NAZARBAYEV UNIVERSITY

Bachelor of Engineering
(Mechanical)

DESIGN OF LOAD-AND-MISALIGNMENT-INSENSITIVE GEARING FOR AEROSPACE, AUTOMOTIVE AND NAVAL APPLICATIONS

Final Capstone Project Report

By

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DECLARATION

We hereby declare that this report entitled "Design of load-and-misalignment-insensitive gearing for aerospace, automotive and naval applications" is the result of our own project work except for quotations and citations which have been duly acknowledged. We also declare that it has not been previously or concurrently submitted for any other degree at Nazarbayev University.

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ABSTRACT

The operation of gears is usually accompanied with noise and vibrations that are caused by various sources. Two of the major ones are load and misalignment, which induce high magnitudes of stress on gear teeth. Hence, the main aim of this project is to mitigate their effects by proposing new design topology, which will address both issues without compromising performance of power transmission. The approach to be used in reaching load-insensitivity is to apply tooth profile modifications along with preloading and torque splitting topology, while, possible solution to the misalignment issue is implementation of gear web modification. In both cases, parametric study is used to check behavior of different systems: in part of load analysis, mesh stiffness variation is analyzed using Kisssoft software and dynamics of entire system is analyzed in Matheamatica, while misalignment effects are studied using ANSYS static structural studies. The above studies led to several important findings. Firstly, during the studies of mesh stiffness we found that loaded transverse contact ratio is more important than theoretical one for obtaining mesh curve with minimal fluctuations. Namely when loaded TCR is slightly less than integer it is possible to achieve desirable constant mesh stiffness, while being exactly integer causes spikes in the curve. Secondly, applying tip relief to spur gears changes the relationship between theoretical and loaded TCR: in case of gears with tip relief, loaded TCR is less than theoretical one, while for ordinary gears loaded TCR is always higher than theoretical. Thirdly, when helical gears have high (over than 2) integer overlap contact ratio (OCR) the effect of TCR is almost negligible. Finally, real-time self-preloading was proved to enable achieving constant mesh stiffness for wide range of loads. Also, split-torque topology helps to reduce overall level of vibrations. To find a solution for gear misalignment, 4 different gear web designs were studied: ordinary, spoked, split-pinion (modification of spoked pinion) and multicomponent topologies. Several observations were made during the study. As it was expected, stress in ordinary gears gets more concentrated with increase of misalignment. Spoked pinion topology showed complicated behavior, but still was effective in terms of stress redistribution. Fragmented pinion and multicomponent topologies helped to achieve the best stress distribution, but have some limitations, such as high structural complexity of multicomponent pinion and high stress in the spokes of fragmented pinion. More studies should be conducted to find their optimal configurations.

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1. Introduction

1.1. Problem statement

Gear is an essential mechanical element almost of any transmission of mechanical energy systems used in many industrial applications. Along with its high utility in many industries, it presents some challenges to be solved by engineers as well. Even though there are many concerns associated with gear operations, the scope of this project covers two problems, encountered mainly in three large industries (aerospace, naval and automotive), which are dynamic loads and misalignment. Dynamic loads are mostly related to aerospace applications, since some aircrafts utilize turboprop and turbofan engines, which includes reduction gearboxes in their operation. These gearboxes are exploited in wide range of torques, causing undesirable noise and vibrations. In turn, misalignment issue is mainly important for naval and heavy industry applications due to difficulties in manufacturing of large gears. Also, their size implies higher sensitivity to stress concentration, which is mainly caused by gear misalignment. Development of the design, which solves these problems simultaneously, will be of high utility for automotive applications due to relevance of both to this area. The combined design is still to be developed in future works.

1.2. Load-insensitive gearing

It was proved by many researchers in the field of gear technology (Emmanuel, 1999) that transmission error is treated as one of the major contributor to dynamic loads generated during the transmission. By definition, transmission error is the difference between actual position of the output gear and the position it would occupy if the gear drive were perfect (Welbourn, 1979). Its main source during the operation is variation of mesh stiffness of gear pair. So as its name suggests the mesh stiffness is highly dependent on meshing of two teeth and particularly on their specific profiles and length of contact line, which varies as tooth deflection occur. Devendiran et al.(2016) found that mesh stiffness is greatly affected by tooth profile modifications. Out of various types of profile modifications, our work is going to investigate effect of tip relief on variations of mesh stiffness and in turn on magnitude of dynamic loads. The parametric study will cover spur gears as well as helical with the following varying parameters: load, amount and length of tip relief, helix angle, face width and addendum of gears. It will be shown that last three parameters can be reduced to contact ratio, which is usually used in analysis of gears and particularly in investigation of vibration issue. For spur gears, there is only transverse contact ratio (TCR), which is number of teeth engaged simultaneously i.e. if TCR is 1.5, it means that 1 tooth is always in contact, while second tooth is engaged 50% of time and disengaged other 50% of time. For helical gears, there are both transverse contact ratio and overlap contact ratio (OCR), presence of latter can be explained by the difference in engaging of teeth between spur and helical gears. With parallel helical gears, each pair of teeth first make contact at a single point at one side of the gear wheel; a moving curve of contact then grows gradually across the tooth face to a maximum, then recedes until the teeth break contact at a single point on the opposite side. In spur gears, teeth suddenly meet at a line contact across their entire width, causing stress and noise. (Khurmi et.al., 2005) Scope of the project implies finding the best configuration of load, contact ratio and tip relief for constant mesh stiffness, then integrate this design into the system along with real-time self-preloading mechanism and split-torque topology.

1.3. Misalignment insensitive gearing

As it was mentioned before, one of the major problems associated with use of industrial gears is gear misalignments. Gear misalignment reduces the contact zone between meshing gears and causes the load to concentrate on some particular region of each gear tooth. According to Barone et al. (2004), that leads to reduction of their fatigue life and produces additional noise and vibration. They also claim that extra heat is generated due to load concentration, which means that more significant part of transmitted energy is lost and increased temperature can cause thermal degradation of vulnerable parts. Action of lubricants is also affected by gear misalignment. There are multiple reasons for gear misalignment including deflection of shafts, bearings and housings, manufacturing errors and errors generated during the assembling. Other factors, such as thermal expansion of components also make their contribution, but their effect is negligible. This problem is particularly important for large gears because they carry larger loads that cause more significant consequences when concentrated. In addition to that, large gears can be subjected to larger misalignments because their manufacturing and assembling are more complicated and incompatible with some of precise technologies due to large size and weight. As this problem is not new, some solutions were already suggested. However, most of them are dealing with tooth profile modifications, such as crowning. They help to solve this problem, but some limitations are also inherent to them. Another way to deal with misalignments is to modify web geometry of the gear, i.e. to edit the region below the dedendum circle. Following figure shows where the web of ordinary gear is located:

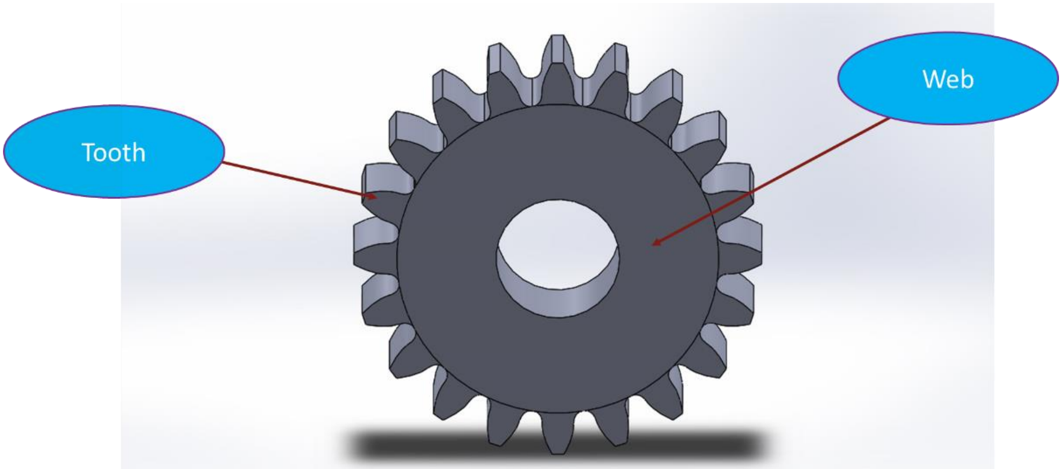


Figure 1.1 *Gear web and teeth*

This part of the project is mainly concerned with gear web modification. To find an appropriate solution, following four different gear web designs will be studied: ordinary, spoked, split-pinion (modification of spoked pinion) and multicomponent topologies. Modification suggested by Ganti et al in 2016 should also be checked if the sufficient amount of time remains. After that, it will be possible to make conclusions and choose successful web configurations. The successful designs are those that have uniform stress distribution with low magnitudes of peak stress.

1.4. Design integration

After developing two different designs that provide load insensitivity and misalignment insensitivity to gears, last task will be to combine these two properties in one single system. In the case of success there will be a design that will resist deviations in load and orientation with equal easiness.

1.5. Contributions

Basically, the work on this project was distributed in the following way:

- Each activity related to load insensitivity was performed by Ali Akiltayev
- Each activity related to misalignment insensitivity was performed by Ilyas Beisekenov
- Common activities, such as writing common chapters of report and solving some challenging problems during the project were done jointly

2. Literature review & Background material

2.1 Dynamic loads

The topic of gear noise and vibration is one of the frequently investigated topic among gear specialists and researchers due to importance of vibration on overall smooth operation of systems involving gears and essential role of noise level being low for comfortable exploitation of machines with gearing systems. It has been well validated by gear technology researchers(Palmer et. al., 2012) that transmission error is one of the major sources of vibration in operation of gears. Moreover, it has been clearly shown that transmission error can be significantly reduced by applying appropriate profile modifications such as tip relief and crowning. In his work Palmer has investigated amount and extent of tip relief which led to conclusion that long tip relief generally gives more desirable transmission error reduction compared to short one. Finally, he concludes that correct application of relief is able to substantially decrease excitations. The decreasing trend of vibrations has been observed as result of increasing crowning magnitude. Moreover, the reduction of mesh stiffness due to increase of tip relief has been proved, indicating that depending on application only sufficient tip relief should be applied to reach less vibrations of gears.(Devendiran et. al., 2016). Since the transmission error is a function of mesh stiffness, the effect of various gear parameters such as pressure angle, helical angle, addendum and face width on mesh stiffness was well studied(Liu et. al., 2015). This study makes several important observations pertinent to our study. Firstly, contact ratio value is a key factor influencing variation of mesh stiffness. The transverse or overlap contact ratio being integer led to minimum fluctuation of mesh stiffness, while total contact ratio being integer showed its maximum fluctuation. Secondly, investigating load variations on contact lines showed that length of contact lines fluctuates only when total contact ratio is integer. Thirdly, the amplitude of fluctuation of contact lines total length has indicated that it corresponds to amplitude of mesh stiffness variation, hence, rough approximation of mesh stiffness fluctuation can be obtained by observing total length of contact lines. Exact amount of tip relief of the teeth has been specified in British Standard (BS 1970) and (ISO/DIS 1983) and the effect of different magnitudes of permissible tip relief amount and length on tooth stress concentrations were investigated (Shanmugasundaram et.al., 2014). The illustration of tip relief parameters is shown on Figure 1:

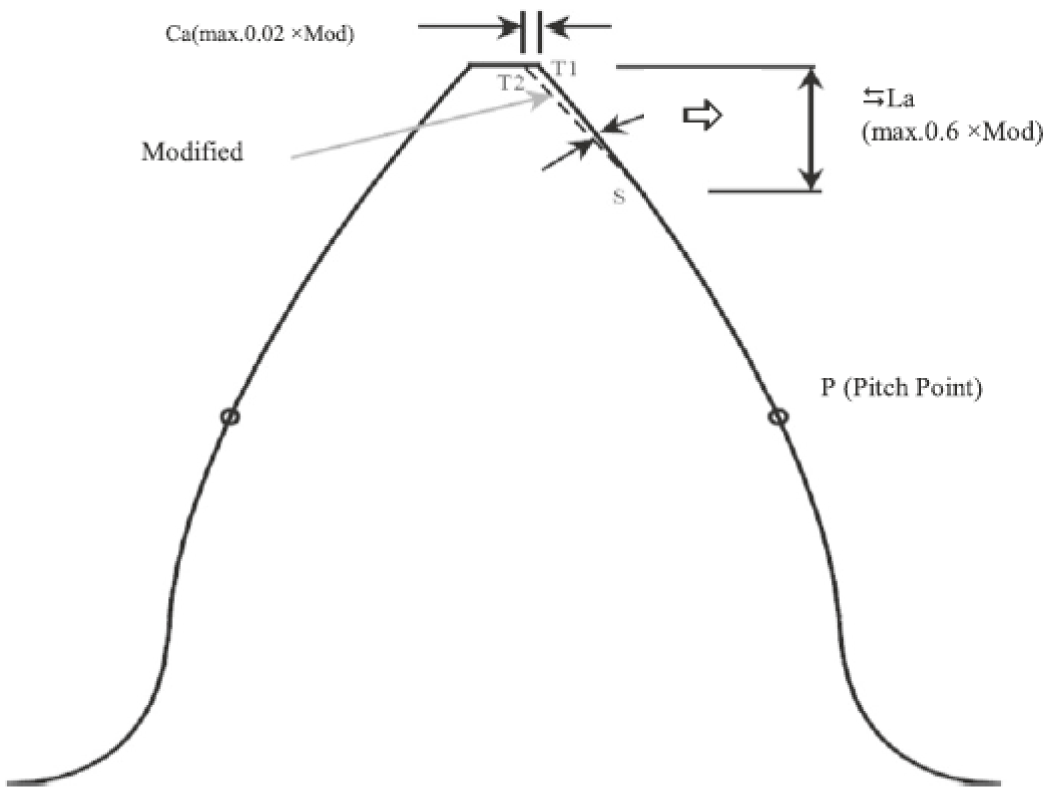


Figure 2.1 *Gear profile with tip relief*

All aforementioned research works greatly contribute to the progress of our project, since some of their results were incorporated into our investigations. However, there is still much space for our research in this sphere, since up to now there is no comprehensive study done for integrating profile modifications with preloading mechanism and two parallel gears. The problem that has not been sufficiently investigated is that profile modifications are only adjusted for only one type of load and not for wide range of loads, which is commonly encountered in aerospace applications. Hence, the introduction of preloading mechanism helps to adjust positive effect of profile modifications for broad range of loads, while implementing two parallel gears will substantially reduce overall excitations by splitting the load between two gear pairs.

2.2 Misalignment

Also, there are many studies covering gear misalignments. Most of them at the beginning focuses on evaluation of the effects of misalignments and then apply tooth profile modification. After that, they compare stress distribution of conventional misaligned gears with that of misaligned gears with tooth profile modification. Barone et al. (2004) do it for spur gears. They also use finite element analysis, and verify their results using numerical methods. Example of more recent paper considering spur gears is “An Investigation on the Influence of Misalignment on Micropitting of a Spur Gear Pair” by Li (2015). The differences are that in this work only FEA is used to validate the solutions, and, in addition to effects of misalignment and tooth profile modification, they estimate fatigue life of the gears. One more study that should be mentioned is “The Stress State Of Heavy Loaded Open Gearing With Incomplete Tooth Contact” by Vinogradov and Fedin (2016). They estimate lifetime and stress distribution of the gears using FEA, but one interesting feature that differ this work from previous ones it that they study large gears, which are of the interest of this Capstone Project, instead of small ones. In particular, they estimate durability of the gears working in “МШРГУ 4500 X 6000” ball mill. Unfortunately, the solution is not proposed in this paper. There are also papers covering bevel gears, such as “Compensation of Errors of Alignment Caused by Shaft Deflections in Spiral Bevel Gear Drives” by Fuentes et al. (2016) and “Design for straight bevel gear based on low installation error sensitivity and experiment tests” by Cao et al. (2016) They also study effect of misalignment and suggest tooth profile modification to reduce it. These papers are interesting and useful, but they consider only tooth profile modification, which has certain limitations. For example, Fuentes et al. (2016) shows that during tooth profile modification microgeometry of the gear tooth is adjusted to misalignment of particular magnitude based on results of parametric study. Far more convenient solution would be to have such gear pair where both components can adjust their relative position by themselves to increase contact area as much as possible. It would be very beneficial because in this case system will show a good behavior even for varying misalignment. Fortunately, there is a hope that such design is possible: Ganti et al. (2016) suggest new approach to the problem. Their idea is to modify web geometry in such way that extra material is removed from some particular regions. The result is that bending stiffness is reduced about X-axis and remain the same in remaining directions. It will partially improve contact area between two gears and reduce need in tooth profile modification as strongly as possible. In this Capstone Project attempt will be made to go even further and modify not only geometry of the gear web, but the material, from which it is made. Generally, it can be said that there are many researches dedicated to problem of misalignment. They help to properly understand the topic and decide in which direction to move.

3. Methods, tools and results description

3.1 Load-insensitivity

3.1.1 Milestones

The dynamic load part of the project has been divided into three main milestones. As a first milestone, mesh stiffness of spur gears is analyzed by varying three main parameters: load, amount and length of tip relief and contact ratios. At this stage, only pair of gears will be investigated in order to obtain the most optimal design in terms of these parameters i.e. finding combination of parameters, which will lead to the smoothest mesh stiffness curve. Second milestone is concerned with studying helical gears namely the same study as with spur gears but extending it with results of varying helical angle and playing with three contact ratios (transverse, overlap, total). As key factors leading to constant mesh stiffness are obtained, the third milestone will be to implement these findings on the system level. The concept of locked-in-torque topology (two pairs of gears) with real-time self-preloading mechanism will be investigated using dynamic equations in Mathematica software. The idea of the concept is not only to reduce transmission errors on both pairs, but also to reduce transmission error between two shafts by controlling preloading angle and synchronizing the pairs with each other.

3.1.2 Methodology

Since our goal was to identify effects of different system characteristics on mesh stiffness variation with applied load, computational and modelling tools have been chosen as essential tools of this project to perform parametric studies. The investigation of mesh stiffness has commenced from creation of 3D geometry shown on Figure 3.1 in Kisssoft software. The shown model has been taken from case study done by Shanmugasundaram as a preliminary model for estimating effects of tip relief and crowning on mesh stiffness variation.

The parameters used for this model are actually for Wind Turbine Application study, however, since this study also investigates wide range of loads, it was reasonable to choose it for our study.

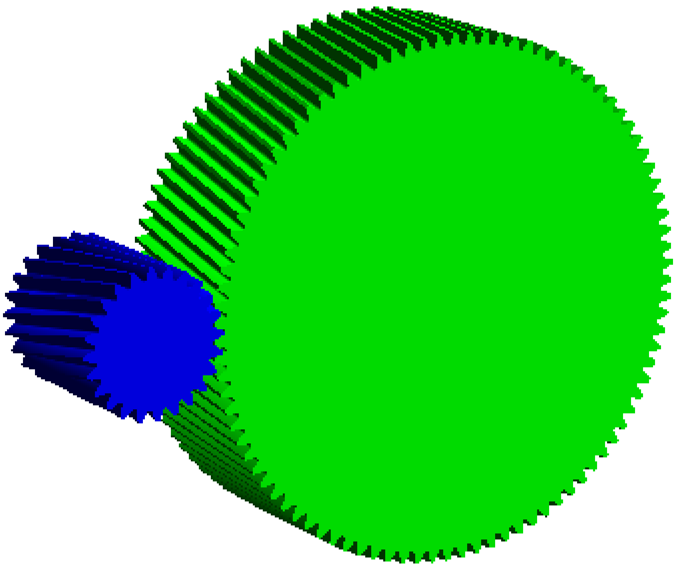


Figure 3.1 *3D Geometry of preliminary model*

3.1.3 Mesh stiffness results for 1st semester

Then, mesh stiffness calculation feature of Kisssoft has been utilized to obtain mesh stiffness variation graphs for different 6 cases of parametric study: integer overlap contact ratio; integer transverse contact ratio; integer total contact ratio; each one with maximum tip relief versus varying loads from 50% up to 150%. The graphs as shown on Figure 3.3 have been obtained, however, only one of them is shown in this part and all other results are shown in Appendices. The following initial gear design specifications shown on Figure 3.2 have been used:

Number of teeth (z)	Pinion	24
	Gear	94
Normal module(M_n)	8 mm	
Pressure angle (α)	20°	
Helix angle (β)	10°	
Face width (b)	245 mm	
Hand of helix	Right	
Reference diameter (d)	194.96 mm	
Base diameter (d_b)	182.872 mm	
Tooth quality (Q-DIN3961)	6	
Center distance (a)		485.00 mm
Tip diameter (d_a)		219.52 mm
Addendum modification co-efficient (x)	Pinion	0.56 mm
	Gear	0.18 mm
Root diameter (d_f)	184.00 mm	
Addendum (h_a)	12.278 mm	
Dedendum (h_f)	5.481 mm	
Effective chordal tooth thickness	15.744/15.694	
Total contact ratio	2.948	
Tip relief (C_a)	0.002 mm	

Figure 3.2 *Initial gear design specifications*

The specifications shown on Figure 3.2 have been changed to perform parametric study with the following parameters:

Table 3.1 *Parametric study data*

Integer	Pressure	Face Width	Profile	Addendum	Tip relief

Contact Ratio	angle (degrees)	(mm)	shift x	Coefficient	(mm)
Overlap = 2.00	20	289.47	0.5170	1.00	D1 = 209.15 D2 = 773.17 Ca = 0.16
Transverse = 2.00	9	217.1	0.6038	1.01	D1 = 211.18 D2 = 771.78 Ca = 0.16
Total = 3.00	20	217.1	0.5170	1.00	D1 = 209.15 D2 = 773.17 Ca = 0.16

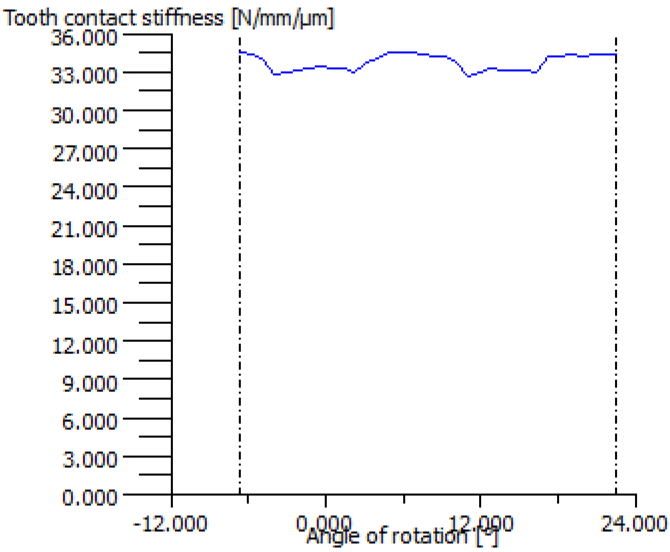


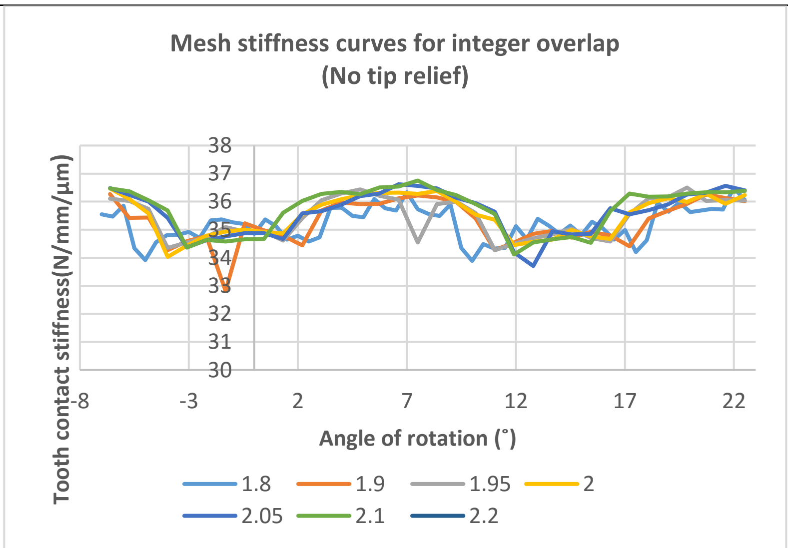
Figure 3.3 *Mesh stiffness variation graph*

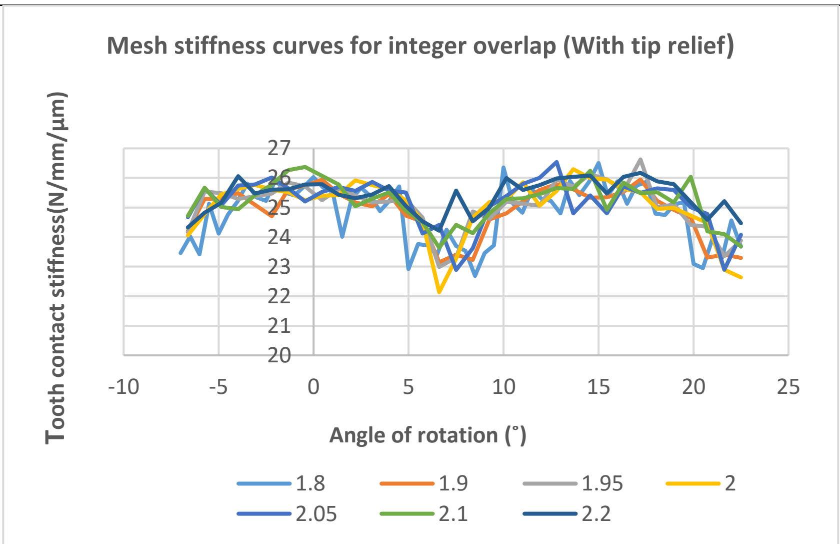
Thorough observation of all obtained graphs (in Appendices) leads to important conclusion to be made. The smoothest operation in terms of mesh stiffness fluctuation has been obtained on gear pair with integer overlap contact ratio of 2 and applied maximum relief amount of Ca = 0.16mm and ΔLa = 4.8mm. Therefore, this result validates the work of Lui et.al., which was discussed in Literature Review part, that minimum variation of mesh stiffness is achieved with overlap ratio being integer.

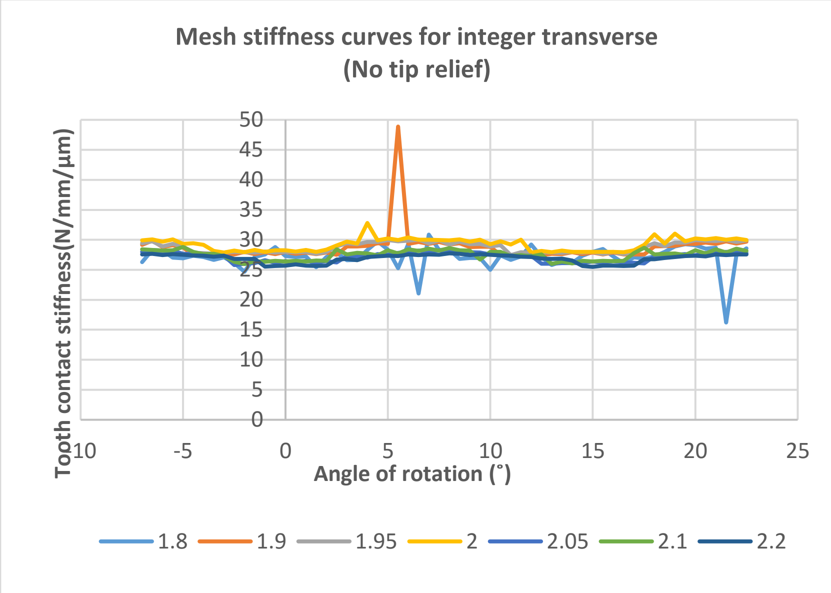
3.1.4. Final mesh stiffness results

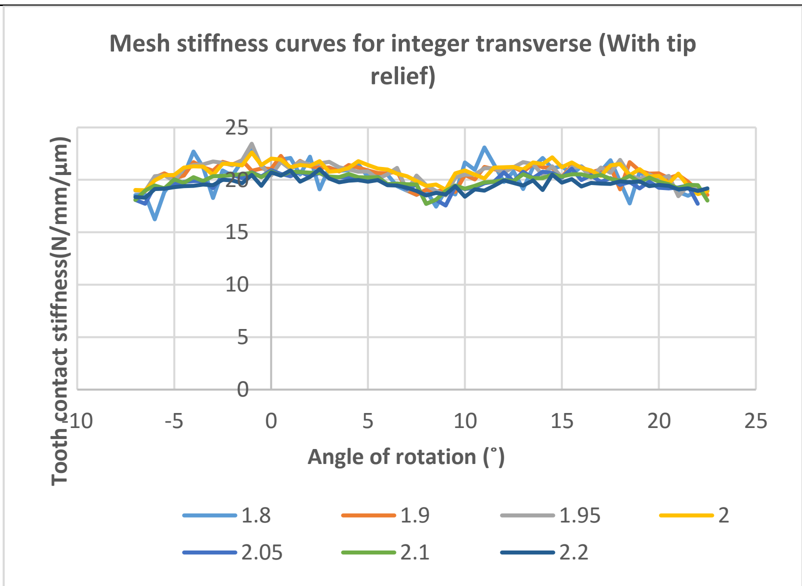
The second semester commenced from obtaining sensitivity results for mesh stiffness of helical gears. Both overlap and transverse contact ratios were investigated and as it can be seen from Table X1 below there is no significant difference between integer and other results either for transverse or overlap case. However, the mesh stiffness curve slightly flattens in case of overlap contact ratio with tip relief being more than 2.

Table 3.2 *Sensitivity analysis for integer overlap, transverse with and without tip relief*



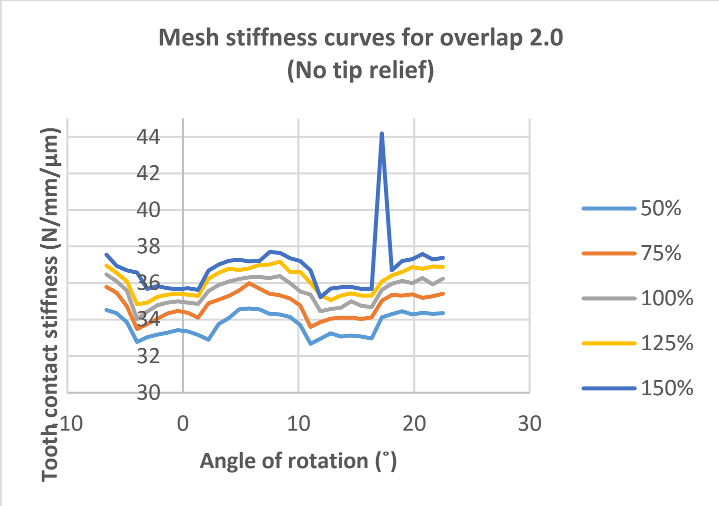


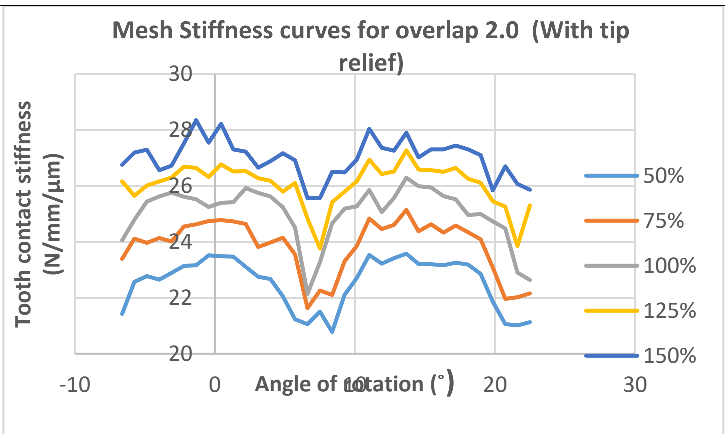


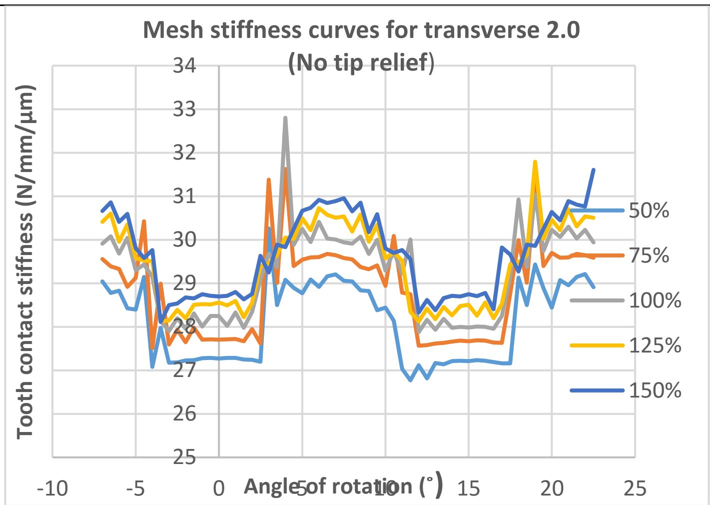


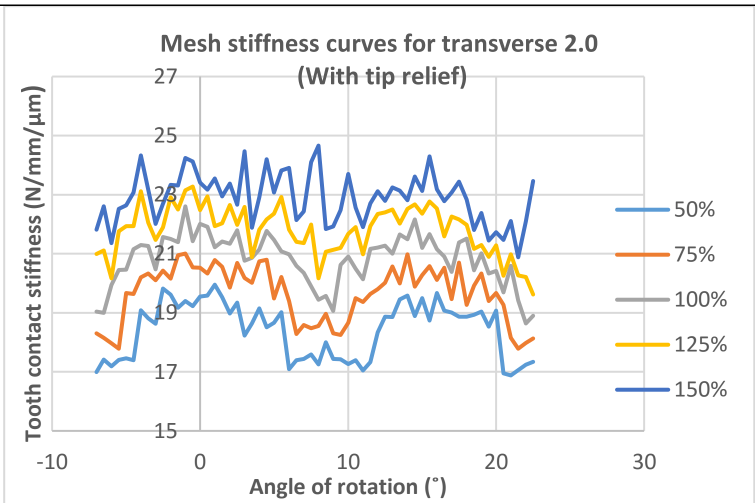
Then our study continues with investigation of relationship between load variation and mesh stiffness of overlap, transverse and total being 2 with/without tip relief(Table 3.3). Firstly, it worth to mention that in all 6 cases as load increases the average mesh stiffness increases accordingly. Secondly, although in case of total integer there is no effect of increasing load on reducing fluctuations and spikes either with or without tip relief, in case of overlap and transverse integer there is slight decrease in depth of large troughs on gears with tip relief. Therefore, these results lead to conclusion that application of tip relief helps to reduce fluctuations of mesh stiffness with increasing the load.

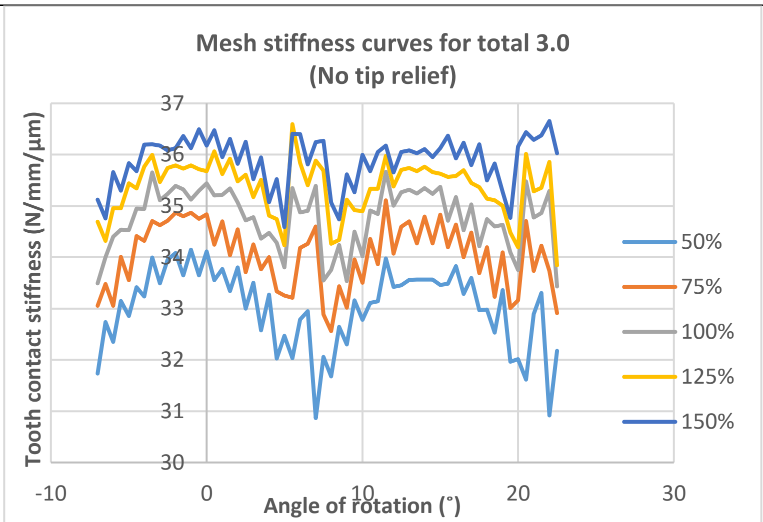
Table 3.3 *Mesh stiffness graphs for various loads for overlap, transverse and total 2.0 with and without tip relief*

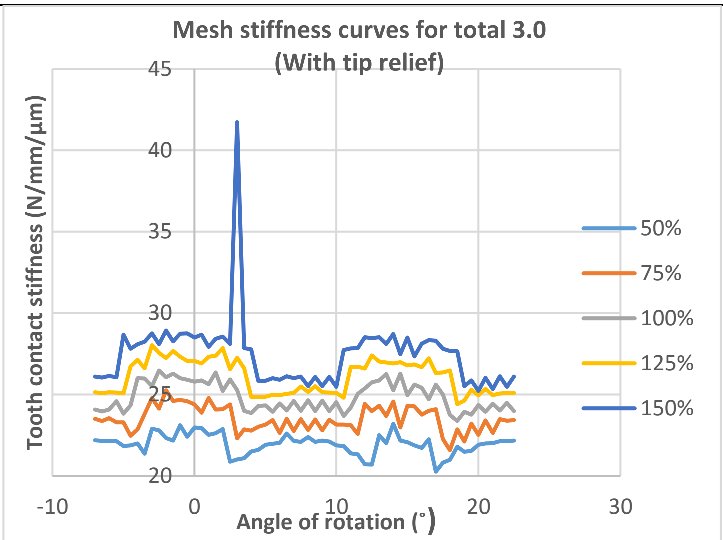





Mesh stiffness curves for transverse 2.0
(No tip relief)
Tooth contact stiffness (N/mm/µm)
Angle of rotation (˚)
50%
75%
100%
125%
150%


Mesh stiffness curves for transverse 2.0
(With tip relief)
Tooth contact stiffness (N/mm/µm)
Angle of rotation (˚)
50%
75%
100%
125%
150%





Next step was to observe sensitivity of mesh stiffness with transverse contact ratio varying from 1.58 up to 2. While in case of increasing overlap there was an improvement in reducing the fluctuations of mesh stiffness, inspecting the Figure 3.4 shows that transverse contact ratio of 1.8 produces better result than those with higher contact ratio. This result can be explained by considering an effect of load on transverse contact ratio. This effect will be investigated later in this work.

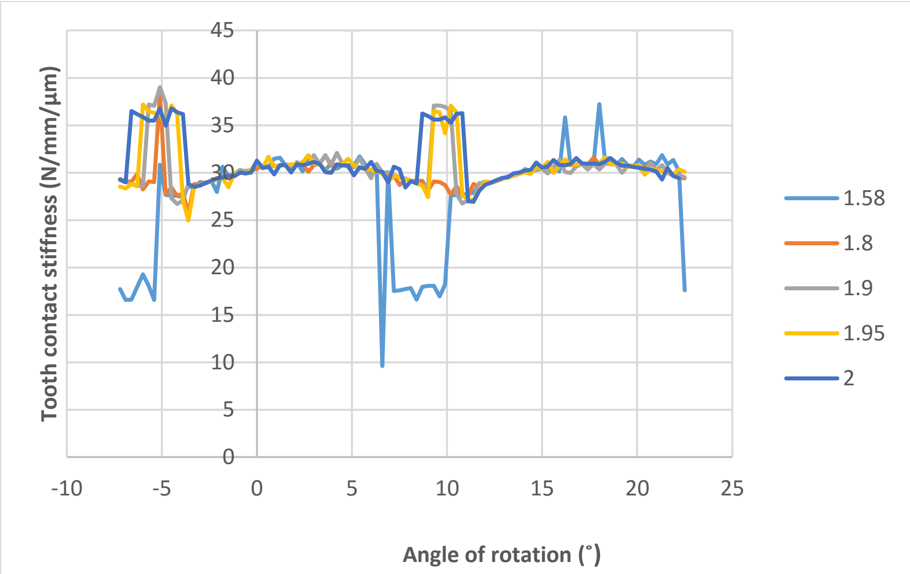


Figure 3.4 *Sensitivity analysis of transverse contact ratio*

As one of the main parameters of helical gears the helix angle was also investigated by increasing it from 0 up to 20(Figure 3.5). Generally, it can be deduced from inspecting this figure that increase in helix angle positively affects reduction of mesh stiffness fluctuations up to certain degree. As helix angle reaches 15 degrees, the mesh stiffness curve is almost straight and increasing it further has no effect on fluctuations, but on average mesh stiffness. However, in our case average value of mesh stiffness doesn't present as much significance as its fluctuations, since our study focuses on reducing vibrations, which are functions of these fluctuations.

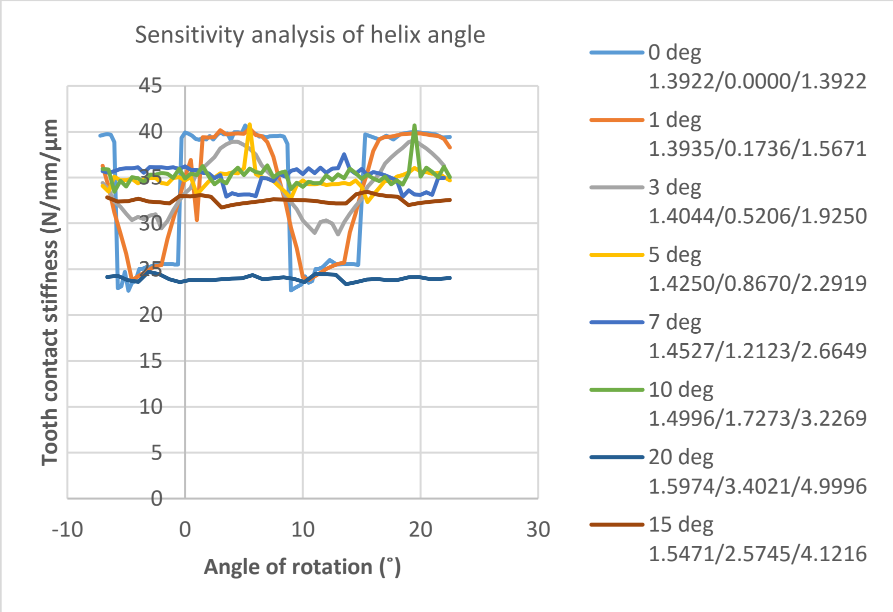
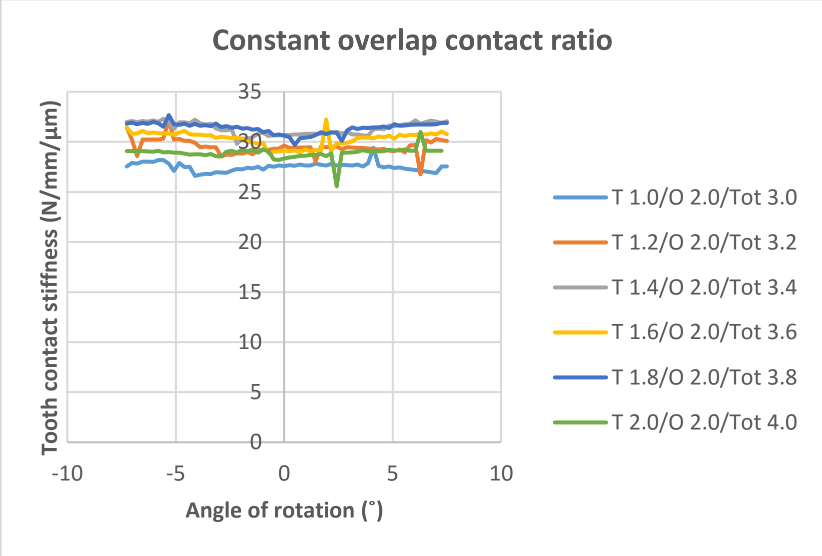


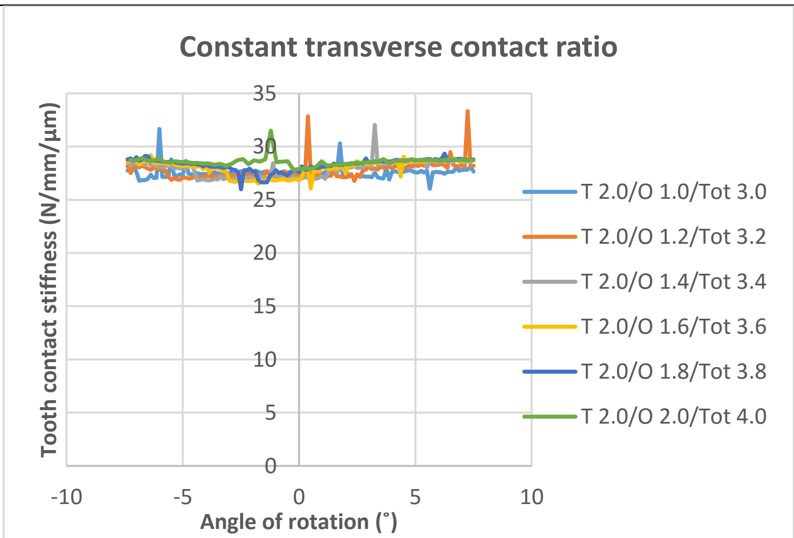
Figure 3.5 *Sensitivity analysis of helix angle*

Table X3 shows parametric study of overlap, transverse and total contact ratios without tip relief. The main aim of this analysis was to observe how keeping constant one of them, while changing another one affects mesh stiffness of the system and identify key trends. Firstly, in case of constant overlap, the effect of increasing transverse contact ratio is almost negligible when overlap is integer and equals 2. Secondly, in case of constant transverse, the curve is becoming smoother as overlap contact ratio increases from 1 to 2 and eventually becomes almost straight line with one large spike. It worth to mention that this spike is considered as computational error, since the dynamics of gears can not physically produce such large rise in mesh stiffness. Therefore, this leads to conclusion that overlap contact ratio is more crucial than transverse one for reducing mesh stiffness fluctuations when it is close to or more than 2. This result can be clearly observed from 3rd graph, where both are changing simultaneously. Although transverse contact ratio increases up to 1.8 with overlap being 1.2, it produces mesh stiffness curve with more fluctuations than the curve with transverse contact ratio 1 and overlap 2(light blue curve)

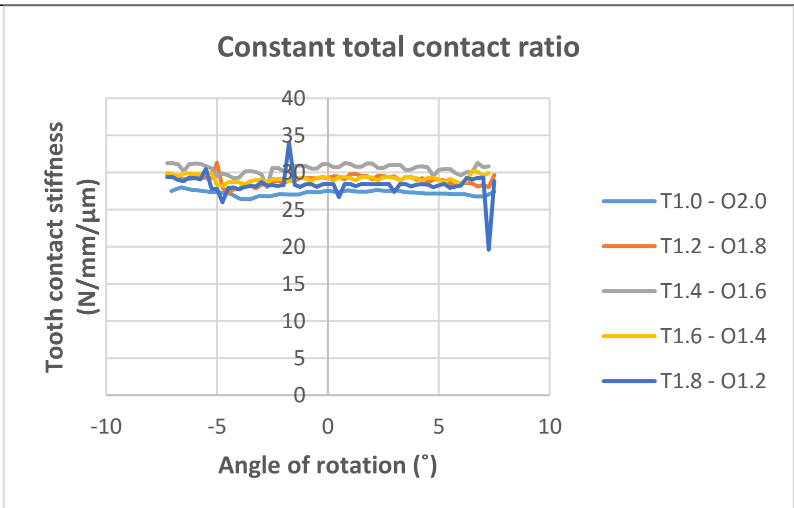
Table 3.4 *Mesh stiffness for constant Overlap/Transverse/Total, varying Transverse/Overlap contact ratio and varying both at 100% load without tip relief*



Constant overlap contact ratio
Tooth contact stiffness (N/mm/μm)
35
30
25
20
15
10
5
0
-10
-5
0
5
10
Angle of rotation (˚)
T 1.0/O 2.0/Tot 3.0
T 1.2/O 2.0/Tot 3.2
T 1.4/O 2.0/Tot 3.4
T 1.6/O 2.0/Tot 3.6
T 1.8/O 2.0/Tot 3.8
T 2.0/O 2.0/Tot 4.0



Constant transverse contact ratio
Tooth contact stiffness (N/mm/μm)
35
30
25
20
15
10
5
0
-10
-5
0
5
10
Angle of rotation (˚)
T 2.0/O 1.0/Tot 3.0
T 2.0/O 1.2/Tot 3.2
T 2.0/O 1.4/Tot 3.4
T 2.0/O 1.6/Tot 3.6
T 2.0/O 1.8/Tot 3.8
T 2.0/O 2.0/Tot 4.0



Constant total contact ratio
Tooth contact stiffness (N/mm/μm)
40
35
30
25
20
15
10
5
0
-10
-5
0
5
10
Angle of rotation (˚)
T1.0 - O2.0
T1.2 - O1.8
T1.4 - O1.6
T1.6 - O1.4
T1.8 - O1.2

All previous results during 1st semester and up to middle of 2nd semester were obtained using older 2012 version of KISSsoft, which was constantly producing small spikes and fluctuations of mesh stiffness despite that it was not supposed to according to basic dynamics of gears. At this point of the project we have installed KISSsoft 2016 to compare its results with 2011 version(Figure 3.6)

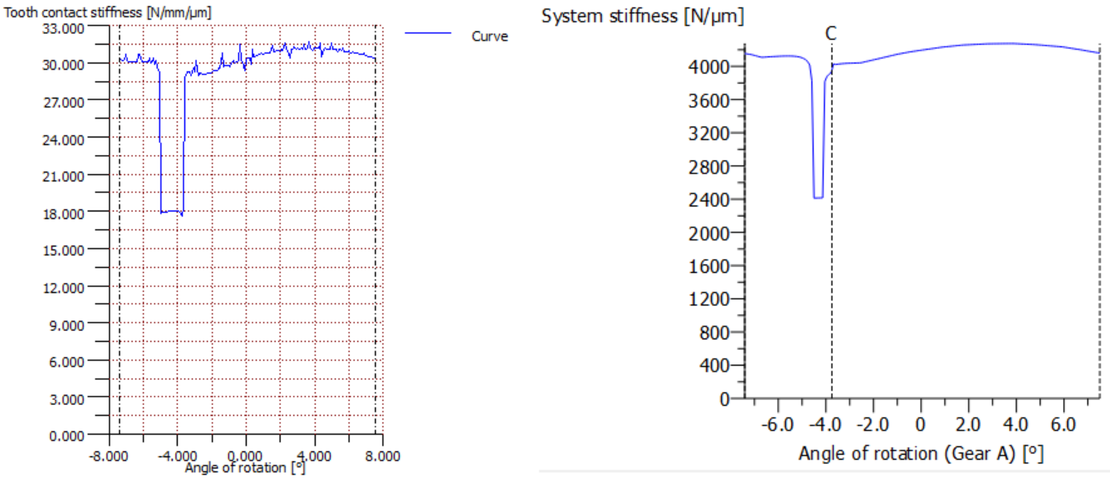


Figure 3.6 *Difference of mesh stiffness result between KISSsoft 2011 and 2016*

The figure above clearly shows the difference in quality of results between these two versions of the software. Even though all previous results were flawed with these small fluctuations, overall trends were identified correctly. However, from this point on the project was continued with newer version to avoid any misconception regarding those computational inaccuracies and other errors.

One important finding along with using newer version of KISSsoft was importance of transverse contact ratio under load for mesh stiffness curve. As it can be seen from Figure 3.7 the mesh stiffness of spur gear is highly dependent on load and being more specific on loaded transverse contact ratio(TCR). It is clear that the maximum load cannot be 2 times larger than maximum allowable, however, this analysis is made only to find theoretical trends in correlation between mesh stiffness and loaded TCR. Therefore, main conclusion to be drawn from this result is that for constant spur gear mesh stiffness the loaded TCR is more critical than theoretical one and it has to be very close to integer, but not exactly integer.

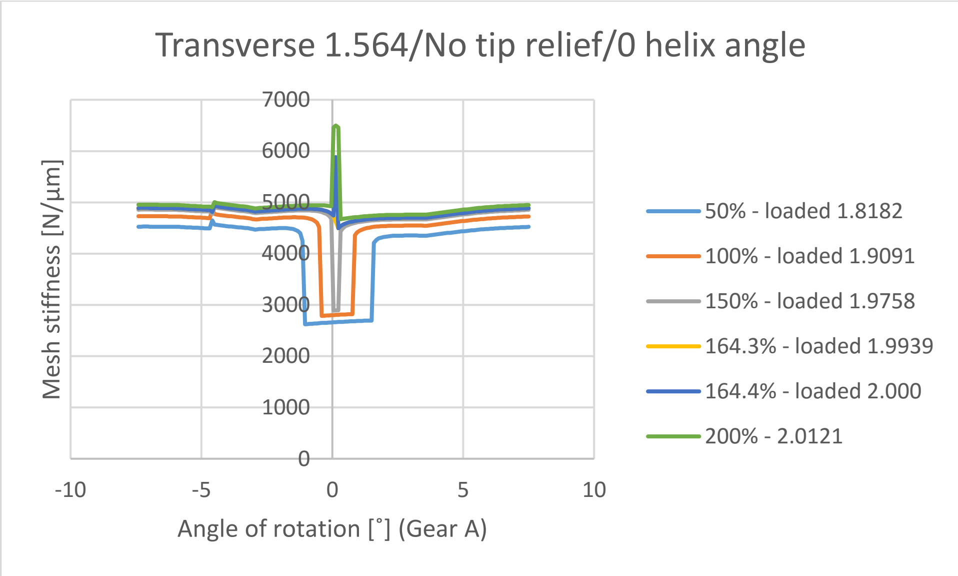


Figure 3.7 *Mesh stiffness versus angle of rotation for various loads*

Figure 3.8 illustrates the result of the same analysis but with tip relief. The maximum load applied is only for theoretical purposes, hence, it cannot be utilized in real practice. However, the fundamental trend can be still utilized. Comparing this graph to previous one, in this case the load needed to achieve the loaded TCR of 2 is much larger(359%). Therefore, these results lead to conclusion that tip relief doesn't alter the theoretical TCR, however, it changes loaded TCR and as a result smoother mesh stiffness curve can be obtained at higher loads.

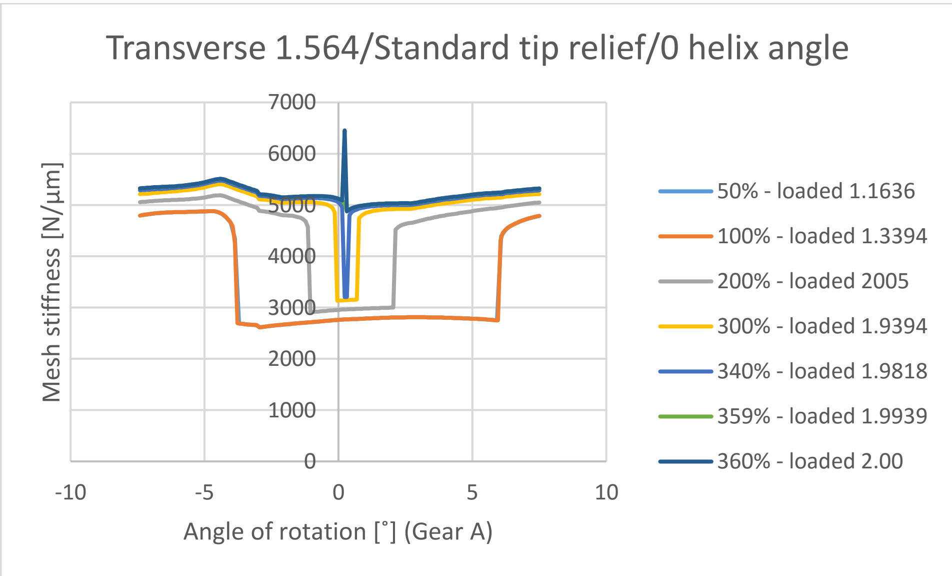


Figure 3.8 Mesh stiffness versus angle of rotation with tip relief for various loads

As it was mentioned before it is common among gear specialists to apply profile modification for reducing gear noise and vibrations. In our case, various amounts and lengths of tip relief are investigated to obtain desirable level of vibrations for certain load. Tip relief itself has two main parameters: amount(Ca) and length(La) as it was shown on Figure 1. Each of them has its own effect on mesh stiffness results, therefore each is studied separately. Firstly, by increasing amount of tip relief applied for both gears and keeping the length at maximum (according to British Standard BS 1970 and ISO/DIS 1983) the loaded TCR is gradually decreasing, however, as the amount reaches the values of 0.5 mm the loaded TCR doesn't decrease further than 1.2 (Figure 3.9). This trend can be observed for different tip relief length as well. However, in case of 6.3m length the loaded TCR still slightly decreases even further than 0.5 mm of tip relief amount. The slop of this decrease is reducing and eventually reaches stable region.

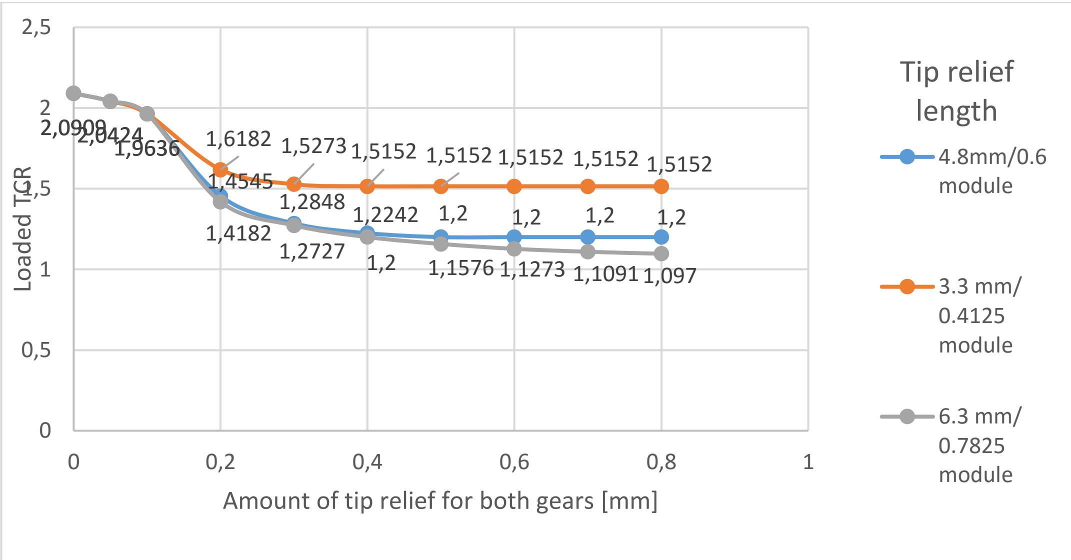


Figure 3.9 Variation of loaded TCR versus amount of tip relief for both gears at 100% load

Unlike amount of tip relief, length is not decreasing asymptotically till certain value, but also starts increasing after tip relief length of 5 and 6 mm for tip relief amount of 0.16 and 0.22 mm respectively (Figure 3.10). However, at 0.1 mm amount of tip relief the loaded TCR doesn't change significantly as the length is being increased. This observation leads to conclusion that length of tip relief is not in linear relationship with loaded TCR and as it was shown by Devendiran(2016) too much of it results in adverse effect for reduction of vibrations. In other words, in order to obtain integer loaded TCR only sufficient length of tip relief has to be applied.

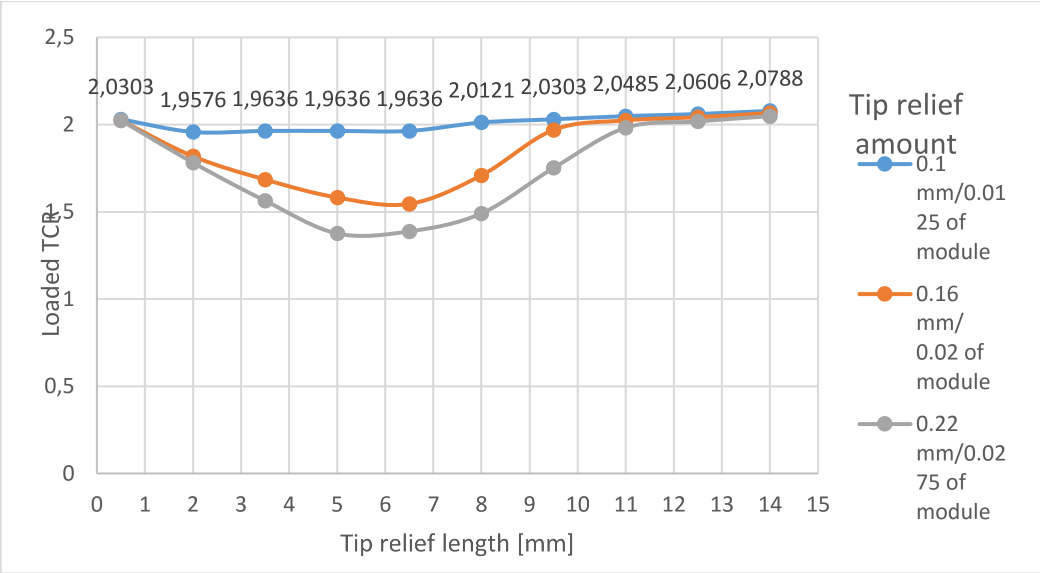


Figure 3.10 *Variation of loaded TCR versus length of tip relief for both gears at 100% load*

3.1.5 Transmission error results for entire system

The study done on obtaining mesh stiffness curve with the most optimum configuration of load, tip relief and contact ratio is to be integrated into the system with torque-splitting topology and real-time self-preloading mechanism(Fiugre 3.11)

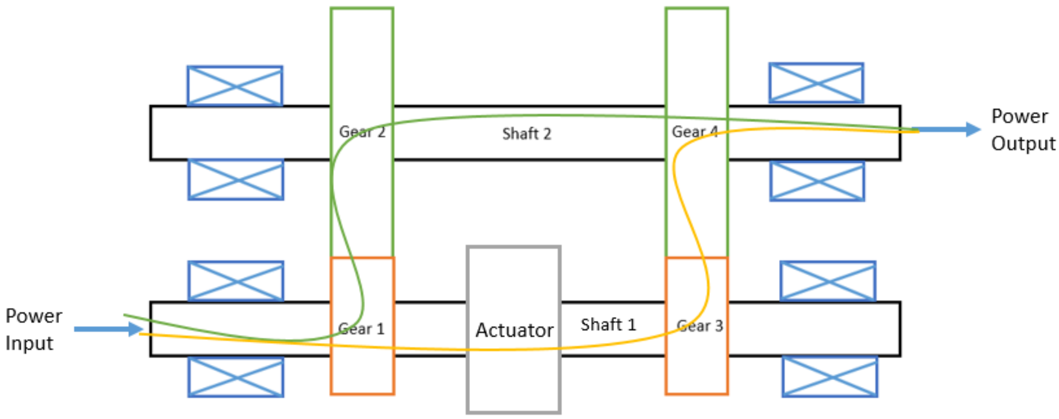


Figure 3.11 *Overall system schematics*

This topology is being utilized to decrease overall level of vibrations by splitting the torque into two pair of gears. The model implies 80% of load being transmitted through the first pair(Gear 1&2) and rest 20% being transmitted through the second pair(Gear 3&4). Actuator is a preloading mechanism used to adjust actual load on teeth to desirable level, so that this pair maintains constant mesh stiffness throughout wide range of loads. For preloading mechanism, the principle of FZG test rig is going to be utilized. Basically, FZG test rig is mainly used for testing lubricant characteristics, however, in our case we need only preloading mechanism of this test rig in order to observe the result of preloading on mesh stiffness and in turn on level of noise and vibrations. It can be seen from Figure 3.12 that preloading mechanism consists of load clutch(4), locking pin(5), load lever and weights(6). This mechanism is called static preloading one, since it cannot change the load in real-time i.e. simultaneously with varying input load. Although our study requires real-time self-preloading, working principles of the preloading mechanism of FZG test rig can be still employed for testing mesh stiffness for various loads by changing the lever load manually for each load.

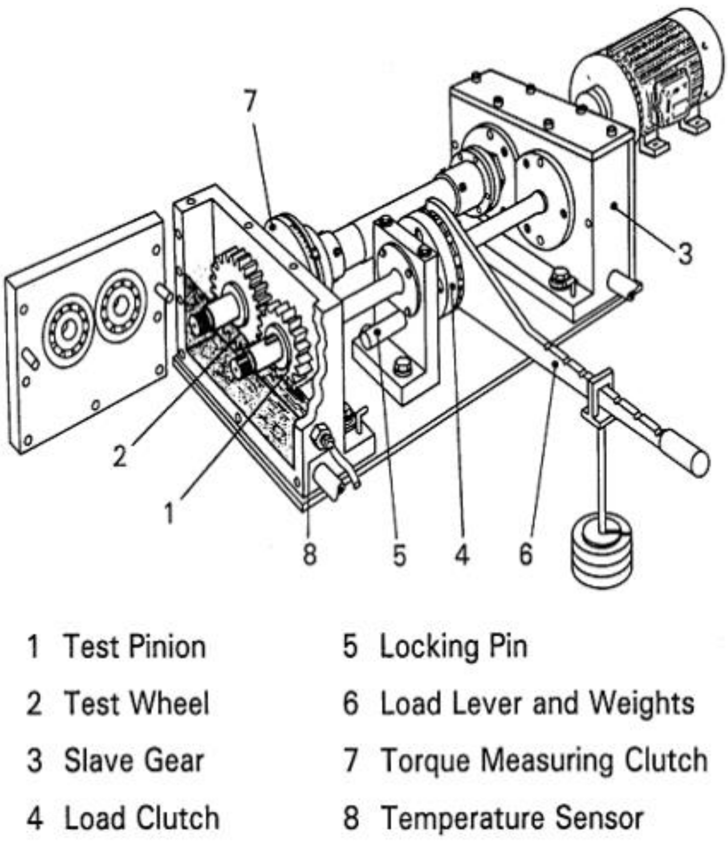


Figure 3.12 *FZG back-to-back gear test rig*

For system level the dynamic model is developed in Mathematica software using standard system dynamic equations as shown below:

$$J_1\,\ddot{\theta}_1 + C_1\dot{\theta}_1 + r_{g1}^2\,k\,\theta_{21}^* = T_1 \tag{3.1}$$

$$J_2\,\ddot{\theta}_2 + C_2\dot{\theta}_2 + r_{g2}^2 r_{g1}^2\,k\,\theta_{21}^* = T_2 \tag{3.2}$$

Where,

$\mathbf{J_1}$	moment of inertia of 1st gear
$\mathbf{J_2}$	moment of inertia of 2nd gear
$\ddot{\boldsymbol{\theta}}_{\mathbf{1}}$	acceleration of 1st gear
$\ddot{\boldsymbol{\theta}}_{\mathbf{2}}$	acceleration of 2nd gear
$\mathbf{C_1}$	damping coefficient of 1st gear
$\mathbf{C_2}$	damping coefficient of 2nd gear
$\mathbf{r_{g1}}$	base radius of 1s gear
$\mathbf{r_{g2}}$	base radius of 2nd gear
k	mesh stiffness
$\boldsymbol{\theta}_{\mathbf{21}}^*$	transmission error
$\mathbf{T_1}$	input torque
$\mathbf{T_2}$	output torque

Mesh stiffness and angles for these equations are obtained using KISSsoft i.e. optimized mesh stiffness results from previous part of the study are substituted into these equations. These equations are solved in Mathematica and plot of Transmission Error for 1st gear pair is obtained(Figure 3.13).

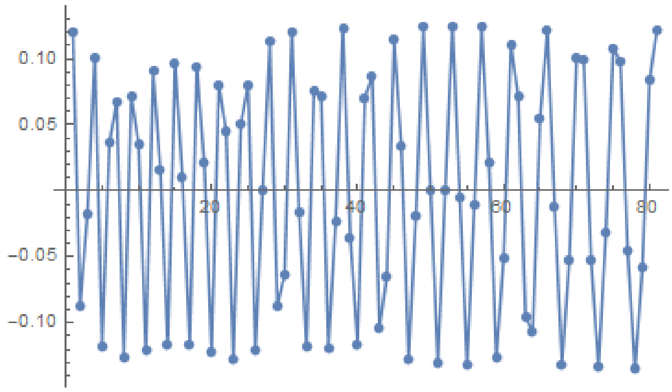


Figure 3.13 *Plot of Transmission Error for 1st pair with time increment of 0.005 in the range [0.1, 0.5]*

The transmission error plot above illustrates its variation only for one pair of gear. The second pair will have similar transmission error plot, but with lower magnitude. The reason for that is that it has 2 times less module (4 mm) than first pair. The mesh stiffness results showed that increase/decrease of actual load changes mesh stiffness. Since preloading mechanism enables correction of actual load on teeth, this will allow keeping transmitted load constant and varying actual load by imposing angular displacement using actuator (Figure 3.12). The splitting-torque topology in turn helps to reduce overall level of vibrations by taking some part of transmitted load. It is of high usefulness for low vibration mode, when the load being transmitted is quite high and actual load correction from actuator is at its maximum capacity. Therefore, utilizing high integer overlap contact ratio along with slightly less than integer transverse contact ratio under load gives almost constant mesh stiffness curve, while implementing splitting-torque topology with preloading mechanism

3.2 Misalignment insensitivity

3.2.1 Milestones

With regards to gear misalignment, there are three milestones. At first, the behavior of conventional gears must be studie using finite element analysis. For this step, it will be necessary to conduct a parametric study, i.e. to learn, how stress distribution changes with the magnitude of misalignment. Also, at this step, influence of misalignment in two different directions is to be studied. After the first milestone is achieved, the topology of one of the gears will be changed in such way that it will be able to adjust its orientation depending on the misalignment. This will be done by splitting the pinion to multiple components that can slide relative to each other and joined together by spring-like connectors. The system will be illustrated more clearly later in the chapter 3. Such system can be good in terms of tolerating misalignment, but at the same time it may have high complexity. When the second milestone is done, another web modifications, such as spoked pinion, fragmented pinion (that are described in details in corresponding sections) will be studied. If sufficient amount of time remains, solution suggested by Ganti et al. (2016) will also be studied for comparison. After completion of all three milestones, it will be possible to make conclusions and suggest a solution for gear misalignment problem.

3.2.2 Justification for use of static structural studies

For the part of the project related to gear misalignment static structural studies were used to obtain the stress distribution patterns. It should be noticed that gears existing in real life obey dynamic behavior rather than static. Despite this, results obtained for this project still can be trusted, because stress distribution occurs with the speed of sound, which is much faster than speed of vibrations inherent to dynamic behavior. It means that before each oscillation affects the performance of the gears in any way, stress gets completely distributed. This difference between speed of stress distribution and frequency of oscillations becomes even more significant for large gears, which transmit high torques and, therefore, rotate with small angular velocity. Design solutions related to gear misalignments are also valid, provided that deformation of material happens faster than oscillations.

3.2.3 Parameters

For all simulations relevant to milestones completion, following parameters were applied

Table 3.5 *parameters used for all simulations relevant to milestones*

Gears type	Spur gears (not helical)
Module of the gears	60 mm
Number of teeth in the pinion	20
Number of teeth in the large gear	80

Torque	95 kN*m
Pressure angle	20 degrees

3.2.4 Model creation for finite element analysis

All the studies related to misalignments are performed in three steps. At the beginning gear geometry is created in KISSsoft. All important parameters including module, number of teeth, pressure angle, width and backlash are adjusted here. Also, different tooth profile modifications, such as crowning, which relieves impact of misalignment, can be applied there. Geometries generated by such way can be then exported to SolidWorks for further treatment. Figure 3.14 shows typical gear geometry when it is just generated in KISSsoft:

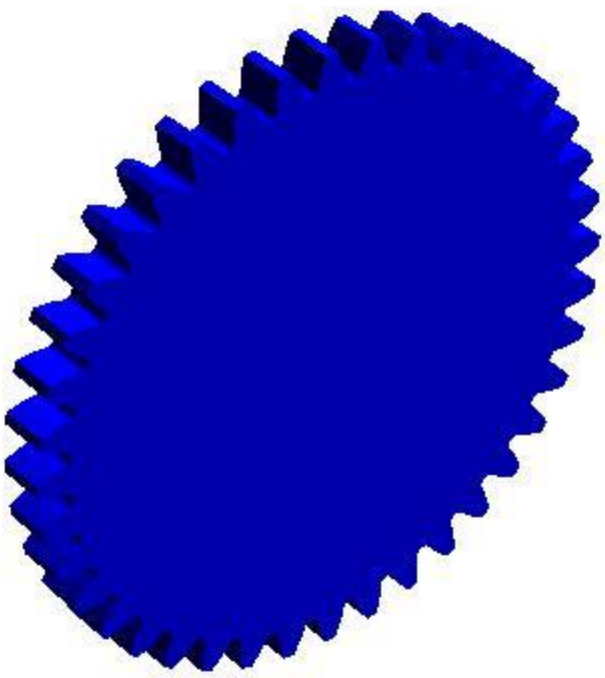


Figure 3.14 *Gear generated in KISSsoft before been exported to SolidWorks*

When the gears are exported to SolidWorks, creation of 3D model is brought to its logical completion. For this part model is two meshing gears with misalignment in two directions: about Y-axis and about X-axis. Holes for the shaft, meshing of the gears and web modifications are performed there. In addition to that, any misalignment in any direction can be applied here. It is particularly important to ensure that in all case studies gears contact at the same position, which basically means that each time stress occurs at the same region of meshing tooth. The reason for this restriction is that when gears rotate and change their meshing position, stress magnitude may also change. Fortunately, SolidWorks gives opportunity to avoid this by fixing one of the gears. For the clarity figure 3.15 shows pair of meshing gears with exaggerated misalignment about X axis:

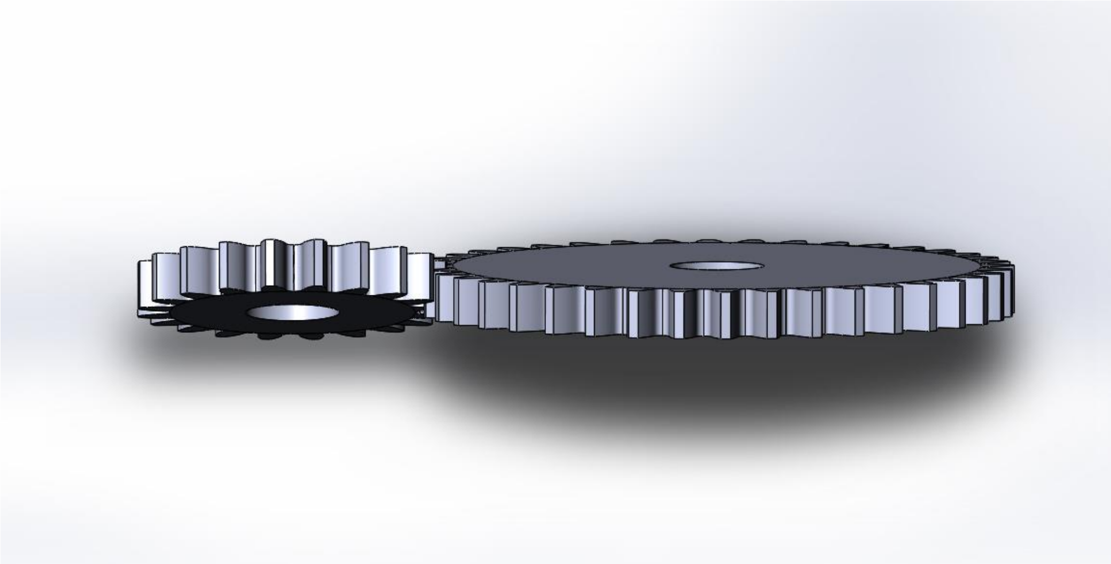


Figure 3.15 *Exaggerated gear misalignment created in SolidWorks*

When 3D model is properly created, it can be further exported to ANSYS to conduct static simulations, which are much more simple than dynamic simulations described in methodology for load insensitive gear design. In this case static simulations make sense because studied gears carry high loads and rotate at relatively low speeds. Simulation starts from conventional preparations like mesh generation, torque application and choice of boundary conditions. When preparations are complete, the solver is started. After solution is finished, it is possible to look at results. For this study mainly stress distribution and deformations are of interest. Figures 3.16 and 3.17 show examples of boundary conditions and initial mesh for gear pair exported to ANSYS:

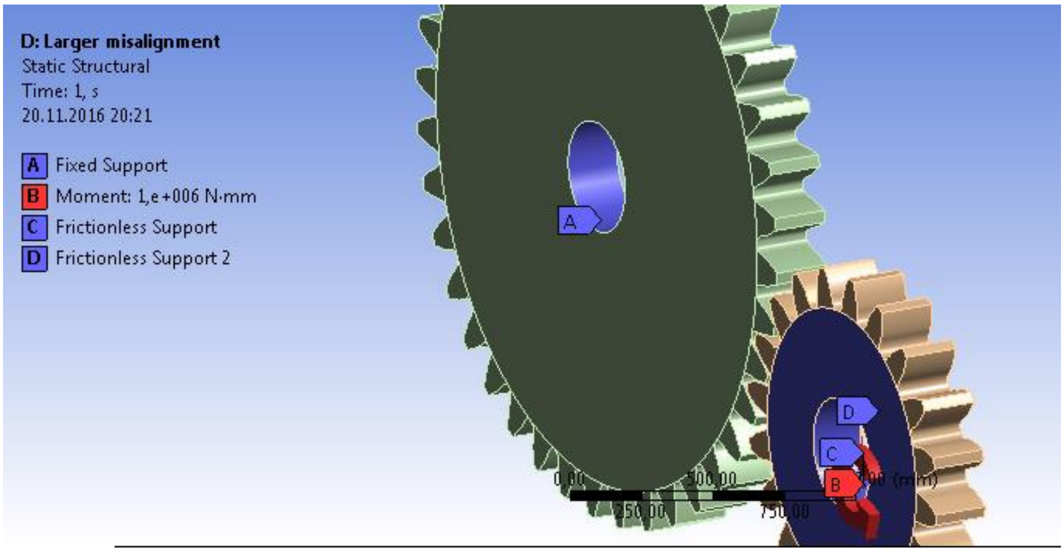


Figure 3.16 *Boundary conditions for gear pair exported to ANSYS*

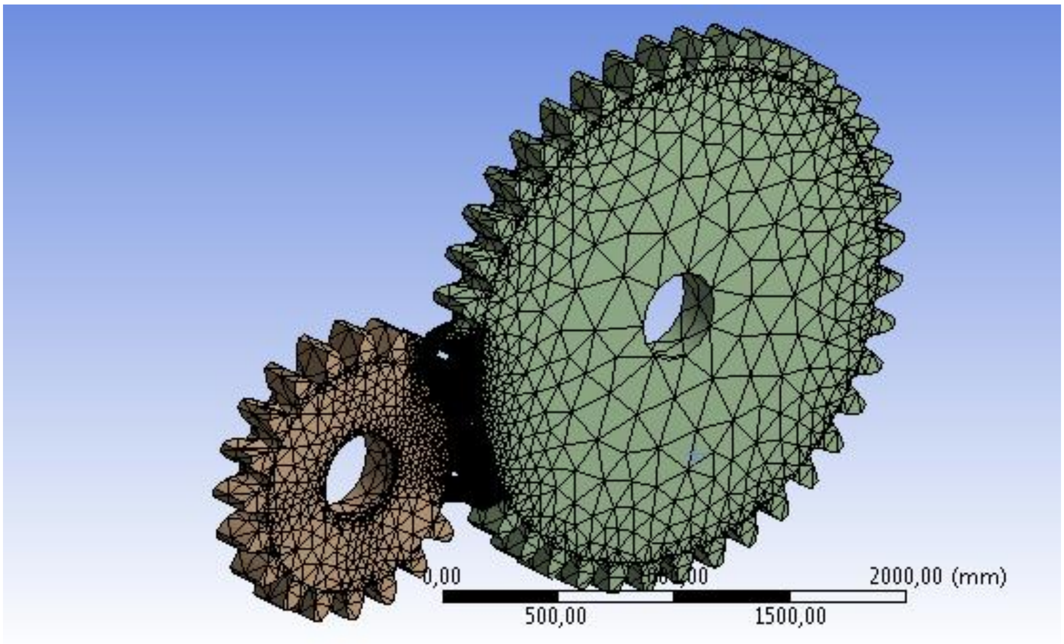


Figure 3.17 *Draft mesh of the gear pair exported to ANSYS*

As it turned out during the studies, having two pairs of teeth in contact, when gears interact with each other, causes additional difficulties, because solving the model with fine mesh generated in both pairs of teeth requires more resources in terms of RAM and productivity of the hardware. To solve this problem, some teeth were removed from the pinion to ensure that only one pair of teeth is always in contact. In addition to that, large gear was cut to avoid extra mesh generation. Figure 3.18 illustrates these measures, which were applied in all studies performed in the second semester (except fragmented pinion topology):

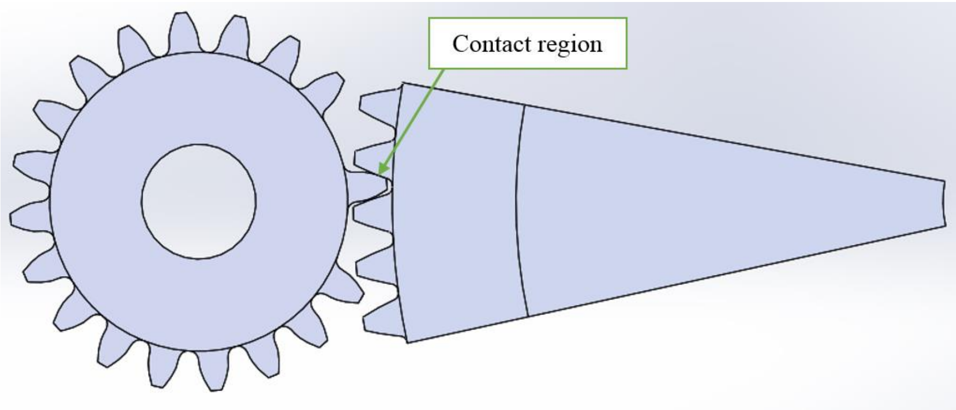


Figure 3.18 *Measures taken to simplify the mesh*

The mesh also was improved during the second semester. Firstly, mesh sizing of 12 mm was applied to contacting teeth and to adjacent surfaces, because all interactions and stresses occur there. After that, contact surfaces of the meshing teeth were subjected to 3rd level mesh refinement, which made the mesh finer in that area. Finally, contact regions of these surfaces, which are the most important, were distinguished and common mesh sizing was replaced by another one having 5 mm cell size instead of 12 mm. In the case of the pinions having the spokes additional mesh sizing of 15 mm was applied to the spokes closest to contacting tooth. Figures 3.19 and 3.20 show the typical mesh:

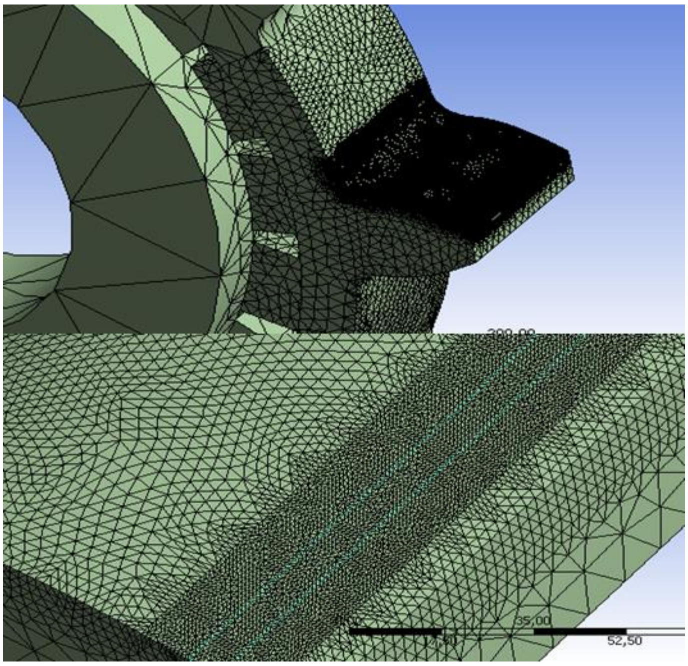


Figure 3.19 *Improved mesh on pinion*

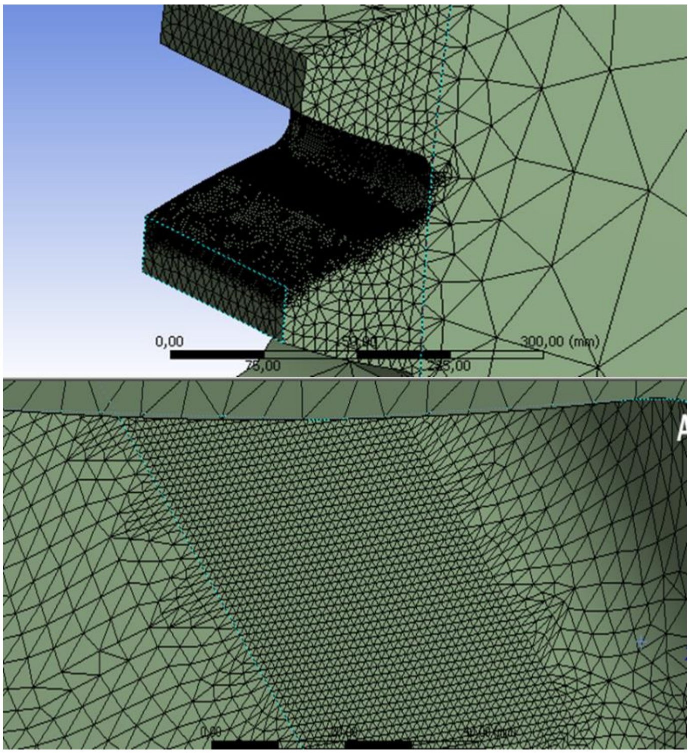


Figure 3.20 *Improved mesh on large gear*

Simulations performed during the first semester are shown below. Following three figures represent how stress distribution among the contacting teeth changes with misalignment about X-axis (horizontal axis perpendicular to the shaft):

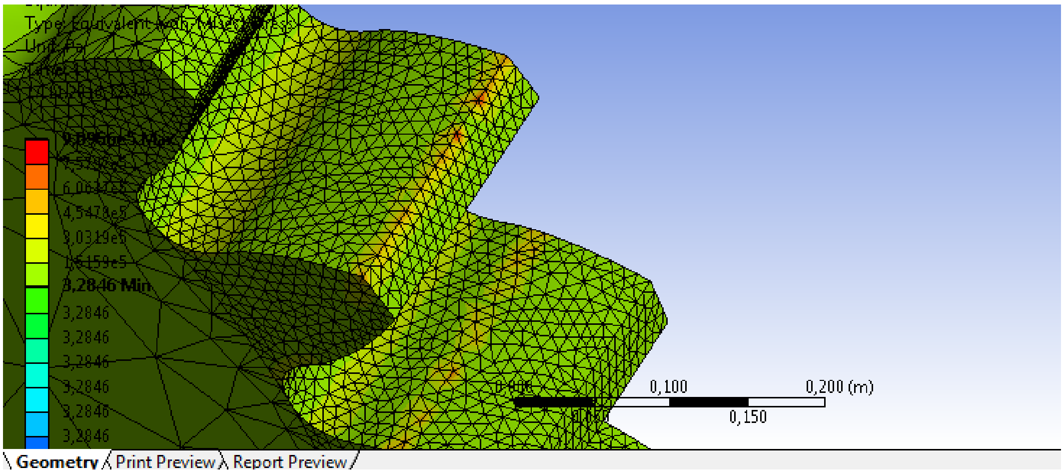


Figure 3.21 *Stress distribution for 0 degree misalignment*

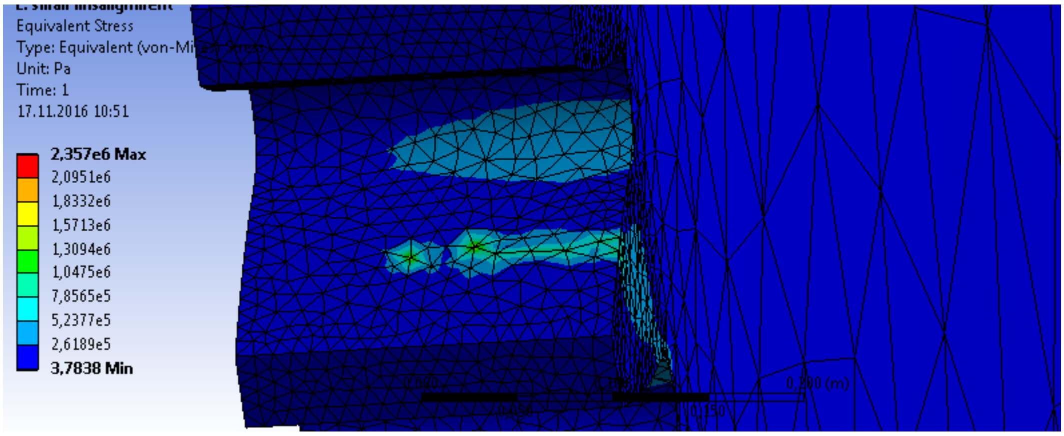


Figure 3.22 *Stress distribution for 0,01 degree misalignment*

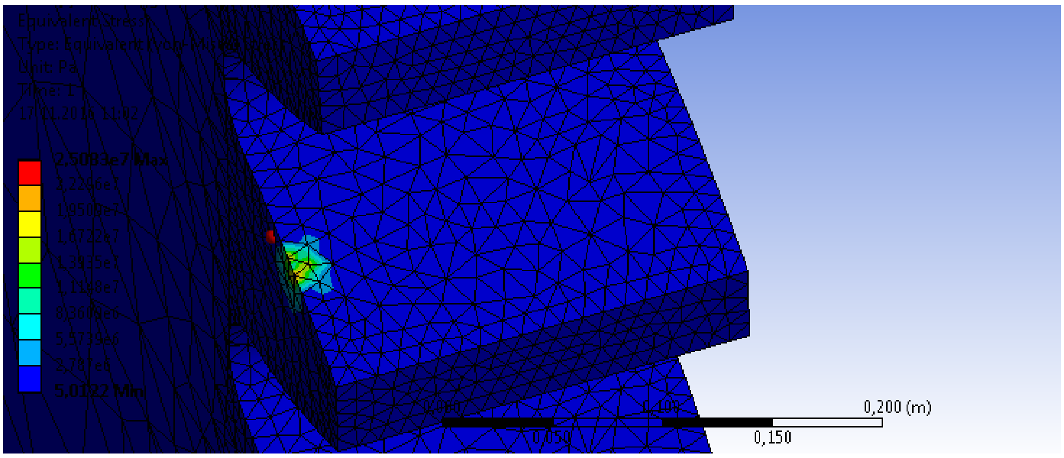


Figure 3.23 *Stress distribution for 0,2 degree misalignment*

It should be noticed, however, that reliability of these results is compromised by coarse mesh selection. Also, parameters mentioned in section 3.2.3 were not applied to these gears: number of teeth in large gear is 40 instead of 80 and load is only 1 kN. Despite this, obtained results still clearly represent the trend: larger misalignment about horizontal axis perpendicular to perfectly aligned shafts reduces contact area and makes the stresses more concentrated.

3.2.5 Milestone 1: parametric study of ordinary gears

When the model was relieved from extra teeth, orientation of the gears was fixed and the mesh was improved, parametric study for ordinary gears was conducted. For convenience, horizontal axis perpendicular to the perfectly aligned shaft axes was called X-axis and vertical axis perpendicular to the perfectly aligned shaft axes was called Y-axes.

Figure 3.24 show typical results of the simulations:

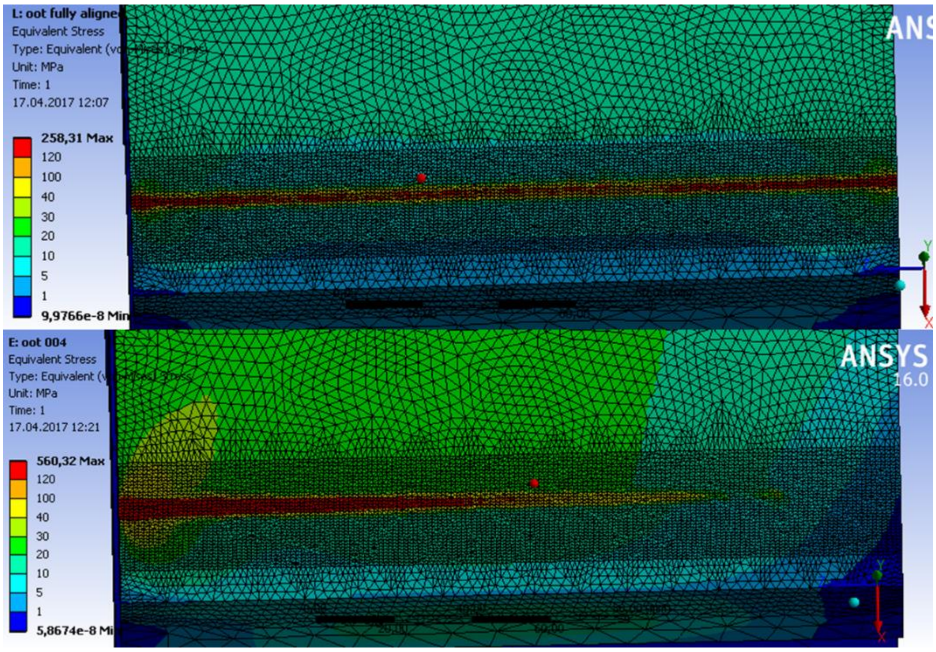


Figure 3.24 *Stress distribution for zero misalignment (upper picture) and 0,04-degree misalignment about X-axis (lower picture)*

It can be clearly seen from the figure above that misalignment creates stress concentration in ordinary gears. Complete parametric study considering misalignment about X-axis (with zero misalignment about Y-axis) is shown on the table 3.6 and graph 3.25:

Table 3.6 *Highest stress versus misalignment about X-axis*

Misalignment about X-axis, degree	Maximum stress, MPa
0	258,31
0,01	338,25
0,02	417,71
0,03	491,33
0,04	560,32
0,05	629,41
0,06	699,46
0,07	768,34
0,08	840,14

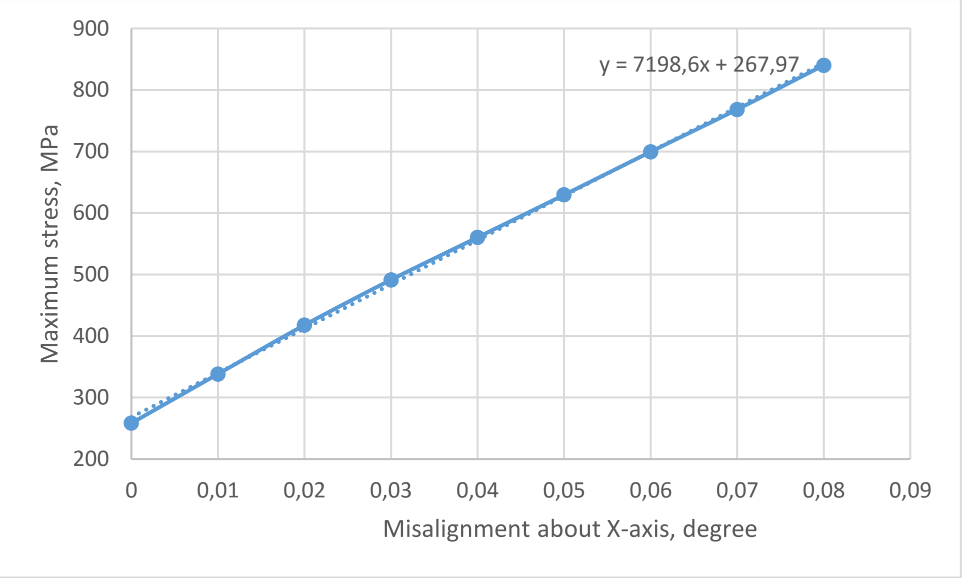


Figure 3.25 *Highest stress versus misalignment about X-axis*

It can be seen from the figure that there is approximately linear dependence between misalignment and peak stress. Even though trend line does not look too vertical, extracting trend line equation in MS Excel gives a slope magnitude of 7198,6 MPA per degree. It can be concluded from here that increase in peak stress, which is representative of stress concentration, at this case is too large to be neglected, especially for large misalignments.

One more purpose of this milestone is to check effect of misalignment about Y-axis on stress distribution. At the beginning of the project suggestion was made that this kind of deviation is not as important as misalignment about Y-axis. To prove this, couple of studies were made:

Table 3.7 *Investigating effect of misalignment about Y-axis*

Misalignment about X-axis, degree	Maximum stress without misalignment about Y axis, Mpa	Maximum stress with 0,04-degree misalignment about Y axis, Mpa
0	258,31	411,14
0,04	560,32	624,7

Table 3.7 shows that imposing additional 0,04-degree misalignment about Y-axis to both aligned and having 0,04-degree misalignment about X-axis systems increase peak stress to approximately 60 MPa. This value is large, but not very significant in comparison with increase detected in previous set of studies.

Also, figure 3.26 shows stress distribution in perfectly aligned gears and those having 0,04-degree misalignment about Y-axis:

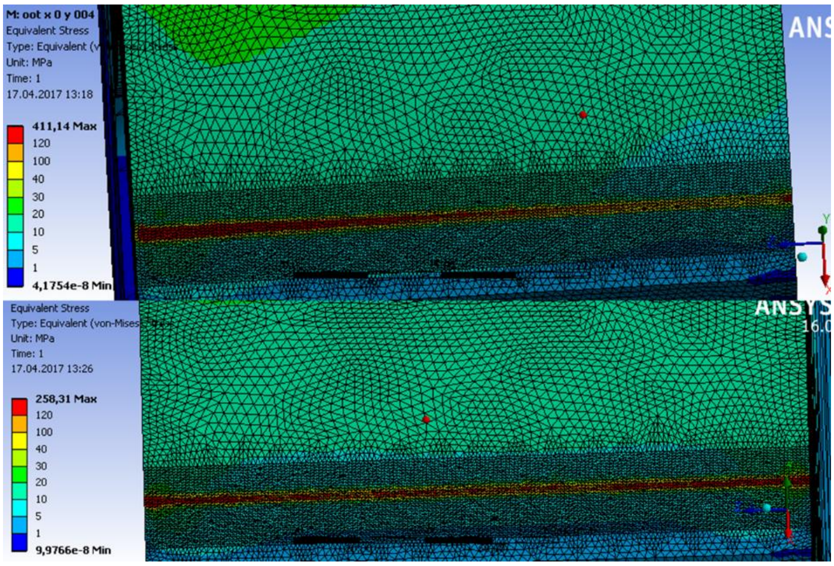


Figure 3.26 *Stress distribution for 0,04-degree misalignment about Y-axis (upper picture) and perfect alignment (lower picture)*

As it can be seen, noticeable difference in stress distribution still exist, but it is far less dramatic than for misalignment of the same magnitude about X-axis.

Therefore, it can be stated that misalignment about X-axis is more important.

3.2.5 Milestone 2: multicomponent pinion

Second milestone of the project was to create multicomponent web for the pinion and study its behavior. In this case pinion consist of two components: internal spherical ring and external component with teeth. They should be able to slide relative to each other and linked by special connectors behaving like springs. Actually, these connectors are just pieces of metal and their spring-like properties are caused by resistance of metal to deformation. Important moment is that they should be inclined from the radial lines connecting center of the pinion with central plane of each tooth, otherwise, their resistance will be low. Such configuration is supposed to allow this pinion to adjust itself when it is subjected to concentrated load caused by gear misalignment. The geometry of the pinion with multicomponent web is shown on the figure 3.27:

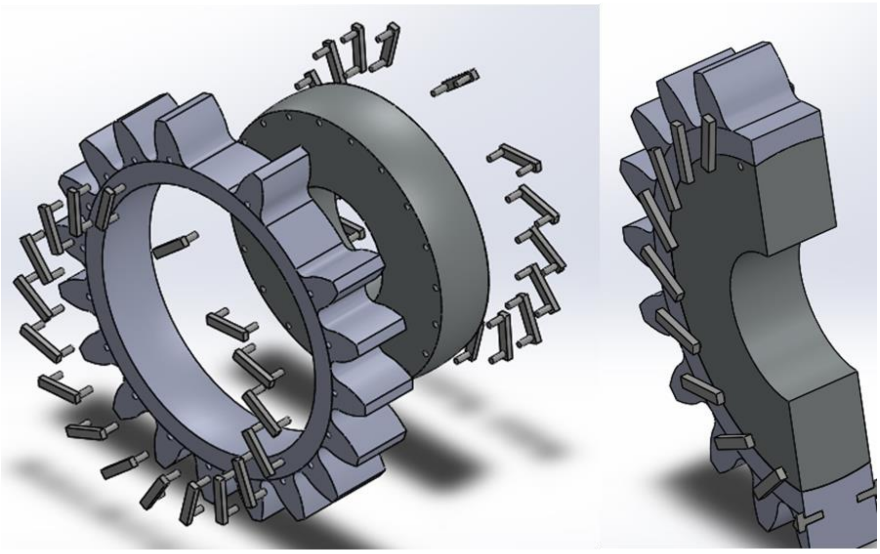


Figure 3.27. Pinion with multicomponent web

Due to complexity of the system, only one simulation was made with this topology, which is shown on the figure 3.28 (misalignment is 0,04 degree about X-axis):

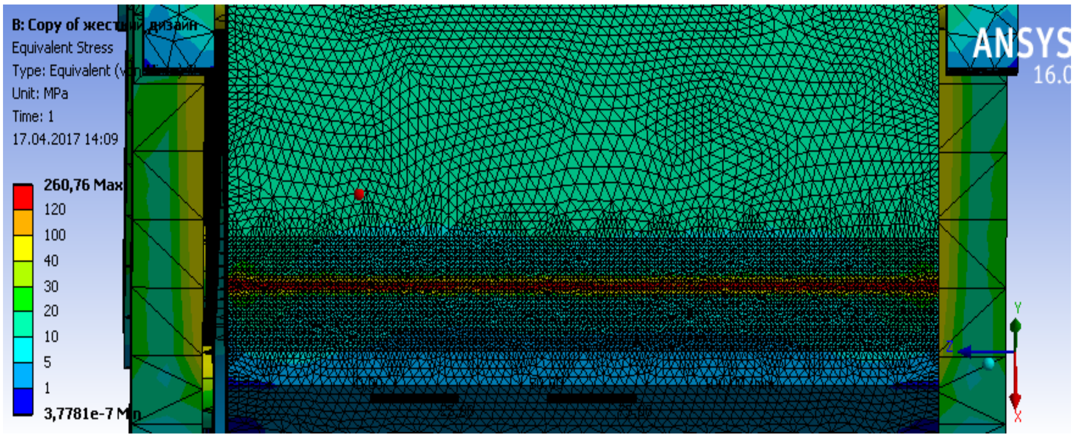


Figure 3.28 *Stress distribution on misaligned pinion with multicomponent web*

As the figure shows, this topology allows to get smooth stress distribution and extremely low peak stress even under misalignment (260 MPa is almost equal to stress in perfectly aligned gears). However, some of limitations of this design should be kept in mind. Firstly, complicated manufacturing would make this design expensive and reduce its feasibility. In addition to that, such pinion can safely rotate only in one direction. Appropriate direction of rotation is those at which connectors will be subjected to tension. If gear rotates in opposite direction, there will be risk of buckling, which harmful for gear performance. If these disadvantages are neglected, this design can be called successful.

3.2.6 Milestone 3: spoked pinion

Spoked pinion topology is just a modification of the solution, suggested by Ganti et al. in 2016: as in their solution, material removed from the web to reduce stiffness of the pinion in direction of misalignment and keep it approximately the same in other directions. Main difference is that more material is taken to form the spokes. Solution suggested by Ganti et al. and spoked pinion can be seen on the figure 3.29:

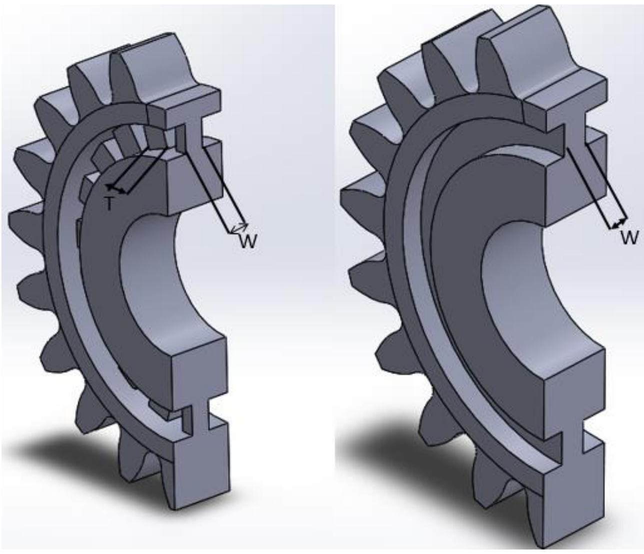


Figure 3.29. *Spoked Pinion (at the left) and solution suggested by Ganti et al (at the right)*

To study performance of spoked pinion, two parameters were chosen: width of the spoke, W, and thickness of the spoke, T. These parameters are also shown on the figure 3.29. Number of spokes is equal to number of teeth, so that there is one spoke behind each tooth. Misalignment for this study is kept constant and equal to 0,04-degree about X-axis. Results for the parametric study of spoked pinion are shown on the table 3.8 and figures 3.30 and 3.31

Table 3.8 *parametric study of spoked pinion*

Study #	W	T	H	Maximum stress, MPa
1	5cm	10cm	9cm	429,9
2	5,5 cm	10cm	9cm	421,2
3	6cm	10cm	9cm	435,5
4	6,5cm	10cm	9cm	445,9
5	7cm	10cm	9cm	450,6
6	7,5cm	10cm	9cm	495,8
7	8cm	10cm	9cm	519,7
8	5cm	8cm	9cm	424,2
9	5,5cm	8cm	9cm	518,5

10	6cm	8cm	9cm	443,3
11	6,5cm	8cm	9cm	421,3
12	7cm	8cm	9cm	424,9
13	7,5cm	8cm	9cm	465,2
14	8cm	8cm	9cm	445,8
15	5cm	6cm	9cm	398,9
16	5,5cm	6cm	9cm	412
17	6cm	6cm	9cm	439
18	6,5cm	6cm	9cm	449
19	7cm	6cm	9cm	434,1
20	7,5cm	6cm	9cm	515,1
21	8cm	6cm	9cm	419,8
22	5cm	4cm	9cm	396,1
23	5,5cm	4cm	9cm	452,6
24	6cm	4cm	9cm	508,2
25	6,5cm	4cm	9cm	416,3
26	7cm	4cm	9cm	470,9
27	7,5cm	4cm	9cm	422,1
28	8cm	4cm	9cm	427,1

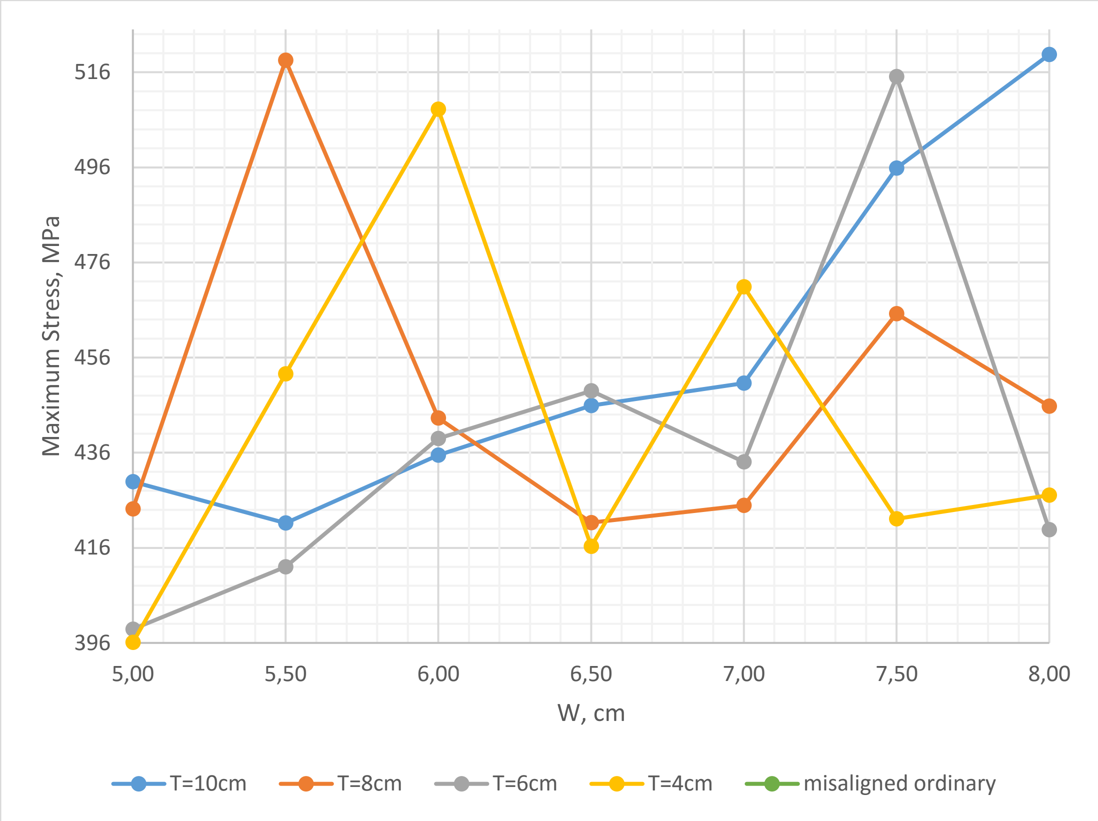


Figure 3.30 *Maximum stress versus W for different values of T*

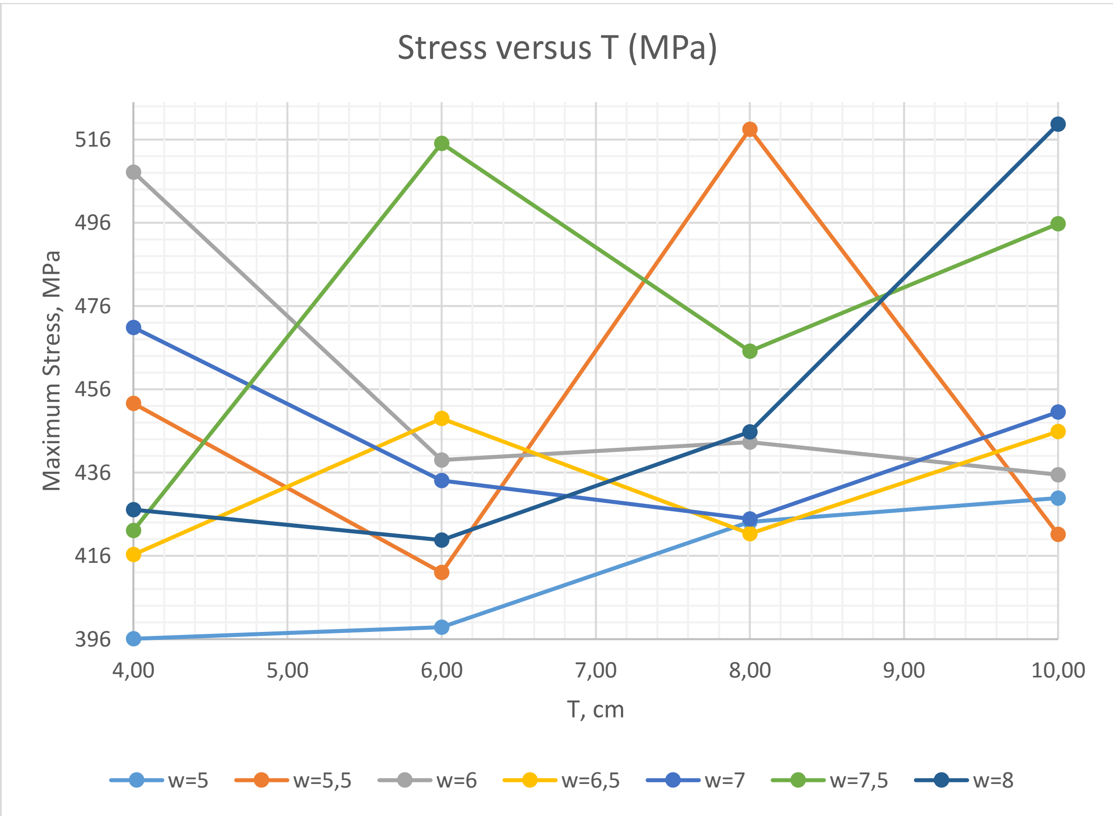


Figure 3.31 *Maximum stress versus T for different values of W*

One of the observations that can be made from these results is that even highest peak stress is lower than in ordinary gear having the same misalignment (519,7 MPa versus 560,32 MPa). Also, for T=10 cm there is quite clear trend of increase of maximum stress with W. However, due to high level of numerical noise, there is need in more careful study and more accurate results to decide if this design is successful.

3.2.7 Milestone 3: fragmented pinion

One more design that was checked in the scope of this Capstone Project is the fragmented pinion. It is basically modification of spoked pinion, performed in such way that teeth are not connected with each other. This is achieved by removal of small amount of material from the outer ring of spoked geometry, creating the gaps between the teeth. Figure 3.32 shows the fragmented pinion with exaggerated distance between the teeth:

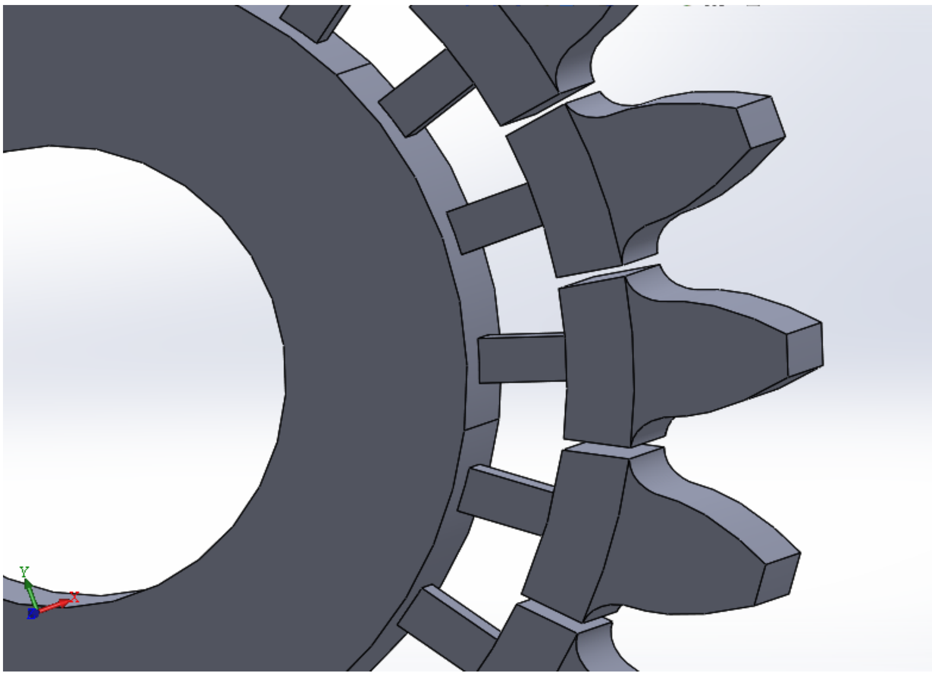


Figure 3.32 *Fragmented pinion with exaggerated space between the teeth*

The result for this geometry is shown on the figure 3.33:

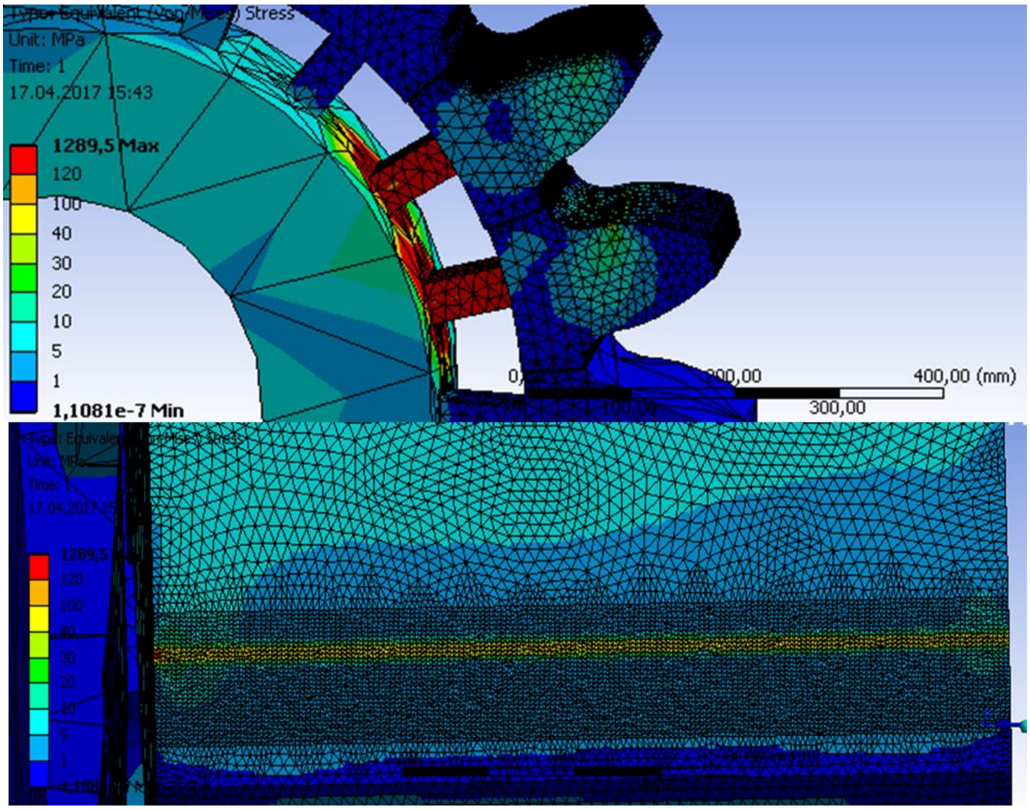


Figure 3.33 *Stress distribution on the split pinion*

This design shows low uniformly distributed stress on the meshing surfaces, but still cannot be considered as successful due to extremely high stress on the spokes (over 1200 MPa). There are several measures that can be taken to improve this situation: to increase width and thickness of the spokes (which probably will not be effective) and to add some deformable material between the teeth (which may help). Before these measures are taken and proven to improve performance of the gear, this design should be considered as inappropriate.

4. Implementation and Integration

Despite that it looks like dynamic and static parts of this study are not connected with each other, they serve the same purpose: to eliminate imperfections in the behavior of the gears, such as noise, vibrations, heat generation and enhancement of fatigue failure caused by impossibility of creating perfect designs satisfying zero tolerance. Difference in these parts of the project is dictated by specific context conditions in different areas of gear application. For example, when they carry moderate torques and rotate at high speeds, like in Aerospace applications, lack of load insensitivity causes vibrations of the system.

At the same time, when large gears carrying extremely high loads and rotating at low speeds, like in Naval applications, are involved, misalignment is the major issue, inducing high stresses on teeth. Therefore, it is still useful to solve two problems separately. However, there are still some applications, like Automotive, where load and misalignment insensitivity are desired to equal extent, thus, combined solution can be applied there. Therefore, one more challenge for future work is to create torque splitting topology, which will be compatible with gears having modified web and teeth with tip relief, so that it will be insensitive to both load and misalignment.

5. Conclusion

The aim of this project was to create load-and-misalignment-insensitive gearing for Aerospace, Automotive and Naval applications. This topic is important because both dynamic loads and deviation in orientation bring about noise, vibrations, enhanced fatigue failure and energy losses. Since these issues are commonly encountered in wide range of applications, many articles have thoroughly investigated this topic. However, there has not been a study that implemented tip relief, real-time self-preloading and torque-splitting to address dynamic load and web modification to handle misalignment issues. The work was divided into two parts, one of which was concerned with load deviations, while another one was related to misalignment. In both cases finite element analysis was used to conduct parametric studies. For the study of dynamic loads, contact ratio, load and tip relief were chosen as the main parameters for obtaining the most optimal configuration leading to constant mesh stiffness curve. Moreover, the effect of preloading and torque-splitting topology was analyzed by solving dynamic equations in Mathematica. The above studies led to several important findings. Firstly, during the studies of mesh stiffness we found that loaded transverse contact ratio is more important than theoretical one for obtaining mesh curve with minimal fluctuations. Namely when loaded TCR is slightly less than integer it is possible to achieve desirable constant mesh stiffness, while being exactly integer causes spikes in the curve. Secondly, applying tip relief to spur gears changes the relationship between theoretical and loaded TCR: in case of gears with tip relief, loaded TCR is less than theoretical one, while for ordinary gears loaded TCR is always higher than theoretical. Thirdly, when helical gears have high (over than 2) integer overlap contact ratio (OCR) the effect of TCR is almost negligible. Finally, real-time self-preloading was proved to enable achieving constant mesh stiffness for wide range of loads. Also, split-torque topology helps to reduce overall level of vibrations.

For the study of misalignment, four different web configurations were analyzed: ordinary, multicomponent, spoked and fragmented pinion configurations. Parametric study of ordinary gears led to two conclusions: firstly, misalignment of horizontal axis perpendicular to perfectly aligned shaft is the most important, and, secondly, relationships between its magnitude and peak stress are close to linear. Multicomponent web was successful in terms of uniform stress distribution, but its complexity might cause additional difficulties with manufacturing. Moreover, to avoid risk of buckling in connectors, pinions with multicomponent web should rotate only in one direction. Despite all this, this design successfully solves the problem of misalignment. Spoked pinion topology showed complicated behavior, but still was effective in terms of stress redistribution. Also, even the highest stress achieved there has lower value than in ordinary pinions having the same misalignment. However, more studies are required to judge about this design with confidence. Fragmented pinion helped to achieve better stress distribution, but there is one important limitation: incredibly high stress at the spokes, which make this configuration inapplicable. It is still possible, however, that some particular measures, such as inserting soft material in the gaps of the fragmented pinion, will solve this issue and make a valid solution from this design. Generally, solution from the second milestone is suggested as most appropriate for solving misalignment issue.

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Appendix

Table 2. Mesh stiffness variation graphs for Integer Overlap Contact Ratio

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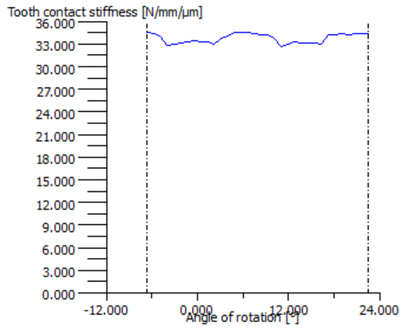
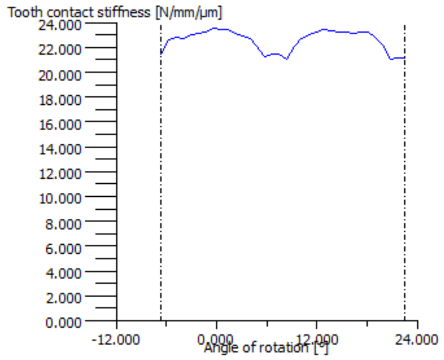
<tr><td>50%</td><td>
</td><td>
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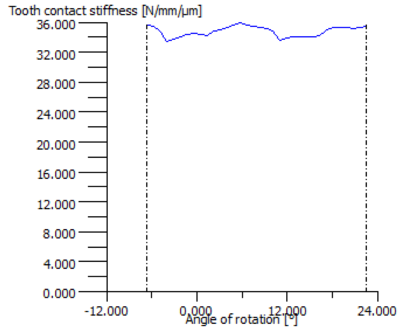
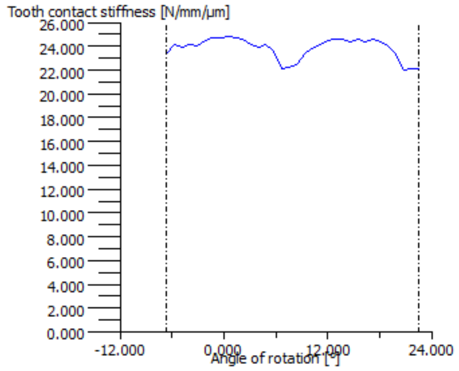
<tr><td>75%</td><td>
</td><td>
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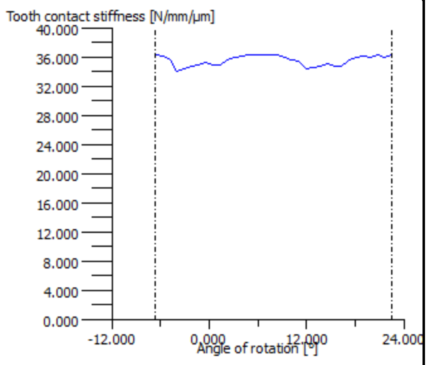
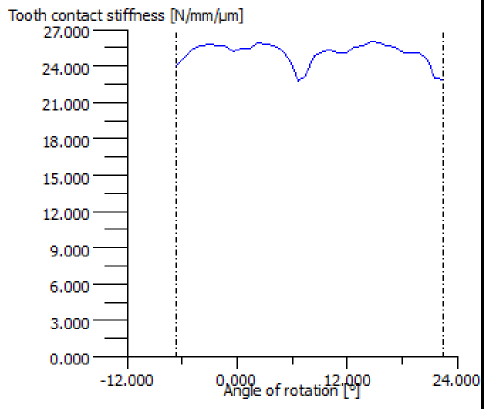
<tr><td>100%</td><td>
</td><td>
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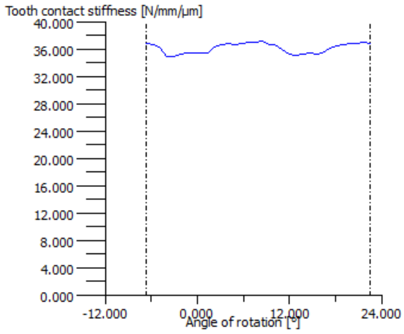
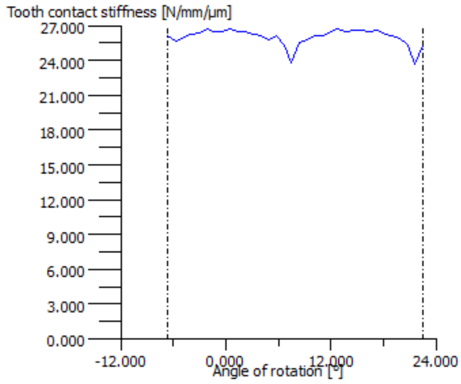
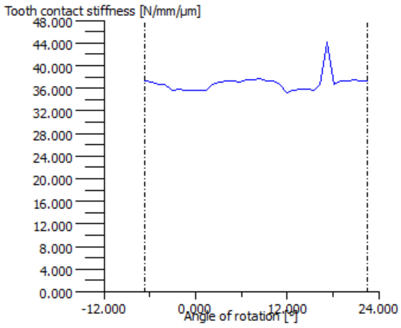
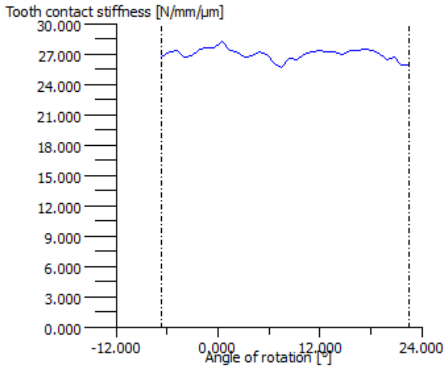
125%		
150%		

Table 3. Mesh stiffness variation graphs for Integer Transverse Contact Ratio

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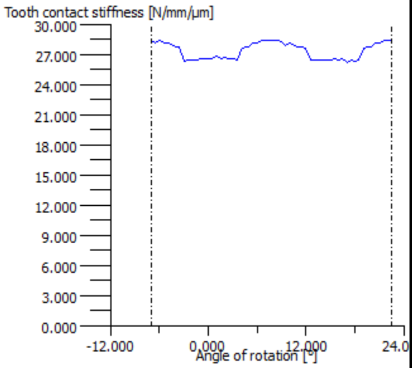
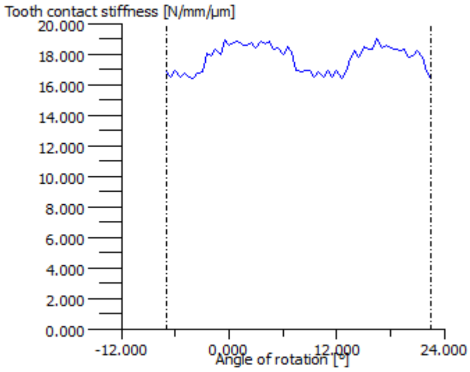
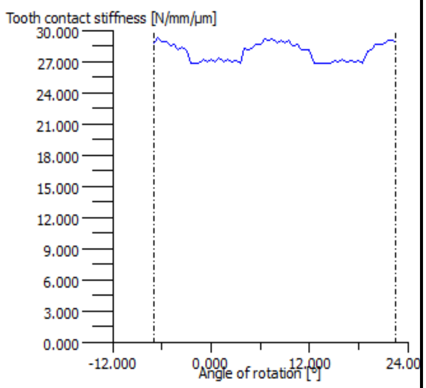
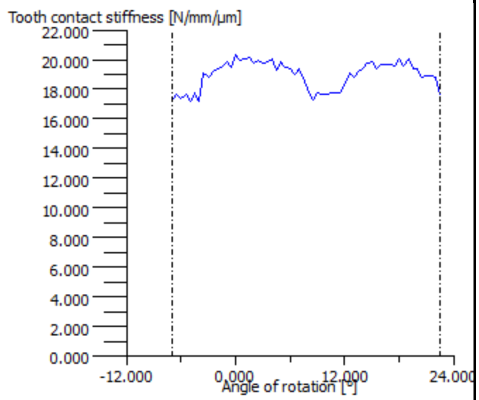
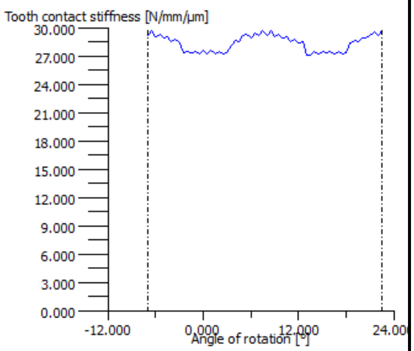
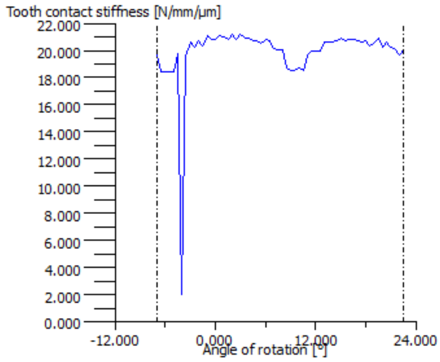
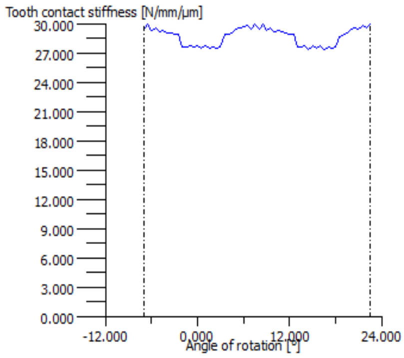
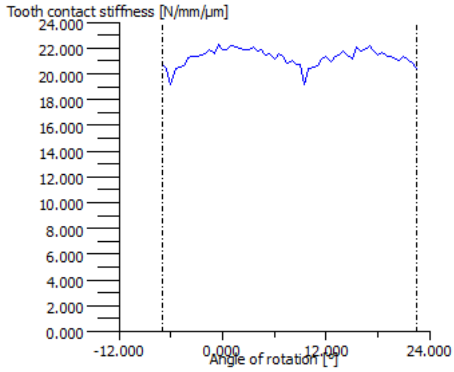
50%		

75%	 Tooth contact stiffness [N/mm/µm] Angle of rotation [°]	 Tooth contact stiffness [N/mm/µm] Angle of rotation [°]
100%	 Tooth contact stiffness [N/mm/µm] Angle of rotation [°]	 Tooth contact stiffness [N/mm/µm] Angle of rotation [°]
125%	 Tooth contact stiffness [N/mm/µm] Angle of rotation [°]	 Tooth contact stiffness [N/mm/µm] Angle of rotation [°]

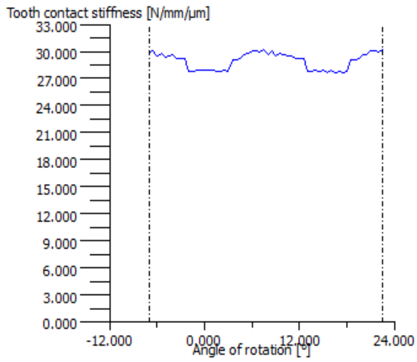
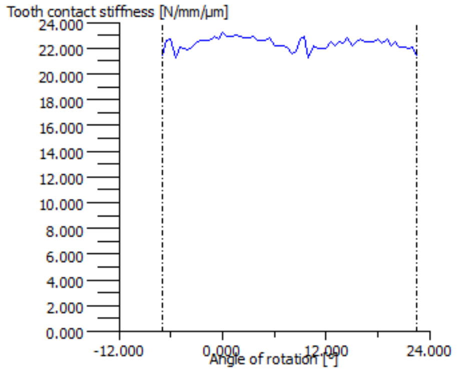
150%		

Table 4. Mesh stiffness variation graphs for Integer Total Contact Ratio

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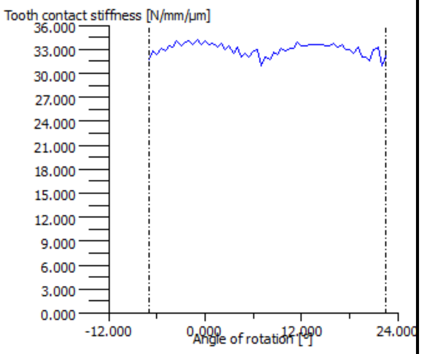
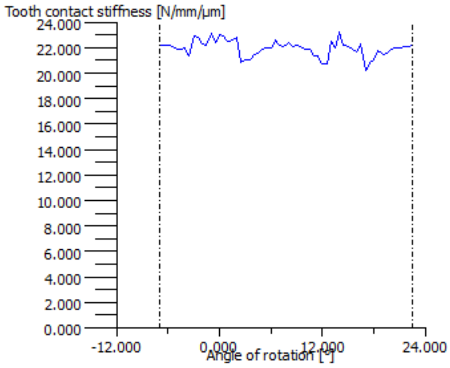
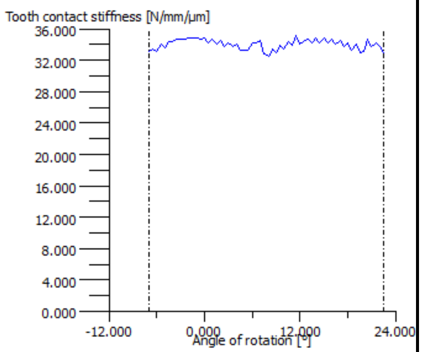
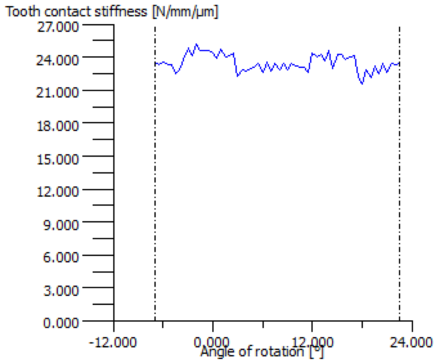
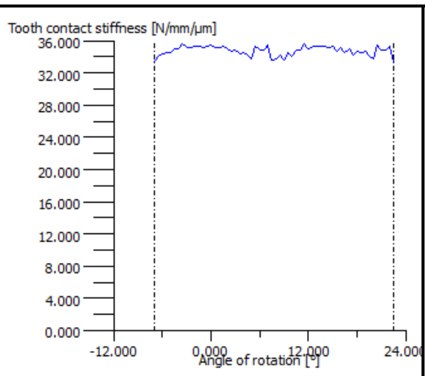
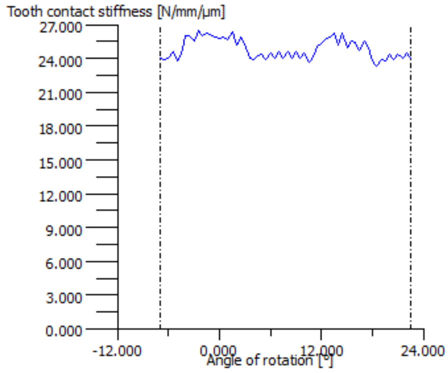
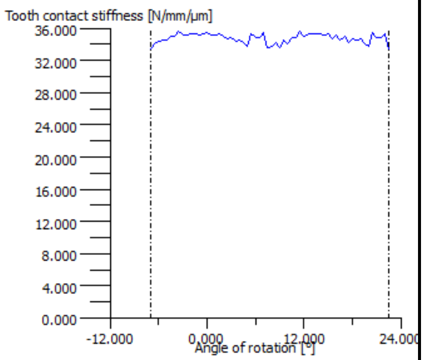
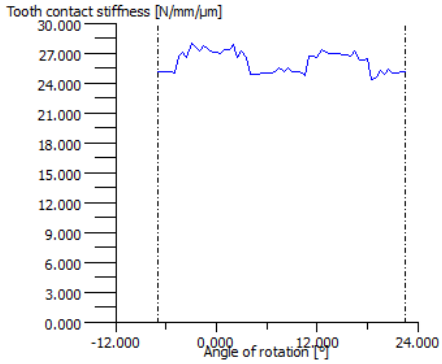
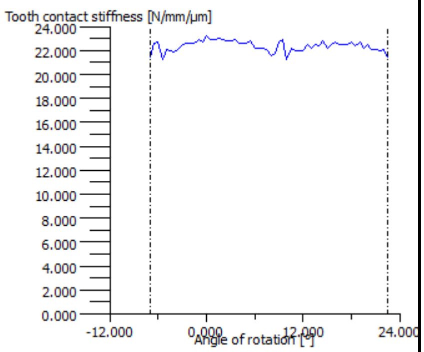
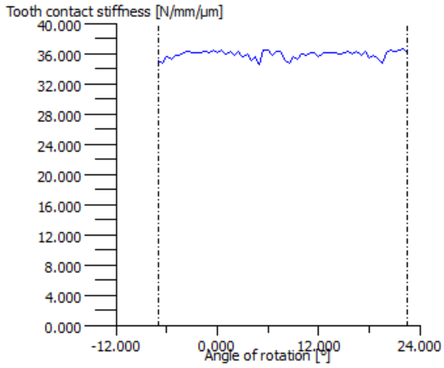
50%		
75%		

100%	Tooth contact stiffness [N/mm/µm] Angle of rotation [°]	Tooth contact stiffness [N/mm/µm] Angle of rotation [°]
125%	Tooth contact stiffness [N/mm/µm] Angle of rotation [°]	Tooth contact stiffness [N/mm/µm] Angle of rotation [°]
150%	Tooth contact stiffness [N/mm/µm] Angle of rotation [°]	Tooth contact stiffness [N/mm/µm] Angle of rotation [°]