.51	1.69	1.77	1.33
	Std. Derivation	0.58	0.45	0.66	0.37
	COV	0.38	0.27	0.37	0.28
	Mean	1.78	1.74	2.29	2.14
EC2	Std. Derivation	0.54	0.58	0.80	0.62
	COV	0.31	0.34	0.35	0.29
CSA-A23.3	Mean	1.37	1.33	2.25	1.54
	Std. Derivation	0.36	0.24	0.90	0.55
	COV	0.26	0.18	0.39	0.35
Modified SNiP	Mean	1.43	1.31	1.55	1.30
	Std. Derivation	0.43	0.26	0.47	0.34
	COV	0.30	0.20	0.30	0.26

7.1 Compressive strength of concrete

According to Moody et al. [37], the shear contribution of concrete (V_c) is significantly affected by the compressive strength of concrete (f_c) . The design code models adopt different power rules for the compressive strength of concrete. For example, the ACI318 and CSA-A23.3 models consider that the shear capacity is directly proportional to $\sqrt{f_c}$, while Eurocode 2 adopt $(f_c)^{1/3}$. On the contrary, the

existing SNiP and modified SNiP codes directly use the tensile strength of concrete (f_t) , for which SNiP code provide tabulated tensile strengths depending on the compressive strength of concrete cubes (f_{cu}), as shown in Table 2.1, and the interpolations are allowed if the concrete strength is located between the specified values in the Table 2.1. In addition, the design codes generally limit the maximum permissible compressive strengths of concrete, except for Eurocode 2. Although each code specifies a different limit on the compressive strength of concrete, all the models considered in this study provided good prediction results on the shear strength of the test specimens cast with high strength concretes even including Eurocode 2 with a limit of the applicable compressive strength. Figures from appendices shows the shear strength ratios $(V_{\scriptscriptstyle test}\,/\,V_{\scriptscriptstyle cal})$ of the test specimens estimated from each model according to the compressive strength of concrete (f_c)). All the shear strength estimation models for reinforced concrete members relatively well estimated the shear strengths of the test specimens without significant biased tendency. However, there is overestimation tendency as concrete strength increases in presetressed concreted members for all standard codes. Moreover, they overestimated the shear strengths of some test specimens with low compressive strengths of concrete.

7.2 Longitudinal reinforcement ratio

Figures from appendices shows the comparisons of the shear strengths estimated from each shear design models according to the longitudinal reinforcement ratio (ρ). Behavior of code results are almost same for both reinforced and prestressed concrete members. All the code models including the modified SNiP model generally overestimated the shear strength of lightly-reinforced members whose reinforcement ratio are less than 2.0 %, and they also showed the conservative tendencies as the reinforcement ratio increases. However, these trends were much clearly observed in SNiP and ACI318 compared to CSA-A23.3, EC2 and modified SNiP models. This is because EC2 and modified SNiP assume that shear capacity is directly proportional to $\rho^{1/3}$, and, for the CSA-A23.3 model, the effect of longitudinal reinforcement ratio is indirectly considered in the longitudinal strain ($\varepsilon_{\rm r}$) expressed in Eq. (6.9). For better estimations of the shear strengths, it seems that the effect of the longitudinal reinforcement ratio (ρ) should be considered in SNiP and ACI318 models by introducing another factor.

7.3 Shear span-to-depth ratio

The shear span-to-depth ratio (a/d) is associated with the ratio between the shear force and flexural moment in the critical section. As shown in Figures from

appendices, the shear strengths of the test specimens with a/d between 2.5 and 3.0 were clearly underestimated by all the shear strength models considered in this study. The CSA-A23.3 code model well takes into account the combined effect between the shear force and flexural moment, as shown in Eq. (6.7). Even though the SNiP code model directly considers the shear span-depth ratio (a/d), it overestimated the shear strengths of the test specimens with a/d between 2.5 and 3.0 as can be seen in those estimated by the ACI318 model with no consideration of a/d. This result indicates that the effect of the shear span-depth ratio (a/d) could be marginal compared to the longitudinal reinforcement ratio (ρ) .

7.4 Effective member depth

The shear strengths of members without transverse reinforcement tends to decrease as the effective member depth (d) increases, which is known as the size effect. The effect of the effective depth (d) is considered in EC2 and CSA-A23.3. In EC2, the effective member depth is directly taken into account by the semi-empirical factors. These factors is $\sqrt{1/d}$ for EC2, and EC2 has no limitation of the effective member depth in practical designs. For the CSA-A23.3 code model, the size effect is considered by the crack spacing parameter (s_{ze}) . Meanwhile, for the ACI, SNiP and modified SNiP code models, there is no variable taking into account the size

effect mechanism. Figures from appendices shows the shear strength ratios (V_{test}/V_{cal}) estimated from the shear design models according to the effective member depth. The EC2 and CSA-A23.3 code models showed relatively small scatters regardless of the member size, while other models showed underestimating tendencies as the effective member depth increases. However, size effect factor is eliminated as shear reinforcement introduced. It can be seen that calculation results for members with shear reinforcement is not influenced by effective depth of member.

Chapter 8 – Conclusions

In this study, the shear design model for RC and PC members with and without shear reinforcement specified in SNiP code was thoroughly reviewed, and its physical limitations were discussed in detail. On this basis, it can be confirmed that the existing SNiP code model can overestimate the shear strengths of RC members without shear reinforcement. Moreover, model underestimate shear strength capacity of prestressed concrete members. To improve the analytical accuracy and safety level, the modified factor (β_{ρ}) was proposed for the SNiP code equation based on the currently available large shear database. Moreover, coefficient for considering prestess effect was replaced with adding square root of axial concrete stress to concrete strength. In addition, the comparative studies were carried out to

verify the modified SNiP code model, and its analytical accuracy and safety level were compared with the shear design equations specified in current international building codes. On this basis, the following conclusions are drawn:

- 1. The CSA-A23.3 and EC2 code models provided good analytical accuracy compared to other code models, and EC2 especially provided the most conservative analysis results for reinforced concrete beams. The SNiP code model provided mostly conservative but poor prediction performances in estimations of the shear strengths of the RC test specimens. However, PC test specimens showed that ACI cod model provided good accuracy and level of safety.
- 2. Since the SNiP and ACI code equations cannot consider the effect of the longitudinal reinforcement ratio, consequently, the shear strengths of the lightly-reinforced members were underestimated. On the other hand, most of the code models already considers the effect of the member depth, except for the SNiP and ACI318 code models.
- 3. The proposed model can provide a higher margin of safety level compared to existing SNiP code model, and this is because the proposed model can consider the longitudinal reinforcement ratio of members failed in shear with the modification factor. Moreover, underestimation of prestressed concrete by SNiP was eliminated.

- 4. By introducing the simple modification factors, the modified SNiP model presented the accurate analysis results on the shear strengths of the RC beam specimens, and its analytical accuracy was significantly enhanced compared to the existing SNiP code model.
- 5. In addition, the form of the proposed model is simple enough to be adopted in practices.

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Appendices

Appendix A: Reinforced concrete beams without transverse reinforcement

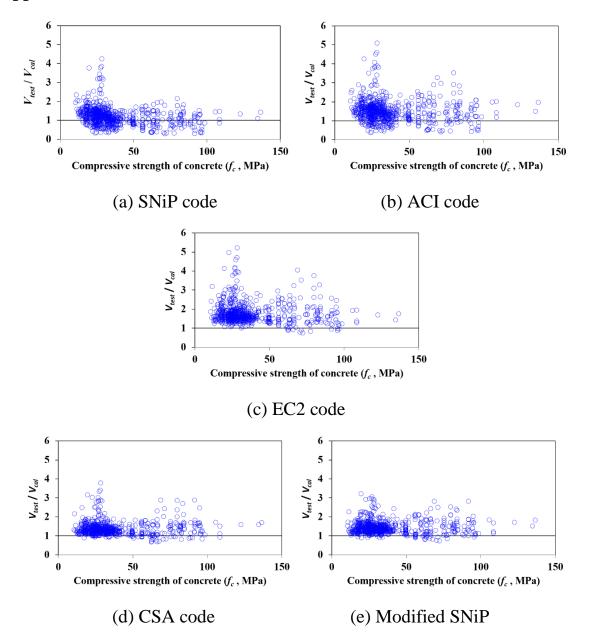


Fig. A1 – Verification results against compressive strength of concrete

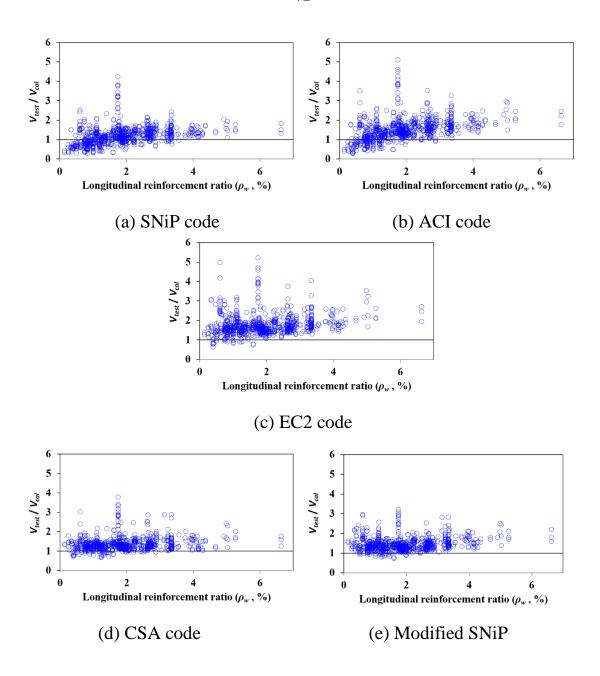


Fig. A2 – Verification results against longitudinal reinforcement ratio

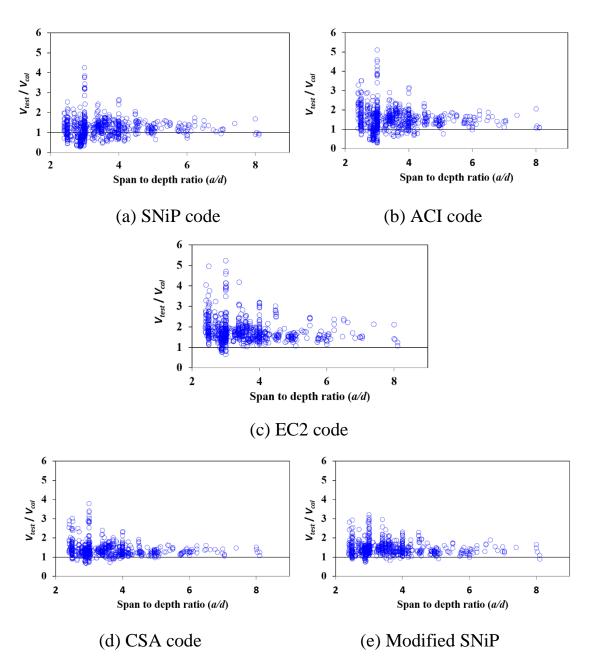


Fig. A3 – Verification results against shear span-depth ratio

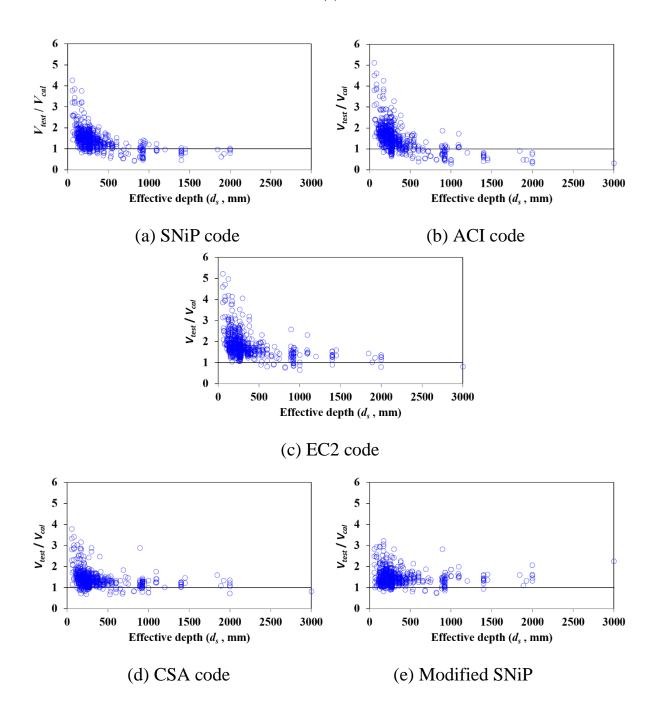
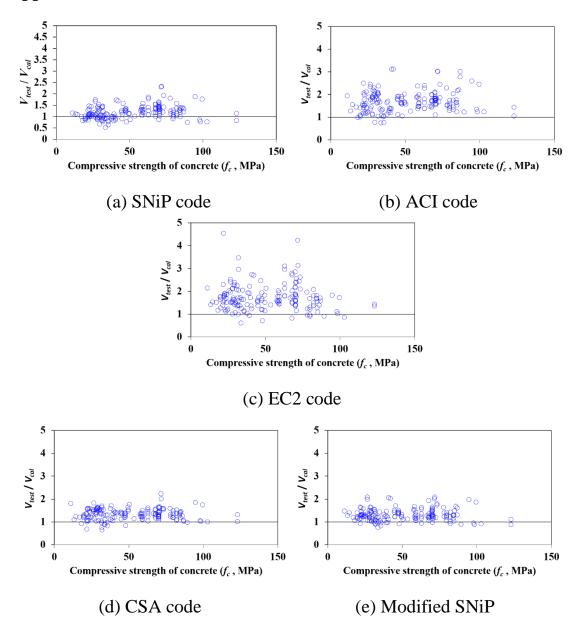


Fig. A4 – Verification results against effective member depth





 $\textbf{Fig.}\ \textbf{B1}-\text{Verification results against compressive strength of concrete}$

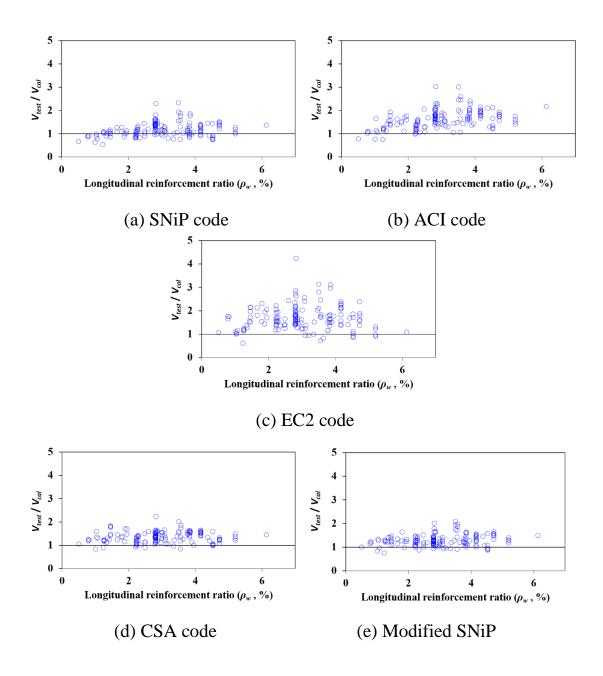


Fig. B2 – Verification results against longitudinal reinforcement ratio

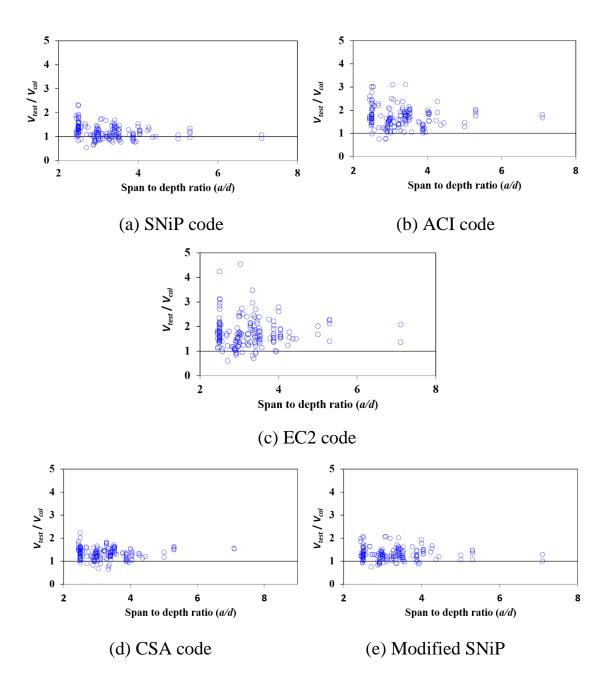


Fig. B3 – Verification results against shear span-depth ratio

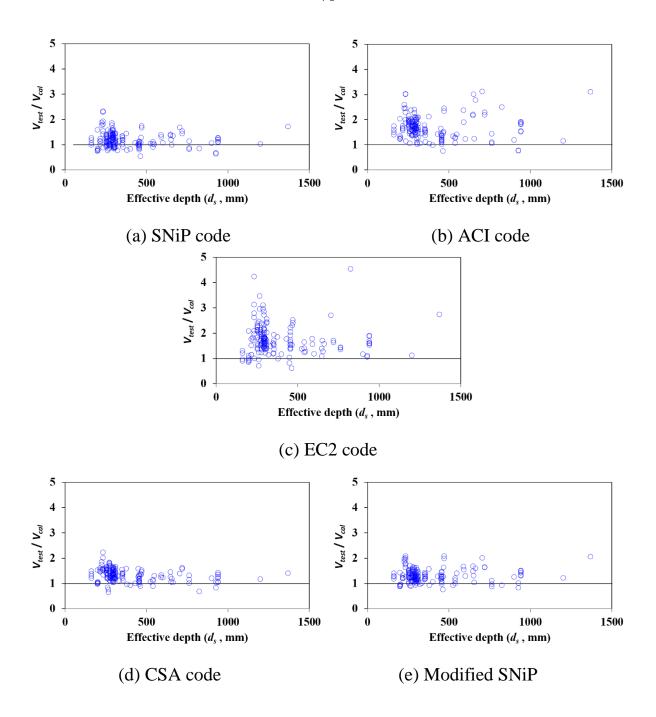


Fig. B4 – Verification results against effective member depth

Appendix C: Prestressed concrete beams without transverse reinforcement

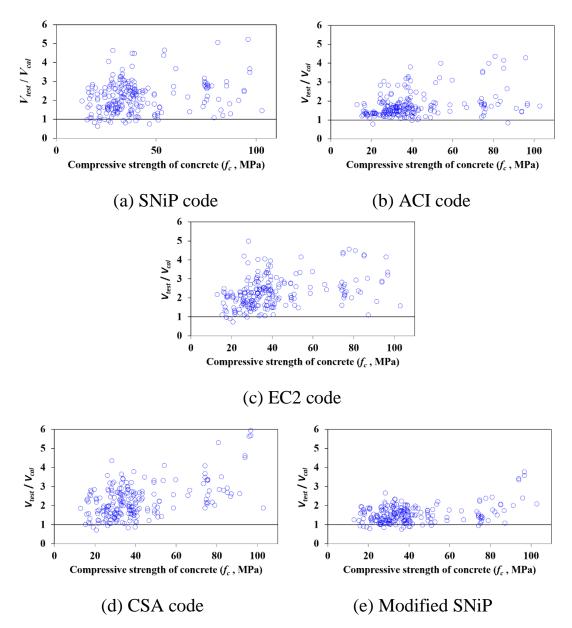


Fig. C1 – Verification results against compressive strength of concrete

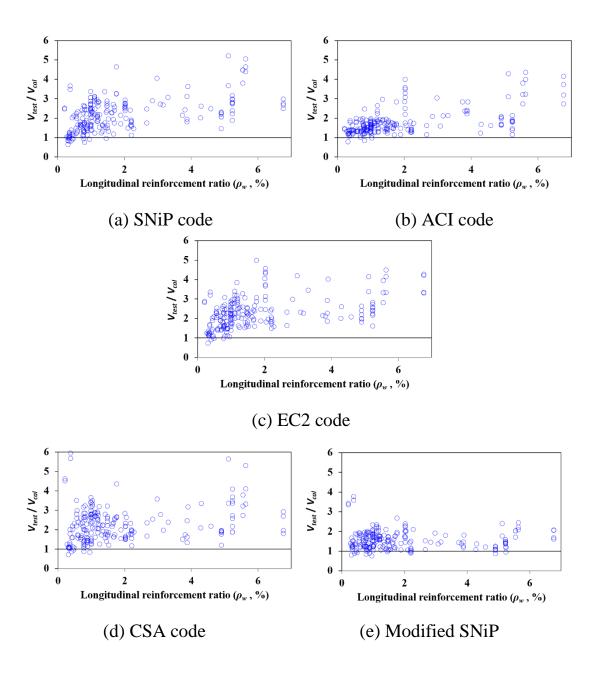


Fig. C2 – Verification results against longitudinal reinforcement ratio

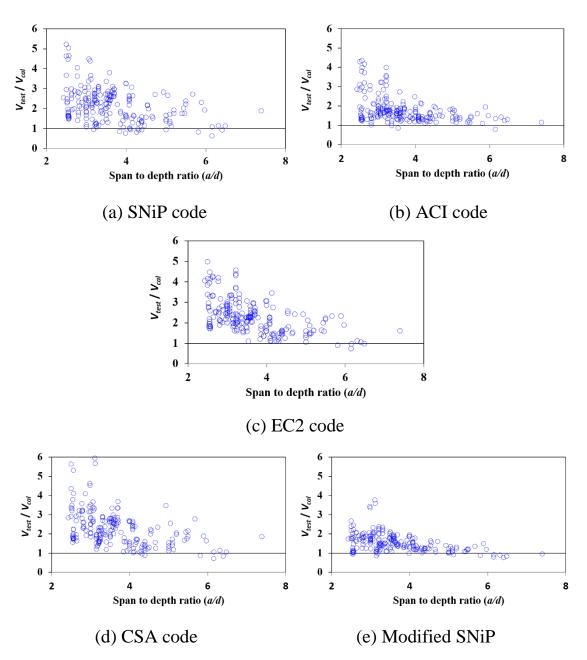


Fig. C3 – Verification results against shear span-depth ratio

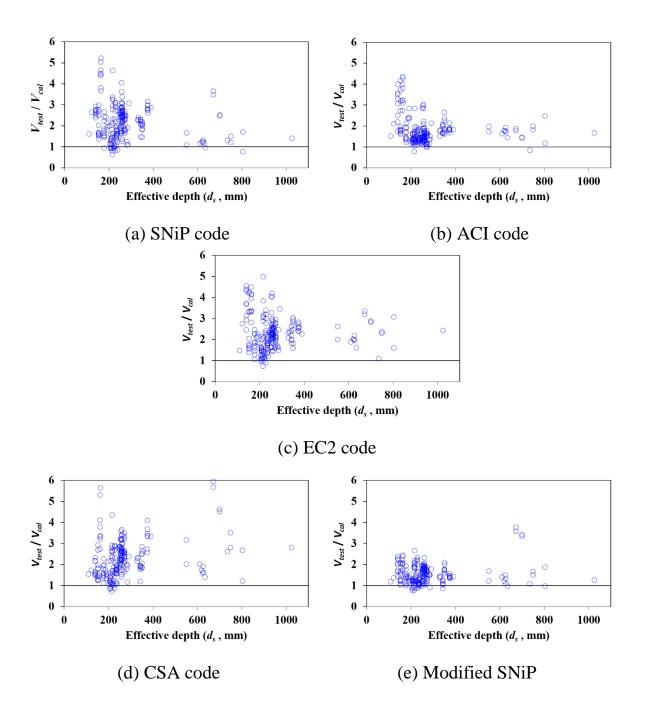
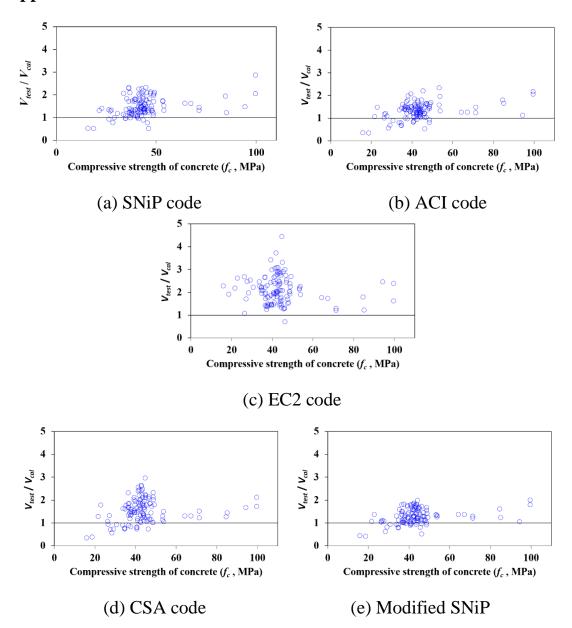


Fig. C4 – Verification results against effective member depth

Appendix D: Prestressed concrete beams with transverse reinforcement



 $\textbf{Fig.}\ \textbf{D1}-\text{Verification results against compressive strength of concrete}$

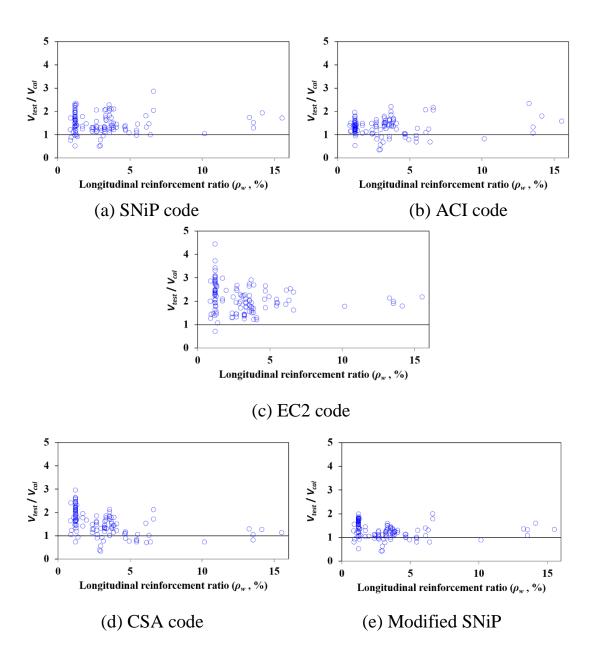


Fig. D2 – Verification results against longitudinal reinforcement ratio

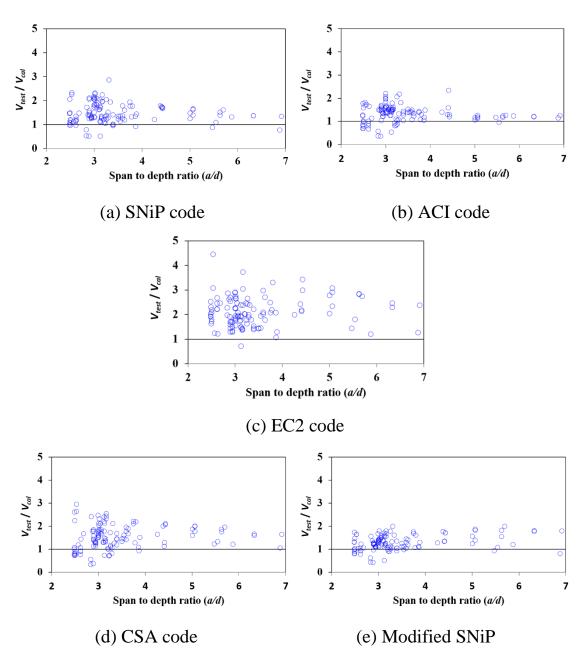


Fig. D3 – Verification results against shear span-depth ratio

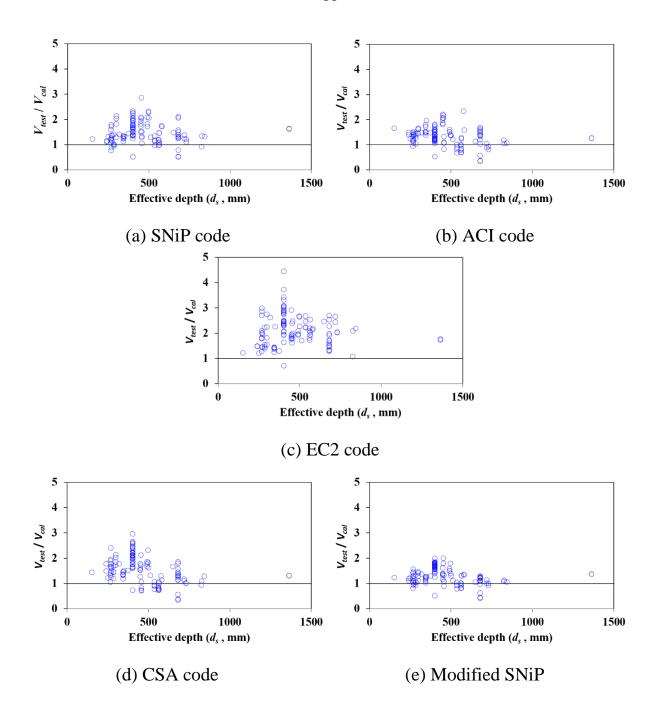


Fig. D4 – Verification results against effective member depth