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Interaction of Turbulence with Shock Wave

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As members of the Thesis Committee, we certify that we have read the thesis prepared by Azamat Zhaksylykov entitled

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Abstract

At the final stages of the lives of massive stars supernova explosion occurs. Before that process nuclear shell burning happens and strong turbulent convection arises in between iron core and silicon shell. Turbulence by amplifying flow in the post-shock region aid supernova explosion. In this work the physical mechanism behind this amplification using a linear perturbation theory was investigated. Shock wave was modeled as one-dimensional planar discontinuity. Vorticity and entropy perturbations in the upstream flow considered as turbulence. Amplification of turbulent kinetic energy of flow crossing shock wave by a factor of 2 was obtained.

Contents

Chapter 1

Background Theory

1.1 Introduction

The largest explosion that takes place in space is explosion of stars and it is called supernova. The term supernova firstly was introduced by Walter Baade and Fritz Zwicky in their joint paper, which was published in 1933. Theoretical studies of many supernovae indicate that there are main two types of supernova. One is thermonuclear and white dwarf is example for it. Another type is core-collapse supernova (CCSN) and it constitutes one of the most outstanding problems in modern astrophysics.In this work only core-collapse supernova was considered. The main mechanism of CCSN is sudden gravitational collapse of the core of massive star. Nevertheless, the problem of supernova explosion mechanism are still not fully solved and reason for that is this explosions have diverse physical ingredients and processes that play role in core of dying star. Early pioneers of the field like Dave Arnett, Jim Wilson, Hans Bethe, Gerry Brown, Steve Bruenn, Wolfgang Hillebrandt, Jim Lattimer, and David Schramm have spended more that four decades to figure out those ingredients and processes. They found that magnetohydrodynamic (MHD) effects, fluid instabilities and turbulent flows, neutrino transport and neutrino-matter interactions, and general relativistic gravity play vital role in explosion of CCSNe [7]. At the end of live massive stars undergo strong convective shell burning. Deviation from associated radial dynamics and asphericities can grow further during collapse [1]. Recent works by Couch and Ott (2013, 2015), Muller and Janka (2015) demonstrate that non spherical instabilities responsible for the asymmetric character of the explosion. According to Couch and Ott (2015); Muller and Janka (2015), this is a result of increased turbulent activity in the post-shock region driven by the passage of the upstream fluctuations through the shock.

In this paper, linear perturbation theory commonly known as the linear interaction approximation (LIA) theory was used to investigate the physics of the interaction of the upstream turbulence with the supernova shock and its effect on the post-shock flow. In the LIA, the shock is modeled as a planar discontinuity with no intrinsic scale and the flow is decomposed into the mean and fluctuating parts. Both components can be specified arbitrarily in the upstream flow. Once the upstream field is specified, the downstream field can be fully determined using the Rankine-Hugoniot jump conditions at the shock[5]. The present analysis extends Ribners and co-works' work on shock-turbulence interaction include the nuclear dissociation at the shock.

1.2 The Linear Interaction Approximation

The LIA employs the Kovasznay (1953) decomposition of the fluctuating field, according to which any small fluctuations in a turbulent flow can be decomposed individual Fourier modes that are characterized by their type, wavenumber, and frequency. There are three types of modes: vorticity, entropy, and acoustic modes. The vorticity mode has no pressure or density fluctuations. It has only a solenoidal velocity field that is advected with the mean flow. The entropy mode is convected by the mean flow and it represents density and temperature fluctuations; it has no pressure or velocity variations. The acoustic mode represents waves that travel relative to the mean flow at the speed of sound. It has all types of fluctuations; pressure and density fluctuations and irrotational velocity field [5].

Shock wave assumed as one dimensional planar discontinuity which lies on xy -plane. X-axis was chosen to be perpendicular to the shock front. Shock was placed at x=0 and the mean flow directed toward positive x direction.The quantities U, $\bar{\rho}$, $\bar{p}$, $\bar{T}$ and M represent the mean velocity, density, pressure, temperature and Mach number. The quantities u',v', ρ', p' and T' denote the perturbation respectively in the x and y components of velocity, density, pressure and temperature. Shock wave divides flow into two states, upstream and downstream which are respectively denote by subscripts 1 and 2. In this work acoustic waves in the pre-shock region was ignored, which means that p_1'=0. The upstream vorticity wave was modeled as planar wave with wavenumber (mk, lk) and angular frequency kmU_1.

$$\frac{u_1'}{U_1} = lA_v e^{i(kx+ly-mU_1 t)} \tag{1.1}$$

$$\frac{v_1'}{U_1} = -mA_v e^{i(kx+ly-mU_1 t)} \tag{1.2}$$

On the other hand upstream entropy wave was defined as another planar sinusoidal wave with the same frequency and wavenumber.

$$\frac{\rho_1'}{\bar{\rho}_1} = A_e e^{ik(mx+ly-mU_1 t)} \tag{1.3}$$

$$\frac{T_1'}{\bar{T}_1} = -\frac{\rho_1'}{\bar{\rho}_1} \tag{1.4}$$

Here $m = \cos\psi_1$ and $l = \sin\psi_1$, where ψ_1 is the angle between k vector and x-axis. A_e is amplitude of incident entropy wave and A_v is amplitude of incident vorticity wave.

Shock wave changes its position and shape when incident vorticity and/or entropy waves hit it. LIA allows to calculate those changes and deformed shock wave has following form.

$$\xi(y,t) = -\frac{L}{ikm} e^{ik(ly-mU_1 t)} \tag{1.5}$$

Where $\xi(y,t)$ is the position of shock at time t and ordinate y, $\frac{L}{ikm}$ is amplitude of the shock oscillations.

Downstream perturbation field consisting of vorticity, entropy, and acoustic wave was derived from interaction of the vorticity and entropy wave with the shock [8].

$$\frac{u_2'}{U_1} = Fe^{i\tilde{k}x}e^{ik(ly-mU_1t)} + Ge^{ik(mrx+ly-mU_1t)} \tag{1.6}$$

$$\frac{v_2'}{U_1} = He^{i\tilde{k}x}e^{ik(ly-mU_1t)} + Ie^{ik(mrx+ly-mU_1t)} \tag{1.7}$$

$$\frac{p_2'}{P_2} = Ke^{i\tilde{k}x}e^{ik(ly-mU_1t)} \tag{1.8}$$

$$\frac{\rho_2'}{\bar{\rho}_2} = \frac{K}{\gamma}e^{i\tilde{k}x}e^{ik(ly-mU_1t)} + Qe^{ik(mrx+ly-mU_1t)} \tag{1.9}$$

$$\frac{T_2'}{\bar{T}_1} = \frac{\gamma-1}{\gamma}Ke^{i\tilde{k}x}e^{ik(ly-mU_1t)} - Qe^{ik(mrx+ly-mU_1t)} \tag{1.10}$$

Amplidutes of acoustic component is defined by coefficients F, H, and K, while coefficients G, I and Q are entropy and vorticity components. All components of turbulent wave have same angular frequency kmU_1 and different wavenumber. For acoustic wave wavenumber is $(\tilde{k}, lk)$, entropy and vorticity wave have (mCk, lk) wavenumber.

1.3 Analysis

In this section LIA formalism will be presented. Rankine-Hugoniot conditions at the shock represented by following three equations.

$$\rho_1 v_1 = \rho_2 v_2 \tag{1.11}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{1.12}$$

$$\frac{v_1^2}{2} + \frac{\gamma p_1}{(\gamma - 1)\rho_1} = \frac{v_2^2}{2} + \frac{\gamma p_2}{(\gamma - 1)\rho_2} \tag{1.13}$$

Instantaneous velocity ψ_t and inclination ψ_y is given by following equations.

$$\psi_t(t, y) = U_1 L e^(ik(ly - U_1 mt)) \tag{1.14}$$

$$\psi_t(t, y) = U_1 L e^(ik(ly - U_1 mt)) \tag{1.15}$$

After solving wave equation for pressure in the post-shock region. Two regimes of solution was found depending on critical angle ψ_c When $\psi_1 > \psi_c$, $\tilde{k}$ is real and is given by

$$\frac{\tilde{k}}{k} = \frac{U_1}{U_2} \frac{M_2}{1 - M_2^2} \Big[- mM_2 + l\sqrt{\frac{m^2}{l^2} - \frac{U_2^2}{U_1^2}(\frac{1}{M_2^2} - 1)} \Big] \tag{1.16}$$

This describes a simple sinusoidal planar wave. When $\psi_1 < \psi_c$,$\tilde{k}$ is complex, real and imaginary part of $\tilde{k}$ is given respectively.

$$\frac{\tilde{k}_r}{k} = -m \frac{U_1}{U_2} \frac{M_2^2}{1 - M_2^2} \tag{1.17}$$

$$\frac{\tilde{k}_i}{k} = l \frac{U_1}{U_2} \frac{M_2}{1 - M_2^2} \sqrt{\frac{U_2^2}{U_1^2}(\frac{1}{M_2^2} - 1) - \frac{m^2}{l^2}} \tag{1.18}$$

In this regime, the solution represents exponentially-damping planar wave. Amplitudes of the post-shock solution was calculated by substituting equations 2.6 -2.10 to linearized Euler equations for the perturbation field. We get

$$U_1(-FikmU_1) + U_2 U_1 F i\tilde{k} = -\frac{1}{\bar{\rho}_2} P_2 K i\tilde{k} \tag{1.19}$$

and

$$U_1(-HikmU_1) + U_2 U_1 H i\tilde{k} = -\frac{1}{\bar{\rho}_2} P_2 K ikl \tag{1.20}$$

Then after simplifying above two equations for brevity new variable α and β were introduced such that $F = \alpha K$ and $H = \beta K$.

$$\alpha = \frac{a_2^2}{\gamma U_1^2} \frac{\frac{\tilde{k}}{k}}{m - \frac{\tilde{k}}{kr}} \tag{1.21}$$

$$\beta = \frac{a_2^2}{\gamma U_1^2} \frac{l}{m - \frac{\tilde{k}}{kr}} \tag{1.22}$$

Relation of I and G was found from condition that velocity field is solenoidal for vorticity waves:

$$U_1 Gikmr + U_1 Iikl = 0 \tag{1.23}$$

from which

$$I = -\frac{mr}{l} G \tag{1.24}$$

The Rankine-Hugoniot conditions at the shock the following equations for downstream filed:

$$\frac{u_2' - \xi_t}{U_1} = B_1 \frac{u_1' - \xi_t}{U_1} + B_2 \frac{T_1'}{\bar{T}_1} \tag{1.25}$$

$$\frac{\rho_2'}{\bar{\rho}_2} = C_1 \frac{u_1' - \xi_t}{U_1} + C_2 \frac{T_1'}{\bar{T}_1} \tag{1.26}$$

$$\frac{p_2'}{\bar{P}_2} = D_1 \frac{u_1' - \xi_t}{U_1} + D_2 \frac{T_1'}{\bar{T}_1} \tag{1.27}$$

$$\frac{v_2'}{U_1} = \frac{v_1'}{U_1} + E_1 \xi_y \tag{1.28}$$

here coefficients A,B,C,D, E are functions of only two parameters. First one is upstream Mach number M_1 and second is nuclear dissociation ϵ. After substituting equations 2.6 - 2.10 into these equations, system of algebraic equations for amplitudes was found.

$$F + G - L = B_1(lA_v - L) - B_2 A_e \tag{1.29}$$

$$\frac{K}{\gamma} + Q = C_1(lA_v - L) - C_2 A_e \tag{1.30}$$

$$K = D_1(lA_v - L) - D_2 A_e \tag{1.31}$$

$$H + I = -mA_v - E_1 \frac{l}{m} L \tag{1.32}$$

Using LIA differential equation simplified to linear algebraic equations and by solving those equations solution was found. The solution depends on the Mach number M_1 of the mean flow, the nuclear dissociation ϵ and the ration of amplitudes of upstream entropy and vorticity wave.

1.4 Results

Results of this work can be found in a paper *Shock-Turbulence interaction in Core-Collapse Supernovae* written by Abdikamalov and co-workers in 2016 [9]. In this section brief summarization of that results are presented. The turbulent kinetic energy defined as

$$E' = \frac{1}{2}(< u'u'^{\star} > + < v'v'^{\star} >) \tag{1.33}$$

here * denotes complex conjugate term and <> stands for averaging over t and y. An incident vorticity-entropy wave has following energy

$$E'_1 = \frac{1}{2}U_1^2|A_v|^2 \tag{1.34}$$

In far-field region downstream vorticity field has following energy

$$E'_2 = \frac{1}{2}U_1^2(|G|^2 + |I|^2) \tag{1.35}$$

Then ratio of these turbulent kinetic energies are

$$\frac{E'_2}{E'_1} = |\frac{G}{A_v}|^2 + |\frac{I}{A_v}|^2 \tag{1.36}$$

After calculation using our fiducial parameters it was found that amplification of turbulent kinetic energy is about 2 [9].

Chapter 2

Conclusion

2.1 Summary of Thesis Achievements

A study of a phenomenon of core-collapse supernova with turbulent convection from nuclear shell burning is presented in this thesis. The Linear Interaction Approximation, a first order perturbation theory, has been exploited in order to include nuclear dissociation inside of the shock wave. A Mathematical background of LIA and a shock wave model used in the investigation are explained. According to result analysis, turbulent eddies shrink in size crossing the shock by a factor 2 due to shock compression.

2.2 Future Work

Relativistic version this problem is one of the possible future works. It has potential application in study of Gamma-ray burst.

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