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# Mispriced index option portfolios

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**Abstract**

In model-free out-of-sample tests, we find that the optimal portfolio of a utility maximizing investor trading in the S&P500 Index, cash, and index options bought at ask and written at bid prices stochastically dominates the optimal portfolio without options and yields returns with higher mean and lower volatility in most months from 1990 to 2013. Unlike earlier claims of overpriced puts, our portfolios include mostly short calls and are particularly profitable when maturity is short and volatility is high. Similar results are obtained with the CAC and DAX indices. Neither priced factors nor a nonmonotonic stochastic discount factor explains the excess returns.

## 1 | INTRODUCTION

Index option anomalies are mentioned for the first time in Rubinstein (1994) who documents the existence of the implied volatility (IV) smile in S&P 500 Index options for the post-1987 crash data. Rubinstein (1994) advances several conjectures regarding the sources of the smile and points out that out-of-the money (OTM) put options may have been overpriced after the crash. Several subsequent studies claim overpricing in both OTM puts and at-the-money (ATM) straddles. A parallel line of research, starting with Jackwerth (2000), argues that the stochastic discount factor derived from the observed equilibrium prices in both the underlying and option markets is U-shaped and also leads to put mispricing relative to monotonic stochastic discount factors. The parameters of the asset dynamics of the index yielding the real distribution of the returns are derived by fitting specific models to the entire time series of index values. Frictions, such as margins and bid–ask spreads, are ignored, put–call parity is assumed, and the prices of the in-the-money (ITM) options are derived from the midpoint quotes of their OTM counterparts.[1]

These assumptions are unrealistic and their empirical impact is major for short-term options, as we discuss further on in this section. Bates (2003) notes that attempts to relax them in a no-arbitrage model have not been particularly successful. For this reason, we use a different methodology in this paper where we relax the assumptions of the frictionless economy and apply model-free tests of the mispricing in the S&P 500 Index options. We apply a stochastic



[1]An exception is Santa Clara and Saretto (2009) in which margins play a central role in explaining the observed mispricing. They also consider option bid–ask spreads and find that they reduce, but do not eliminate, put overpricing. Their data, however, do not cover the 2008 financial crisis.

dominance (SD) framework introduced by Constantinides and Perrakis (2002) and tested in sample by Constantinides, Jackwerth, and Perrakis (2009) and out of sample by Constantinides, Czerwonko, Jackwerth, and Perrakis (2011).

In particular, we consider an investor who holds a risk-free bond and a fund tracking the S&P 500 Index, the latter subject to proportional transaction costs. The investor maximizes the expectation of their increasing and concave utility function of cash wealth at the end of a horizon longer than the maturities of any traded options, which can be infinite.[2] We call this portfolio the "Index Trading" (IT) portfolio. We then consider overlaying a zero-net-cost portfolio on this portfolio consisting of long and short positions in European-style call and put options of 28-, 14-, or 7-day maturities on the index. We call the IT portfolio, overlaid with the options portfolio, the "Option Trading" (OT) portfolio. We account for transaction costs in the trading of options by buying options at their ask price and selling them at their bid price. Unlike earlier SD studies, the OT portfolio does not consist of a single option and is not predetermined at portfolio formation time. This generalization has a major impact on the empirical results, particularly for the shorter term options.

We select the zero-net-cost option portfolio at the start of each 28-, 14-, or 7-day maturity from the entire universe of available options filtered by limits on moneyness and liquidity. In all cases, the options are kept until maturity and the OT investor is not allowed to close positions. We develop a linear programming (LP) algorithm that identifies all of the option portfolios such that the OT portfolio stochastically dominates (SD) the IT portfolio in the second degree if both are liquidated at the options' maturity. The SD conditions built into the LP indicate that the total excess payoff of the OT portfolio over the IT portfolio is nonnegative at low values of the index support, intersects the support at a single value, becomes nonpositive at high values, and has a positive expected payoff. We use only observables at the time the portfolios are formed in order to ensure that our strategies can be executed by any option end user and that any excess profits are anomalous. We find these portfolios for almost every month of our data for all three maturities from 1990 to the end of 2012.

Once we identify the set of SD portfolios at each date, we select one from the set by optimizing a given criterion, either the Sharpe ratio or a similar criterion.[3] The resulting portfolios are of variable composition and contain both call and put options with 28, 14, or 7 days to maturity. Their realized excess returns over the IT holdings are very similar for all of the optimization criteria. Using these realized returns, we then confirm with out-of-sample tests that irrespective of the selection criterion, the options portfolios would have increased, on average, the utility of any risk-averse IT investor. The results are stronger for shorter maturity options than for their longer term counterparts in terms of both profitability and the significance of the SD tests. Our results also hold in even stronger form if we assume that there is no bid–ask spread and execute the OT option trades at the midpoint of the spread, but without distorting the data by imposing put–call parity and eliminating ITM options. As a robustness check, we repeat our tests using options on the CAC and DAX indices, as well as weekly options of the S&P 500 Index, and obtain similar results.

It is important to note that the out-of-sample SD test does not depend upon the portfolio selection criteria that establish SD. The SD test compares two time series drawn from two different distributions and examines the null of nondominance. The only requirement is that the observations be serially uncorrelated, a requirement that is verified for all of the series used in our tests. Because the portfolios are chosen using only observables at every point of the resulting time series, the out-of-sample test results identify a tradable anomaly insofar as an investor holding an index-tracking tradable fund, such as SPDR, can increase their returns without incurring additional volatility risk or costs. In fact, in all of the cases, the total volatility of the OT portfolios is lower than that of the volatility of the IT portfolios, thereby precluding the possibility that the excess return is compensation for volatility risk. It is in this sense that we claim that there exist mispriced index options.

Although most options in the OT portfolios are OTM, more than 37% of the portfolios contain ITM calls and more than 32% contain ITM puts. The portfolios include more than double the number of calls than puts and the call positions are overwhelmingly short positions, consistent with the practice of writing covered calls and contradicting the common

[2] For an infinite horizon under transaction costs, the IT investor maximizes the utility of the flow of consumption (Constantinides, 1986).

[3] Specific portfolio selection criteria are imposed as we have many portfolios that satisfy our SD conditions.

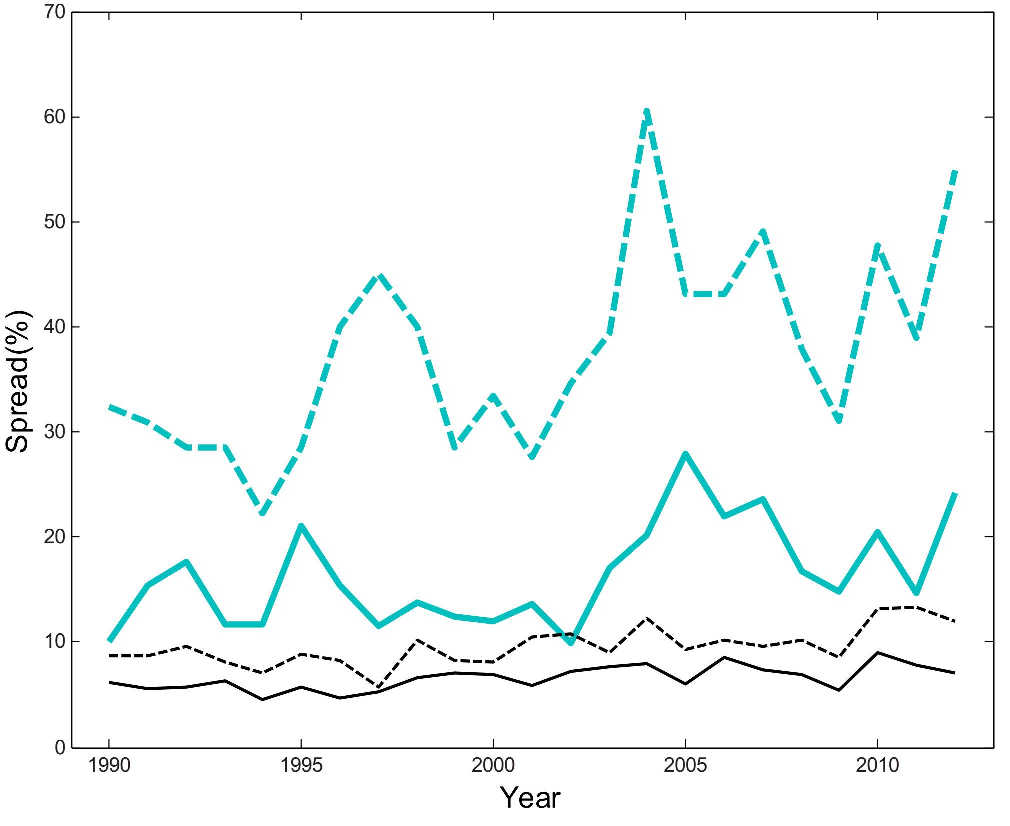


**FIGURE 1** Median of proportional spreads around the midpoint price for ATM ($K/S_t = 1$) and OTM ($K/S_t = 0.93$) puts with 28 and 7 days to maturity [Color figure can be viewed at wileyonlinelibrary.com]
*Note:* Solid darker (lighter) lines correspond to 28-day ATM (OTM) options. Dashed darker (lighter) lines correspond to 7-day ATM (OTM) options.

belief that puts rather than calls are overvalued.[4] An exception is the 2008–2009 period of the financial crisis when there appears to be investor overreaction in the form of inflated put prices.

The potential errors implied by assuming away frictions are large. In Figure 1, we illustrate the observed bid–ask spread, as a percentage of its midpoint, for selected put options for each year of our dataset. The data for this important variable, widely used as an indicator of option market illiquidity, clearly show two effects, a moneyness effect and a maturity effect, with OTM and 7-day maturity options as the least liquid. For the least liquid 7-day OTM options, the spread rarely dips below 30% and can be as high as 60% of its midpoint. Furthermore, there are clear indications that the spread has increased over time for all maturities and degrees of moneyness, as shown by the regressions of all spreads data (not just the annual medians) for the four time series in Figure 1 against a constant and a time trend.

In the absence of friction and market segmentation, put–call parity implies that if OTM puts are overpriced and short positions are profitable after adjusting for risk, then ITM calls are also overpriced. Yet, Bondarenko (2014, table 1) reports that long positions in ATM puts yield a negative and highly significant average monthly return of –0.39%, whereas long positions in ATM calls yield a positive, but insignificant average monthly return of 0.04%. Many studies present evidence that the option market is at least partially segmented. Chen, Joslin, and Ni (2019, figure 2) find that calls and puts with the same moneyness are not substitutes for each other. Constantinides and Lian (2018) report that

[4] In unreported results, we allow the OT portfolios to short an optimally chosen quantity of the underlying. Short puts appear in very few dates, while the preponderance of short calls is maintained.

**TABLE 1** Average number of filtered option contracts

| Period | Number of months | Average number of calls | Average number of calls in first pass portfolios | Average number of calls in second pass portfolios | Average number of puts | Average number of puts in first pass portfolios | Average number of puts in second pass portfolios |
|---|---|---|---|---|---|---|---|
| 28-Day options | | | | | | | |
| January 1990–December 2000 | 132 | 7.1 | 2.1 | 1.7 | 9.3 | 1.5 | 1.7 |
| January 2001–June 2004 | 42 | 12.7 | 2.6 | 2.1 | 18.8 | 1.0 | 0.8 |
| July 2004–February 2013 | 102 | 21.8 | 2.0 | 2.0 | 44.6 | 2.1 | 1.8 |
| January 1990–February 2013 | 276 | 17.7 | 2.0 | 2.0 | 34.1 | 1.8 | 1.6 |
| 14-Day options | | | | | | | |
| January 1990–December 2000 | 132 | 6.2 | 2.0 | 1.6 | 8.2 | 1.5 | 1.7 |
| January 2001–June 2004 | 42 | 10.9 | 2.2 | 2.1 | 16.9 | 1.1 | 1.6 |
| July 2004–February 2013 | 102 | 21.0 | 2.2 | 2.0 | 37.2 | 1.6 | 1.7 |
| January 1990–February 2013 | 276 | 16.9 | 2.1 | 1.9 | 29.0 | 1.6 | 1.7 |
| 7-Day options | | | | | | | |
| January 1990–December 2000 | 132 | 5.5 | 1.8 | 1.7 | 7.2 | 1.1 | 1.9 |
| January 2001–June 2004 | 42 | 9.6 | 1.9 | 1.7 | 14.4 | 0.8 | 1.6 |
| July 2004–February 2013 | 102 | 20.0 | 2.3 | 2.1 | 32.7 | 1.1 | 1.4 |
| January 1990–February 2013 | 276 | 15.8 | 2.1 | 2.0 | 25.2 | 1.1 | 1.5 |

*Notes*: The table reports the averages of the number of calls and puts in cross-sections and averages of the number of options entering zero net cost portfolios. First pass options are options selected when all filtered options are available. Second pass options are those selected when the first pass options are discarded from the opportunity set.

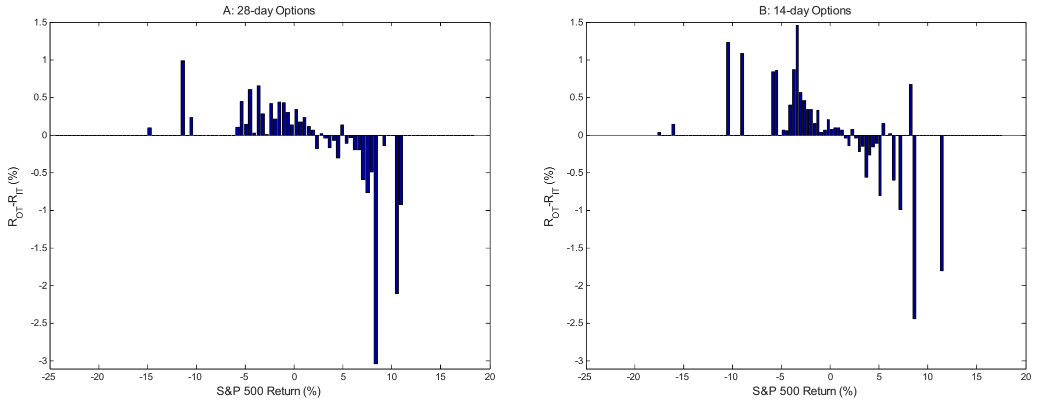


**FIGURE 2** Difference in the realized returns between the OT and IT 28-day portfolios [Color figure can be viewed at wileyonlinelibrary.com]

*Notes*: The returns are measured in the 278 28-day and 14-day periods from January 1990 to February 2013 and sorted by the contemporaneous S&P 500 returns. Bars in the graph correspond to means for 100 equally spaced S&P returns. Consistent with the objective of constructing OT portfolios that stochastically dominate the IT portfolios, the difference in returns is generally decreasing in the S&P 500 Index return.

from 1996 to 2016, public customers are net buyers of OTM and ATM puts with the intermediary market makers having the opposite net positions, consistent with the view that public customers buy puts as insurance and market makers write puts for profit. Finally, Czerwonko and Perrakis (2018) determine that public customers are net writers of calls.

The positive excess returns of short positions in OTM puts and straddles rely on specific option pricing models in a frictionless economy and leave open the possibility that the overpricing is model specific. We illustrate these distortions of the frictionless economy by applying our portfolio selection algorithm to the universe of frictionless options as defined in most studies by using only the OTM call and put bid–ask spread midpoints. The OT portfolios again stochastically dominate the IT portfolios and have significant excess returns. However, the short calls that used to predominate are now a small fraction of the portfolios and are replaced by short puts combined with short positions in the underlying index.

In robustness tests, we examine whether our OT portfolio results are explained by adopting the conventional approach to option valuation by adding the options as an asset class to an existing portfolio. We find that a U-shaped stochastic discount factor chosen from the observed index and option prices does not explain the excess returns in a frictionless setting.

The literature on the pricing of options is voluminous. The ingenious idea behind the option pricing model of Black and Scholes (1973) and Merton (1973) (BSM) is that, in the absence of arbitrage, the price of an option equals the cost of setting up a judiciously managed portfolio with a payoff that replicates the option payoff. The central premise of the BSM model supports the existence a self-financing dynamic trading policy of the stock and risk-free accounts that renders the market dynamically complete. This requires that the market be complete and frictionless. Two assumptions of the BSM model make the market complete. First, the price of the underlying security has continuous sample paths at the exclusion of jumps. In addition, the stock return volatility is constant.[5] Finally, the assumption of the BSM model that renders the market frictionless is the absence of trading costs. In the BSM model, the volume of trading over any finite time interval is infinite and the trading costs associated with the replicating dynamic trading policy would be infinite.

Following the October 1987 stock market crash, limitations of the BSM model became evident. Rubinstein (1994) finds the existence of the IV smile in S&P 500 Index options for the post-1987 crash data. In response, most of the

[5] In the BSM model, the volatility is constant. Markets are also complete if the volatility is solely a function of the stock price.

ensuing literature recognizes that volatility is stochastic and/or price changes include jumps, but retains the assumption of a frictionless market. Under these assumptions, the market is dynamically incomplete and a dynamic trading strategy that replicates the payoff of an option no longer exists. Formally, the absence of arbitrage no longer determines the relation between the real and risk-neutral probability measures.

The most common approach is to exogenously set the prices of volatility and jump risks and estimate them from the option market assuming that the options are correctly priced. Prominent examples include Bates (1991) and Heston (1993). An alternative approach that allows for stochastic volatility and jumps but maintains the assumption of a frictionless market is to price options in an equilibrium model that introduces investors with specific utility functions. Examples include Chen et al. (2019), Constantinides and Lian (2018), and Garleanu, Pedersen, and Poteshman (2009).

In a frictionless market, several empirical studies of index options find that options are incorrectly priced. Ait-Sahalia and Lo (2000), Bakshi, Madan, and Panayotov (2010), Beare and Schmidt (2016), and Jackwerth (2000) estimate a U-shaped stochastic discount factor from stock index option prices that challenges equilibrium asset pricing models based on a risk-averse representative investor in which the index return is the only state variable. We discuss this possibility in Section 8 and find that it cannot account for our results. Bollen and Whaley (2004), Bondarenko (2003), Driessen and Maenhout (2007), Garleanu et al. (2009), Jones (2006), Santa Clara and Saretto (2009), and Chambers, Foy, Liebner, and Lu (2014) find that strategies that involve writing put options on the S&P 500 Index offer very high Sharpe ratios. Alternatively, Broadie, Chernov, and Johannes (2009) claim that these anomalous results are explained by jump risk premia and estimation risk.

Levy (1985), Perrakis (1986, 1988, 2019), Perrakis and Ryan (1984), Ritchken (1985), and Ritchken and Kuo (1988) replace the assumption that investors have specific utility functions with a weaker assumption that investors have unspecified utility functions that are increasing and concave in wealth and apply the methods of SD to derive upper and lower bounds on option prices. Oancea and Perrakis (2014) demonstrate that both bounds tend to the continuous time limit of the BSM option price when there are no transaction costs and the underlying security follows a diffusion process.

Constantinides and Zariphopoulou (1999, 2001), Davis, Panas, and Zariphopoulou (1993), and Hodges and Neuberger (1989) introduce proportional transaction costs in trading the underlying security and derive bid and ask prices of options from the perspective of an investor maximizing a given utility function. Constantinides and Perrakis (2002, 2007) generalize this approach by replacing the assumption that investors have specific utility functions with the assumption that investors have unspecified utility functions that are increasing and concave in wealth and applying the methods of SD. They derive a tight upper bound on the reservation write price of a call and a tight lower bound on the reservation purchase price of a put. Constantinides et al. (2009) test these bounds in sample and Constantinides et al. (2011) test these bounds out of sample.

The theoretical contribution of the current paper over the results in Constantinides and Perrakis (2002, 2007) is to allow the investor to include options of a different type, long and short calls and puts of different moneyness in a zero-net-cost portfolio, not just one type of options. We identify a set of OT portfolios (zero-cost option portfolios added to the IT portfolio) that stochastically dominate the IT portfolio. Recently, Post and Longarella (2018) provide a complete characterization of the set of portfolios that stochastically dominate the index.

The first empirical contribution of the current paper over the results in Constantinides et al. (2009, 2011) is to identify the mispricing of options in almost every sample month, as opposed to the earlier results that identify mispricing only in approximately one-third of the sample months. The second empirical contribution of the current paper is to identify the characteristics of the most mispriced options.

There is a close relation between the results of the current paper and the U-shaped stochastic discount factor estimated by Ait-Sahalia and Lo (2000), Bakshi et al. (2010), Beare and Schmidt (2016), and Jackwerth (2000). A U-shaped stochastic discount factor implies SD in a single period model in which the index return is the only state variable. Christoffersen, Heston, and Jacobs (2013) note that a U-shaped stochastic discount factor can arise if volatility is a second state variable. Note, however, that a U-shaped stochastic discount factor does not identify the particular options that are responsible for the nonmonotonicity of the stochastic discount factor.

The third empirical contribution of the current paper over these earlier results is to identify SD in the presence of realistic trading costs, identify the characteristics of the most mispriced options, and demonstrate the existence of such mispricing in almost every month of our sample. Specifically, the most mispriced options are calls and short maturity options. Our last contribution is to demonstrate that other major index option markets exhibit patterns of mispricing similar to the market for S&P 500 options.

In the next section, we discuss the theoretical foundations of SD and present the portfolio selection algorithm. Section 3 presents the data. We provide the methodology in Section 4 and the empirical results in Section 5. In Section 6, we discuss the relation between SD and the smile. Section 7 describes the characteristics of the dominating portfolios. In Section 8, we present evidence that risk factors do not explain the excess return of the option portfolio returns. Section 9 presents the robustness tests, while Section 10 provides our conclusions.

## 2 | STOCHASTICALLY DOMINATING PORTFOLIOS

We consider an investor who trades at discrete dates $t = 0, 1, \dots, T$ in an index portfolio and a risk-free asset (cash) subject to proportional transaction costs for the index. We denote the index price at date $t$ by $S_t$. At the terminal date, $T$, the wealth is converted into cash, net of transaction costs. The investor maximizes the expectation of their utility of terminal cash wealth. We assume that the utility function is von Neumann-Morgenstern, strictly increasing, and concave. We denote by $V(x_t, y_t, t)$ the value function (indirect utility) at date $t$, $t \leq T$, where $(x_t, y_t)$ denote the holdings in the cash and index accounts, respectively. We assume that the equity premium is positive so that the investor optimally invests a nonnegative amount in the index portfolio at each date.

Constantinides (1979) addresses this problem under general conditions and proves two properties that are useful for the problem at hand. First, he proves that $V(x_t, y_t, t)$ is strictly increasing and concave in $(x_t, y_t)$. This implies that the marginal utility of the value function with respect to the index account, $V_y(x_t, y_t, t)$, is monotone decreasing in $y_t$ at any date $t, t \leq T$. In addition, he proves that there exists a region of no-transactions such that the investor refrains from transacting at date $t$ if the portfolio holdings lie in it. In the special case of constant relative risk aversion, this region is a cone with no transactions if $\underline{\lambda}_t \leq y_t/x_t \leq \overline{\lambda}_t$. Constantinides (1986) finds that the no-transactions region is wide, even for a very small transactions cost rate, and the investor refrains from trading most of the time in the sense that the utility losses from not adjusting the portfolio to its frictionless optimal proportions are low for all realistic parameter values.

For our purposes, we do not make the limiting assumption that the relative risk aversion is constant or that the dynamics are limited to diffusion. Nevertheless, we assume that an investor who starts at the beginning of the month, somewhere in the middle of the no-transactions region, optimally refrains from trading in the short time span of 28, 14, or 7 days until option maturity assumed shorter than $T$. Even if the investor refrains from trading for longer periods, such as several months, our results remain approximately correct as the width of the no-transactions region stipulates little trading.[6] Combined with the first result that $V_y(x_t, y_t, t)$ is monotone decreasing in $y_t$ and the fact that the investor has a positive investment in the index, we conclude that $V_y(x_{t+1}, y_{t+1}, t+1)$ is monotone decreasing in the stock price, $S_{t+1}$. Hereafter, we assume without loss of generality that the options mature at date $t$+1.

Let $A(S_{t+1})$ denote the payoff of the zero-net-cost portfolio at date $t+1$, where the net cash flow from the option exercise is converted into units of the index account, net of transaction costs. If:

$$E_t \left[A(S_{t+1}) V_y \left(x_{t+1}, y_{t+1}, t+1\right)\right] > 0, \tag{1}$$

[6] We verify this using the numerical approach of Czerwonko and Perrakis (2016) that assumes a constant relative risk aversion investor, jump diffusion asset dynamics, and a finite fixed horizon. The observed intermediate trading in numerical simulations under all realistic parameter values is insignificant even for a 2-year horizon.

then the investor increases their expected utility by overlaying this zero-net-cost portfolio over their original investment in the index and the risk-free asset. The following Lemma provides sufficient conditions for Equation (1) to hold.

**Lemma.** *A sufficient condition for Equation* (1) *to hold is that (a)* $E_t[A(S_{t+1})] \geq 0$ *and (b) there exists a number* $\hat{S}$ *such that* $A(S_{t+1}) > 0$ *for* $S_{t+1} \leq \hat{S}$ *and* $A(S_{t+1}) \leq 0$ *for* $S_{t+1} > \hat{S}$.

*To see this, note that Conditions (a) and (b) imply:*

$$
\begin{aligned}
& E_t\left[A(S_{t+1})V_y\left(x_{t+1}, y_{t+1}, t+1\right)\right] \\
& > E_t\left[A(S_{t+1})\right] E\left[\left\{V_y\left(x_{t+1}, y_{t+1}, t+1\right)\right\}_{S_{t+1}=\hat{S}}\right] \\
& (\text{because } V_y\left(x_{t+1}, y_{t+1}, t+1\right) \text{ is decreasing in } S_{t+1} \text{ and } A(S_{t+1}) > (\leq)\ 0 \text{ as } S_{t+1} \leq (>)\ \hat{S}) \\
& > 0.
\end{aligned}
$$

This completes the proof.[7] Note that for an investor with linear utility, $V_y(x_{t+1}, y_{t+1}, t+1)$ is a positive constant and Inequality (1) implies that $E_t[A(S_{t+1})] \geq 0$ . The Lemma states that when $A(S_{t+1})$ has the shape indicated by Condition (b), then Condition (a) is necessary for SD. Intuitively, for the chosen shape of$A(S_{t+1})$, an increase in the utility of the risk neutral investor implies an increase in the utility of all risk-averse investors.

We denote by OT the investor who holds the same portfolio as the IT plus the zero-net-cost option portfolio with payoff $A(S_{t+1})$ at maturity. Our empirical methodology involves two steps. In the first step, for each maturity, we construct zero-net-cost portfolios at dates on which we have options with such maturity. In the second step, we verify out of sample SD of the OT investor's terminal wealth relative to the IT investor's terminal wealth by applying SD tests over the entire sample period. Based on the Lemma, the IT and OT comparisons are done on the basis of the time series of the corresponding index values in their portfolios at the option expiration dates. Both steps are independent of any assumptions about the IT investor.

The IT investor is an index fund holding an unspecified number of index units. We present our method for a scale of trading of one option per unit index. The actual scale of trading will depend upon the depths of the quotes of individual options that will determine the IT wealth for comparison with OT.

At time $t$, we build a grid of feasible values of $\hat{S}$, $\hat{S} > 0$ that allows us to find a zero-net-cost portfolio from the universe of options such that the payoff $A(S_{t+1})$ at maturity$_{t+1}$ is as follows: $A(S_{t+1}) > 0$ for $0.6S_t \leq S_{t+1} \leq \hat{S}$; $A(S_{t+1}) \leq 0$ for $S_{t+1} > \hat{S}$; and $E_t[A(S_{t+1})] > 0$. For each value of $\hat{S}$, we choose the portfolio of options that maximize the expected payoff, $E_t[A(S_{t+1})]$, subject to the conditions $A(S_{t+1}) > 0$ for $0.6S_t \leq S_{t+1} \leq \hat{S}$ and $A(S_{t+1}) \leq 0$ for $S_{t+1} > \hat{S}$. Thus, we form a set $\Omega(\hat{S})$ of OT portfolios that ex ante stochastically dominate their IT counterparts.

We find the set $\Omega(\hat{S})$ by solving the following linear program. Let $w_i \geq 0, i = 1, \ldots, 2n$ denote the number of options $C_i$ (both calls and puts) entering into the OT portfolio from the $n$ available options in a given cross-section ordered in ascending strike price. We treat long and short option positions as separate options allowing us to linearly restrict the total option position. Also, let $\Pi$ denote the initial value of the OT portfolio. We have:

$$
0 \leq \sum_{1}^{2n} w_i \leq 1, \quad \Pi = \sum_{1}^{2n} w_i C_i. \tag{2}
$$

Then, if $g_i(S_{t+1})$ denotes the payoff of the $i^{\text{th}}$ option, the total payoff at option expiration equals $-\Pi R + \sum_1^{2n} w_i g_i(S_{t+1})$. Then,

$$
\begin{aligned}
A\left(S_{t+1}\right) &= -\Pi R + \sum_{1}^{2n} w_i g_i(S_{t+1}) / (1+k), \ S_{t+1} \leq \hat{S} \\
&= -\Pi R + \sum_{1}^{2n} w_i g_i(S_{t+1}) / (1-k), \ S_{t+1} > \hat{S}.
\end{aligned} \tag{3}
$$

[7] The Lemma depends upon the condition that the trader's wealth at the end of each period is weakly monotone increasing in the index return. This condition is trivially satisfied under the position limits we apply later on.

Observe that $A\left(S_{t+1}\right)$ is piecewise linear with a constant slope $\frac{\partial A}{\partial S_{t+1}}$ within any interval $[K_j, K_{j+1})$ of two successive strike prices $K_j, j = 1, \ldots, n$ of the available strike prices in the option cross-section. We add the fundamental SD constraints:

$$A\left(S_{t+1}\right) > 0 \text{ for } 0.6S_t \leq S_{t+1} \leq \hat{S};\ A\left(S_{t+1}\right) \leq 0 \text{ for } S_{t+1} > \hat{S};\ \text{and } E_t\left[A\left(S_{t+1}\right)\right] \geq 0. \tag{4}$$

These constraints need only be verified at the strike prices to the left of $\hat{S}$, while at the right, we simplify the search by adding the constraint that the payoff be nonincreasing. Finally, we find the OT portfolio by solving the following LP:

$$\max_{w_i} E_t[A(S_{t+1})] \quad \text{given} \quad \hat{S}, \tag{5}$$

subject to Equations (2)–(4). If this program is feasible, then the set of optimal weights and corresponding options $\{w_i^* \neq 0,\ C_i^*\}$ belongs to the ex-ante stochastically dominant set $\Omega(\hat{S})$ of OT portfolios.[8]

In our search, we vary $\hat{S}$ until the LP becomes infeasible for some maximum value of $S_{t+1} = \hat{S}$, arbitrarily restricted to $1.15 \times S_t$. We restrict our search to the segment $[S_t,\ 1.15S_t]$. Once this maximum feasible value for $\hat{S}$ is found, we partition the segment $[S_t, \hat{S}]$ and maximize the excess return to OT for each value of this partition to find the complete set $\Omega(\hat{S})$. Finally, the optimal portfolio is defined as the one for which a given selection criterion reaches its supremum.

We try several alternative criteria to choose from the set $\Omega(\hat{S})$ the "best" portfolio and find broadly consistent results in terms of the composition of this portfolio. As our base case criterion, we select the portfolio that maximizes the Sharpe ratio $E_t[S_{t+1} + A(S_{t+1}) - S_t R]/std_t(S_{t+1} + A(S_{t+1}))$. In robustness tests, we replace the maximization of the Sharpe ratio with the maximization of the gain/loss ratio, $E_t[A(S_{t+1})^+]/E_t[-A(S_{t+1})^-]$, the maximization of the Sortino ratio, $E_t[A(S_{t+1})]/std_t([S_{t+1} + A(S_{t+1})]^-)$, or simply choose the maximum feasible $\hat{S}$.

We stress that the restriction $0.6S_t \leq S_{t+1} \leq \hat{S}$ is imposed on the construction of the portfolios, but is not imposed on the realized prices $S_{t+1}$. In unreported robustness tests, we also consider the case where we allow the index to become worthless in 1 month, by replacing this restriction with $S_{t+1} \leq \hat{S}$, and obtain similar results. We also consider time-varying volatility following well-known daily GARCH processes. These are discussed in our robustness checks. The construction of these portfolios relies on information that is available at the beginning of the 28-day (or 14- or 7-day) maturity period.

## 3 | DESCRIPTION OF THE DATA

The main empirical results are based on monthly and weekly options on the S&P 500 Index. We obtain prices of monthly S&P 500 European puts and calls 28, 14, and 7 days to maturity from the Chicago Board Options Exchange (CBOE) tape with intraday quotes from January 1990 to February 2013 yielding 278 dates.[9] The prices of the 14-day options are the prices of the same 28-day options with respect to moneyness and expiration date, but observed 14 days to expiration. The prices of the 7-day options are the prices of the same 28-day options with respect to moneyness and expiration date, but observed 7 days to expiration. We also present the results over the shorter time period in which weekly options are traded. For robustness, we replicate our results on the French CAC and German DAX indices for which we use OptionMetrics data with end-of-day prices for their European options.[10]

We delete obvious data entry errors, such as multiple or missing data or bid prices exceeding ask prices. We filter the data by checking that the put–call parity and convexity with respect to the strike price under transaction costs in the index and bid–ask prices of options hold. We conservatively use 10 basis points as a one-way transactions cost

[8] Because the LP has a trivial solution of all zero weights, we constrain the sum of weights in the first constraint of (2) to be above some low threshold, $10^{-4}$.

[9] Unreported results for 21-day options are generally similar to the ones for 28-day options.

[10] Other optioned indices, such as EURO STOXX 50 and FTSE, are eliminated either because of lack of data or settlement terms that make the results noncomparable to our base case.

rate for index trades.[11] We also apply liquidity filters to guarantee that only options that can be traded under realistic conditions enter our choice set. We include call prices with bid prices of at least \$0.15 and moneyness within \$0.96–\$1.08. For put options, we discard all options more than 4% in the money, but admit all options with bid prices of at least \$0.15. This asymmetry in admitting put options is justified by the relatively higher liquidity of OTM puts. Finally, we only admit quotes updated within the past 15 min. After applying our filters, we exclude four dates on which we cannot find at least three call options and three put options available for selection in our portfolios.

In Table 1, we display statistics on the average number of 28-, 14-, and 7-day S&P 500 options that pass the filters each month and are available for inclusion in the zero-net-cost portfolios. In general, the market for puts is more liquid than the market for calls. Thus, the number of puts that pass the filters exceeds the number of calls. In the years 2000 and 2004, the CBOE increased the quote update frequency and the number of strikes at which options are traded generally increased with the index level given a relatively constant increment among pairs of adjacent strikes. As a result, the average number of options that pass the filters doubles after 2004. We take these changes into account when we report the results on the SD tests and when we describe the characteristics of the options that are frequently included in the zero-net-cost portfolios. The table also describes the average number of options included in these portfolios.

We build the zero-net-cost portfolios at 3:00 p.m. SET, 1 hr before market closing, thus avoiding possible distortions of the closing market inherent in end-of-day prices. We execute the trades 15 min later for these same options that are found to be optimal to include in the portfolio, readjusting their weights with the same objective as at the 3 p.m. portfolio derivation time for the data observed 1 min before the actual trade. For a limited number of options in the optimal portfolios, this additional derivation takes a few hundredths of a second leading to little if any distortion due to the additional trade execution time. Because SPX options are exercised at the opening price of the terminal date, we collect the exercise proceeds by using the opening value of the index and ascribe the proceeds to the ending position of the index.

We derive the index price from the cost-of-carry relation between the observed spot index and its nearest-to-maturity futures contract as follows. We use a data set from Tick Data. We estimate the implied index price by recording the implicit cost-of-carry coefficients from the observed spot futures pairs for 1 hr before our estimation or trade time in 1-min intervals. We then use the median value of this coefficient to convert the most recent futures value into the implied spot index price. Note that as of 2006, the increased quality of reporting of the index price renders the difference between the cash index and its derived price negligible. We derive the dividend yield by using cash daily payouts obtained from Standard and Poor. For the interest rate, we use the 3-month constant maturity T-bill rate obtained from the Federal Reserve Economic Data. Finally, we assume a one-way transaction costs rate of 0.25%.

For our base case results, we model the index price as lognormally distributed with an average cum dividend return equal to 4%, plus the annualized risk-free rate, as per the long-term historical average. In the online Table A1, we use 2% and 6%, plus the annualized risk-free rate, as the cum dividend index return. We obtain virtually identical results implying that our portfolios are insensitive to the choice of this parameter for all realistic values. We forecast the index return volatility until the expiration date by using the CBOE VIX volatility adjusted by the mean forecast difference between the VIX and the realized volatility from 1986 to the current date. Both the VIX and the realized volatility of daily returns are measured in 4-week intervals without overlap, with the latter quantity defined as the square root of 252 times the mean squared daily return.[12] The amount by which the VIX exceeds the realized volatility (the negative volatility risk premium) is provided in Figure A1 of our online appendix. The figure indicates that our estimate of the average volatility risk premium is relatively stable over time at about 4.5%–4.8%. Further, in unreported results,

[11] Note that the lower this rate, the more arbitrage violations found. With no transaction costs for the index and with trading in options at the bid–ask midpoint, virtually all options prices are rejected due to arbitrage violations (Ioffe & Prisman, 2013).

[12] Due to the importance of the measurement of this gap in identifying in-sample mispricing, we use two additional variants of the VIX forecast, both in terms of the logarithm of the squared VIX. In the first variant, we measure the average difference between this quantity and a similar quantity for realized returns as above. In the second variant, we regress this quantity for realized returns against a similar one for VIX and, at the trading date, we apply our estimates to the previous day closing value of VIX to form a forecast. In this last variant, we also vary the length of the rolling window used to estimate the regression coefficients.

we verify the VIX as an estimate of the risk neutral volatility of the returns over the path to option expiration for our three maturities by using the Heston (1993) stochastic volatility model with the parameter estimates of Bakshi, Cao, and Chen (1997). The observed VIX is approximately equal to the expected risk neutral path volatility, and this relation is relatively insensitive to the mean reversion speed parameter. We conclude that our volatility estimation method produces unbiased projections of the realized volatility over the path to maturity of the options. In our robustness checks section, we also use GARCH-based volatility estimates and find similar anomalous out-of-sample results. Recall also that the out-of-sample tests are independent of the index return volatility choice and all of the choices produce very similar out-of-sample results.

There are fewer options available for our robustness checks with the CAC and DAX indices. Although closing prices are available from January 2002 for DAX and April 2004 for CAC, bid and ask option prices for both indices exist only since 2006. We use the dataset that contains bid and ask prices to construct the trades and both datasets to construct the volatility projections. In constructing our portfolios, we use the data from January 2006 to February 2013, with the latter date corresponding to the end of our S&P 500 data. This results in 86 potential dates for options maturing on the third Friday of each month, reduced to about 80 dates due to data availability and our requirement of choosing from at least three call and put options after deletions due to violations of the no arbitrage conditions. For the latter, we use the same approach and a 0.1% transaction cost rate as in the S&P 500 options. Because the resulting sample has less than one-third of the 278 dates available for the full sample of S&P 500 options, we also present results with the January 2006–February 2013 S&P 500 data to maintain comparability among indices. The data selection and volatility projections, which differ from the S&P 500 options, are described in detail in part C of our online appendix.

## 4 | METHODOLOGY

We denote by IT (index trader) the investor who holds an optimal portfolio of the index and cash and by OT (option trader) the investor who holds the same portfolio as the IT plus the zero-net-cost portfolio with payoff $A(S_{t+1})$ at maturity. Our empirical methodology involves two steps. In the first step, for the portfolios with options 28 days to maturity, we construct in-sample zero-net-cost portfolios at dates on which we have options with 28 days to maturity by applying Equations (2)–(5). This step is independent of any assumptions about the IT investor. We follow a similar procedure to construct portfolios with options 14 and 7 days to maturity. In the second step, we verify out-of-sample the SD of the OT investor's terminal wealth relative to the IT investor's terminal wealth by applying SD tests over the entire sample period. Based on the Lemma stated in Section 2, the IT and OT comparisons are done on the basis of the time series of the corresponding index values in their portfolios at the option expiration dates. The procedure is described in detail in Section 2 and is only summarized here.

In our base case, the IT investor is an index fund holding 100,000 index units corresponding to approximately \$150 million in index holdings in our sample period. In this case, we consider the scale of trading in one option per unit index as realistic. For instance, writing 0.5 calls with strike $K_1$ and buying 0.5 calls or puts with strike $K_2$ per unit index exhaust our limit.[13]

At the beginning of the 28-day (or 14- or 7-day) period, $t$, we build a grid of feasible values of $\hat{S}, \hat{S} > 0$ that allows us to find a zero-net-cost portfolio from among the universe of 28-day (or 14- or 7-day) options such that the payoff $A(S_{t+1})$ in 28 days (or 14 days or 7 days) satisfies the conditions described in Equation (4) for an interval of values of $S_t \leq \hat{S} \leq 1.15S_t$. For each such value of $\hat{S}$, we choose the portfolio of options that maximizes the expected payoff, $E_t[A(S_{t+1})]$ as in Equation (5), subject to the Conditions (4). Thus, we form a set $\Omega(\hat{S})$ of OT portfolios that ex ante stochastically dominate their IT counterparts. The procedure follows the LP formulation described in detail in Section 2. Finally, we choose from the set $\Omega(\hat{S})$ the "best" portfolio according to a given criterion as described in Section 2.

[13] In unreported results, we also consider trade sizes not easily available, even for investors with large holdings, allowing trading for up to 1,000 options per unit index. In this case, the expected return of the OT investor is higher.

The search for the potentially mispriced portfolios takes place on a subset of the available options within the moneyness range of the options that has already been limited by liquidity considerations according to the criteria specified in Section 2. Essentially, our choice of portfolios incorporates the conditions of the Lemma at every single point of an appropriately defined lower range of the support and imposes the maximization of the expected payoff. Specifically, we assume lesser support for the index generally independent of the span of a given discrete return space. In the last step, we select the portfolio corresponding to the value $\hat{S}$ that maximizes the given criterion, the Sharpe ratio in the base case.

We carefully distinguish the period of the financial crisis, which we define as the 12 months after October 2008. In the 1-month period before mid-October 2008, the index lost some 25% of its value that resulted in several unusual opportunities in the options market in the following months.

In the second phase, we compare the IT and OT portfolios at the option maturity and generate two time series of realized returns. We compare the performance of the IT and OT portfolios in several ways. First, we derive bootstrap *p*-values for a negative mean excess return. In addition, we apply the Davidson-Duclos (2013) (DD) test for restricted second-order SD. This test is based on the null hypothesis of nondominance as opposed to several other tests where the null is dominance and would provide relatively weak evidence, by finding a high *p*-value for the null because, by construction, they do not reject anything. DD demonstrates that the null of nondominance cannot, in principle, be rejected over the entire joint support for the two examined prospects even if it exists in the population. Therefore, some points in the tails of this joint support are removed from the search for the minimal *t*-stat that forms the basis of the bootstrap procedure in the DD test.[14]

As noted in the introduction, the out-of-sample DD tests do not depend upon the portfolio selection criteria that establish in-sample SD, namely, the VIX, the mean index return, or the lognormality assumption. The only requirement for the null of nondominance tests is that the two time series of observations be serially uncorrelated, a requirement that is verified for all of the series used in our DD tests. For our base case with the Sharpe selection criterion, the 28-day IT and OT portfolio returns and their difference have first-order autocorrelations of –0.039 (*p*-value .515), –0.062 (*p*-value .301), and –0.051 (*p*-value .396), respectively. The autocorrelations for the 14- and 7-day portfolios and for the CAC and DAX indices are also statistically insignificant.

## 5 | EMPIRICAL RESULTS ON SD

In Table 2, we present the results over the entire sample period from January 1990 to February 2013 for four different portfolio selection criteria: (a) the Sharpe ratio, (b) the gain/loss ratio, (c) the Sortino ratio, and (d) the maximization of $\hat{S}$. $\mu$ is the mean and $\sigma_{OT-IT}$ is the volatility of the difference in the annualized percentage return between the OT and IT portfolios. In the top panel, we present the results for the portfolios constructed 28 days prior to the options' maturity. Of the 278 dates, there are 270 dates with feasible portfolios. In the middle panel, we report the results for the portfolios constructed 14 days prior to the options' maturity. Of the 278 dates, there are 272 dates with feasible portfolios. The results are provided in the bottom panel for the portfolios constructed 7 days prior to the options' maturity. Of the 278 dates, there are 272 dates with feasible portfolios. Statistical tests are performed based on the total number of dates. The *p*-values for the difference in means are derived via bootstrap with 10,000 draws. For the DD test, 10% trimming (deleting the sequentially lowest outcomes in either return set) in the left tail is uniformly performed, while similar trimming in the right tail is as shown. The results of the DD tests without trimming in the right tail are not provided as they are qualitatively the same as the *p*-values for the difference in means.

[14] The DD test considers a minimal *t*-stat in the restricted support. If there is no restriction in the left tail, a minimal *t*-stat is equal to one by construction. Without any restriction in the right tail, the minimal *t*-stat in cases like ours will usually correspond to the difference in means whose statistical significance is too strong a condition for SD. See Constantinides, Czerwonko, Jackwerth, and Perrakis (2011) where the application of the DD test in a similar situation is described in detail and for evidence that the test is conservative in rejecting a false null.

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**TABLE 2** Portfolio returns and stochastic dominance tests: January 1990–February 2013

| Portfolio selection criterion | $\mu$ | $p$-Value for $\mu \leq 0$ | $\sigma_{OT}$ | $\sigma_{OT-IT}$ | DD test $p$-value 5% Trim | DD test $p$-value 10% Trim |
|---|---|---|---|---|---|---|
| 28-Day options | | | | | | |
| Sharpe ratio | 0.50 | .112 | 15.89 | 1.97 | .039 | 0 |
| Gain/loss ratio | 0.92 | .029 | 15.81 | 2.19 | .008 | 0 |
| Sortino ratio | 0.45 | .128 | 15.90 | 1.97 | .045 | 0 |
| max $\hat{S}$ | 0.66 | .057 | 15.92 | 1.89 | .008 | 0 |
| 14-Day options | | | | | | |
| Sharpe ratio | 2.07 | .062 | 15.68 | 3.99 | 0 | 0 |
| Gain/loss ratio | 2.55 | .026 | 15.64 | 3.79 | 0 | 0 |
| Sortino ratio | 1.82 | .041 | 15.71 | 3.88 | 0 | 0 |
| max $\hat{S}$ | 2.15 | .051 | 15.76 | 3.73 | 0 | 0 |
| 7-Day options | | | | | | |
| Sharpe ratio | 1.90 | .062 | 17.43 | 2.79 | 0 | 0 |
| Gain/loss ratio | 2.72 | .008 | 17.36 | 2.49 | 0 | 0 |
| Sortino ratio | 2.24 | .038 | 17.45 | 2.82 | 0 | 0 |
| max $\hat{S}$ | 2.22 | .007 | 17.61 | 2.06 | 0 | 0 |

*Notes*: The table presents the results for four different portfolio selection criteria: (a) the Sharpe ratio, (b) the gain/loss ratio, (c) the Sortino ratio, and (d) the maximization of $\hat{S}$. $\mu$ is the mean and $\sigma_{OT-IT}$ is the volatility of the difference of the annualized percentage return between the OT and IT portfolios. In the top panel we present the results for portfolios constructed 28 days to the option maturity. Of the 278 dates, there are 272 dates with feasible portfolios. In the middle panel, we report the results for portfolios constructed 14 days to the option maturity. Of the 278 dates, there are 270 dates with feasible portfolios. In the bottom panel, we provide the results for portfolios constructed 7 days to the option maturity. Of the 278 dates, there are 270 dates with feasible portfolios. The volatility of the return of the 28-, 14-, and 7-day IT portfolios is 16.48%, 17.15%, and 18.12%, respectively. Statistical tests are performed on the basis of the total number of dates. The $p$-values for the difference in means are derived under via bootstrap with 10,000 draws. For the DD test, 10% trimming (deleting the sequentially lowest outcomes in either return set) in the left tail is uniformly performed, while similar trimming in the right tail is as shown. The results of the DD tests without trimming in the right tail are not shown as they are qualitatively the same as the $p$-values for the difference in means.

For the portfolios constructed 28 days prior to the options' maturity and for all of the selection criteria, the annualized mean return of the OT portfolio exceeds the mean return of the IT portfolio by about 0.5%. With the exception of the gain/loss ratio criterion, this difference is not statistically significant at conventional levels as we conservatively restrict the scale of the zero-net-cost portfolio. The volatility of the annualized return of the IT portfolio is 16.48% and is higher than the volatility of the OT portfolios. In Figure 2, we present the difference in returns between the OT and IT portfolios as a function of the S&P 500 Index return for 28- and 14-day options. Consistent with the objective of constructing OT portfolios that stochastically dominate the IT portfolios, the difference in returns is positive for the low values of the index return and negative for the high values.

For the portfolios constructed 14 days prior to the options' maturity and for all of the selection criteria, the annualized mean return of the OT portfolio exceeds the mean return of the IT portfolio by well over 1% and this difference is statistically significant at conventional levels for all of the portfolio selection criteria with the exception of the Sharpe ratio. It is especially large and significant under the gain/loss ratio and the maximization of $\hat{S}$ criteria. The volatility of the annualized return of the IT portfolio is 17.15% and is higher than the corresponding volatility of the OT portfolios. This improvement in performance for the 14-day options occurs in spite of higher proportional transaction costs with the bid–ask spread at 8.7%, on average, for the ATM options compared to 6.5% for the 28-day options for similar average levels for ATM IV for both maturities. We look at possible drivers of this difference in trading results in the next section.

For the portfolios constructed 7 days prior to the options' maturity and for all of the selection criteria, the annualized mean return of the OT portfolio exceeds the mean return of the IT portfolio by well over 1%. This difference is statistically significant at conventional levels for all of the portfolio selection criteria. It is especially large and significant under the gain/loss ratio and the maximization of $\hat{S}$. The volatility of the annualized return of the IT portfolio is 18.12% and is higher than the corresponding volatility of the OT portfolios. This improvement in performance for 7-day options occurs in spite of the 9.8% average ATM proportional spreads for the 7-day options. We look at possible drivers of this difference in trading results in the next section.

In unreported tests, we relax the assumption of lognormality used to derive the OT portfolios. For the underlying distribution, we use an Edgeworth tree as in Rubinstein (1998) with the excess kurtosis and skewness set at 0.5 and –0.5, respectively, roughly corresponding to the long-term empirical quantities for the S&P 500 Index. The increased accuracy in approximating the underlying distribution of the index improves, as expected, the results in comparison to the ones in Table 2, but the improvement in profitability is limited to about 0.3% per annum in the most extreme case of the 7-day options. For these reasons, we maintain lognormality as our base case.

The formal tests of SD resoundingly reject the null hypothesis of nondominance for the 28-, 14-, and 7-day portfolios and for all of the portfolio selection criteria with the weakest results for the 28-day options for which the null is rejected only at 5% significance for 5% trimming in the right tail. In the 28-day option portfolios, the dominance is achieved by keeping the same mean as IT, but reducing risk by shifting weight to the low return states. In most of the 14- and 7-day portfolios, the mean return significantly increases as well.

Note that the flexibility in portfolio choice achieved by the approach used in this paper finds dominating portfolios for almost all cross-sections. In contrast, Constantinides et al. (2011) identify OT dominance in only about one-third to one-half of the sample months. We conclude that the 28-, 14-, and 7-day OT portfolios stochastically dominate their IT counterparts. Furthermore, this conclusion is robust to all of the portfolio selection criteria.

These results remain unchanged when we drop the restriction on the maximal loss and allow the index to become worthless at its lowest support. In unreported results, we find that the realized returns significantly increase for all decision criteria for the 28-day options and become significant at the 5% level or better for all but the gain/loss ratio. Exactly the opposite takes occurs for the 14- and 7-day options for which the results worsen and become nonsignificant for all but the max $\hat{S}$ criterion. The realized returns remain essentially unchanged in terms of the SD tests and demonstrate OT dominance in all cases.

An indicator of the mispricing of each option cross-section is the Sharpe ratio of the OT portfolio. In Panel A of Figure 2, we present the time series of the Sharpe ratio of the OT portfolio for both the 28- and 14-day options in the case where we maximize the Sharpe ratio. The Sharpe ratio is persistent and follows the same pattern for both maturities. The similarities of the two graphs are remarkable given that they are derived 2 weeks apart and suggest that SD opportunities in the S&P 500 options are persistent.

Another indicator of the mispricing of each option cross-section is the annualized expected gain, $E_t[A(S_{t+1})^+]$. In Panel B of Figure 3, we present the time series of the expected gain for both the 28- and 14-day options in the case where we maximize the Sharpe ratio. The expected gain is evidently less persistent and is higher for the shorter maturity portfolios.

In the online Table A.2, we divide the option portfolios into terciles based on the expected gain, $E_t[A(S_{t+1})^+]$. The table demonstrates that the difference in the realized return between the OT and the IT portfolios is highest and SD is most prevalent when the expected gain is the highest.

## 6 | RELATION BETWEEN SD AND THE SMILE

In the online Tables A3–A5, we consider separately the SD tests for the low and high terciles of a given smile characteristic (ATM IV, left skew, and right skew) for 28-, 14-, and 7-day options, respectively. Overall, the power of the tests

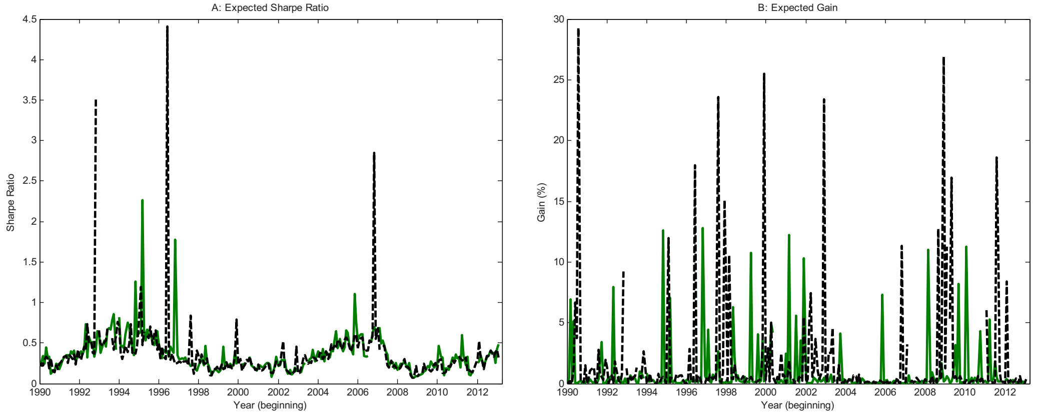


**FIGURE 3** Time series of maximized Sharpe ratio and expected gain [Color figure can be viewed at wileyonlinelibrary.com]

*Notes*: The figure displays the time series for expected Sharpe ratio and the corresponding expected gain. The solid lighter line corresponds to 28-day options, while the dashed darker line corresponds to 14-day options.

is diminished by the fact that we have only 92 observations in each tercile instead of 278 observations for the main results in Table 2.

For all maturity options, the mean difference of the annualized percentage return between the OT and IT portfolios ($\mu$) is significantly higher when the ATM IV is in the high tercile of the ATM IV. The formal tests of SD resoundingly reject the null hypothesis of nondominance for all maturity options and for all of the portfolio selection criteria. We conclude that SD is prevalent when the ATM IV is high. Similarly, we conclude that SD is prevalent when the right skew is low. Alternatively, the classification of portfolios according to the steepness of the left skew does not provide consistent results.

These results motivate our examination of the characteristics of the options that are included in the dominating portfolios. We find that most of the trading occurs in call options, consistent with the above conclusion that the right skew, but not the left one, influences our results.

# 7 | CHARACTERISTICS OF THE DOMINATING PORTFOLIOS

## 7.1 | Characteristics of the options in the zero-net-cost portfolios

In Table 3, we describe the composition of the zero-net-cost portfolios. The total number of contracts in each category is the sum of the absolute values of the number of long and short contracts. We present the results for the entire available sample period and before, during, and after the financial crisis. We provide the results for the Sharpe ratio portfolio selection criterion, defined in Section 4, as the unreported results for the other criteria are qualitatively similar.

For the 28-, 14-, and 7-day options over the whole sample period and in the subperiod before the financial crisis, the total number of call contracts is more than double the number of put contracts. In the subperiod after the financial crisis, the total number of call contracts is more than double the number of put contracts for the 14- and 7-day options, but the total number of call contracts is about the same as the number of put contracts for the 28-day options.

For the 28-, 14-, and 7-day options over the whole sample period and in the subperiods before and after the financial crisis, the call positions are overwhelmingly short positions. Thus, the OT investor creates the dominating portfolio by primarily writing calls, consistent with the observation that portfolio managers often write covered calls. The put positions are evenly divided between the long and short positions before the crisis and are primarily short positions

**TABLE 3** Composition of option portfolios

| Option maturity (days) | Total number of call contracts | Number of short call contracts | Number of long call contracts | Total number of put contracts | Number of short put contracts | Number of long put contracts |
|---|---|---|---|---|---|---|
| January 1990–February 2013 ($N = 278$) | | | | | | |
| 28 | 0.72 | 0.60 | 0.12 | 0.27 | 0.17 | 0.10 |
| 14 | 0.79 | 0.69 | 0.10 | 0.18 | 0.10 | 0.08 |
| 7 | 0.86 | 0.72 | 0.13 | 0.12 | 0.06 | 0.06 |
| January 1990–October 2008 ($N = 220$) | | | | | | |
| 28 | 0.79 | 0.65 | 0.14 | 0.21 | 0.11 | 0.10 |
| 14 | 0.81 | 0.70 | 0.11 | 0.17 | 0.09 | 0.08 |
| 7 | 0.87 | 0.73 | 0.14 | 0.12 | 0.06 | 0.06 |
| November 2008–October 2009 ($N = 12$) | | | | | | |
| 28 | 0.28 | 0.25 | 0.03 | 0.72 | 0.67 | 0.05 |
| 14 | 0.47 | 0.41 | 0.06 | 0.44 | 0.36 | 0.09 |
| 7 | 0.73 | 0.65 | 0.09 | 0.17 | 0.12 | 0.05 |
| November 2009–February 2013 ($N = 46$) | | | | | | |
| 28 | 0.50 | 0.46 | 0.04 | 0.44 | 0.32 | 0.12 |
| 14 | 0.78 | 0.69 | 0.09 | 0.15 | 0.08 | 0.08 |
| 7 | 0.83 | 0.71 | 0.12 | 0.08 | 0.04 | 0.04 |

*Notes*: We present results for the entire available sample period and before, after, and during the financial crisis. The total number of contracts in each category is the sum of the absolute values of the number of long and short contracts. We present the results when the selection criterion is the Sharpe ratio.

after the crisis. Thus, calls are more overpriced than puts despite the steep IV skew and consistent with the earlier findings in Constantinides et al. (2009, 2011).

The 12-month period from November 2008 to October 2009 of the financial crisis is different. The total number of put positions is double the number of call positions. Furthermore, the put positions are overwhelmingly short positions. Our interpretation is that during the crisis, prices overreacted to the prospect of a financial disaster and the slope of the skew steepened to the point that it became attractive to the OT investor to write overpriced puts rather than calls.

For the 14- and 7-day options, we observe a gradual decrease in put trading as the maturity gets shorter. This decrease is especially pronounced during the period of the financial crisis when we observe the majority of trading in put options for 28 days to maturity, about equal trading in calls and puts for 14 days to maturity, and only a small fraction of trading in put options for 7 days to maturity. We also find a gradual increase in short call positions as the maturity gets shorter. OT put usage goes down sharply with maturity, especially during the crisis. Because we keep the lowest support at 0.6 for all maturities, it is obvious that going down that far is going to be much less probable in shorter time resulting in lower priced protective puts.

In Table 4, we present the percentage of the months out of the total number of sample months in which zero, one, two, or three types of 28-day options are included in the optimal zero-net-cost portfolios. For example, in the first row of the table, no OTM short calls are included in 11.7% of the sample months. OTM short calls with two different strike-to-price ratios are included in 5.5% of the sample months. ATM calls are included in the category of ITM calls and ATM puts are included in the category of ITM puts. We present the results when the selection criterion is the Sharpe ratio. We obtain similar results when we use the other selection criteria.

Over the entire sample period and in the subperiods before and after the financial crisis, most of the calls and puts in the optimal zero-net-cost portfolios are OTM options as opposed to ITM options (with ATM options included in the

**TABLE 4** Frequency of 28-day options in the OT portfolios

| Option type | 0 | 1 | 2 | 3 | >0 |
|---|---|---|---|---|---|
| January 1990–February 2013 (*N* = 278) | | | | | |
| Short calls OTM | 11.7 | 82.8 | 5.5 | 0.0 | 88.3 |
| Short calls ITM | 69.2 | 28.6 | 2.2 | 0.0 | 30.8 |
| Long calls OTM | 43.2 | 41.4 | 15.4 | 0.0 | 56.8 |
| Long calls ITM | 93.4 | 6.6 | 0.0 | 0.0 | 6.6 |
| Short puts OTM | 34.4 | 59.0 | 6.6 | 0.0 | 65.6 |
| Short puts ITM | 89.7 | 9.9 | 0.4 | 0.0 | 10.3 |
| Long puts OTM | 31.5 | 57.9 | 10.3 | 0.4 | 68.5 |
| Long puts ITM | 78.0 | 21.6 | 0.4 | 0.0 | 22.0 |
| January 1990–October 2008 (*N* = 220) | | | | | |
| Short calls OTM | 7.0 | 87.4 | 5.6 | 0.0 | 93.0 |
| Short calls ITM | 71.6 | 26.0 | 2.3 | 0.0 | 28.4 |
| Long calls OTM | 35.3 | 46.5 | 18.1 | 0.0 | 64.7 |
| Long calls ITM | 91.6 | 8.4 | 0.0 | 0.0 | 8.4 |
| Short puts OTM | 40.9 | 57.7 | 1.4 | 0.0 | 59.1 |
| Short puts ITM | 87.4 | 12.1 | 0.5 | 0.0 | 12.6 |
| Long puts OTM | 36.7 | 53.5 | 9.3 | 0.5 | 63.3 |
| Long puts ITM | 80.9 | 18.6 | 0.5 | 0.0 | 19.1 |
| November 2008–October 2009 (*N* = 12) | | | | | |
| Short calls OTM | 22.2 | 71.1 | 6.7 | 0.0 | 77.8 |
| Short calls ITM | 55.6 | 42.2 | 2.2 | 0.0 | 44.4 |
| Long calls OTM | 71.1 | 24.4 | 4.4 | 0.0 | 28.9 |
| Long calls ITM | 100.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Short puts OTM | 13.3 | 68.9 | 17.8 | 0.0 | 86.7 |
| Short puts ITM | 97.8 | 2.2 | 0.0 | 0.0 | 2.2 |
| Long puts OTM | 13.3 | 73.3 | 13.3 | 2.2 | 88.9 |
| Long puts ITM | 77.8 | 22.2 | 0.0 | 0.0 | 22.2 |
| November 2009–February 2013 (*N* = 46) | | | | | |
| Short calls OTM | 58.3 | 41.7 | 0.0 | 0.0 | 41.7 |
| Short calls ITM | 75.0 | 25.0 | 0.0 | 0.0 | 25.0 |
| Long calls OTM | 83.3 | 8.3 | 8.3 | 0.0 | 16.7 |
| Long calls ITM | 100.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Short puts OTM | 0.0 | 41.7 | 58.3 | 0.0 | 100.0 |
| Short puts ITM | 100.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Long puts OTM | 40.0 | 55.6 | 2.2 | 4.4 | 62.2 |
| Long puts ITM | 77.8 | 22.2 | 0.0 | 0.0 | 22.2 |

*Notes*: We present the percentage of the months out of the total number of sample months in which zero, one, two, or three types of options are included in the optimal zero-net-cost portfolio. For example, in the first row of the table, OTM short calls with two different strike-to-price ratios are included in 5.1% of the sample months. ATM calls (puts) are included in the category of ITM calls (puts). We present the results when the selection criterion is the Sharpe ratio.

category of ITM options). This is consistent with the observation that OTM options are more liquid than ITM options and the volume of trading in OTM options are higher than the volume of ITM options.

Over the whole sample period and in the subperiods before and after the financial crisis, the OT investor primarily transfers payoffs from the high market return states to the low market return states by writing OTM calls. In most months, the OT investor writes only one and, at most, two types of OTM calls. In the subperiod before the financial crisis, second in frequency of options included in the optimal zero-net-cost portfolios are long positions in OTM puts and calls. In the subperiod after the financial crisis, second in frequency of options included in the optimal zero-net-cost portfolio are long positions in OTM calls and long and short positions in OTM puts. Here, again, most often only one type of option in each category is included in the portfolio. Of interest is also the observation that long and short positions in OTM puts are approximately equal in size implying that the net OTM put position is hedged at the left tail of the index support as required for SD. The put options appear to be aimed primarily at achieving a desirable payoff pattern at low and intermediate parts of the support.

As we observed earlier, the period of the financial crisis is different. The OT investor primarily writes OTM puts. Second in frequency of options included in the optimal zero-net-cost portfolio are short ITM and OTM calls and long OTM puts. The long puts, both OTM and ITM, combine with the OTM puts to establish the net nonnegative position at the left tail.

Over the entire sample period and in all of the subperiods, the option portfolios are parsimonious. For example, over the whole sample period, OTM short calls of only one strike-to-price ratio appear in 82.8% of the sample months, but only in 5.5% of the sample months do OTM short calls with two or more different strike-to-price ratios appear in the option portfolios. The same observation applies to all of the other categories of options, ITM short calls, OTM long calls, and so on. The results for the 14- and 7-day options are similar to the results reported in Table 4 for the 28-day options and are not shown.

In unreported results, we allow the OT portfolio to include a long or short position in the index, over and above the initial position of one index unit, and include it as part of the payoff $A(S_{t+1})$ at option maturity. A priori, we do not expect this variant of the program to result in additional acquisitions of the index as it would increase the left tail risk, contrary to the objectives of SD. It may, however, potentially increase the attractiveness of writing puts.[15] This relaxation of our program brings in fact little change in the OT portfolio composition. Depending upon the portfolio selection criterion, the OT portfolio does include a short position in the index at one or two dates in our sample with these dates corresponding to the period of the financial crisis. We also estimate the portfolios after eliminating the 0.25% transaction cost in the index. The main result is that the OT portfolios now contain positions in short index units that are about 10%–13%, on average, which allow some increase in the number of puts. The latter remain, however, much lower than the calls in the OT composition. Otherwise, the excess returns of the OT portfolios remain approximately the same even though their standard errors decrease and they become significantly positive everywhere.

## 7.2 | Option characteristics in relation to the IV Smile

In Table 5, we present the characteristics of calls in relation to the ATM IV, left skew, and right skew from January 1990 to February 2013 period. We present the results for the Sharpe ratio portfolio selection criterion. Left (right) skew is 5% OTM IV (5% ITM IV) net of the ATM IV for a given cross-section. The total number of contracts in each category is the sum of the absolute values of the number of long and short contracts. Each tercile for a given smile characteristic corresponds to 92 observations.

In Table 4, we demonstrated that most of the calls in the optimal zero-net-cost portfolios are short contracts as opposed to long ones. In Table 5, we establish that this conclusion is robust to the ATM IV and the size of the left and right skew. The number of short calls is higher when the ATM IV is low and/or the right skew is high. Furthermore, the

[15] In principle, an increased quantity of the underlying may increase the attractiveness of writing calls. However, with our restriction of at most one call, there is no need to carry more of the underlying to arrive at the stochastically dominating payoff.

**TABLE 5** Characteristics of calls in relation to the smile: January 1990–February 2013

| | Lowest tercile | | | Highest tercile | | |
|---|---|---|---|---|---|---|
| Option maturity (days) | Total number of contracts | Number of short contracts | Number of long contracts | Total number of contracts | Number of short contracts | Number of long contracts |
| ATM IV | | | | | | |
| 28 | 0.92 | 0.72 | 0.20 | 0.44 | 0.40 | 0.04 |
| 14 | 0.90 | 0.78 | 0.12 | 0.64 | 0.57 | 0.06 |
| 7 | 0.88 | 0.74 | 0.14 | 0.83 | 0.71 | 0.12 |
| Left skew | | | | | | |
| 28 | 0.79 | 0.65 | 0.14 | 0.64 | 0.52 | 0.11 |
| 14 | 0.78 | 0.69 | 0.09 | 0.76 | 0.67 | 0.09 |
| 7 | 0.89 | 0.74 | 0.15 | 0.82 | 0.69 | 0.13 |
| Right skew | | | | | | |
| 28 | 0.48 | 0.43 | 0.06 | 0.91 | 0.73 | 0.18 |
| 14 | 0.63 | 0.57 | 0.06 | 0.92 | 0.79 | 0.14 |
| 7 | 0.81 | 0.70 | 0.12 | 0.90 | 0.74 | 0.15 |

*Note:* Left (right) skew is 5% OTM IV (5% ITM IV) net of the ATM IV for a given cross-section. The total number of contracts in each category is the sum of the absolute values of the number of long and short contracts. Each tercile for a given smile characteristic corresponds to 92 observations. We present the results when the selection criterion is the Sharpe ratio.

number of calls in the OT portfolio, both long and short, increases or stays the same in all cases as the maturity gets shorter.

In Table 6, we present the corresponding characteristics for puts. The number of short and long puts is higher when the ATM IV is high and/or the right skew is low. Here, the number of puts in the OT portfolio varies inversely with the number of calls decreasing or staying the same in all cases for shorter maturities. This indicates that put overpricing is much less pronounced for shorter maturities because the probability that there will be a crash until option expiration is correspondingly lower. It is probable that the parallel increase in the number of calls noted in Table 5 takes place because the constraint in the total number of options in the OT portfolio is now less binding on the calls that remain overpriced.

Because over the whole sample period and in the subperiods before and after the financial crisis the OT investor primarily transfers payoffs from the high market return states to the low market return states by writing OTM calls, we address the question whether all types of options are needed for profitable OT portfolios. First, we allow trades in only short calls, but not in long calls, long puts, and short puts without changing the program objectives. Then, we allow trades in both short and long calls, but not in puts. In unreported results, we find that with only short calls, the results are weak. The inclusion of long calls improves the results, but there are still fewer feasible portfolios (234, 264, and 260, respectively, for the 28-, 14-, and 7-day options) than in Table 2. Thus, trading in long calls and in puts significantly enhances the performance of the OT portfolios, even though these options are relatively less important than OTM calls in most months.

## 7.3 | Are the options in the OT portfolios special?

Our results beg the question as to whether the options in the optimal portfolios are special. Recall that in the “first pass” columns of Table 1, we find that the average number of separate contracts in the option portfolios is low exceeding four in only one case. This is surprising given the number of available options in each cross-section. For the 28-day options, this number rises from 16.4 in the beginning of the sample period to more than four times as much in the later periods.

**TABLE 6** Characteristics of puts in relation to the smile: January 1990–February 2013

| | Lowest tercile | | | Highest tercile | | |
|---|---|---|---|---|---|---|
| Option maturity (days) | Total number of contracts | Number of short contracts | Number of long contracts | Total number of contracts | Number of short contracts | Number of long contracts |
| ATM IV | | | | | | |
| 28 | 0.08 | 0.04 | 0.04 | 0.54 | 0.37 | 0.17 |
| 14 | 0.10 | 0.05 | 0.05 | 0.29 | 0.16 | 0.13 |
| 7 | 0.10 | 0.05 | 0.06 | 0.15 | 0.08 | 0.07 |
| Left skew | | | | | | |
| 28 | 0.20 | 0.12 | 0.08 | 0.35 | 0.24 | 0.12 |
| 14 | 0.19 | 0.11 | 0.08 | 0.22 | 0.12 | 0.10 |
| 7 | 0.08 | 0.04 | 0.03 | 0.14 | 0.07 | 0.08 |
| Right skew | | | | | | |
| 28 | 0.51 | 0.34 | 0.17 | 0.09 | 0.05 | 0.04 |
| 14 | 0.29 | 0.17 | 0.13 | 0.08 | 0.04 | 0.04 |
| 7 | 0.15 | 0.08 | 0.07 | 0.07 | 0.03 | 0.04 |

*Notes*: Left (right) skew is 5% OTM IV (5% ITM IV) net of the ATM IV for a given cross-section. The total number of contracts in each category is the sum of "the absolute values" of the number of long and short contracts. Each tercile for a given smile characteristic corresponds to 92 observations. We present the results when the selection criterion is the Sharpe ratio.

In other words, the option portfolios contain more than 20% of the available options in the earlier cross-sections, but a much lower proportion in the later ones.

We address whether our search for mispriced options exhausts the options that produce dominating OT portfolios by removing the options that are included in the optimal portfolio, the first pass options from each cross-section, and repeating the search with the remaining options in the cross-section. Only the cross-sections with at least three calls and puts are admitted in the sample. The results are shown in online Part B. They are weaker than the main results in Table 2, yet they remain strong for the 14-day and 7-day options. The statistical tests reject the null of nondominance in all cases for these two sets of options. However, for the 28-day options, the null of nondominance is rejected only for 10% trimming in the right tail with the low overall means indicating little stability in the results.

## 7.4 | Does the OT investor achieve higher returns by simply trading outliers from the IV Smile?

A potential criticism of the SD of the OT portfolios over the IT portfolios is that the OT investor essentially adopts the common trading strategy of buying options that lie below the IV smile and writing options that lie above it. Recall that a higher mean return of the OT portfolio over the IT portfolio is a necessary, but not sufficient condition for SD. We examine whether the observed higher mean return of the OT portfolios over the IT portfolios is attained by the OT investor's buying options that lie below the IV skew and writing options that lie above it.

In each cross-section, we regress the spread midpoint of the $IV_i$ of all options that pass all our filters except for the moneyness filter on each option's moneyness, $K_i/S_t$, and its squared value: $IV_i = a + b(K_i/S_t) + c(K_i/S_t)^2 + e_i$. We run separate regressions for puts and calls.

In Table 7, we report the means and $t$-stats of the residuals of the options that enter the optimal portfolios in units of the annualized IV in percentage points. For example, for short calls in the first line of the table, the mean residual 0.09 implies that short calls in the optimal portfolio have, on average, annualized IV 0.09% above the skew. For all three maturities, we observe similar average positive residuals. However, the corresponding standardized residuals

**TABLE 7** Regression residuals of options in the optimal portfolios: January 1990–February 2013

| Option type | Number of observations | Residuals | | Standardized residuals | |
|---|---|---|---|---|---|
| | | Mean | t-stat | Mean | *t*-stat |
| 28-Day options | | | | | |
| Short call | 346 | 0.09 | 1.47 | −0.02 | −0.35 |
| Long call | 469 | −0.17 | −3.37 | −0.12 | −2.48 |
| Short put | 226 | 0.19 | 2.89 | 0.13 | 1.46 |
| Long put | 278 | −0.35 | −8.14 | −0.33 | −7.68 |
| 14-Day options | | | | | |
| Short call | 371 | 0.07 | 1.38 | −0.04 | −0.52 |
| Long call | 482 | −0.18 | −3.85 | −0.27 | −3.76 |
| Short put | 203 | 0.17 | 2.74 | 0.17 | 1.38 |
| Long put | 225 | −0.32 | −5.74 | −0.35 | −5.78 |
| 7-Day options | | | | | |
| Short call | 340 | 0.08 | 1.39 | −0.07 | −0.58 |
| Long call | 512 | −0.15 | −3.38 | −0.30 | −3.24 |
| Short put | 135 | 0.18 | 2.41 | 0.10 | 0.69 |
| Long put | 157 | −0.33 | −5.35 | −0.37 | −3.70 |

*Notes*: The table presents the means and t-stats of the residuals of the options that enter the optimal portfolios in units of the annualized IV in percentage points. For example, for short calls in the first line of the table, the mean residual 0.09 indicates that short calls in the optimal portfolio have, on average, annualized IV 0.09% above the skew. The standardized residuals are standardized each month by the standard deviation of the residuals in that month.

are slightly below zero implying that the main ingredient of the zero net-cost portfolios is, on average, located almost exactly at the smile. For all three maturities, short puts have, on average, positive residuals, whereas long calls and puts have, on average, negative residuals. For all three maturities, residuals are generally larger for short puts. However, as we argued previously, these categories of options bear less weight in zero-net-cost portfolios than short calls. The residuals are standardized each month by the standard deviation of the residuals in that month.

For all three maturities, the mean values of the residuals are positive for short positions and negative for long ones, consistent with the above conjecture. The standardized residuals are statistically significant for the long calls and long puts. However, we argue below that these results are economically insignificant.

Consider first the short calls in the 28-day option portfolios. These calls have a mean residual 0.09. Assuming that the index price is one, the annualized volatility of the index is 18%, the dividend yield is 2%, the risk-free rate is 2%, and the Vega of a 28-day ATM European call is 0.0011. That is, the call is overpriced by $0.09 \times 0.0011 = 0.000099\%$. If the OT investor were to write the maximum allowed number of such calls (one call per index unit), the sale of these calls would increase the annualized excess return of the OT portfolio by $12 \times 0.000099 = 0.12\%$.

We perform the corresponding calculations for the long calls, short puts, and long puts. If the OT investor were to buy the maximum allowed number of calls (one call per index unit), the purchase of these calls would increase the annualized excess return of the OT portfolio by 0.2345%. If the OT investor were to buy the maximum allowed number of puts, the purchase of these puts would increase the annualized excess return of the OT portfolio by 0.4827%. Additionally, if the OT investor were to write the maximum allowed number of puts, the writing of these puts would increase the annualized excess return of the OT portfolio by 0.2650%. From the first row of Table 6, we know that the OT investor writes, on average, 0.60 calls, buys 0.12 calls, writes 0.17 puts, and buys 0.10 puts. Therefore, the contribution to the higher annual mean return of the OT portfolios in buying options that lie below the IV skew and writing options that lie above it is about $0.60 \times 0.1188 + 0.12 \times 0.2345 + 0.17 \times 0.2620 + 0.10 \times 0.4827 = 0.19\%$. This contribution is small compared to the difference between the mean return of the OT and IT portfolios, $\mu$, in Table 2.

Repeating these calculations for the 14- and 7-day options produces the respective contributions of 0.21% and 0.29%, which yield even smaller fractions of the overall excess return of the OT portfolios. For the 7-day options, the contribution is 0.79%, which is large, but still a fraction of the overall excess return of the OT portfolios. We conclude that even though "expensive" or "cheap" options contribute to the excess return of the OT portfolios, they are not the main driver of the SD results.

Apart from the fact that our portfolios incorporate realistic trading prices, which are not represented by the mid-point prices used in the above exercise, they are chosen strategically in order to shift the probability weight to the "low" states of the index return and attain superior risk-adjusted expected returns. A policy of trading outliers of the IV skew, even if it can be achieved at zero cost, is unlikely to achieve ex post OT dominance without such features.

# 8 | EXCESS RETURN OF THE OPTION PORTFOLIO REWARD FOR RISK

Our SD tests are based on the maintained hypothesis that the index return is the only priced factor and the stochastic discount factor is monotone decreasing in the index return. This maintained hypothesis further implies that the documented excess returns of the OT portfolios, adjusted for market risk, are positive. In this section, we test whether the excess returns of the OT portfolios are rewards for risks that are not taken into account in our theoretical setup. We consider the three factors in Fama and French (1993), "Jump," "Volatility Jump," "Volatility," and "Liquidity," which have been shown in Constantinides, Jackwerth, and Savov (2013), to explain the cross-section of S&P 500 option returns, and the Christoffersen et al. (2013) extension of the Heston and Nandi (2000) stochastic discount factor, which is U-shaped in index returns.

## 8.1 | Construction of option portfolio returns over calendar months

We construct 28-day option portfolio returns with a holding period that approximately coincides with the calendar months.[16] At the beginning of a month, we construct the OT options portfolio by buying and writing options that were originally issued as 28-day options. One, 2, and 3 weeks later, we close out our positions (or exercise them if they expire and are in the money) and construct a new OT options portfolio by buying and writing options that were originally issued as 28-day options. The 1-month excess return of the OT options portfolio, $r_{\mathrm{OT},t}$, is the sum of the cash flows of these trades divided by the index value at the end of the previous month.

The OT portfolios are chosen in the presence of transaction costs as in the previous sections and then the bid or ask price at which an option is written or purchased is replaced by the corresponding bid–ask midpoint, as is commonly done in this literature. Unlike earlier studies, we do not assume that the put–call parity holds as to not to interfere with the option market data. Because the short option positions now have higher prices and the long positions have lower prices, the resulting realized excess OT payoffs are higher by about one-half the bid–ask spread.

## 8.2 | Factor-adjusted option portfolio returns

For the first test, we adjust the excess returns of the OT options portfolio with the three factors in Fama and French (1993) by running the time series regression:

$$r_{\mathrm{OT},t} = \alpha + \beta_{\mathrm{M}} r_{\mathrm{M},t} + \beta_{\mathrm{SMB}}\, r_{\mathrm{SMB},\,t} + \beta_{\mathrm{HML}}\, r_{\mathrm{HML},\,t} + \varepsilon_t,$$

[16] For our earlier results, we construct the OT option portfolios of the 28-day options by buying or writing the options when they are 28 days to maturity that is at the end of the third week of the month and holding them until expiration at the end of the third week of the following month. Thus, the holding period of the 28-day options does not coincide with a calendar month. We also construct the OT option portfolios of the 21-, 14-, and 7-day options in a similar fashion. The OT option portfolios of the 21- and 14-day options may or may not expire within the same calendar month, but the 7-day options expire within the same calendar month. For our earlier results, it is immaterial that the holding period does not coincide with a calendar month.

**TABLE 8** Excess returns adjusted for risk with the Three Fama–French factors: January 1990–February 2013

| $\alpha$ | $\beta_M$ | $\beta_{SMB}$ | $\beta_{HML}$ |
|---|---|---|---|
| 0.64 | −0.09 | −0.02 | −0.06 |
| (0.07) | (0.02) | (0.01) | (0.03) |

*Notes*: The excess returns of the OT options portfolio are adjusted for risk with the three factors in Fama and French (1993) by running the time-series regression $r_{OT,t} = \alpha + \beta_M r_{M,t} + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \varepsilon_t$, where $r_{OT,t}, r_{M,t}, r_{SMB,t}$,and $r_{HML,t}$are the excess returns of the OT, market, small-minus-big, and high book-to-market minus low book-to-market portfolios in month $t$. Standard errors are in parentheses and Newey-West (1987) corrected using four lags.

**TABLE 9** Excess returns adjusted for risk with the Constantinides et al. factors: January 1990–February 2013

| Jump | Volatility jump | Volatility | Liquidity |
|---|---|---|---|
| Risk premia estimated from equities | | | |
| 0.75 | 0.41 | 0.67 | 0.65 |
| (0.07) | (0.07) | (0.07) | (0.07) |
| Risk premia estimated from options | | | |
| 0.88 | 0.35 | 0.69 | 0.72 |
| (0.07) | (0.07) | (0.07) | (0.07) |

*Notes*: The excess returns of the OT options portfolio are adjusted for market risk and one of the factors "Jump," "Volatility Jump," "Volatility," or "Liquidity" as $\overline{r_{OT,t}} - \beta_M \overline{r_{M,t}} - \beta_f \lambda_f$, where the risk prices of these factors, $\lambda_{Jump}, \lambda_{VolJump}, \lambda_{Vol}$, and $\lambda_{Liq}$, are estimated in Constantinides et al. (2013) either from the universe of equities or from the universe of index options. Standard errors are in parentheses and Newey-West (1987) corrected using four lags.

where $r_{OT,\,t}$, $r_{M,\,t}, r_{SMB,\,t}$, and $r_{HML,\,t}$ are the excess returns of the OT, market, small-minus-big (SMB), and high book-to-market minus low book-to-market (HML) portfolios in month $t$ and $\alpha$ is the risk-adjusted average excess return of the OT portfolio. Table 8 indicates that the risk-adjusted average excess return of the OT portfolio is positive and highly significant partly because the factor loadings on the SMB and HML factors are small and marginally significant. We conclude that the three-factor model in Fama and French (1993) does not explain away the average excess return of the OT portfolio.

For the second set of tests, we adjust the excess returns of the OT options portfolio with the market and, each one in turn, the factors Jump, Volatility Jump, Volatility, and Liquidity that have been shown in Constantinides et al. (2013) to explain the cross-section of S&P 500 option returns. In the first stage, we estimate the factor loading $\beta_f$ from the time series regression $r_{OT,t} = \alpha + \beta_M r_{M,t} + \beta_f f_t + \varepsilon_t$, where $r_{OT,t}$ and $r_{M,t}$ are the excess returns of the OT and market portfolios and $f_t$ is the realization of the factor Jump, Volatility Jump, Volatility, or Liquidity. The risk prices of these factors, $\lambda_{Jump}$, $\lambda_{VolJump}, \lambda_{Vol}$ , and $\lambda_{Liq}$, are estimated in Constantinides et al. (2013) over the same time period either from the universe of equities or from the universe of index options. In the second stage, we estimate the risk-adjusted average excess portfolio return, $\overline{r_{OT,t} - \beta_M r_{M,t} - \beta_f \lambda_f}$. Table 9 indicates that the risk-adjusted average excess return of the OT portfolios is positive and highly significant in all cases. We conclude that the Constantinides et al. (2013) model does not explain away the average excess return of the OT portfolios.

## 8.3 | Option portfolio returns adjusted for risk and valued with the Christoffersen et al.'s stochastic discount factor

Jackwerth (2000) was the first to introduce a nonmonotonic stochastic discount factor to explain option prices. Nonmonotonicity under the form of a U-shaped stochastic discount factor is, in fact, the explanation given for the anomalous in-sample results for individual options in Beare (2011), Constantinides et al. (2009) by Bakshi et al. (2010), and Beare and Schmidt (2016). A U-shaped stochastic discount factor captures an important empirical observation for

**TABLE 10** Excess returns adjusted for risk with the Christoffersen et al. stochastic discount factor: January 1990–February 2013

| Option maturity (days) | Excess return | | | | Risk-adjusted excess return | | | |
|---|---|---|---|---|---|---|---|---|
| | $\mu$ | *t*-stat | *t*-stat 95% LCI | *t*-stat 95% UCI | $\mu$ | *t*-stat | *t*-stat 95% LCI | *t*-stat 95% UCI |
| | A: Index returns | | | | | | | |
| 28 | 0.78 | 2.68 | −1.89 | 2.05 | $-2.2 \times 10^7$ | −1.00 | −1.16 | $2.3 \times 10^5$ |
| 21 | 0.93 | 2.72 | −1.83 | 2.11 | $-1.5 \times 10^7$ | −1.00 | −1.16 | $6.8 \times 10^5$ |
| 14 | 0.93 | 2.17 | −1.90 | 2.03 | $-2.3\text{E} \times 10^4$ | −1.00 | −1.16 | $2.8 \times 10^3$ |
| 7 | 1.65 | 2.58 | −1.90 | 2.03 | $-1.9 \times 10^4$ | −1.00 | −1.16 | $1.5 \times 10^3$ |
| | B: OT portfolios | | | | | | | |
| 28 | 5.53 | 2.06 | −2.53 | 1.65 | $5.8 \times 10^6$ | 1.00 | $-5.2 \times 10^4$ | 1.16 |
| 21 | 7.40 | 4.74 | −2.52 | 1.70 | $2.1 \times 10^6$ | 1.00 | $-1.1 \times 10^5$ | 1.16 |
| 14 | 6.77 | 2.63 | −1.56 | 2.66 | $1.3 \times 10^4$ | 1.01 | $-2.1 \times 10^2$ | 1.17 |
| 7 | 18.00 | 1.23 | −5.47 | 1.34 | $2.1 \times 10^5$ | 1.00 | $-9.4 \times 10^3$ | 1.16 |

*Notes*: The table displays the average monthly excess return and the risk-adjusted excess return of the index and the OT portfolio in percent. Confidence intervals on *t*-statistics are derived via a bootstrap procedure as in Christoffersen et al. (2013, table 2).

extreme positive index returns. These returns occur when the overall volatility is very high in situations after a market crash, as it happened for about a year following the recent financial crisis.

We examine whether the U-shaped stochastic discount factor correctly prices the identified OT portfolios. We apply the Christoffersen et al. (2013) extension of the Heston and Nandi (2000) stochastic discount factor, which is U-shaped in index returns and may potentially explain away the average excess return of the OT portfolio. This is the only such model available in the literature containing closed-form expressions for both options and the stochastic discount factor. We present the essential features of this model and defer the technical details to the appendix. The key feature of this model is that only one parameter, $\xi$, is defined by option market data, while the remaining parameters are extracted from the underlying market.

We start by estimating the parameters of the real distribution with GARCH on the daily returns. Because the U-shaped stochastic discount factor depends upon the entire volatility path from the beginning of the month until the option expiration, we filter out the realized volatilities from the realized index returns given that they both depend on the same random factor. We then determine the extra volatility pricing parameter $\xi$ needed to price the entire universe of options from the closed-form expressions and the observed option prices. This parameter is chosen by maximizing the likelihood for our data. The results for the smaller value of this parameter found in Christoffersen et al. (2013) are qualitatively similar. To compute the realized stochastic discount factor at time $t$, $\mathrm{SDF}_t$, we multiply the realized volatility by $\xi$.

Table 10 presents the average risk-adjusted excess return of the OT portfolios, $\overline{\mathrm{SDF}_t \times r_{\mathrm{OT},t}}$, where $r_{\mathrm{OT},t}$ is the realization of the excess return of the OT portfolios at time $t$. The risk-adjusted excess returns on the OT portfolios assume extreme positive values due to the extreme variation in the SDF for the realized paths of the conditional volatility whose variation results in all of the $t$-statistics in the proximity of one. Moreover, the stochastic discount factor is unable to correctly price the index whose risk-adjusted returns assume extreme negative values. We conclude that the Christoffersen et al. (2013) stochastic discount factor does not explain away the average excess return of the OT portfolios.

Finally, we examine whether the observed OT option prices are consistent with the U-shaped stochastic discount factor as extracted from the entire option market data. For this, it suffices to compare the predicted model prices with the realized ones in an exercise that parallels the one conducted by Christoffersen et al. (2013, table 2 and pp. 1992–

**TABLE 11** Frictionless returns on optimal portfolios and straddles: January 1990–February 2013

| Option maturity (days) | $\mu_{\text{MARKET}}$ | $\mu_{\text{MODEL}}$ | 95% LCI | 95% UCI |
|---|---|---|---|---|
| OT portfolios | | | | |
| 28 | 5.59** | 20.35*** | 0.64 | 11.13 |
| 21 | 7.77*** | 20.55*** | 4.55 | 10.65 |
| 14 | 7.36*** | 15.02*** | 1.20 | 11.37 |
| 7 | 18.28* | 43.91*** | −3.49 | 50.76 |
| Straddles | | | | |
| 28 | 0.61*** | 2.08** | 0.23 | 0.95 |
| 21 | 0.48** | 1.92*** | 0.04 | 0.87 |
| 14 | 0.83*** | 1.99*** | 0.33 | 1.28 |
| 7 | 1.38*** | 1.94*** | 0.65 | 2.12 |

*Notes*: The table displays monthly frictionless average excess returns on OT portfolios and straddles, as well as 95% bootstrap confidence intervals for market returns in percent. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively, via bootstrap with 10,000 draws. *p*-Values for this test are consistent with bootstrapping *t*-statistics as in Christoffersen et al. (2013, table 2).

1994) for a policy of shorting straddles that is found to be highly profitable in several studies.[17] In that exercise, the authors consider short straddles worth 10% of the underlying index and conclude that the straddle returns at model prices are consistent with those observed in the market as they are within the 95% bootstrapped confidence interval around the market returns.

Table 11 presents the realized average returns at market and model prices for the OT portfolios and straddles, as well as the 95% confidence intervals around the market returns bootstrapped as in Christoffersen et al. (2013, table 2). Because these returns consist of the initial portfolio prices and the realized payoffs that are identical in both cases, the difference between Columns 2 and 3 in all of the panels comes solely from the market and model prices. Recall also that the model prices are the expected payoffs of the options in the portfolios evaluated by the risk neutral$Q$ distribution of the returns and volatilities and the same value of the parameter $\xi$ as in Table 10. To preserve comparability with the Christoffersen et al. (2013, table 2) results, we normalize the OT portfolios to yield the same initial premium as the 10% of the underlying index collected via the short straddles at each date, while the portfolio excess returns are derived relative to one unit of index as before.

The highly significant straddle returns in our table can be compared to those of Christoffersen et al. (2013, p. 1982 and table 2) only for the 28-day options for which the Christoffersen et al. (2013) results are more than twice as high reflecting the post-crisis years in our data. In contrast, their model returns at 1.82% are of the same order of magnitude as ours in Column 3 of Panel B. Alternatively, it is clear that for the same collected premium, the optimal OT portfolios yield higher returns, but not necessarily lower *p*-values than the corresponding straddles. Similarly, in Panel C of the table, we find that the difference in means between the OT portfolios and straddles is always significant at 10% or better.

Most important, however, is the fact that the model prices lie far above the 95% confidence interval around the market price in the second column for the OT portfolios and the straddles, as well as their differences. This is in sharp contrast to the consistency between the market and model prices observed in the earlier study for the 28-day options. This inconsistency holds for the 28-, 21- and 14-day OT portfolios and straddles, and only for the highly volatile 7-day maturity options do the OT portfolio and straddle confidence intervals encompass the model prices. We conclude

[17] The literature on the anomalous straddle returns includes Broadie, Chernov, and Johannes (2007, 2009), Christoffersen, Heston, and Jacobs (2013), Coval and Shumway (2001), Driessen and Maenhout (2007), and Santa Clara and Saretto (2009).

that the observed option prices in the OT portfolios are not consistent with the Christoffersen et al. (2013) U-shaped stochastic discount factor.[18]

# 9 | ROBUSTNESS TESTS

## 9.1 | Mispriced OT portfolios for the CAC and DAX indices

In the online Table C1, we provide the results for the mispriced 28-, 14-, and 7-day OT portfolios extracted from options on the CAC and DAX indices, together with the contemporaneous sample of the January 2006–February 2013 S&P 500 options. For the weekly options, the sample starts in June 2006. Weekly options (i.e., S&P 500 and DAX options) that are listed about 1 week before the maturity on Fridays other than the third Friday of a given month are considered separately. We use the Sharpe ratio criterion and show all of the information presented for that criterion in Table 2, together with the sample sizes in each case and the dates where feasible dominating OT portfolios could be extracted.

The table indicates that the results for all of the indices are qualitatively similar, although the feasibility is somewhat reduced for the two European indices from the impressive 272 of 278 (approximately 98%) in the entire sample of the S&P 500 Index, which is virtually the same in the shorter sample. Still, the feasibility is more than 90% in all but one of the six cases in which it is 82%. The strong maturity effect on in-sample OT profitability is clear for all three indices, with the 7-day options recording an impressive excess return of over 5% everywhere, much higher than for the entire S&P 500 sample in Table 2. The similarity of the results for all three indices extends to the out-of-sample DD tests, which are not significant for the 28-day maturity OT portfolios, but strongly significant for the other two maturities. The size of the time series sample accounts for the difference between the first panel of Table 2 and the last panel of the online Table C1. We conclude that the mispricing effect of the OT portfolios dominating the index extends to the two other indices that are examined.

The results for the weekly options for the S&P 500 and DAX are similar to each other, but distinct from the results for the 7-day options. Weekly options exhibit both lower profitability and lower feasibility in relation to their 7-day counterparts. The profitability decreases from over 5% to about 2% per annum, whereas the feasibility decreases from well over 90% to about 75% for the S&P 500 and to 50% for DAX.

## 9.2 | Alternative volatility projections for the return distribution

Volatility projections are a key element of our SD tests. Our base case method is forward-looking in the sense that it uses the VIX, which is corrected on the basis of the observed average error for its well-known upward bias. Here, we also consider alternative projection methods based on two GARCH models, the Glosten, Jagannathan, and Runkle (1993) and the exponential EGARCH model of Nelson (1991), as well as an ad hoc random walk volatility model.

The following expressions indicate the assumed dynamics under the two daily GARCH models with $t$ and $t+1$ indicating two successive days and $h_t$ the variance at $t$.

GJR:

$$\ln\frac{S_{t+1}}{S_t} = \mu + \sqrt{h_t}\varepsilon_{t+1}, h_t = \omega + (\alpha + \gamma I_{t-1})\varepsilon_{t-1}^2 + \beta h_{t-1}, I_{t-1} = 0 \quad \text{if} \quad \varepsilon_{t-1} \geq 0, I_{t-1} = 1 \quad \text{if} \quad \varepsilon_{t-1} < 0,$$

and EGARCH:

$$\ln\frac{S_{t+1}}{S_t} = \mu + \sqrt{h_t}\varepsilon_{t+1}, \quad \ln \quad (h_t) = \omega + \alpha(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|) + \gamma\varepsilon_{t-1} + \beta \quad \ln \quad (h_{t-1}),$$

where $\varepsilon_{t+1}$ is the standard normal conditional on the information set at time $t$. For the GJR model, the following recursive expression allows the estimation of total return volatility over the life of the option: $h_{t+1} = \omega + (\alpha + \frac{\gamma}{2} + \beta)h_t$.

[18] In unreported results, we reverse the exercise and determine that the value of $\xi$ that correctly prices the OT portfolios generates highly biased model prices for the entire cross section.

**TABLE 12** Portfolio returns and stochastic dominance tests in relation to volatility forecast: January 1990–February 2013

| Option maturity (days) | Number of feasible dates | $\mu$ | $p$-Value for $\mu \leq 0$ | $\sigma_{OT}$ | $\sigma_{OT-IT}$ | DD test $p$-value | |
|---|---|---|---|---|---|---|---|
| | | | | | | 5% Trim | 10% Trim |
| A: GJR | | | | | | | |
| 28 | 218 | 0.45 | .170 | 15.50 | 2.57 | .029 | .000 |
| 14 | 215 | 1.23 | .019 | 16.34 | 1.91 | .000 | .000 |
| 7 | 222 | 1.53 | .082 | 17.04 | 2.60 | .000 | .000 |
| B: EGARCH | | | | | | | |
| 28 | 224 | 0.32 | .235 | 15.79 | 2.11 | .125 | .007 |
| 14 | 225 | 1.25 | .018 | 16.39 | 1.92 | .000 | .000 |
| 7 | 222 | 2.08 | .053 | 17.11 | 2.85 | .000 | .000 |
| C: Random walk | | | | | | | |
| 28 | 243 | −0.30 | .682 | 15.38 | 2.85 | 1.000 | 1.000 |
| 14 | 235 | 1.27 | .005 | 16.61 | 1.51 | .010 | .000 |
| 7 | 239 | 0.93 | .176 | 17.33 | 2.25 | .000 | .000 |

*Notes*: $\mu$ is the mean and $\sigma_{OT-IT}$ is the volatility of the difference in the annualized percentage return between the OT and IT portfolios. The volatility of the return of the 28-, 14-, and 7-day IT portfolios is 16.48%, 17.15%, and 18.12%, respectively. Statistical tests are performed on the basis of the total number of dates. The $p$-values for the difference in means are derived via bootstrap with 10,000 draws. For the DD test, 10% trimming (deleting the sequentially lowest outcomes in either return set) in the left tail is uniformly performed, while similar trimming in the right tail is as shown. The results of the DD tests without trimming in the right tail are not reported as they are qualitatively the same as the $p$-values for the difference in means. The table presents the results when the selection criterion is the Sharpe ratio.

For each GARCH application, we estimate the model coefficients over a rolling window of 3,800 daily observations corresponding to approximately 15 years of data. For the GJR model, we project the volatility by summing the forecasted conditional variances given the estimated coefficients and the above recursive expression. For the EGARCH model, we sum the forecasted conditional variances evaluated by simulating 100,000 return paths. Random walk projected volatility is simply the sample volatility realized just before each trading date over the same number of days as the option maturity. Table 12 presents the results for the Sharpe ratio criterion for the 28-, 14-, and 7-day maturities, which should to be compared to our base case adjusted VIX method in Table 2.

We find evidence of mispricing in all of the panels that is confirmed ex post at all maturities for either the GJR or EGARCH forecast methods with somewhat lower profitability compared to our base case except for the 7-day options under the EGARCH forecasts. The results are generally weaker for the random walk case insofar as they exhibit no evidence of SD for the 28-day options either in-sample or ex post. All three models, especially the two GARCH ones, also yield a significantly lower number of feasible dates than our base case.

In unreported results, we further analyze the relationship between the quality of the forecasts and the SD results by estimating basic statistics, such as the bias and the variability of the forecast error $\varepsilon_{t,P} \equiv \sqrt{h_{t,\mathrm{PRED}}} - \sqrt{h_{t,\mathrm{OBS}}}$, predicted minus observed volatility. We find that VIX-adjusted is at par with both GARCH models with respect to the forecast quality, but both GARCH models produce positive biases of similar magnitude that are similar to the negative bias for the adjusted VIX. The random walk produces a small bias, but its forecast error dispersion measures are clearly inferior to the other three methods. These results explain the lower number of feasible dates for the two GARCH methods because the net short position of our OT portfolios tends to produce lower in-sample expected payoffs when the predicted volatility is high. We conclude that the mispricing of the OT portfolios exists in all of the volatility prediction models and that the adjusted VIX method is the most efficient of all candidate methods in identifying them.

**TABLE 13** Trading results with restricted option choices: January 1990–February 2013

| Option maturity (days) | $\mu$ | $p$-Value for $\mu \leq 0$ | $\sigma_{OT}$ | $\sigma_{OT-IT}$ | DD test $p$-value 5% Trim | 10% Trim |
|---|---|---|---|---|---|---|
| Optimal portfolios ex call upper bound violations | | | | | | |
| 28 | 0.04 | .458 | 15.81 | 1.88 | .241 | .003 |
| 14 | 0.70 | .175 | 16.13 | 2.46 | .009 | .000 |
| 7 | 1.31 | .096 | 17.15 | 2.39 | .000 | .000 |
| Optimal portfolios composed of calls | | | | | | |
| 28 | −0.10 | .531 | 14.79 | 4.84 | 1.000 | 1.000 |
| 14 | 0.93 | .295 | 14.71 | 6.00 | .007 | .007 |
| 7 | 1.33 | .298 | 16.17 | 6.06 | .005 | .000 |
| Optimal portfolios composed of puts | | | | | | |
| 28 | −0.24 | .843 | 16.31 | 0.98 | 1.000 | 1.000 |
| 14 | −0.27 | .797 | 16.83 | 1.24 | 1.000 | 1.000 |
| 7 | −0.22 | .764 | 17.90 | 1.30 | 1.000 | 1.000 |

*Notes*: The first panel presents the results when we exclude calls that violate the Constantinides and Perrakis (2002) upper bound. The second panel reports the results when puts are excluded. The third panel provides the results when calls are excluded. $\mu$ is the mean and $\sigma_{OT-IT}$ is the volatility of the difference in the annualized percentage return between the OT and IT portfolios. Statistical tests are performed on the basis of the total number of dates. The $p$-values for the difference in means are derived via bootstrap with 10,000 draws. For the DD test, 10% trimming (deleting the sequentially lowest outcomes in either return set) in the left tail is uniformly performed, while similar trimming in the right tail is as shown. The results of the DD tests without trimming in the right tail are not provided as they are qualitatively the same as the $p$-values for the difference in means.

## 9.3 | Trading results with restricted option choices

We find that the OT portfolios uncover violations that go beyond the violations of the Constantinides-Perrakis (2002) call upper bounds by eliminating these violating call options from the choice set. This involves eliminating them from 212, 183, and 199 cross-sections, respectively, for the 28-, 14-, and 7-day maturities. The results are presented in the first panel of Table 13. The OT portfolios stochastically dominate the IT portfolios for all of the maturity options, but we reject the hypothesis that the expected return of the IT portfolios is lower than the expected return of the OT portfolios only for the 7-day options. In unreported results, we choose the OT portfolios after replacing the violating call options with their upper bounds and find that the expected return of the OT portfolios is significantly higher than the IT return for both the 14- and 7-day options.

We then consider some popular and allegedly profitable strategies: covered calls, put vertical spreads, and butterfly spreads. We restrict the choice set for the OT portfolios to calls only. The results are reported in the second panel of Table 13. The OT portfolios stochastically dominate the IT portfolios for 14- and 7-day options, but we cannot reject the hypothesis that the expected return of the IT portfolios is lower than the expected return of the OT portfolios for any of the maturity options. Finally, we restrict the choice set for the OT portfolios to puts only. The results are provided in the third panel of Table 13. The OT portfolios do not stochastically dominate the IT portfolios for any of the maturity options and we cannot reject the hypothesis that the expected return of the IT portfolios is lower than the expected return of the OT portfolios for any of the maturity options. In unreported results, we restrict the OT choice set to OTM options for both calls and puts. The excess returns decrease by a factor of two for all three maturities and are significant only for the 7-day options.

We conclude that the OT portfolios are profitable because they contain both OTM and ITM calls and puts and both long and short positions. The portfolios cannot be explained by call or put butterflies or vertical spreads and are not an extension of the profitable covered calls identified in Constantinides et al. (2011). Further, the observed ITM option prices that are ignored in most frictionless studies are significant contributors to the portfolios' profitability.

**TABLE 14** Frictionless portfolio returns and stochastic dominance tests: January 1990–February 2013

| Costless trading in the index and options | | | | | | | |
|---|---|---|---|---|---|---|---|
| Option maturity (days) | $\mu$ | $p$-Value for $\mu \leq 0$ | $\sigma_{OT}$ | $\sigma_{OT-IT}$ | | DD test $p$-value 5% Trim | 10% Trim |
| 28 | 0.41 | 0.00 | 16.39 | 0.49 | | 0.00 | 0.00 |
| 14 | 0.63 | 0.00 | 16.97 | 0.43 | | 0.00 | 0.00 |
| 7 | 0.88 | 0.00 | 17.96 | 0.38 | | 0.00 | 0.00 |
| **Portfolio composition for costless trading in the index and options** | | | | | | | |
| | Total number of calls | Number of short call contracts | Number of long call contracts | Total number of puts | Number of short put contracts | Number of long put contracts | Number of short index |
| 28 | 0.46 | 0.10 | 0.36 | 0.50 | 0.43 | 0.07 | 0.36 |
| 14 | 0.49 | 0.10 | 0.39 | 0.51 | 0.48 | 0.03 | 0.45 |
| 7 | 0.50 | 0.12 | 0.37 | 0.50 | 0.49 | 0.01 | 0.37 |

*Notes*: $\mu$ is the mean and $\sigma_{OT-IT}$ is the volatility of the difference in the annualized percentage return between the OT and IT portfolios. In the top panel, we present the results for portfolios constructed under costless trading in the index and options. For all of the dates, there are feasible portfolios. The volatility of the return of the 28-, 14-, and 7-day IT portfolios is 16.48%, 17.15%, and 18.12%, respectively. Statistical tests are performed on the basis of the total number of dates. The $p$-values for the difference in means are derived via bootstrap with 10,000 draws. For the DD test, 10% trimming (deleting the sequentially lowest outcomes in either return set) in the left tail is uniformly performed, while similar trimming in the right tail is as shown. The results of the DD tests without trimming in the right tail are not provided as they are qualitatively the same as the $p$-values for the difference in means. In the bottom panel, we present the results for these portfolios' composition. We report the results for the entire available sample. The total number of contracts in each category is the sum of the absolute values of the number of long and short contracts. The table provides the results when the selection criterion is the Sharpe ratio.

## 9.4 | SD in a frictionless world

Earlier studies that ignore transaction costs find that it is puts rather than calls that are typically overpriced. In contrast, we find that the options portfolios contain far more short calls than long calls, short puts, and long puts suggesting that it is calls rather than puts that are overpriced, except during the financial crisis from November 2008 to October 2009 (Table 3).

We explain this difference by applying our portfolio selection algorithm to the frictionless universe of options as defined in most studies. First, we calculate the average of the bid and ask prices ($P_{av}$) of OTM calls and OTM puts. We then calculate the corresponding prices ($P_{av}$) of ITM calls and ITM puts from the average prices of the corresponding of OTM calls and OTM puts through the put–call parity. The ITM options that appear in more than 30% of the cross-sections play no role in this frictionless data. Finally, we obtain the set of adjusted prices ($P_{adj}$) by minimizing the sum of squared deviations $(P_{adj} - P_{av})^2$ subject to the condition that there are no convexity violations. The resulting adjustments never exceed a few cents.

The results of our algorithm for the Sharpe ratio criterion are shown in in the first panel of Table 14 for all three maturities. The average OT excess payoffs have much lower means than the equivalent Table 2 entries, but also very low volatilities and are strongly significant. The major difference with the results under frictions lies in the composition of the OT portfolios shown in the bottom panel. Compared to our base case in Table 3, we find that the shifting of payoffs to the low end of the support of the index by shorting calls that take 65% or more of the portfolio in all three maturities now takes only about 10%, while the long calls significantly increase. The payoff shifting now takes place by the adoption of relatively large positions in the short index and a large increase in short puts. Thus, put options appear overvalued in frictionless data, even though trading in them to eliminate the over valuation is not feasible under realistic conditions that account for the bid and ask prices. This resolves the differences with earlier studies that ignore transaction costs.

## 10 | CONCLUDING REMARKS

We demonstrate that at least some S&P 500 options are significantly mispriced relative to the index. A utility-maximizing investor holding the S&P 500 Index and a risk-free bond, subject to proportional transaction costs, stochastically dominates their portfolio by overlaying a zero-net-cost portfolio of European S&P 500 options of 28-, 14-, or 7-day maturities bought at their ask price and written at their bid price in almost every month from 1990 to 2013. The mispricing is strongest in short maturity options. The portfolios include about twice the number of calls than puts and the call positions are overwhelmingly short positions, consistent with the practice of writing covered calls. This contradicts the common belief that puts, but not calls, are overvalued, which we attribute to the neglect of trading costs and the methodology of creating a frictionless option universe. Similar results are obtained with options on the CAC and DAX indices. The mispricing is explained by neither priced factors nor a nonmonotonic stochastic discount factor.

Our SD methodology uses the same assumptions as the recent working paper by Post and Longarella (2018) who find mispriced zero-net-cost option portfolios that produce in-sample SD for OT when added to the index. The derivation of the OT option portfolios uses a different LP, which produces higher expected out-of-sample expected returns that were, however, more volatile and did not survive the DD out-of-sample tests. A reconciliation of the two studies' methodologies transcends the subject of this paper.

There are a number of possible reasons as to why this mispricing persists. Index funds and ETFs minimize tracking errors and the inclusion of options in their portfolios would likely increase tracking errors. Passive mutual funds may find it difficult to communicate to their investors the benefits of SD and may be unsuitable for investors seeking quick returns. Other active mutual funds and hedge funds may not hold the market portfolio as they have different targets, such as picking winners or enhancing portfolio returns by skewing their holdings toward small capitalization, value, or high profitability stocks. Finally, option traders' and intermediaries' credit constraints and funding liquidity may distort the prices of index options. In any case, it remains to be seen whether this paper's demonstration of the large OT excess returns will alter investors' behavior and eliminate the documented anomaly.

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## SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

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## APPENDIX: THE CHRISTOFFERSEN ET AL.'s (2013) EXTENSION OF THE HESTON AND NANDI's GARCH PROCESS

We modify our notation by denoting the option maturity time by $T$ instead of $t+1$ and estimate a GARCH model, the physical or $P$-distribution of daily index return data over a time period that covers the entire dataset and includes all option maturities. The risk-neutral or $Q$-distribution is derived from the stochastic discount factor that transforms the parameters of the physical distribution and includes parameters reflecting investor preferences with respect to return and volatility. The asset dynamics are given by:

$$\ln(S_\tau) = \ln(S_{\tau-1}) + r + (\mu - \frac{1}{2}h_\tau) + \sqrt{h_\tau}\varepsilon_\tau$$

$$h_\tau = \omega + \beta h_{\tau-1} + \alpha(\varepsilon_{\tau-1} - \gamma\sqrt{h_{\tau-1}})^2, \tau = 1, \dots, T, \tag{A1}$$

where $T$ denotes the upper range of the return data.

Because the conditional density of the daily GARCH returns is normal, the log-likelihood function is[19]:

$$\log L^P = -\frac{1}{2}\sum_{t+1}^{T}\left(\ln(h(\tau)) + \frac{[\ln S_\tau - \ln S_{\tau-1} - r - (\mu - \frac{1}{2}h(\tau))]^2}{h(\tau)}\right). \tag{A2}$$

[19] See Christoffersen et al. (2013, p. 1986).

The corresponding risk neutral process has the same form as (A1), but has an instantaneous expected return equal to $r$ and a volatility parameter set $\Omega_Q = \{\omega*, \beta*, \alpha*, \gamma*\}$, with parameters transformed via the stochastic discount factor taking the form:[20]

$$\frac{M_T}{M_t} = \left(\frac{S_T}{S_t}\right)^{\varphi} \exp(\delta T + \eta \sum_{t+1}^{T} h_\tau + \xi(h_{T+1} - h_{t+1})). \tag{A3}$$

The stochastic discount factor parameters ($\delta$, $\eta$, $\varphi$, and $\xi$) are linked to the $P$-distribution set $\Omega_P$ and to each other by the following relations:

$$\delta = -(\varphi + 1)r - \xi\omega + \frac{1}{2}\ln(1 - 2\xi\alpha); \varphi = -(\mu - \frac{1}{2} + \gamma)(1 - 2\xi\alpha) + \gamma - \frac{1}{2}$$
$$\eta = -(\mu - \frac{1}{2})\varphi - \xi\alpha\gamma^2 + (1 - \beta)\xi - \frac{(\varphi - 2\xi\alpha\gamma)^2}{2(1 - 2\xi\alpha)}. \tag{A4}$$

Given the parameter values $\Omega_P$, the stochastic discount factor has exactly one extra parameter, $\xi$, as the other three are given by (A4) once $\xi$ is given. This parameter accounts for the U-shaped stochastic discount factor if $\xi > 0$, a requirement for the stochastic discount factor to potentially account for our OT results.[21] The set $\Omega_Q$ is given by the following system:

$$h_t^* = \frac{h_t}{1 - 2\xi\alpha}; \omega^* = \frac{\omega}{1 - 2\xi\alpha}; \alpha^* = \frac{\alpha}{(1 - 2\xi\alpha)^2}$$
$$\gamma^* = \gamma - \varphi; \varphi = -(\mu - \frac{1}{2} + \gamma)(1 - 2\xi\alpha) + \gamma - \frac{1}{2}. \tag{A5}$$

This system yields a unique set $\Omega_Q$ consistent with the $P$-distribution set $\Omega_P$ and the parameter $\xi$. In turn, given this $\Omega_Q$, the OT portfolio model value can be easily found from the option pricing expressions in Christoffersen et al. (2013, appendix D) for each one of the four maturities $[\tau_i, T]$, $i = 1, 2, 3, 4$of the 28-, 21-, 14-, and 7-day options, respectively. We estimate $\xi$ by maximizing the log-likelihood for all options defined as:[22]

$$\log L^Q = -\frac{1}{2}\sum_{i=1}^{N}\left[\log\left(s_e^2\right) + \frac{e_i^2}{s_e^2}\right], \tag{A6}$$

where $e_i = (\mathrm{IV}^{\mathrm{Mod}} - \mathrm{IV}^{\mathrm{Mkt}})/\mathrm{IV}^{\mathrm{Mkt}}$ and in place of $s_e^2$, its sample analog is used, $\hat{s}_e^2 = \frac{1}{N}\sum_{i=1}^{N} e_i^2$.

The parameter set $\Omega_P = \{\mu, \omega, \beta, \alpha, \gamma\}$ of the $P$ distribution contains the risk premium parameter $\mu$ and four volatility parameters and is common to the four maturities $[t, T_i]$, $T_i - t = 28, 21, 14,$ and 7 for $i = 1, 2, 3, 4$, respectively. As in Christoffersen et al. (2013), it is estimated by filtering the daily index returns for a time period bracketing the option dataset on both sides. We use return data from 1984–2014 that contain the option data and apply the sequential likelihood estimation of Broadie, Chernov, and Johannes (2007) starting from the returns and then proceeding to the option market for any additional parameters. Maximizing the log-likelihood function $\log L^P$ given in relation (A2), we find the $P$-distribution parameter set $\Omega_P$: $\omega = 0$, $\alpha = 4.1931 \times 10^{-6}$, $\beta = 0.8467$, $\gamma = 165.17$, $\mu = 2.8161$, and $\log L^P = 32{,}584.9$.[23]

[20] For a single GARCH period, the logarithm of the stochastic discount factor can be expressed as a quadratic function of the random stock return only. See Christoffersen et al. (2013), Corollary 2, p. 1970). Unfortunately, no such closed form expression exists for maturities greater than 1 day.

[21] See Corollary 3 of Christoffersen et al. (2013, p. 1970).

[22] Expression (B6) is virtually identical to expression (24) of Christoffersen et al. (2013, p. 1986) with the difference that the numerator of the error $e_i$ is the difference in actual prices rather than their IV's.

[23] These correspond to a higher return premium and volatility than the corresponding results in Christoffersen et al. (2013, table 5), most probably due to our much longer return dataset.

We then estimate $\xi$ and the corresponding $Q$distribution as in Christoffersen et al. (2013) or, indeed, as in most, if not all asset pricing models, by using the entire cross-section of available option prices for the four maturities under consideration.[24] Specifically, put–call parity is imposed for every option in the cross-section implying that it no longer matters whether puts or calls are used for the estimation with OTM calls and puts in each cross-section. The likelihood function $logL^Q$ given by (A6) is then evaluated as a function of $\xi$ and its maximum, $\log L^Q = 84,704$, is found at $\xi = \xi^* = 58,933$.[25] This value is significantly higher than the value $\xi = 32,839$ implied by the results of Table 5 in Christoffersen et al. (2013, p. 1992) reflecting the different maturities and the different span of the data in our sample.[26] Our corresponding $Q$-distribution parameters are $(1 - 2\alpha\xi^*)^{-1} = 1.9771$, $\omega = 0$, $\alpha = 1.6391 \times 10^{-5}$, $\beta = 0.8467$, and $\gamma = 85.21$.

The stochastic discount factor is equal to the stochastic discount factor $M_T/M_t$, with the parameters given by (A4), the above estimates of the parameter set $\Omega_P$, the value $\xi = \xi^*$ as above, and the implied realized volatility path $(h_{t+1}, \ldots, h_{T+1})$ extracted from the observed daily index returns (A1) and the corresponding observed errors $\varepsilon_\tau$. If $N_{T_i}$ denotes the number of feasible cross-sections for the corresponding maturity $T_i = 28$, 21, 14, and 7 days, the average risk-adjusted excess return $\overline{\mathrm{SDF}_t \times r_{\mathrm{OT},t}}$ of the OT portfolio in Table 10 is given by:

$$\overline{\mathrm{SDF}_t \times r_{\mathrm{OT},t}} = \frac{1}{N_{T_i}} \sum_{t=1}^{t=N_{T_i}} \frac{M_T}{M_t}(h_{t+1}, \ldots, h_{T+1}) r_{\mathrm{OT},t}. \tag{A7}$$

As for the returns in Table 11, at model prices for both OT portfolios and straddles, the initial values of the portfolios are estimated as in Christoffersen et al. (2013, appendix D) with the above $Q$ distribution parameter set.

[24] Note that there are significantly fewer maturities in these estimations that were used by Christoffersen et al. (2013, p. 1975), because the latter included all of the available maturities between 2 weeks and 1 year.

[25] Unreported results show that $\log L^Q$ is a parabolic function of $\xi$, first increasing and then decreasing after reaching $\xi^*$.

[26] Christoffersen et al. (2013, p. 1975) include all of the available maturities between 2 weeks and 1 year.