Mitigating Snell's-Law Reflection and Transmission with Metasurfaces of Ordinary Dielectrics

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Abstract — **Obtaining anomalous reflection and transmission (namely propagation along directions different from those predicted by Snell's law) is investigated for plane waves impinging on a planar periodic metasurface composed of two dielectric layers. The associated boundary-value problem is treated by a rigorous entire-domain integral equation methodology. Optimizations with respect to the configuration's parameters are performed which reveal that it is possible to obtain significantly enhanced anomalous transmission and reflection depending on the color of the incident light. The optimal parameter values correspond to metasurfaces which can be easily realizable by low-loss dielectric materials.**

1 INTRODUCTION

Directing the reflection and transmission of an incident plane wave by a gradient metasurface towards propagation angles not predicted by Snell's law is a challenging problem, which can prove to be very useful in several applications, including antireflection coatings [1], polarization beam splitters [2], microwave surface plasmon coupling [3], and acoustic wavefront engineering [4]. Hence, it becomes important to design the parameters of the metasurface such that the major part of the reflected or transmitted power is guided to certain prescribed unusual directions; such phenomena are usually referred to as *anomalous reflection and transmission*.

In this work, we consider a simple planar all-dielectric metasurface, containing periodically alternating rectangular rods of two different dielectrics. The plane wave scattering problem is solved semi-analytically by implementing a fast and highly efficient entire-domain Galerkin integral equation methodology. The results of this methodology are compatible with energy conservation and have already been checked against the ones obtained with different numerical techniques. Our main purpose concerns the determination of suitable geometrical and electromagnetic parameters of the metasurface in order for it to exhibit significant anomalous reflection and transmission.

By following a certain systematic optimization scheme, we show that it is indeed possible to guide a significantly large portion of the input power of light in the visible frequency range towards the region of plane where the incident field propagates. More precisely, we consider metasurfaces admitting only two diffracted propagating orders (namely along two directions), and determine the metasurfaces' width and period such that the powers of the -1-reflected or transmitted order is significantly enhanced, while the corresponding powers of the 0-reflected and transmitted order (ordinary Snell's law directions) are annihilated. These properties are achieved for metasurfaces composed of conventional low-loss dielectric materials with specific characteristics. For each optimized configuration, we also examine the variations of the reflected and transmitted powers of the -1 order with respect to the angle of incidence as well as the operating wavelength, and derive conclusions on the associated anomalous reflection and transmission effects.

It is expected that such optimized all-dielectric structures can be used in components and devices that use the mechanism of anomalous reflection or transmission to enhance their efficiency. The developed fast and accurate integral equation methodology supplemented by the established selection rules for the problem's parameters can provide an efficient optimization platform for designing metasurfaces which generate the above described effects.

2 INTEGRAL EQUATION ANALYSIS OF THE SCATTERING PROBLEM

Fig. 1 depicts the considered two-dimensional gradient dielectric metasurface composed of a Λ-periodic slab with two dielectric materials alternating inside each unit cell. The refractive indices of the two materials are n_1 and n_2, respectively, while their common thickness is w and their common width is $\Lambda/2$. Thus, the metasurface has the form of a binary grating (i.e. characterized by 50% duty cycle); binary gratings are easier to fabricate than other types of gratings like multilevel or graded-index ones [5]. The metasurface is placed in vacuum with refractive index $n_0 = 1$.

A transverse-electric (TE)-polarized plane wave with angle of incidence θ (see Fig. 1) impinges on

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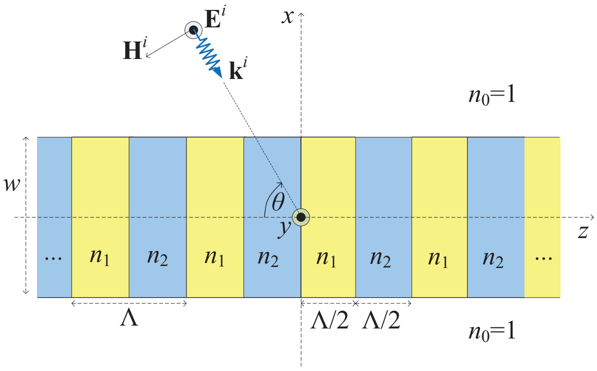


Figure 1: The considered all-dielectric Λ-periodic metasurface composed of two alternating dielectric materials with refractive indices n_1 and n_2.

the metasurface. The total electric field induced in every region of the scattering problem is of the form $\mathbf{E}(x,z) = \Psi(x,z)\hat{\mathbf{y}}$, and the unknown electric field factor Ψ admits the integral representation [6]-[8]

$$\Psi(x,z) = \Psi_0(x,z) + k_0^2(n_2^2 - n_1^2) \iint_S G(x,z;x',z')\Psi(x',z')\mathrm{d}x'\mathrm{d}z', \quad (1)$$

where Ψ_0 and G are the electric field factor and the Green's function both induced on the respective homogeneous slab structure due to the plane incident wave and a line-source excitation inside the dielectric slab, respectively. Furthermore, S denotes the total transverse cross-section of the inclusions with refractive index n_2 and k_0 the vacuum wavenumber.

The electric field factor is expressed as

$$\Psi(x,z) = \exp(-ik_0 \cos\theta\, z)u(x,z), \quad (2)$$

where $u(x,z)$ is a Λ-periodic function of z. We reformulate the integral representation (1) with respect to function u and apply a highly efficient entire-domain Galerkin technique. More precisely, function $u(x,z)$ is expanded on the cross section of the inclusions on the unit cell in the Fourier series

$$u(x,z) = \sum_{n=-\infty}^{+\infty} \left(c_n^+ e^{g_n x} + c_n^- e^{-g_n x}\right) e^{-i\frac{2\pi n}{\Lambda} z}, \quad (3)$$

where $g_n = \sqrt{(k_0\cos\theta + \frac{2\pi n}{\Lambda})^2 - k_0^2 n_2^2}$, while $c_n^\pm$ are determinable coefficients. Next, by restricting the observation vector (x,z) on the domain of the inclusions and considering the inner products of both sides of the resulting equation with the conjugates of the expansion functions in (3), we formulate an infinite non-homogeneous linear system of equations with respect to $c_n^\pm$, $n \in \mathbb{Z}$. This infinite system is solved numerically by truncation, as described in [9], yielding the coefficients $c_n^\pm$ for $n = -N_t, \ldots, N_t$, where N_t is the truncation order.

Then, the reflected and transmitted electric field factors are, respectively, expressed by

$$\Psi^r(x,z) = \sum_{p=-\infty}^{+\infty} r_p \exp\left[i\left(k_{x,p}\, x - k_{z,p}\, z\right)\right], \quad (4)$$

$$\Psi^t(x,z) = \sum_{p=-\infty}^{+\infty} t_p \exp\left[-i\left(k_{x,p}\, x + k_{z,p}\, z\right)\right], \quad (5)$$

where r_p and t_p denote, respectively the complex reflection and transmission coefficients (explicitly determined by means of $c_n^\pm$), and $k_{x,p}$ and $k_{z,p}$ the components of the reflected and transmitted wavevectors, which are given by

$$k_{x,p} = i\sqrt{\left(k_0\cos\theta + \frac{2\pi p}{\Lambda}\right)^2 - k_0^2}, \quad (6)$$

$$k_{z,p} = k_0\cos\theta + \frac{2\pi p}{\Lambda}. \quad (7)$$

Each term in (4) and (5), indexed by $p \in \mathbb{Z}$, represents a specific field component referred to as the p-reflected or p-transmitted order. The 0-order fields are always propagating (Snell's law). For orders $p \neq 0$, we define the thresholds

$$p^\pm = \pm\frac{\Lambda}{\lambda_0}(1 \mp \cos\theta). \quad (8)$$

For $p^- < p < p^+$, the component $k_{x,p}$ is real and the p-order reflected and transmitted fields are propagating along the x-axis. For $p > p^+$ or $p < p^-$, the component $k_{x,p}$ becomes purely imaginary and the p-order fields are evanescent.

3 NUMERICAL RESULTS AND DISCUSSION

We seek to determine suitable parameters of the metasurface so that it generates *anomalous* reflection and transmission in which cases most of the scattering response is confined to directions in the same region of space with the incident field; thus light propagates towards the side of incidence.

To this end, we consider that *only* the 0- and the -1-reflection and transmission orders propagate, which lead by (8) to the following conditions

$$\max\left\{1-\cos\theta, \frac{1+\cos\theta}{2}\right\} < \frac{\lambda_0}{\Lambda} < 1+\cos\theta. \quad (9)$$

The latter implies also that the -1-order is reflected back in the same (second) quadrant of the plane with that where the incident field impinges on the metasurface, as well as to be transmitted back in the corresponding (third) quadrant [9].

The angle of incidence is fixed at $\theta = 60^o$, since, as analyzed in [9], then the reflection angle (measured as the angle between the negative z-axis and the wavevector of the -1-reflected order) is close to the angle incidence θ. The propagation direction of the -1-transmission order is symmetric with respect to the z-axis to that of the -1-reflected order.

Furthermore, we consider two different operating wavelengths λ_0 each one corresponding to a different color of the visible spectrum. The range of the period Λ is that dictated by (9). The two dielectric regions of the slab are considered to be occupied by Teflon AF fluoropolymers [10] with $n_1 \cong 1.35$ and lossless Silicon with n_2 obeying the frequency-varying models of [11]. The thickness w of the slab is required to be suitably small permitting the characterization as a metasurface; the optimal w will turn out to be of the order of a few hundred nanometers (smaller than the effective wavelength).

The main aim is to select suitable metasurface's parameters such that the powers carried from the -1-reflected and transmitted orders are enhanced while simultaneously the powers carried by the respective 0-orders are suppressed. To this end, we devise the following metric in order to quantify the degree of anomalous reflection and transmission

$$m = \frac{P_{-1}^r + P_{-1}^t}{P_0^r + P_0^t}, \qquad (10)$$

where

$$P_p^r = \frac{|r_p|^2}{\sin\theta}\sqrt{1 - \left(\cos\theta + p\frac{\lambda_0}{\Lambda}\right)^2}, \qquad (11)$$

$$P_p^t = \frac{|t_p|^2}{\sin\theta}\sqrt{1 - \left(\cos\theta + p\frac{\lambda_0}{\Lambda}\right)^2} \qquad (12)$$

are the *efficiencies* of the diffracted orders, which are defined as the ratios of the flux of the Poynting vector associated with the p-diffracted order over the corresponding flux of the incident field [12]. Large values of m indicate significant steering of the reflected/transmitted waves in an unusual direction, totally different from that dictated by Snell's law. Hence, m denotes how significant are the powers carried by the waves of the -1-diffracted order as compared to the ones of 0-diffracted order.

Optimization is now performed with respect to the thickness w and the period Λ of the metasurface in order to maximize the metric m. The optimization is based on successive computations of the reflection and transmission coefficients by means of the above described semi-analytical integral equation methodology. The beneficial characteristics of the methodology, i.e. its superior numerical stability, controllable accuracy, and rapid convergence aid substantially the fast and efficient implementation of the optimizations required. The contour plots of the metric m in the plane of w and Λ for the violet color ($\lambda_0 = 415$ nm) and the blue color ($\lambda_0 = 470$ nm) are depicted in Fig. 2.

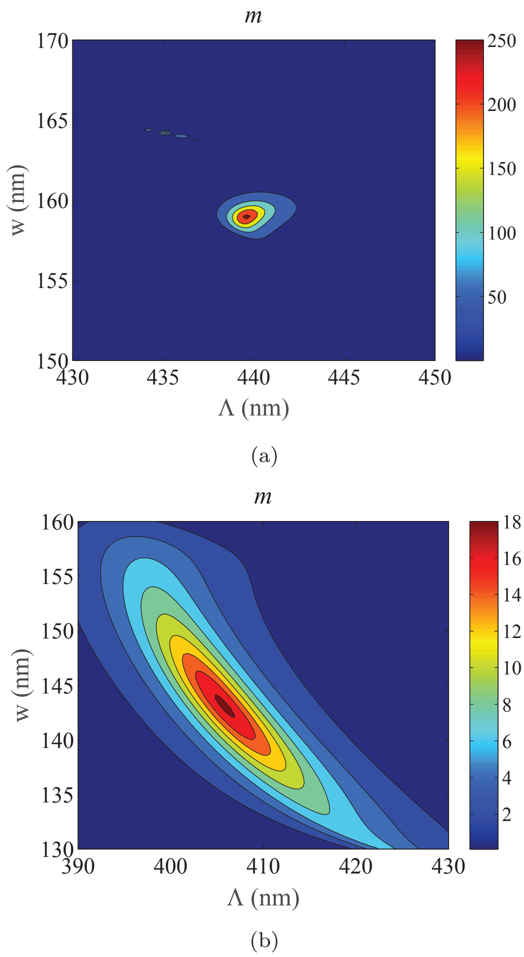
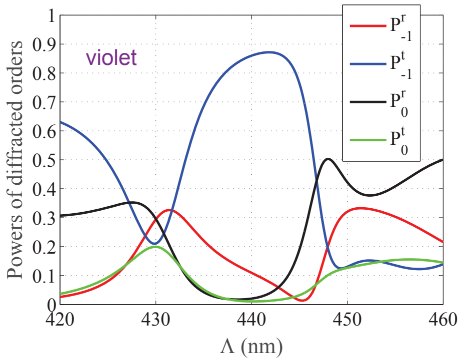
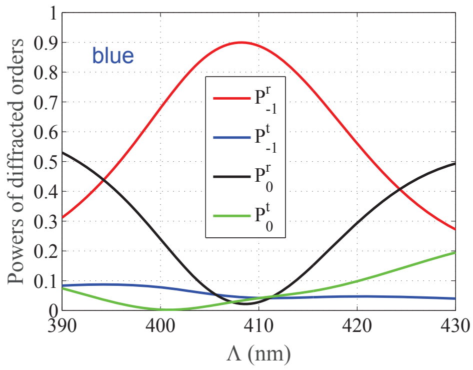


Figure 2: Contour plots of metric m vs w and Λ for (a) $\lambda_0 = 415$ nm (violet) and (b) $\lambda_0 = 470$ nm (blue).

In the sequel, we proceed to a secondary optimization where we select a narrow interval of Λ and then search for the specific value of w in the region of large m (as predicted by Fig. 2) so that P_{-1}^r and P_{-1}^t are significantly larger than P_0^r and P_0^t. In Fig. 3, we depict the variations of the powers P_{-1}^r, P_{-1}^t, P_0^r, and P_0^t versus the period Λ for the two examined colors in the visible range. For each color, the optimal value of the thickness w of the metasurface was determined according to the secondary optimization scheme described above and



(a)



(b)

Figure 3: Powers of −1- and 0- orders vs Λ for (a) $\lambda_0 = 415$ nm (violet) and (b) $\lambda_0 = 470$ nm (blue).

found to be $w = 160$ nm for violet and $w = 141$ nm for blue. For violet, maximum transmission in the −1-order is obtained with attained value P^t_{-1}=87%. Such a result corresponds to substantial negative refraction into free-space with no use of active or exotic/artificial media. The thorough study of this effect would be the topic of a future work. On the other hand, for blue, maximum reflection in the −1-order is obtained with value P^r_{-1}=90%. These significantly large values observed for the two examined colors make the considered designs very interesting in terms of anomalous transmission and reflection. Moreover, the value of the angle in the −1-transmitted order is 64° for violet and the angle in the −1-reflected order is 50° for blue, namely close to the incidence angle $\theta = 60°$.

Additional numerical results on achieving significantly enhanced reflection and transmission from realizable all-dielectric metasurfaces have been obtained and will be presented at the conference.

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