Convergence rates for inverse-free rational approximation of matrix functions

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Abstract

This article deduces geometric convergence rates for approximating matrix functions via inverse-free rational Krylov methods. In applications one frequently encounters matrix functions such as the matrix exponential or matrix logarithm; often the matrix under consideration is too large to compute the matrix function directly, and Krylov subspace methods are used to determine a reduced problem. If many evaluations of a matrix function of the form f(A)v with a large matrix A are required, then it may be advantageous to determine a reduced problem using rational Krylov subspaces. These methods may give more accurate approximations of f(A)v with subspaces of smaller dimension than standard Krylov subspace methods. Unfortunately, the system solves required to construct an orthogonal basis for a rational Krylov subspace may create numerical difficulties and/or require excessive computing time. This paper investigates a novel approach to determine an orthogonal basis of an approximation of a rational Krylov subspace of (small) dimension from a standard orthogonal Krylov subspace basis of larger dimension. The approximation error will depend on properties of the matrix A and on the dimension of the original standard Krylov subspace. We show that our inverse-free method for approximating the rational Krylov subspace converges geometrically (for increasing dimension of the standard Krylov subspace) to a rational Krylov subspace. The convergence rate may be used to predict the dimension of the standard Krylov subspace necessary to obtain a certain accuracy in the approximation. Computed examples illustrate the theory developed.

Original language	English
Pages (from-to)	291-310
Number of pages	20
Journal	Linear Algebra and Its Applications
Volume	510
State	Published - Dec 1 2016

Jagels, C., Mach, T., Reichel, L., & Vandebril, R. (2016). *Convergence rates for inverse-free rational approximation of matrix functions*. *Linear Algebra and Its Applications*, *510*, 291-310. DOI: 10.1016/j.laa.2016.08.029