Geostatistical Simulation of Cross-Correlated Variables: a Case Study through Cerro Matoso Nickel-Laterite Deposit

Nasser Madani[1], Julian Ortiz[2]

[1] Department of Mining Engineering, School of Mining and Geosciences, Nazarbayev University, Astana, Kazakhstan

[2] The Robert M. Buchan Department of Mining, Queen's University, Kingston, ON, Canada

Abstract— Geostatistical methods have been increasingly used as powerful techniques for predicting spatial attributes and modelling the uncertainty of predictions in un-sampled locations, especially through multi-element deposits. Independent Gaussian simulation constructs precise outputs over each variable, in most cases by simulating using the multi-Gaussian assumption. However, this approach does not consider the underlying correlations between the variables. Spatial uncertainty can also be quantified by co-simulation, where the relationship of the co-regionalized variables is accounted for and the spatial relationships between variables are reproduced. In this study, we apply the two aforementioned approaches (independent simulation and co-simulation) for modelling two correlated elements (Fe & MgO) at Cerro Matoso S.A. Nickel laterite deposit located in Colombia. Results show that co-simulation provides a reasonable outcome in regards to the correlation coefficient parameter and relative error as expected.

Keywords: Multivariate geostatistics, Co-simulation, Nickle-Laterite Deposits.

1. Introduction

Geostatistical methods have been increasingly used as powerful techniques for predicting spatial attributes and modeling the uncertainty of predictions in un-sampled locations, especially through multi-element deposits, which are important in mineral resource estimation and ore reserve evaluation [1, 2, 3, 4, 5, 6, 7, 8, 9 and 10]. Independent Gaussian simulation construct precise outputs over each variable [11, 12, 13], and most of them can be simulated separately by transformation to the Gaussian (or multi-Gaussian) distribution. But the problem of applying this approach in multi-element deposits is that it does not consider the intrinsic correlation coefficients between co-regionalized variables. In a nutshell, ore body evaluation in multi-element deposits requires considering the characterization of cross-correlated variables observed at available datasets. Quantifying the uncertainty at un-sampled locations encourage geostatistical modeling of these co-variables. This modeling can be divided into two parts. The first one uses co-kriging methods [14, 15] and the second one assessing the local uncertainty by applying co-simulation. The later generates some realizations, in which they can reproduce the desired spatial continuity and the desired correlation. The objective of this work is to assess the performance and check the accuracy of independent simulation and co-simulation for modeling two co-regionalized attributes (Fe & MgO) meanwhile they are cross-correlated significantly, in a nickel-laterite actual case study located in Colombia.

2. Material and Methods

2.1. Methodology

Several Gaussian simulation algorithms have been developed. Generally speaking, they are divided into two types; exact and approximate algorithms [16]. The applied simulation and co-simulation algorithm in this paper is turning band proposed by Emery (2008) following Matheron (1973) and Mantoglou (1987). This method was first introduced by Chentsov (1957) in a special case of Brownian random functions, but has been extended for the Gaussian simulation of stationary and intrinsic random functions by Emery and Lantuejoul (2006) for independent simulation and by Emery (2008) for co-simulation. These methods aim at simplifying the Gaussian simulation problem in multidimensional spaces, using simulations in one dimension and spreading them to 2-D or 3-D spaces. This method is extremely fast with parallelizable computations and one can simulate as many locations as desired. The Gaussian simulation also exactly reproduces the desired covariance model [21, 22, 23, 24 and 25]. In co-simulation, the relationship is being characterized by examining the cross-variogram together with direct-variogram. There exist a range of various methodologies for modeling such a variogram [26, 27, 28, 29, 30, 31 and 32]. In this research, the linear model of coregionalization has been proposed of the following form [14]:

$$C(h) = \sum_{n=1}^{N} B_n \rho_n(h) \quad (1)$$

where $\{\rho_n, n = 1 \dots N\}$ is a set of positive semi-definite covariance functions and $\{B_n, n = 1 \dots N\}$ is a set of symmetric positive semi-definite matrices.

2.2. Presentation of dataset

The Cerro Matoso S.A. Nickel laterite deposit is an important resource of Ni in the world located in northwest Colombia [32 and 33]. Cauca ophiolite complex belonging to Cretaceous age shows some peridotite outcrops in the region which is the main house of the Cerro Matoso deposit [34]. The principal tectonic feature of this deposit is Romeral fault system with approximately 500 km that hosted this ophiolitic complex in the time of Pre-Andean orogeny. For instance, the regional boundary among accreted pre-Tertiary ophiolitic sequences and polymetamorphic core of the Central Cordillera can be detected through this structural discontinuity. Furthermore, the Geophysical surveys confirm that the Romeral fault system separate continental crust to the east from oceanic crust to the west. This deposit manifests itself through a hill range about 2.5 km length and 1.5 width and is evolved during a variably serpentinized ultramafic body [35]. The mineralogical and chemical alteration system is Lateritization and Saprolitization according to the geographical extent of the deposit. The dataset is composed of 3000 records of blasting holes belongs to this deposit. In which seven variables are assayed for each sample. In this research, two co-regionalized variables (Fe and MgO) out of these seven variables have been selected due to the presence of the satisfactory correlation coefficient. The sampling zone covers approximately 180×200×70 meters. Table 1 summarizes statistical parameters of these variables through collecting the corresponding samples in entire deposit. The frequency of each variable has been also depicted in Figure 1 to assess the shape and spread of the sample data. This bar chart is necessary before or during any analysis. As can be seen from the figure, Fe shows a distribution with two peaks, pretending some bimodality. The MgO seems to bear lognormal distribution with one peak which demonstrates that the low values are further fluctuating in the region.

Correlation coefficient as another important yardstick in multivariate analysis turns out the linear relationship (proportionality relationship) enclosed by Fe and MgO through a value between -1 and 1. This coefficient for Fe and MgO is -84.04%, which means that there is a considerable relationship between these two variables. Scatter plot (Figure 2) is an intuitively determination of the dependence relationship between two variables that can also be used to detect the possible anomalous data.

Table 1: Statistical parameters of two variables

Statistical Parameter	Fe	MgO
Mean	27.95%	8.95%
Median	26.1	3.6
Std. Deviation	15.74	10.23
Variance	247.839	104.699
Minimum	5.2	0.1
Maximum	57.2	39.43

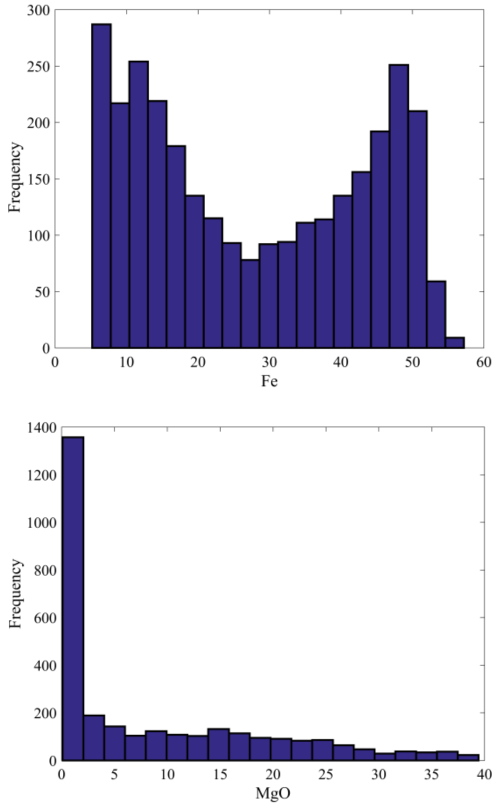

Figure 1. Histogram of Fe and MgO distributions

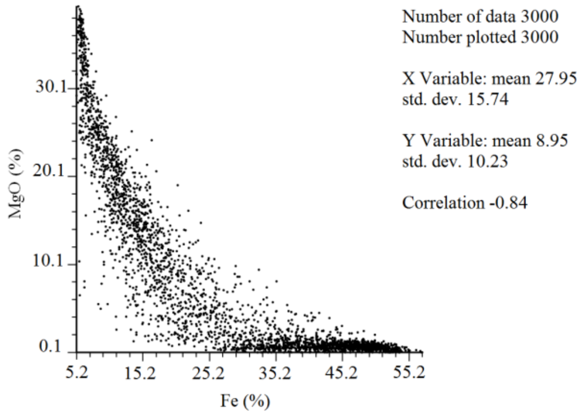

Figure 2. Scatter plot between Fe and MgO

2.3. Geostatistical Modeling

The initial analysis parts of the modeling are, first, implementing independent simulation for each variable and, second, co-simulating both variables simultaneously. For this purpose, the primary data should be declustered and transformed to the normal score Gaussian distribution [1]. The technique allows us assign each datum a weight based on closeness to surrounding data for alleviating the high pseudo frequency occurring in high graded areas [36]. These declustered data are then should be transformed to the standard normal distribution. So the analysis data (Fe & Mgo) have been transformed to normal score N(0,1) by nscore subroutine of Gslib [11]. Each variogram-based geostatistical modeling including these two simulation and co-simulation approaches require learning the model of spatial continuity [1]. In simulation, direct-variogram plays an important role and in co-simulation; the cross-variogram is needed to be modeled as well. Hence, the initial step is to analyze the direct experimental variogram for deriving the potential isotropy or anisotropy of the variables of interest in the region. One technique is to calculate the direct experimental variogram in alternative directions with narrow tolerances. The differences in range values through the variograms give the idea of geometric anisotropy. So, for this purpose, the experimental variograms are calculated along the specific directions. The results showed that there is isotropy variability in the horizon and anisotropy in the vertical direction. As mentioned earlier, spatial continuity structure for employing the co-simulation can be represented by cross-variogram associated with the information obtained from the direct-variograms for co-regionalized variables. The cross-variogram was first introduced by Matheron (1965) as the natural generalization of the variogram [1]. This coregionalization matrix can be modeled by several methods. The most common and widespread approach is known as “Linear Coregionalization Model (LCM)” applicable to any multivariate spatial data analysis [37, 14]. In this model, the sample variograms should be fitted by semidefinite coregionalization matrices which indeed are mathematically consistent [14]. The difficulty of obtaining a semidefinite model has been covered somewhat by automatic or semi-automatic fitting method which are often used for modeling this type of spatial correlation structure [38]. This protocol makes the process of fitting somehow convenient. In this study, direct and cross-variogram model for Fe and MgO are calculated and depicted in Fig. 3. The theoretical model of LCM is also fitted to the experimental ones:

$$\text{Cross-Variogram: } \begin{pmatrix} \gamma_{Fe}(h) & \gamma_{Fe-Mgo}(h) \\ \gamma_{Fe-Mgo}(h) & \gamma_{Mgo}(h) \end{pmatrix} = \begin{pmatrix} 0.05 & 0.05 \\ 0.05 & 0.05 \end{pmatrix} Nugget + \begin{pmatrix} 0.45 & -0.25 \\ -0.025 & 0.25 \end{pmatrix} Sph(80m, 27m) + \begin{pmatrix} 0.65 & -0.65 \\ -0.65 & 0.75 \end{pmatrix} Sph(\infty, 27m)$$

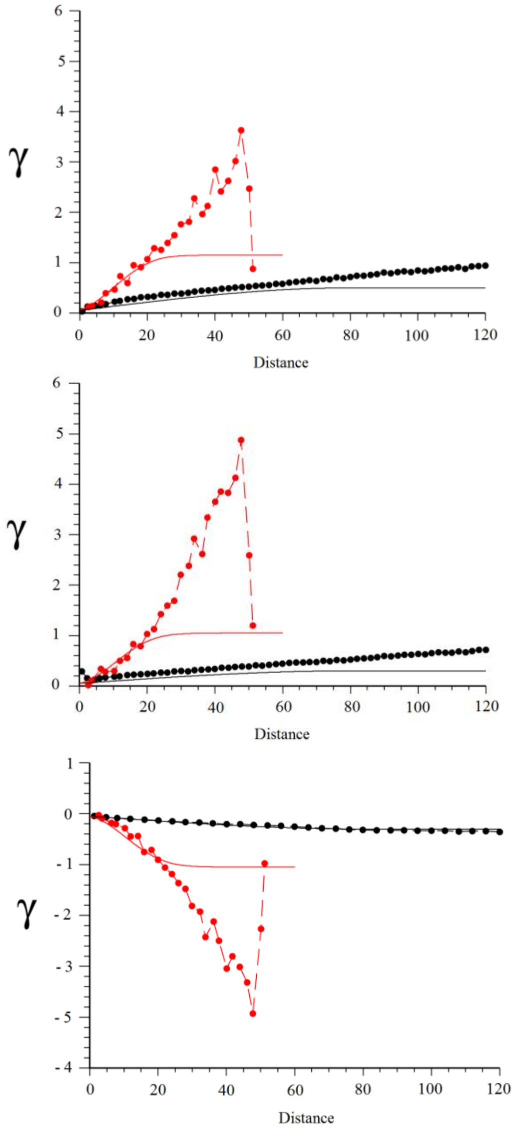

Figure 3. Direct and cross-variogram analysis (top: Fe; middle: MgO; bottom: cross-variogram

In this part, our objective is to model the Fe and MgO via two commented methodologies. The first one considers the independent simulation for each variable and the second one acknowledges the correlation coefficient between two dependent variables by co-simulation. So, for independent simulation, one just needs to apply the direct-variograms of Fe and MgO, while the co-simulation is dealing with the cross-variogram as well as the direct ones. The applied simulation and co-simulation methodologies as explained above; are turning band simulation and co-simulation in which they have priority to other approximate approaches of simulation [39]. In Figure 4 and 5, one realization of simulation and co-simulation for each, have been provided. Visually consideration, one is not able to find out bolded diversity among the simulation and co-simulation results. But in the upcoming sections, we will discuss about the statistical parameters of them.

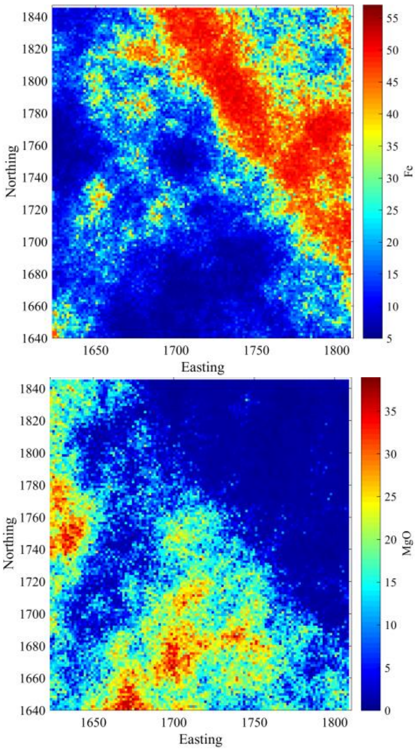

Figure 4. Simulation results (top: Realization No. 1 (Fe); bottom: Realization No. 2 (MgO))

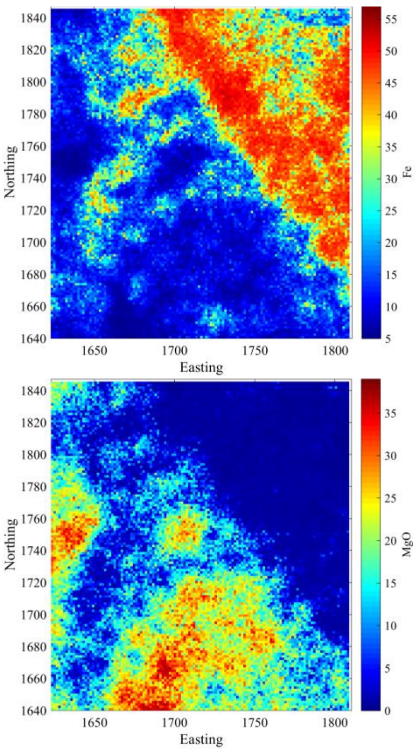

Figure 5. Co-simulation results (top: Realization No. 1 (Fe); bottom: Realization No. 2 (MgO))

3. Results and Discussion

In this part, for making a comparison between two approaches statistically, Table 2 is presented to show the correlation coefficient as an important key factor calculated over 100 realizations for each variable. As can be acquired from this table, the obtained correlation coefficient from co-simulation in average; is closer to the correlation coefficient of primary data.

Table 2: Correlation coefficients

Variable	Primary data	Simulation	Co-Simulation
Fe vs. MgO	-0.84	-0.53	-0.76

Based on the definition of relative error which is an absolute measure of difference between the true value and its approximation divided by the magnitude of that true value, one should consider these true and approximation values. For this purpose, the dataset of this case study are divided into two parts, one analysis dataset (30% of all the primary data) and one test data set (70% of all the primary data). In Geostatistical literatures, this methodology is known as jack-knife [11]. In a nutshell, the test dataset are quantified by these two methodologies. As the true value of these test dataset are known, the approximation values can be obtained from this quantification. As can be seen from the Table 3, the absolute relative error for simulation is higher than the co-simulation.

Table (3): Relative error between the methods for Validation dataset

Variable	Simulation	Co-Simulation
Fe	0.278	0.175
MgO	0.756	0.659

4. Conclusions

Multivariate Geostatistics offers a flexible framework for modeling the continuous variables when the variables convey a satisfactory correlation. The current methodology such as independent simulation in this case suffers from reproducing the expected correlation among the co-regionalized variables. Co-simulation as an alternative flexible technique, is dealing with quantifying linear coregionalization model which hold much acceptable results statistically. This approach overcomes the limitation of independent simulation and increases the model versatility. The priority of this method is attractive because of its capability to accept any number of nested structures. However, this method is concerned with quantifying linear coregionalization model, in which the process of modeling is somewhat time consuming and not very easy to use.

References

[1] Chilès, J.P., Delfiner, P. (2012): Geostatistics: Modeling Spatial Uncertainty. Wiley, New York

[2] Emery, X., (2005). Simple and ordinary kriging multiGaussian kriging for estimating recoverable reserves. Math. Geol. 37, 295–319.
[3] Emery, X., (2012). Co-simulating total and soluble copper grades in an oxide ore deposit. Math. Geosci. 44 (1), 27–46.
[4] Emery, X., González, K.E., (2007). Probabilistic modelling of mineralogical domains and its application to resources evaluation. J. S. Afr. Inst. Min. Metall. 107 (12), 803–809.
[5] Emery, X., Robles, L.N., (2009). Simulation of mineral grades with hard and soft conditioning data: application to a porphyry copper deposit. Comput. Geosci. 13 (1), 79–89.
[6] Emery, X., Bertini, J.P., Ortiz, J.M., (2005). Resource and reserve evaluation in the presence of imprecise data. CIM Bull. 98 (1089) (12 pp.).
[7] Emery, X., Ortiz, J.M., Rodríguez, J.J., (2006). Quantifying uncertainty in mineral resources by use of classification schemes and conditional simulations. Math. Geol. 38 (4), 445–464.
[8] Maleki Tehrani, M.A., Asghari, O., Emery, X., (2013). Simulation of mineral grades and clas- sification of mineral resources by using hard and soft conditioning data: application to Sungun porphyry copper deposit. Arab. J. Geosci. 6 (10), 3773–3781.
[9] Montoya, C., Emery, X., Rubio, E., Wiertz, J., (2012). Multivariate resources modelling for assessing uncertainty in mine design and mine planning. J. S. Afr. Inst. Min. Metall. 112, 353–363.
[10] Ortiz, J.M., Emery, X., (2006). Geostatistical estimation of mineral resources with soft geo- logical boundaries: a comparative study. J. S. Afr. Inst. Min. Metall. 106 (8), 577–584.
[11] Deutsch, C.V., Journel, A.G., (1998). GSLIB: Geostatistical Software Library and User's Guide. Oxford University Press, New York.
[12] Matheron, G., (1973). The intrinsic random functions and their applications. Adv. Appl.
[13] Shinozuka, M., Jan, C.M., (1972). Digital simulation of random processes and its applications. J. Sound Vib. 25, 111–128.
[14] Wackernagel H (2003) Multivariate geostatistics, 3rd rev edn. Springer, Berlin white noise. Theory Probab. Appl. 2, 265–266.
[15] Goovaerts, P. (1997): Geostatistics for Natural Resources Evaluation. Oxford University Press, New York
[16] Emery, X., Lantuejoul, C., (2006). TBSIM: a computer program for conditional simulation of three-dimensional Gaussian random fields via the turning bands method. Comput. Geosci. 32, 1615–1628.
[17] Emery, X., (2008). A turning bands program for conditional co-simulation of cross-correlated Gaussian random fields. Comput. Geosci. 34, 1850–1862.
[18] Mantoglou, A., (1987). Digital simulation of multivariate two- and three-dimensional stochastic processes with a spectral turning bands method. Mathematical Geology 19 (2), 129–149.
[19] Chentsov, N.N., (1957). Lévy Brownian Motion for Several Parameters and Generalized White Noise, Theory Probab. Appl., 2(2), 265–266
[20] Emery, X., Lantuejoul, Ch., (2006). TBSIM: A computer program for conditional simulation of three-dimensional Gaussian random fields via the turning bands method. Comput. Geosci. 32, 1615–1628.
[21] Chilès, J.P., (1977). Géostatistique des phénoménes non stationnaires (dans le plan), doctoral thesis. Université de Nancy I, Nancy (152 pp.).
[22] David, M., (1977). Geostatistical Ore Reserve Estimation. Elsevier, Amsterdam.
[23] Delhomme, J.P., (1979). Spatial variability and uncertainty in groundwater flow parameters: a geostatistical approach. Water Resour. Res. 15, 269–280.
[24] Journel, A.B., Huijbregts, C.J., (1978). Mining Geostatistics. Academic, New York.
[25] Mantoglou, A., Wilson, J.L., (1982). The turning bands method for simulation of random fields using line generation by a spectral method. Water Resour. Res. 18, 1379–1394.
[26] Almeida, A.S., Journel, A.G., (1994). Joint simulation of multiple variables with a Markov-type coregionalization model. Mathematical Geology 26 (5), 565–588.
[27] Grzebyk, M., Wackernagel, H., (1994). Multivariate analysis and spatial/temporal scales: real and complex models. In: Pro- ceedings of the XVIIth International Biometrics Conference, International Biometrics Society, Hamilton, Ontario, Cana- da, pp. 19–33.
[28] Pardo-Iguzquiza, E., Chica-Olmo, M., (1994). Spectralsimulation of multivariable stationary random functions using covariance Fourier transforms. Mathematical Geology 26 (3), 277–299.
[29] Gutjahr, A., Bullard, B., Hatch, S., (1997). General joint conditional simulation using a fast Fourier transform method. Mathematical Geology 29 (3), 361–389.
[30] Oliver, D.S., (2003). Gaussian cosimulation: modelling of the cross covariance. Mathematical Geology 35 (6), 681–698.
[31] De Fouquet, C., (2001). Joint simulation of a random function and its derivatives. In: Kleingeld, W.J., Krige, D.G. (Eds.), Geostats 2000 Cape Town. Geostatistical Association of Southern Africa, Cape Town, pp. 84–93.
[32] Brand, N.W., Butt, C.R.M., and Elias, M., (1998), Nickel laterites: Classification and features: AGSO, Journal of Australian Geology and Geophysics, v. 17, p. 81–88.
[33] Elias, M., (2002), Nickel laterite deposits—Geological overview, resources and exploitation, in Cooke, D., and Pontgratz, J., eds., Giant ore deposits: Char- acteristics, genesis and exploration: CODES Special Publication 4, Hobart, University of Tasmania, p. 205–220.
[34] Mejia, V.M., and Durango, J.R., (1981), Geología de las lateritas niquelíferous de Cerro Matoso: Boletin de Geología, v. 15, p. 117–123.
[35] Glesson, S.A., Herrington, R.J., (2004). The Mineralogy and Geochemistry of the Cerro Matoso S.A. Ni Laterite Deposit, Montelibano, Colombia. Economic Geology, Vol. 9 1197-1213

[36] Rossi, M., Deutsch, C., (2014). Mineral resource estimation. Springer, New York.
[37] Goulard, M., Voltz, M (1992) Linear coregionalization model: tools for estimation and choice of multivariate variograms. Mathematical Geology, Vol. 24, 269- 286
[38] Emery, X., (2010). Iterative algorithms for fitting a linear model of coregionalization. Comput, Geosci. 36, 1150-1160
[39] Paravarzar Sh, Emery X, Madani, N (2015) Comparing sequential Gaussian and turning bands algorithms for cosimulating grades in multi-element deposits, Comptes Rendus Geoscience 347:84-93