Neuro-Spike Communications with Multiple Synapses under Inter-Neuron Interference

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Abstract— The nervous system is a complex intra-body communication system, called neuro-spike communications, for transferring vital information throughout the body. This system consists of unreliable neural components like axon and synapses. The nervous system can deal with the unreliability of its components by making use of multiple synapses. In this paper, we analyze the performance of neuro-synaptic channels consisting of multiple synapses. It is assumed that synaptic channels are subject to synaptic noise, random vesicle release, and inter-neuron interference. The optimal detection for two cases of multiple cooperative synapses and multiple interfering synapses is investigated. The closed-form expressions of the PDF of variable quantal amplitude is derived that is used to calculate the probability of detection error.

I. INTRODUCTION

Molecular communication is an emerging technology for applications requiring nanoscale networks (see, e.g., [1]–[3]). One of the main examples of molecular communication is the human nervous system which is used for transferring vital information throughout the body. In this paper, similar to [4]–[6], the stochastic channel model of neuro-synaptic channels in the central synapses is considered and we study the optimal detection of signals at the postsynaptic terminal. In this paper, in contrast to our former work [6], we consider the multiple synapses case in which multiple synapses are transmitting to a single receiving neuron. This is a practical assumption since the value of the vesicle release probability is low in practice. Hence, to transfer the information reliably, the cooperative transmission by neurons is required. In addition, we consider the case that multiple synapses destructively add up to the desired signal. This kind of interference is more common in central nervous system diseases like multiple sclerosis (MS) [7]. In MS, the myelin sheath, which covers the axons, degenerates, and damaged axons interfere with each other due to ion exchanges during axonal propagation (see, e.g., [8]).

In this paper, we evaluate the neuro-synaptic communication with multiple synapses under different random impairments like synaptic noise, random vesicle release, interference, and random amplitude of the spikes regenerated at the postsynaptic terminal. The goal is to optimally detect the signals at the postsynaptic terminal when there exist multiple transmitting synapses. The optimal detection for two cases of multiple cooperative synapses and multiple interfering synapses are investigated. The closed-form expressions of the probability

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density function (PDF) of variable quantal amplitude is derived that is used to find the probability of detection error.

The rest of the paper is organized as follows: In Section II, the channel model for Neuro-Synaptic system is given. The Neuro-Synaptic channels with multiple synapses are studied in Section III, where two cases of cooperative aggregation and additive interference are investigated. In Section IV, the problems of optimal spike detection for two cases of cooperative and interfering multiple access are formulated and the detection error probability are derived.

II. NEURO-SYNAPTIC CHANNEL MODEL

Fig. 1 depicts the model for a neuro-synaptic communication channel between the presynaptic neuron and postsynaptic neuron in central synapses. In the model, the spike train $x(t)$ in the axon, which is due to the process of neural spike response (see, e.g., [9]) is written as

$$x(t) = \sum_n \delta(t - \tau_n), \qquad (1)$$

where $\delta(t)$ is unit impulse function and τ_n is the time of spike occurrence. The information is carried by the timing of spikes, i.e., inter-spike intervals represented by the occurrence time of the spikes τ_n's. The spike train $x(t)$ can be modeled as a doubly Poisson stochastic process which is an inhomogeneous Poisson process with mean $\lambda(t)$ called average firing rate [10]. A spike is a fluctuation of 100 mV electric voltage Considering the resting potential of around $V_{\text{rest}} = -65$ mV, a neuron typically fires a spike whenever its membrane potential reaches the threshold potential of around $V_{\text{th}} = -50$ mV. The detailed description of the spike train generation can be found in [10].

Due to refractory effect, similar to [4], [6], [9], the spike train in (1) can be divided into bins of size T_f. When $x(t)$ reaches the presynaptic terminal of the neuron, neurotransmitters are released into the synaptic cleft. The release of neurotransmitters from vesicle is a random process which is again modeled as binary random variable W. When a spike is arrived at the presynaptic terminal, a vesicle containing neurotransmitters is released with the probability of p_r and we have $V = 1$.

The next stage in the modeling of the neuro-synaptic channel is variable quantal amplitude denoted by q. This is another form of the uncertainty which is due to the influx of neurotransmitters variably absorbed by receptors at the the receiving neuron. Moreover, this form of the molecular-based communication has a pulse shape at the receiving neuron of
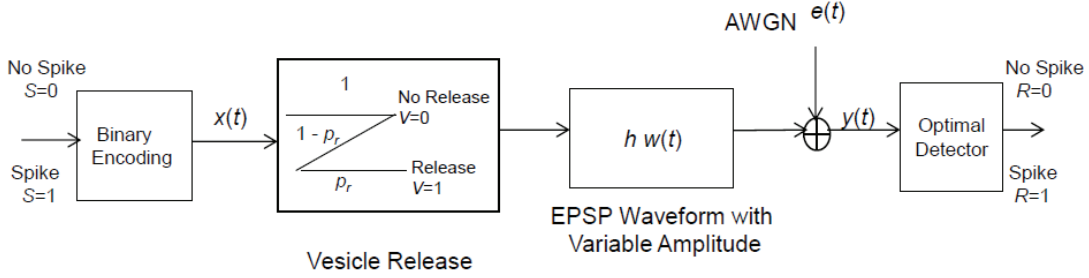
Fig. 1. The mathematical model for the neuro-synaptic communication channel.

the form [4]

$$w(t) = \frac{w_{\max}\, t}{T_{\max}} \exp(1 - t/T_{\max}),\qquad (2)$$

for $t > 0$, where $w_{\max}$ and $T_{\max}$ represent the maximum value of the pulse shape and its corresponding time, respectively.

The last stage in the synaptic channel shown in Fig. 1 is related to adding additive white Gaussian noise available at the postsynaptic terminal. Therefore, the received signal can be written as

$$y(t) = V\, h\, w(t) + e(t),\qquad (3)$$

where $e(t)$ is the synaptic background noise that is white and Gaussian with zero-mean and variance of σ_n^2 [5].

III. NEURO-SYNAPTIC SYSTEM WITH MULTIPLE SYNAPSES

In this section, we consider scenarios that involve multiple synapses at the transmitter side. When presynaptic signals are related to the same information source, they are summing up constructively, and it is called cooperative synaptic communications. On the other hand, the received signals at the postsynaptic neuron could come from different sources, and thus, some neurons may interfere with synaptic communications.

A. Cooperative Synaptic Communications

First, we consider the case that we have multiple synapses cooperatively transmit their data toward a single postsynaptic terminal. Note that this is a practical assumption since the value of the vesicle release probability p_r is low is practice. Hence, to transfer the information reliably, the cooperative transmission by neurons is inevitable.

To get the spatial diversity gain due to the cooperative multiple synaptic case, we calculate the optimal decision strategy. The postsynaptic membrane voltage of the cooperative synaptic communication when a spike is transmitted becomes

$$y_M(t) = \sum_{m=1}^{M} V_m\, h_m\, w(t) + e(t),\qquad (4)$$

where M is the number of independent parallel synapses, and V_m is the vesicle release process at the m-th synapse. The random variable V_m is modeled as a Bernoulli random variable. In addition, the random variable h_m is the variable

quantal amplitude of the m-th synaptic connection. Note that in (4), we assumed that the same type of receptors is used for all synaptic connections. Hence, the EPSP waveform $w(t)$ is the same for all summation terms in (4).

Proposition *1:* Considering a set of independent k-th order Gamma random variables $\mathcal{H} = \{h_1, \cdots, h_M\}$ with mean and variance of λ and σ^2, respectively, and a set of independent Bernoulli random variables $\mathcal{V} = \{V_1, \cdots, V_M\}$ with parameter p_r, the probability distribution function (PDF) of the summation of independent Gamma distributed random variables, i.e., $H_M = \sum_{m=1}^{M} V_m\, h_m$ is given by

$$f_M(h) = (1 - p_r)^M \delta(h)$$
$$+ \sum_{m=1}^{M} \binom{M}{m} \frac{p_r^m\,(1-p_r)^{M-m}\mu^{km}}{(km-1)!}\, h^{km-1}\, \exp(-\mu\, h).\qquad (5)$$

where $\mu = \lambda/\sigma^2$, $k = \lambda^2/\sigma^2$.

Proof: The PDF of h_m is given as $f(h) = \frac{\mu^k}{(k-1)!}\, h^{k-1} \exp(-\mu\, h)$. From [11], it can be shown that, for a fixed M_0, the sum of independent Gamma-distributed random variables with the same parameter μ and of order km is again a Gamma distributed random variable with the order $\sum_{m=1}^{M_0} km$. Therefore, we have

$$f_M(h\,|M_0) = \frac{\mu^{kM_0}}{(kM_0 - 1)!}\, h^{kM_0-1} \exp(-\mu\, h).\qquad (6)$$

Now, by defining $M_0 = \sum_{m=1}^{M} V_m$, it is clear that it has a binomial distribution, i.e., $\Pr\{M_0 = m\} = \binom{M}{m} p_r^m\,(1 - p_r)^{M-m}$. Hence, based on the law of total probability, the PDF of Q_M can be written as

$$f_M(h) = \sum_{m=0}^{M} \Pr\{M_0 = m\} f_M(h\,|m)$$
$$= (1 - p_r)^M \delta(h) + \sum_{m=1}^{M} \Pr\{M_0 = m\} f_M(h\,|m)$$
$$= (1 - p_r)^M \delta(h)$$
$$+ \sum_{m=1}^{M} \binom{M}{m} \frac{p_r^m\,(1-p_r)^{M-m}\mu^{km}}{(km-1)!}\, h^{km-1} \exp(-\mu\, h).\qquad (7)$$
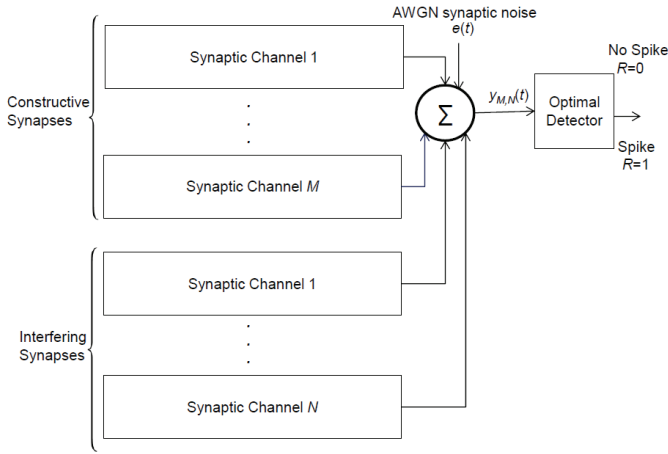
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Fig. 2. The cooperative and interfering multiple access model for the neuro-synaptic communication channel when there are M constructive and N destructive synapses.

B. Synaptic Interference Case

In this subsection, we assume multiple neurons cause interference to the desired signal. Thus, unlike the previous subsection, the interfering synapses destructively add up their signals similar to the synaptic background white noise. For example, in the MS disease, the myelin covering the axon are damaged and it makes the synaptic connections vulnerable to interference from other adjacent neurons.

We assume that the postsynaptic neuron receive signals from N interfering synapses. Thus, the postsynaptic membrane voltage of (4) when a spike is transmitted is changed to

$$y_{M,N}(t) = \sum_{m=1}^{M} V_m h_m w(t) + \sum_{i=1}^{N} V_i' h_i' w(t) + e(t), \quad (8)$$

where V_i' is the binary random variable for modeling the vesicle release mechanism at the i-th interfering synapse and h_i' is the variable quantal amplitude of the i-th interfering synaptic connection. Fig. 2 depicts this scenario in which there are M constructive and N destructive synapses.

IV. SIGNAL DETECTION

In this section, we derive the probability of an erroneous receiver decision in recovering the binary information X based on the received postsynaptic signal $y(t)$. In additive white Gaussian noise (AWGN) channels, the matched filters are the optimal in minimizing the error probability [12]. It is also known that correlation receivers give the same optimal performance, and here we obtain the optimal binary decision by correlating of the postsynaptic signal $y(t)$ and the waveform $w(t)$.

A. Hypothesis Testing for Cooperative Synaptic Communications

We formulate the problem of finding the error probability in detecting S as a hypothesis test. After observing the signal $y(t)$

over one bin of size T_f, the hypothesis test for the decision on a single spike decision is written as

$$\begin{cases} H_0 : y_M(t) = e(t), \\ H_1 : y_M(t) = \sum_{m=1}^{M} V_m h_m w(t) + e(t), \end{cases} \quad (9)$$

where H_0 and H_1 refer to two cases of no spike , i.e., $S = 0$, and spike existence, i.e., $S = 1$, respectively. The choice of T_f should be long enough for one spike only. Due to stochastic nature of $e(t)$, V_m, and h_m, both H_0 and H_1 events become random processes.

By assuming $\underline{y} = [y_M(t_1), y_M(t_2), \ldots, y_M(t_n)]$ in the interval of length T_f, the conditional PDF of $y_M(t)$ over H_0 and H_1 are denoted by $f_Y\{\underline{y}|S = 0\}$ and $f_Y\{\underline{y}|S = 1\}$, respectively.

The likelihood ratio of the problem stated in (9) can be written as [13]

$$\Lambda_S(y) = \frac{f_Y\{\underline{y}|S = 1\}}{f_Y\{\underline{y}|S = 0\}}. \quad (10)$$

For the binary decision, the region H_0 consists of values of $\underline{y}$ for which $\Lambda_S(y) < \Lambda_0$, and H_1 of values of $\underline{y}$ for which $\Lambda_S(y) > \Lambda_0$. The critical threshold Λ_0 is given by $\Lambda_0 = p_{\text{prior}}/(1 - p_{\text{prior}})$ where $p_{\text{prior}} = P\{S = 0\}$ is the prior probability of hypothesis H_0.

Since the aggregated amplitude $H_M = \sum_{m=1}^{M} V_m h_m$ of the received signal in H_1 is stochastic, the composite hypothesis test for the detection of signals with unknown parameters can be utilized [13]. Note that the received signal $\underline{y}$ has jointly normal distribution when conditioned over $\{h_1, \cdots, h_M\}$, for both cases of H_0 and H_1. Therefore, from (10) and by increasing the number of samples, the equivalent continuous time conditional likelihood ratio can be written as [13], we have

$$\Lambda_S(y|h_1, \cdots, h_M) = \exp\left(\frac{2 H_M \int_0^{T_f} w(t)y(t)dt - H_M^2 E_w}{N_0}\right),$$
$$(11)$$

where N_0 is the power spectral density of $e(t)$ and $E_w = \int_0^{T_f} w^2(t)\, dt$ is the energy of the excitatory postsynaptic potential (EPSP) response $h(t)$. By assuming that $w(t)$ is non-zero only in the refractory time slot of duration of T_f, and by using the expression in (2), we have

$$E_w \cong \frac{w_{\max}^2 e^2}{T_{\max}^2} \int_0^{+\infty} \alpha^2 e^{\frac{-2\alpha}{\max}} d\alpha = \frac{e^2}{4} T_{\max} w_{\max}^2. \quad (12)$$

Now, by averaging $\Lambda_S(y|h_1, \cdots, h_M)$ in (11) over random variables $\{h_1, \cdots, h_M\}$, the likelihood ratio can be calculated as in (13), where

$$\Psi_{m,M} = \binom{M}{m} p_r^m (1 - p_r)^{M-m} \mu^{km},$$

and $c(y) = \int_0^{T_f} w(t)y(t)dt$ is the correlation of the postsynaptic signal $y(t)$ and waveform $w(t)$.

$$\Lambda_S(y) = \int_0^\infty f_M(h) \exp\left(\frac{2\,h\,c(y) - h^2 E_w}{N_0}\right) dh = (1-p_r)^M + \sum_{m=1}^M \frac{\Psi_{m,M}}{(km-1)!} \int_0^\infty h^{km-1} \exp(-\mu\,h)\exp\left(\frac{2\,h\,c(y) - h^2 E_w}{N_0}\right) dh,$$
$$(13)$$

From [14], the closed-form solution for the integral in (13) can be obtained as

$$\Lambda_S(y) = (1-p_r)^M + \sum_{m=1}^M \Psi_{m,M}\left(\frac{2E_w}{N_0}\right)^{-km/2}$$
$$\times \exp\left[\frac{(\mu N_0 - 2\,c(y))^2}{8N_0 E_w}\right] C_{-km}\left(\frac{\mu N_0 - 2\,c(y)}{\sqrt{2N_0 E_w}}\right) \quad (14)$$

where $C_k(\cdot)$ is the parabolic cylinder function of order k [14].

The parameter k in the distribution of $f_M(h)$ in Proposition 1 is used to model the variability of h [4]. For the case of $k = 1$ and $M = 1$, the PDF has the maximum variability indicates the worst-case scenario, and from [14, Eq. 3.322], $\Lambda_S(y)$ becomes

$$\Lambda_S(y) = 1 - p_r + \quad (15)$$
$$p_r\mu\sqrt{\frac{N_0\pi}{E_w}} \exp\left(\frac{(\mu N_0 - 2\,c(y))^2}{4N_0 E_w}\right) Q\left(\frac{\lambda N_0 - 2\,c(y)}{2\sqrt{N_0 E_w}}\right)$$

where $Q(.)$ is the q-function of the standard normal distribution.

B. Hypothesis Testing Synaptic Interference Case

Now, we consider the problem of spike detection in a system with interfering neurons discussed in Subsection III-B. Thus, the hypothesis test described in (9) can be rewritten as

$$\begin{cases} H_0 : y_{M,N}(t) = \sum_{i=1}^N V_i' h_i' w(t) + e(t), \\ H_1 : y_{M,N}(t) = \sum_{m=1}^M V_m h_m w(t) + \sum_{i=1}^N V_i' h_i' w(t) + e(t). \end{cases}$$
$$(16)$$

Now, by defining $I_i = V_i' h_i'$, the PDF of the ith interfering component at the receiving neuron can be written as

$$f_{I_i}(\gamma) = p_r \frac{\mu^k}{(k-1)!} \mu^{k-1} \exp\left(-\mu\,\gamma\right) + (1-p_r)\,\delta(\gamma).$$

Similar to (11), we employ composite hypothesis testing. Conditioned on $\{h_1, \cdots, h_M\}$ and $\{I_1, \cdots, I_N\}$, $y(t)$ has jointly normal distribution over both hypotheses. Hence, from (10), the conditional likelihood ratio for hypothesis test in (16) can be written, using [13, p. 65], as in (17), where $\mathbb{E}\{\cdot\}$ denotes the expectation operation and $H_I = \sum_{n=1}^N I_n$ is defined as the aggregated amplitude of the transmitted signal under H_0.

It is easy to check that the summation of two independent random variable with the distribution stated in (5) will again have the same distribution. Thus, $H_{M+N} = H_M + H_I$ which is the summation of two identical and independent random variables has a PDF of $f_{M+N}(h)$.

Now, the conditional likelihood ratio is averaged over unknown variable and we have

$$\Lambda_S(y) = \frac{\int_0^\infty f_{M+N}(h) \exp\left(\frac{2\,h\,c(y) - h^2 E_w}{N_0}\right) dh}{\int_0^\infty f_N(h_I) \exp\left(\frac{2\,h_I\,c(y) - h_I^2 E_w}{N_0}\right) dh_I} \quad (18)$$

where $f_N(h_I)$ is similar to the PDF obtained in (5). By replacing the PDF from Proposition 1 in (18), we have (19), where $\Phi_N = (1-p_r)^N$.

From [14, Eq. 3.462], the closed-form solution for the integrals in (13) can be obtained as in (20), where $\Theta_n^N = \Psi_{n,N}\left(\frac{2E_w}{N_0}\right)^{-kn/2}$. Hence, we obtained a closed-form solution for the optimal detector.

C. Probability of Error

Now, we investigate the probability of error of spike detection at the receiving neuron. The average probability of error of the hypothesis test in (9) or (16) can be written as

$$P_e = p_{\text{prior}} P_{\text{false}} + (1 - p_{\text{prior}}) P_{\text{miss}} \quad (21)$$

The probability P_{false} of selecting hypothesis H_1 when H_0 is valid is named false detection probability and can be written as

$$P_{\text{false}} = \mathrm{P}\{R = 1 | S = 0\} = \mathrm{P}\{\Lambda_S(y) > \Lambda_0 | S = 0\} \quad (22)$$

where $R = 1$ stands for the event that spike is detected by the proposed binary detector. Moreover, the probability P_{miss} of choosing hypothesis H_0 when H_1 is correct is called miss-detection probability and can be found as

$$P_{\text{miss}} = \mathrm{P}\{R = 0 | S = 1\} = \mathrm{P}\{\Lambda_S(y) \le \Lambda_0 | S = 1\} \quad (23)$$

where $R = 0$ stands for the event that spike is not detected at the proposed binary detector.

The likelihood ratio $\Lambda_S(y)$ in (14) is a function of correlator function $c(y) = \int_0^{T_f} w(t)y(t)dt$. Therefore, the CDF of $\Lambda_S(y)$ can be represented in term of the CDF of $c(y)$. If $S = 0$, one can observe that

$$c(y) = \int_0^{T_f} w(t)y(t)dt = e_{\text{out}},$$

where e_{out} is again a white Gaussian noise. In addition, if $S = 1$, we have $c(y) = E_w H_M + n_{\text{out}}$, which is the summation of the random variable with the PDF in (5) and zero-mean Gaussian random variable.

For the case of synaptic channel with interference, the likelihood ratio $\Lambda_S(y)$ in (20) is a function of random variable $c(y)$, and thus, the CDF of $\Lambda_S(y)$ can be expressed in terms of the CDF of $c(y)$. If $S = 1$, one can observe that

$$c(y) = \int_0^{T_f} w(t)y(t)dt = E_w H_N + e_{\text{out}},$$

$$\Lambda_S(y) = \frac{\mathbb{E}\left\{\exp\left(\frac{-y^2(t)-(H_M^2+H_I^2+2H_I\,H_M)E_w+2(H_M+H_I)\int_0^{T_f}w(t)y(t)dt}{N_0}\right)\right\}}{\mathbb{E}\left\{\exp\left(\frac{-y^2(t)+2H_I\int_0^{T_f}w(t)y(t)dt-H_I^2E_w}{N_0}\right)\right\}} = \frac{\mathbb{E}\left\{\exp\left(\frac{2(H_M+H_I)\int_0^{T_f}w(t)y(t)dt-(H_M+H_I)^2E_w}{N_0}\right)\right\}}{\mathbb{E}\left\{\exp\left(\frac{2H_I\int_0^{T_f}w(t)y(t)dt-H_I^2E_w}{N_0}\right)\right\}},$$

(17)

$$\Lambda_S(y) = \frac{\Phi_{M+N}+\sum_{i=1}^{M+N}\frac{\Psi_{i,M+N}}{(ki-1)!}\int_0^\infty h^{ki-1}\exp(-\mu h)\exp\left(\frac{2\,h\,c(y)-h^2E_w}{N_0}\right)dh}{\Phi_N+\sum_{n=1}^N\frac{\Psi_{n,M}}{(kn-1)!}\int_0^\infty h^{kn-1}\exp(-\mu\,h)\exp\left(\frac{2\,h\,c(y)-h^2E_w}{N_0}\right)dh},$$

(19)

$$\Lambda_S(y) = \frac{\Phi_{M+N}+\sum_{m=1}^{M+N}\Theta_i^{M+N}\exp\left(\frac{(\mu N_0-2\,c(y))^2}{8N_0E_w}\right)C_{-ki}\left(\frac{\mu N_0-2\,c(y)}{\sqrt{2N_0E_w}}\right)}{\Phi_N+\sum_{n=1}^N\Theta_n^N\exp\left(\frac{(\mu N_0-2\,c(y))^2}{8N_0E_w}\right)C_{-kn}\left(\frac{\mu N_0-2\,c(y)}{\sqrt{2N_0E_w}}\right)}$$
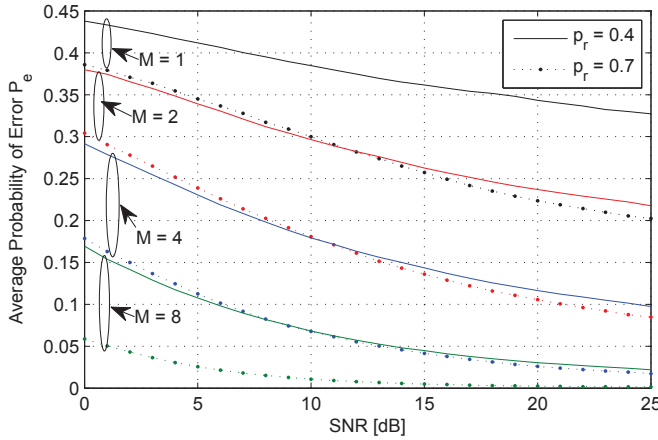
(20)

Fig. 3. The probability of detection error P_e curves versus the SNR for different parameters of the vesicle release and number of synapses M, in a neuro-synaptic communication channel with $k = 1$.

Fig. 4. The probability of detection error P_e curves versus the SNR for different parameters of the variable quantal amplitude and number of synapses M in a neuro-synaptic communication channel with $p_r = 0.4$.

where e_{out} is again a white Gaussian noise and the distribution of H_N is $f_N(h_I)$ given in Proposition 1. In addition, if $S = 1$, we have $c(y) = E_w H_{M+N} + e_{\text{out}}$, which is the summation of the random variable with the PDF of $f_{M+N}(h)$ and zero-mean Gaussian random variable.

V. NUMERICAL RESULTS

In this section, we present our numerical results that show the performance of the proposed hypothesis testing for both cooperative synaptic case and synaptic interference case. For the EPSP waveform, similar to [4], it is assumed that $w_{\text{max}} = 2$ mV and $T_{\text{max}} = 1$ msec. The parameters of trial-to-trial variability, modeled by the Gamma distribution, are assumed to be $\lambda = 1$ and $k = 1, 4$. The refractory period is assumed to be normally distributed with mean 5 msec [15], and hence, we have $T_f = 5$ msec.

In Fig. 3, the performance of a neuro-synaptic communication channel with different conditions is evaluated. The curves
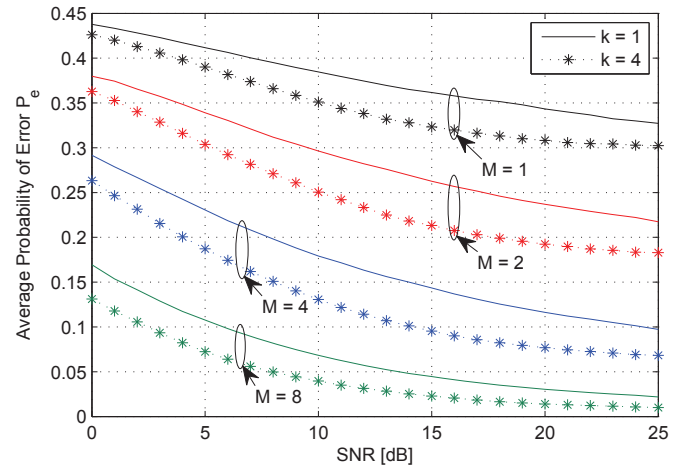
are based on the transmission of 50000 transmitted symbols. The probability of error curves versus SNR, i.e., E_w/N_0, at the receiving neuron are shown for different scenarios such as different number of synapses, $M = 1, 2, 4, 8$, and different values of synaptic release probability $p_r = 0.4, 0.7$. The Gamma distribution parameter k, which is used for modeling the variable quantal amplitude, is assumed to be 1. It is shown that by increasing the number of synapses, the probability of error is significantly reduced in all SNR conditions. However, the rate of decrease in P_e is reducing by increasing the number of cooperating synapses. Thus, it is very efficient if a few number of synapses are collaboratively transmit the data toward the next neuron. The figure also shows the effect of vesicle release probability, p_r, on the system performance. For example, one can observe that at $P_e = 0.05$, around 13 dB more SNR is required for the synaptic release probability of $p_r = 0.4$, compared to a system with $p_r = 0.7$, in a channel
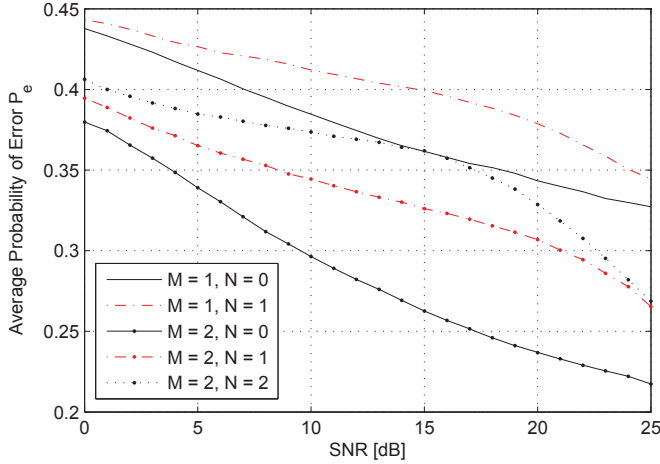
Fig. 5. The probability of detection error P_e curves versus the SNR for a neuro-synaptic communication channel with different number of cooperating and interfering synapses when $p_r = 0.4$ and $k = 1$.

Fig. 6. The probability of detection error P_e curves versus the SNR for a interference-free neuro-synaptic communication channel with different number of cooperating synapses when $p_r = 0.4$ and $k = 1$.

with $M = 8$.

Fig. 4 shows the impact of the parameter k in a system with different number of synapses $M = 1, 2, 4, 8$ and $p_r = 0.4$. It is shown that by increasing the value of gamma distribution variability parameter k, the performance is improved because of the reduction in randomness of postsynaptic amplitude h.

In Fig. 5, we show the impact of the interfering synapses in a neoro-synaptic channel with multiple synapses and $p_r = 0.4$. It can be shown that at SNR of 15 dB, the average probability of error are increased by 0.07 and 0.1 when there are one and two interfering synapses, respectively, in a channel with $M = 2$ constructive synapses. In addition, one can observe that the effect of interference in the performance degradation is more destructive in average SNR conditions, i.e., in the range from 15 dB to 20 dB.

Furthermore, in Fig. 6, we show the effect of increasing the number of cooperating synapses on the performance of a interference-free system with $p_r = 0.4$ and $k = 1$, when we use logarithmic scale for the probability of error. By asymptotic analysis in high SNR conditions, it can be seen that the diversity order of M is achievable when there are M cooperating synapses.

VI. CONCLUSION

In this paper, the performance of the neuro-synaptic communication with multiple synapses has been investigated under different stochastic impairments like synaptic noise, random vesicle release, interference, and random amplitude of the spikes regenerated at the postsynaptic terminal. The impact of interference from adjacent neurons have been analyzed. This type of interference is more common in central nervous system diseases like MS. In addition, the average probability of error at the receiving neuron has been derived for two cases of multiple cooperative synapses and multiple interfering synapses are investigated.
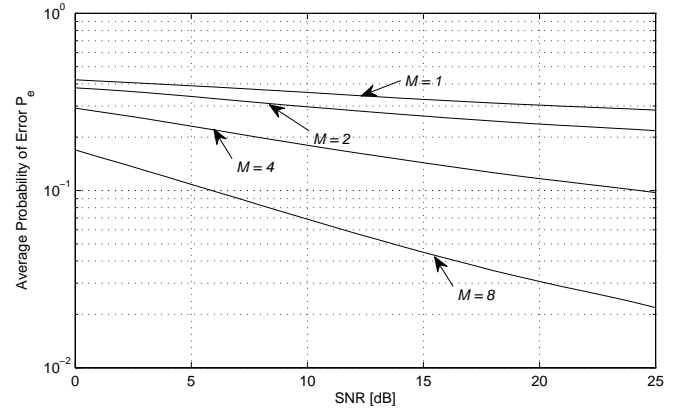
REFERENCES

[1] L. Galluccio, S. Palazzo, and G. E. Santagati, "Characterization of molecular communications among implantable biomedical neuro-inspired nanodevices," *Nano Communication Networks*, vol. 4, no. 2, pp. 53–64, Jun. 2013.

[2] N. Farsad, N.-R. Kim, A. W. Eckford, and C.-B. Chae, "Channel and noise models for nonlinear molecular communication systems," *IEEE J. Select. Areas Commun.*, vol. 32, no. 12, pp. 2391–2401, Dec. 2014.

[3] V. Jamali, A. Ahmadzadeh, C. Jardin, H. Sticht, and R. Schober, "Channel estimation techniques for diffusion-based molecular communications," in *Proc. IEEE Int. Conf. Commun. (ICC)*, (Kuala Lumpur, Malaysia), May 2016.

[4] A. Manwani and C. Koch, "Detecting and estimating signals over noisy and unreliable synapses: information-theoretic analysis," *Neural Computation*, vol. 13, no. 1, pp. 1–33, Jan. 2001.

[5] E. Balevi and O. Akan, "A physical channel model for nanoscale neuro-spike communications," *IEEE Trans. Commun.*, vol. 61, no. 3, pp. 1178–1187, Mar. 2013.

[6] B. Maham, "A communication theoretic analysis of synaptic channels under axonal noise," *IEEE Comm. Letters*, vol. 19, no. 11, pp. 1901–1904, Nov. 2015.

[7] A. Compston and A. Coles, "Multiple sclerosis," *The Lancet*, vol. 372, no. 9648, pp. 1502–1517, Oct. 2008.

[8] L. Steinman, "Multiple sclerosis: a two-stage disease," *Nature immunology*, vol. 2, no. 9, pp. 762–764, Sep. 2001.

[9] D. Malak and O. Akan, "A communication theoretical analysis of synaptic multiple-access channel in hippocampal-cortical neurons," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2457–2467, Jun. 2013.

[10] P. Dayan and L. F. Abbott, *Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems*. Massachusetts, USA: MIT Press, 2001.

[11] A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*. Massachusetts, USA: Addison-Wesley Publishing Company, 1994.

[12] J. G. Proakis and M. Salehi, *Communication Systems Engineering*. Massachusetts, USA: 2nd ed. Prentice Hall, 2001.

[13] H. V. Poor, *An Introduction to Signal Detection and Estimation*. New York, USA: Springer-Verlag, 1994.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. San Diego, USA: Academic, 1996.

[15] W. Bair, C. Koch, W. T. Newsome, and K. H. Britten, "Power spectrum analysis of bursting cells in area MT in the behaving monkey," *Journal of Neuroscience*, vol. 14, 5, pp. 2870–2892, May 1994.