Subpercent Accurate Fitting of Modified Gravity Growth

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Adding to our previous method for dealing with gravitational modifications at redshift $z \gtrsim 3$ through a single parameter, we investigate treatment of lower redshift modifications to linear growth observables. We establish subpercent accurate fits to the redshift space distortion observable $f\sigma_8(a)$ using two parameters binned in redshift, testing the results for modifications with time dependence that rises, falls, is nonmonotonic, is multipeaked, and corresponds to $f(R)$ and braneworld gravity. The residuals are then propagated to cosmological parameter biases for DESI observations, and found to cause a shift in the dark energy joint confidence contour by less than the equivalent of $\sim 0.1\sigma$. The proposed 2–3 parameter modified gravity description also can reveal physical characteristics of the underlying theory.

I. INTRODUCTION

Gravity has been stringently tested on terrestrial, solar system, and astrophysical scales but cosmic scales represent a further 10^6 to 10^{14} extrapolation in length scale (from galaxy scales or solar system scales respectively). Given that cosmic expansion is, surprisingly, accelerating, opposite to the expectation from gravity acting on matter, it is natural to desire tests of gravity on cosmic scales.

The cosmic expansion by itself cannot distinguish between a change in the laws of gravity and in the material contents, i.e. a dark energy component, but the combination with the cosmic growth of large scale structure can. Therefore considerable effort has gone into understanding the effect of modified gravity on cosmic structure growth; for reviews, especially model independent work, see [1–4].

Numerous alternatives to general relativity exist, with many of them falling within the Horndeski class of gravity, or described by an effective field theory approach (see [5] and references therein). These approaches involve four or more free functions of time, in addition to the expansion history, with no prescription for how they should behave. Even next generation data will not be able to constrain four functions, or more than a few parameters. Simple functional forms tend to be highly restrictive and possibly poor approximations [6, 7]. Thus we must either work one at a time with one particular model of gravity, one particular functional form within that model, and one particular parameter set within that functional form (e.g. $f(R)$ gravity, of the Hu-Sawicki [8] form, with $n = 1$), or seek a phenomenological low dimensional model independent approach.

If we follow the data, then in the subhorizon, quasistatic limit (applicable to where precision data will lie) the linear growth of structure is determined by a generalized Poisson equation, as clearly shown by [9]. Here the gravitational strength determining matter density perturbation growth is $G_{\rm matter}(k, a)$ rather than Newton's constant G_N, where the scale factor a represents the time dependence and the wavenumber k the scale dependence. This is a robust treatment for modified gravity under these circumstances [9]. Thus the issue, if one is concerned with using cosmic growth data to test gravity, is how to parametrize $G_{\rm matter}$.

One advance in this direction appears in [10] (hereafter Paper 1). The authors derived, and demonstrated numerically, that modifications to the gravitational strength at early times, $a \lesssim 0.25$ in the matter dominated era, could be modeled with high accuracy in their effects on the growth observables by a single parameter, $G_{\rm hi}$, related to the effective area under the $\delta G_{\rm matter}(a)$ curve. This is accurate to 0.3% or better. The treatment of later time modifications to gravity, however, was left as an unresolved question. The aim here is to address it.

In Sec. II we describe the variety of gravity models that we seek to fit, and the model independent method used. The specific approach and observational data used is described in Sec. III, and the results for the accuracy of the parametrization are presented in Sec. IV. We propagate the fitting residuals to cosmological parameter bias in Sec. V. Section VI discusses how to use the parametrization with data in a practical sense, and how to extract key properties of the gravity theory from the results. We conclude in Sec. VII.

II. GRAVITY MODELS AND FITS

In the quest for a low dimensional parametrization of the effect of modified gravity on linear growth observables, we want not only an accurate parametrization but a broadly model independent one. Functional forms such as power laws tend to be limited, and often bias the results by weighting unfairly parts of the cosmic history, as well as the results being sensitive to the power law, or prior on the power law, assumed, while being unable to constrain the power law well [11, 12]. Assuming a close relation with the effective dark energy density also yields misleading conclusions [6, 7], with the simplest counterargument being that $f(R)$ gravity often shows a gravitational strength that only deviates from general relativity at quite late times, e.g. reaching 1% only at $z \approx 1.5$, when the dark energy density fraction is already greater than 15%. Conversely, assuming that gravitational modifications only occur at late times can miss important aspects of many theories such as the Horndeski class of gravity.

Therefore we turn toward bins in scale factor or redshift as a model independent approach. These have been successfully used in projecting future constraints on modified gravity, e.g. [12, 13]. We will lay out a methodology for deciding on the number of bins, and interpreting the meaning of the results in the remainder of the article. We emphasize that our goal is to fit to the observables, specifically the redshift space distortion (RSD) function $f\sigma_8(a)$, not the theory function $G_{\rm matter}(a)$.

To test the efficacy of binned parameters, we have to compare them to some underlying "true" theory. To robustly explore the comparison of the results of the binned model with the exact theory, we need to "stress test" the binned approximation by comparing it to a wide variety of theoretical behavior. Since our focus is on growth observables and looking for signatures of modified gravity, we use identical expansion histories for the model and the theory case it is attempting to fit.

The theory behaviors should be fairly realistic, with enough complexity and features to provide an adequate test of the binned parametrization. We adopt six different forms of scale factor dependence to test:

1. a nonmonotonic function, taken to be a Gaussian of variable width and location, as in Paper 1, but at late times;
2. a rising function;
3. a falling function (it is obvious that the constant function considered in Paper 1 can be fit by a binned parametrization);
4. a multipeaked function such as seen in some Galileon gravity cases (e.g. see Fig. 3 of [7]), taken to be a sum of Gaussians;
5. braneworld theory given by DGP gravity [14, 15];
6. $f(R)$ gravity.

The Gaussian deviation, normalizing $G_{\rm matter}$ by Newton's constant so that general relativity has $G_{\rm matter} = 1$, is

$$\delta G_{\rm matter} = \delta G\, e^{-(\ln a - \ln a_t)^2/(2\sigma^2)} \ , \quad (1)$$

where we will study the results for various central values a_t and widths σ.

The rising parametrization is

$$\delta G_{\rm matter} = \delta G\, a^{s} \quad \text{for } a > a_\star \ , \quad (2)$$

and otherwise zero, where $a_\star$ is a cutoff scale factor. We might choose $a_\star = 0.25$ ($z = 3$) since from Paper 1 we know how to treat the deviations for $z > 3$. The falling parametrization is

$$\delta G_{\rm matter} = \delta G\, a^{-s} \quad \text{for } a > a_\star \ , \quad (3)$$

and otherwise zero.

The sum of two Gaussians gives either a multipeaked function or a broader deviation, depending on the separation of the Gaussians and their width. We take $a_t = 0.3$ and $a_t = 0.7$, with either $\sigma = 0.25$ (giving multiple peaks) or $\sigma = 0.5$ (giving a broad deviation).

For DGP gravity, the expansion history is given by the modified Friedmann equation

$$\frac{H(a)}{H_0} = \frac{1-\Omega_m}{2} + \sqrt{\frac{(1-\Omega_m)^2}{4} + \Omega_m\, a^{-3}} \ , \quad (4)$$

and the modified gravity strength is

$$\delta G_{\rm matter} = -\frac{1}{3}\,\frac{1-\Omega_m^2(a)}{1+\Omega_m^2(a)} \ . \quad (5)$$

where $\Omega_m(a) = \Omega_m\, a^{-3}/[H(a)/H_0]^2$. At early times, $\Omega_m(a) \to 1$ and the strength restores to the Newtonian value, i.e. $\delta G_{\rm matter} = 0$. In the asymptotic future, the Hubble parameter freezes to a de Sitter state, $H/H_0 \to 1-\Omega_m$ and gravity freezes to $\delta G_{\rm matter} = -1/3$, i.e. $G_{\rm matter} = 2/3$, weaker than Newtonian due to the extra dimensional leakage.

For the $f(R)$ scalar-tensor gravity case, we adopt exponential gravity. See [16] for the relevant equations; the basic features are that the expansion history is close to ΛCDM but with the dark energy equation of state varying slowly around $w = -1$, on both the phantom and normal sides. The gravitational strength is greater than Newtonian, rising from the general relativity value at high redshift (and indeed for $z \gtrsim 1.5$) to 4/3 times the value; i.e. $\delta G_{\rm matter}$ goes from 0 to 1/3.

These are all compared to the results from the binned parametrization. This is simply $G_{\rm matter}$ piecewise constant in two bins of a. These span $a = [0.25, 0.5]$ and $a = [0.5, 1]$, since these are the main observational windows. As discussed in the next section, if the data show the need then we include an early time parameter corresponding to the area parameter of Paper 1, implemented as a constant value in a window $a = [0.1, 0.25]$. We smooth the bin edges with a tanh function; results are insensitive to a smoothing width below $\Delta \ln a = 0.01$.

III. METHOD AND DATA

For the theory and the binned model we solve the growth evolution equation using a fourth order Runge-Kutta method. The background expansion is taken to be a flat, ΛCDM cosmology with $\Omega_m = 0.3$, except for the DGP and $f(R)$ gravity cases where we simultaneously solve the background evolution equations. We then compare the observable RSD quantity $f\sigma_8(a)$ between the input theory and the binned parametrization results and determine the maximum and rms deviation.

The bin values are optimized by minimizing one of these quantities. We find that substantially similar values result from either optimization. For values used below, we nominally minimize the maximum deviation over the range $z = 0.15$–1.9, corresponding to the data used, as discussed below.

Note that a point deviation value, or rms, is not really the key quantity. Neither will pick up particular trends, such as the RSD observable being high for several redshift bins, then low for several, as opposed to random scatter. One possibility is to use some statistic such as the crossing statistic [17, 18] that does identify such patterns. However, what we are really interested in is the propagation of the residual between the theory prediction and the binning approximation to the cosmological parameters. For example, even a moderately large amplitude high frequency oscillation will not affect the cosmological determination since it does not look like a shift in cosmology. Therefore we use the maximum deviation in Sec. IV to determine the bin values, but then propagate the residuals to cosmology in Sec. V with the Fisher bias formalism.

As our mock data we take RSD measurements as projected for the Dark Energy Spectroscopic Instrument (DESI [19]), with the uncertainties on $f\sigma_8(a)$ given in Tables 2.3 and 2.5 of [20], for $k_{\rm max} = 0.1\,h$/Mpc.

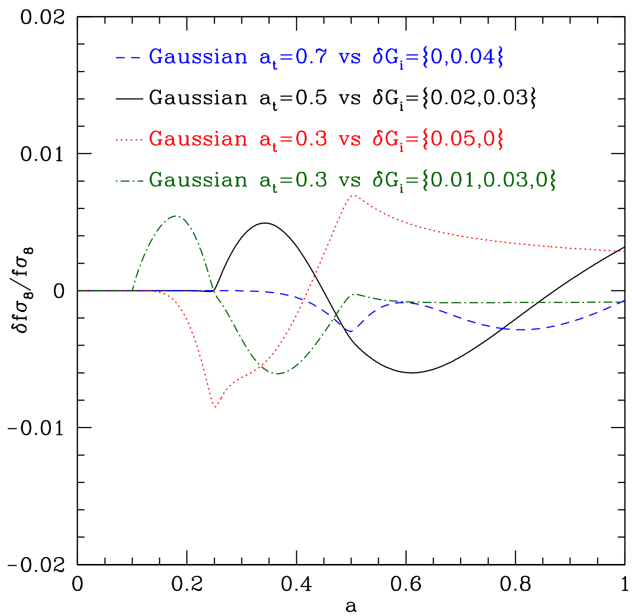

FIG. 1. The accuracy of fitting the observational RSD factor $f\sigma_8$ with two late time bins for modified gravity $\delta G(a)$ is compared to that for the exact theory case. The theory model has a Gaussian $\delta G(a)$ with parameters $\delta G = 0.05$, $\sigma = 0.25$, and $a_t = 0.3$ (dotted red), 0.5 (solid black), 0.7 (dashed blue). The dot dashed green curve shows the $a_t = 0.3$ case fit when allowing for a third, early bin due to the early modification.

IV. RESULTS

For theory Case 1, with nonmonotonic time dependence, we adopt Gaussian modifications $\delta G_{\rm matter}$ with amplitude 0.05 and width $\sigma = 0.25$. Figure 1 shows the deviations in $f\sigma_8(a)$ for the theory models with $a_t = 0.3$, 0.5, 0.7 vs the binned approximations.

We see that two bin parameters achieve subpercent level residuals relative to the exact results over the full redshift range. For the $a_t = 0.3$ case, the modification extends earlier than the bin start at $a = 0.25$. If we wanted to add the early time modification area parameter, or equivalently a third bin at $a = [0.1, 0.25]$, we reduce the maximum deviation from 0.9% to 0.6% (though nearly 0 for $a > 0.5$). There is no particular need for a third parameter even in this case, and this conclusion is verified by the cosmology bias analysis in Sec. V.

Note that for modifications close to the present, e.g. the $a_t = 0.7$ case, even just one parameter, from the bin $a = [0.5, 1]$ gives excellent results. Even if we double the amplitude, to $\delta G = 0.1$, the maximum deviation in $f\sigma_8$ stays under 0.5% as seen in Fig. 2. This also illustrates the possibility of trading off a residual curve that stays closer to 0 for much of its run, but has an overall larger max–min range, with one that is further from 0 but rather flat. We might expect that the latter, though with greater rms deviation, has less cosmological consequence, and indeed this holds true. One could further improve on the fit by allowing the second bin to enter, and then the high amplitude case has only 0.2% maximum deviation.

Turning to theory Case 2 and 3, we consider power law rising and falling time dependences, with $\delta G \propto a^3$ and a^{-3}. (Note that the parametrizations used by [21, 22] are basically within the rising class.) The normalization we use gives a maximum $\delta G = 0.21$, a considerable amplitude, with the rising case reaching this at $a = 1$ and the falling case at its starting point $a = 0.25$. As seen in Fig. 3, the rising case can be easily fit with two bin parameters, and the maximum deviation is less than 0.5% for $a < 0.85$. This is more than satisfactory as the DESI data projects an uncertainty of greater than 12% for $a > 0.85$ due to the small cosmic volume available. (Better measurements may be possible by using peculiar velocities [23].) In any case, one could achieve 0.9% accuracy over all a using $\delta G_3 = 0.075$ instead of 0.06.)

The falling case achieves 1.4% accuracy with two bins, due to its large amplitude and rapid variation in the bin $a = [0.25, 0.5]$. This deviation pattern would be noticeable in the data fits, and would spur an analysis where this bin would be split in two, e.g. $a = [0.25, 0.4]$ and $a = [0.4, 0.5]$. With this three parameter fit, the residuals obtain subpercent accuracy. Either way, this sort of oscillation in residuals does not tend to give a cosmology bias, and thus is mostly harmless. Finally, note that in any case such a falling model is not generally seen in

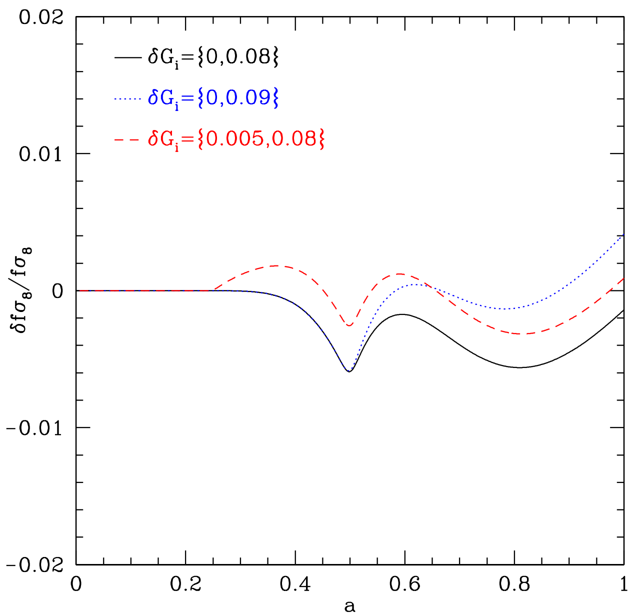

FIG. 2. The Gaussian modification case with larger amplitude $\delta G = 0.1$, $a_t = 0.7$, $\sigma = 0.25$ can still be accurately fit with two bins. Three different possibilities are shown, with different rms residuals, but all give high accuracy fits.

gravity theories commonly investigated.

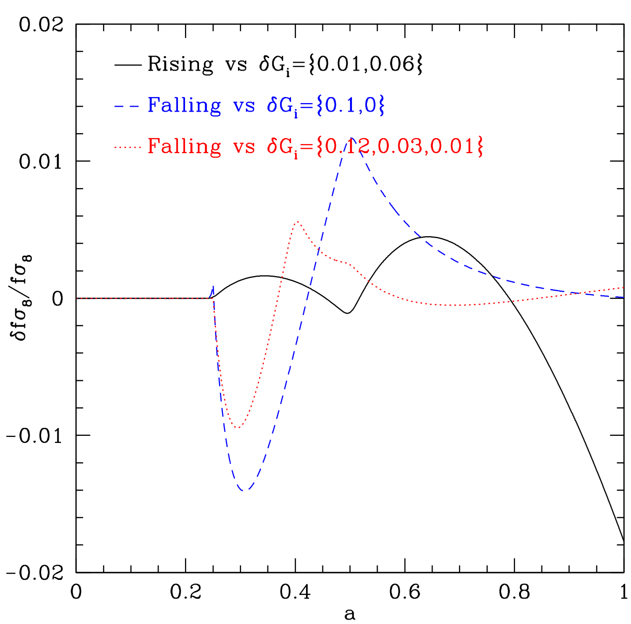

FIG. 3. Modifications that rise or fall monotonically over the range $a = [0.25, 1]$ can also be fit well with just two parameters, though the less realistic falling case benefits from splitting the $a = [0.25, 0.5]$ bin.

For the multipeak case, reminiscent of modifications seen in theories with many terms such as Horndeski gravity, we model this by the sum of two Gaussians, at $a_t = 0.3$ and 0.7. We adopt $\sigma = 0.25$ to obtain a multipeak $\delta G(a)$, and also investigate $\sigma = 0.5$ to give a broad, non-Gaussian $\delta G(a)$. Figure 4 shows the accuracy of the binned approximation.

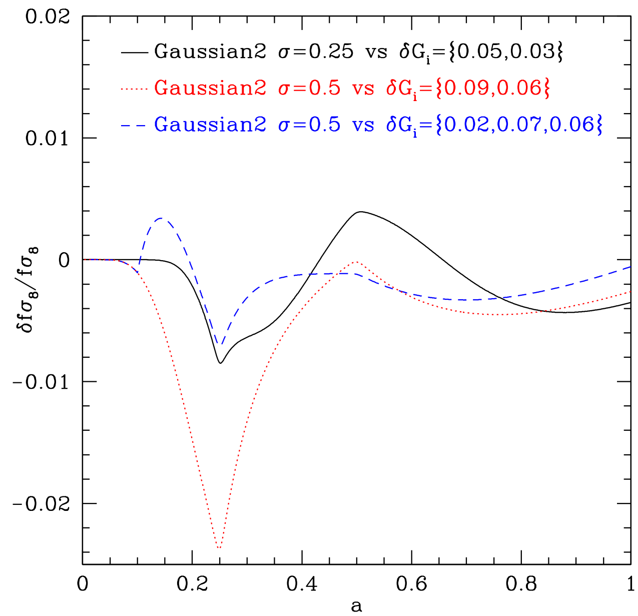

FIG. 4. The multipeak (two Gaussians) model can have subpercent residuals when using two bin parameters. When the early Gaussian has substantial support at $a < 0.25$ (the $\sigma = 0.5$ case) then adding a third, early bin substantially improves the accuracy.

For broad early modification, one needs an early bin for subpercent accuracy, i.e. the area parameter of Paper 1. As discussed in Sec. VI, the need for an early bin makes itself known from the trend of data points with redshift. However, if the precision data extends only to $z \approx 1.9$ ($a \approx 0.34$) then two bins gives 0.8% precision.

Finally, we consider actual gravity theories. Braneworld gravity, specifically DGP gravity, exhibits a significant change in the strength of gravity, with $\delta G \approx -1/3$. Since its deviation from general relativity starts relatively early, i.e. once $\Omega_m(a)$ starts to deviate from 1, we expect to need to include the third, area parameter or early bin. The results appear in Fig. 5.

Fitting to binned δG gives an oscillating residual, reflecting that $\delta G_{\rm DGP}$ is quite smooth and monotonic so each bin fits the average value within its redshift range, under- and overestimating the function in the different halves of the bin. The amplitude of the residuals is 0.6% except at very early or late times. (Again, the DESI measurement precision at $a < 0.2$ or $a > 0.9$ is such that even a 1% residual there needs no improvement.) Since this oscillatory pattern does not look like cosmological

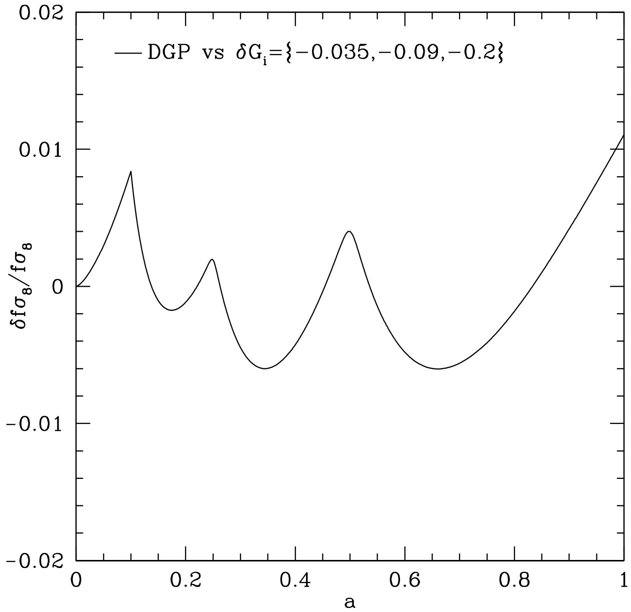

FIG. 5. DGP gravity is well fit by two bin parameters plus the early time modification, or area, parameter.

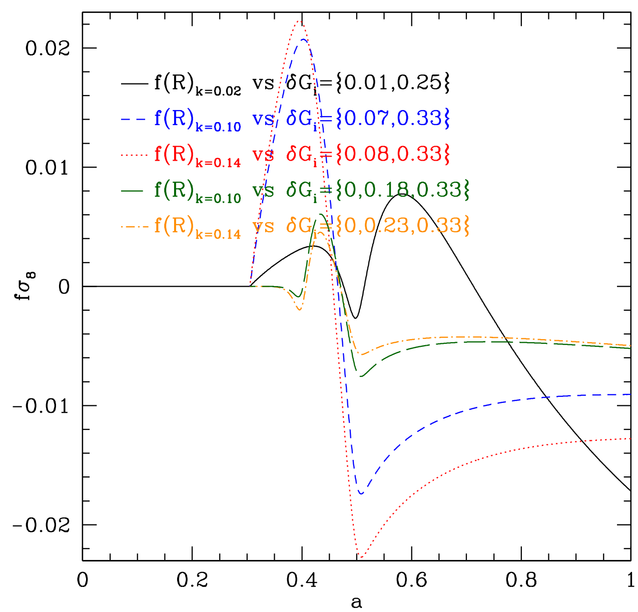

FIG. 6. Exponential $f(R)$ gravity is fit by 2-3 bin values, with different gravitational strengths at different wavenumbers k (i.e. scale dependent). Due to the steepness of the time evolution of the modification, the fit is greatly improved when using a third bin made by splitting the $a = [0.25, 0.5]$ bin (long dashed green and dot dashed orange curves, relative to dashed blue and dotted red curves).

parameter variation, we expect little bias in the three bin case.

Finally, consider $f(R)$ theory. We adopt the exponential gravity form, with $c = 3$, which is consistent with observations [16]. Recall that $f(R)$ gravity also exhibits a significant change in the strength of gravity, with $\delta G \approx 1/3$. It generally has a steep time dependence, with $\delta G(a) = 0$ until quite recent times and then rapidly rising. For example, it reaches 1% deviation from general relativity at $z \approx 1.5$ and has 33% deviation at $z = 0$. In addition, the gravitational strength, and hence growth, is scale dependent. Figure 6 shows the binned gravity values for growth at three separate wavenumbers k.

If we knew the true theory was $f(R)$ gravity then we could scale the bin values according to the predicted scale dependence of $G_{\rm matter}$ in $f(R)$ theory, i.e. $[3 + 4k^2/(aM)^2]/[3 + 3k^2/(aM)^2]$ where $M(a)$ is the scalaron mass [9, 24, 25]. However, we do not know this a priori. (See [26] for a more model independent approach.) Indeed, we discover the scale dependence empirically, when we compare the data to the result from the binned gravity fit and the residuals indicate a discrepancy that can be removed by introducing different binned values for different wavenumbers. Note, however, that the binned values are fairly similar for $k \gtrsim 0.1\,h/{\rm Mpc}$.

The steepness of the evolution of the gravitational strength shows up not only in the two-bin values, but in the strong improvement made when splitting the $a = [0.25, 0.5]$ bin into two parts, $a = [0.25, 0.4]$ and $[0.4, 0.5]$. As mentioned above, there is almost no deviation from general relativity for $a < 0.4$, and the finer early bin value is consistent with zero, while the larger, later split bin value greatly reduces the residuals.

As discussed in Sec. VI, the steepness of the increase in the bin values for any k, the late time value near $1/3$, and the scale dependence would together allow us to deduce – from a model independent analysis method! – that the true gravity theory is likely of the $f(R)$ class.

V. IMPACT ON COSMOLOGY

We established in the previous section that the residuals from fitting the RSD observable $f\sigma_8(a)$ with the two or three gravity parameters are at subpercent accuracy. Since next generation, DESI precision for $f\sigma_8(a)$ will be at the $\gtrsim 2\%$ ($\gtrsim 1\%$ if we used data out to $k_{\rm max} = 0.2\,h/{\rm Mpc}$), this seems sufficient. However, if the residuals coherently combine in their effect, due to a time dependence mimicking a shift in a cosmological parameter, they have the potential to bias the cosmological conclusions.

Therefore we now propagate the residuals in $f\sigma_8(a)$ to the cosmological model parameters. We use the Fisher bias formalism to carry this out [27]. The set of cosmological parameters considered is the present matter density in units of the critical density, Ω_m, the dark energy equation of state parameter today w_0, and a measure of its time variation w_a, where $w(a) = w_0 + w_a(1 - a)$. We use the DESI $f\sigma_8(a)$ data as described in Sec. III and to break background degeneracies we apply a simple Gaussian prior on the matter density $\sigma(\Omega_m) = 0.01$. Our fiducial model is flat ΛCDM with $\Omega_m = 0.3$.

The bias on a parameter p_i due to a misestimation of the observable $\Delta O(a)$ is (see, e.g., [28], including for the case where the error matrix is not diagonal)

$$\delta p_i = \left(F^{-1}\right)_{ij} \sum_k \frac{\partial O_k}{\partial p_j} \frac{1}{\sigma_k^2} \Delta O_k \ , \tag{6}$$

where O_k is the kth observable (i.e. $f\sigma_8(z_k)$) and F is the Fisher matrix.

Once we have the set of $\{\delta p_i\}$ we can quantify the bias statistically. One way is of course simply looking at $\delta p_i/\sigma(p_i)$, the bias relative to the statistical uncertainty. A common statistical quantity that employs this is the risk, which take the square root of the quadrature sum of the bias and dispersion. We can take the ratio of the risk to the statistical uncertainty to find the bloat, or effective increase in the uncertainty on a parameter:

$$B_i = \frac{\sqrt{\delta p_i^2 + \sigma^2(p_i)}}{\sigma(p_i)} = \sqrt{1 + [\delta p_i/\sigma(p_i)]^2} \tag{7}$$

This quantity appears for example in the Rao-Cramér-Frechet bound [29].

Finally, perhaps most useful is the shift induced in the joint parameter fitting, e.g. in the offset of the derived values from the true best fit in the dark energy equation of state plane w_0–w_a. The shift relative to the likelihood contours at some confidence level presents an informative, quantitative assessment of the bias that takes into account parameter degeneracies. This is given by [30, 31]

$$\Delta\chi^2 = \sum_{ij} \delta p_i \, F_{ij}^{(\rm red)} \, \delta p_j \ , \tag{8}$$

where the reduced Fisher matrix $F^{(\rm red)}$ runs over only those parameters p_i, p_j whose bias we are interested in, e.g. w_0 and w_a for the 2D joint likelihood contour plot in the w_0–w_a plane, and is marginalized over all others. This quantity automatically takes into account the *direction* of the shift, i.e. that a bias perpendicular to the degeneracy direction is more damaging than one along the degeneracy direction.

Table I presents the values of $\Delta\chi^2$ for the joint w_0–w_a likelihood, the maximum $\delta p/\sigma(p)$ for any of the cosmological parameters, and the maximum bloat in any of the cosmological parameters. Note that a shift of $\Delta\chi^2 = 2.3$ moves the true values out to the 68% confidence contour, i.e. a joint 1σ bias. A shift smaller than this lies within the contour.

We see that in the results for the whole range of gravity models, using two parameters to represent the bin values, or in rare cases three parameters, keeps $\Delta\chi^2 < 0.18$, i.e. less than a tenth of the distance to the 1σ joint likelihood contour. Alternately, the risk bloat factor is less than 1.08, i.e. the binned approximation only blows up the error bars, taking into account the systematic offset, by at most 8%. Thus the two, or if needed three, parameter description of gravitational strength modification is statistically extremely robust.

Model	$\Delta\chi^2$	$[\delta p/\sigma(p)]_{max}$	Risk$_{\rm max}$
Gaussian ($a_t = 0.7$)	0.02	0.02	1.00
Gaussian ($a_t = 0.5$)	0.13	0.33	1.05
Gaussian ($a_t = 0.3$)	0.16	0.22	1.02
Gaussian ($a_t = 0.7$; $\delta G = 0.1$)	0.09	0.04	1.00
Gaussian$_3$ ($a_t = 0.3$)	0.09	0.22	1.02
Gaussian^2 ($\sigma = 0.25$)	0.03	0.09	1.00
Gaussian^2 ($\sigma = 0.5$)	0.04	0.04	1.00
Gaussian$_3^2$ ($\sigma = 0.5$)	0.03	0.07	1.00
Rising a^3	0.01	0.09	1.00
Falling a^{-3}	0.36	0.25	1.03
Falling$_{3s}$ a^{-3}	0.10	0.23	1.03
DGP	2.28	0.45	1.10
DGP$_3$	0.00	0.02	1.00
$f(R)$ ($k_0 = 0.02$)	0.07	0.06	1.00
$f(R)$ ($k_0 = 0.10$)	1.81	1.34	1.67
$f(R)_{3s}$ ($k_0 = 0.10$)	0.18	0.40	1.08
$f(R)$ ($k_0 = 0.14$)	2.57	1.52	1.82
$f(R)_{3s}$ ($k_0 = 0.14$)	0.12	0.31	1.05

TABLE I. Parameter bias levels corresponding to the binned approximation of $\delta G(a)$. $\Delta\chi^2$ is the shift in the dark energy equation of state parameter w_0-w_a plane due to the bias; recall that $\Delta\chi^2 = 2.3$ corresponds to a 1σ shift in the joint parameter fit. The maximum bias of a parameter relative to its statistical uncertainty is shown in the $\delta p/\sigma(p)$ column. The Risk column shows the maximum "bloat" of the Risk, i.e. the increase in the uncertainty due to the bias. The subscript 3 denotes the three bin fit with an early bin, and $3s$ denotes a three bin fit splitting the mid z bin. The superscript 2 denotes a convolution of two Gaussians, with $a_t = 0.3$ and $a_t = 0.7$. Note the approximate form can be good to $\Delta\chi^2 < 0.18$ for all models.

VI. OBSERVATIONAL SIGNATURES

For any parametrization it is important that it be clear how it can be used to understand the data. That is, it should be of practical use to the observers and data analysts, as well as offering guidance to theorists.

The binned parametrization is simple to apply, readily able to calculate $f\sigma_8(a)$ or other growth quantities with excellent accuracy. The steps in using it are straightforward:

1. Fit the predictions from two bins in $a = [0.25, 0.5]$ and $a = [0.5, 1]$ to the data. If all values are consistent with zero then general relativity is a viable gravity theory. If some values differ from zero with statistical significance, this is an alert that a potential signature of modified gravity has been found.

2. If there are any residuals that show a pattern of exceeding the data error bars in some redshift range,

then

(a) Add the area parameter or equivalently a third bin at $a = [0.1, 0.25]$ if the deviation shows up from early times (note the kink deviation and then slope in the residuals shown in the figures for the early Gaussian and DGP figures), or

(b) Split the $a = [0.25, 0.5]$ bin into two bins over $a = [0.25, 0.4]$ and $a = [0.4, 0.5]$ if the deviation peaks in that range (see the falling and $f(R)$ figures).

3. If the residuals indicate an overall poor fit, and in particular if the time evolution also looks steep (as in the $f(R)$ case), try separating the data into low and high wavenumbers to look for scale dependence.

One could carry the bin refinement to a further level but none of the varied models we have considered require more than three bins, with two bins always sufficient here if data were at the 2% precision level.

The next question is how to interpret the results in terms of gravity theory. Note that the bin values are not a map of $\delta G_{\rm matter}(a)$ per se; they are a combination of the gravitation strength, the redshift weighting of the data and its precision, and a delay due to the convolution windowing of $\delta G_{\rm matter}(a)$ in the integral for $f\sigma_8(a)$. That said, they do provide a coarse guide to $\delta G_{\rm matter}$.

A late bin value near 1/3 inspires closer examination in terms of scalar-tensor gravity, while $-1/3$ would recall DGP gravity. The need for an early bin might lead to theories with early modifications such as the many members of the Horndeski class. Steepness of evolution in the binned value, reflecting steep time evolution of $G_{\rm matter}$, could point to $f(R)$ gravity, especially if splitting the $a = [0.25, 0.5]$ bin led to a significant improvement in the residuals. And of course scale dependence gives theoretically important information. Thus, even though the analysis method is model independent, not assuming any functional form or even that gravitational modification is only a late time phenomenon, we can obtain substantial information about the theory characteristics from the signatures in cosmic growth data.

VII. CONCLUSIONS

Comparing cosmic growth vs cosmic expansion is one of the premier methods for probing the nature of dark energy. Moreover, the details of cosmic growth can test the laws of gravity in the universe, on scales much greater than solar system or astrophysical tests. Given a well defined theory, such tests are straightforward. However, without a compelling theory – not just a class but with a particular functional form and hyperparameters – the comparison with data is more difficult, or at best model dependent.

Allowing the data to play a central role, we demonstrate a model independent approach. We find that only two (or in specific physical cases three) parameters in the form of binned values of $G_{\rm matter}(a)$ deliver subpercent accuracy in fitting the predominant redshift space distortion observable $f\sigma_8(a)$. This extends to all redshifts the previous high redshift parameter method of Paper 1.

We stress tested the approach against a set of six, diverse modified gravity classes with a variety of time dependences, including DGP gravity and $f(R)$ gravity. Residuals against exact behaviors of the observable successfully achieved subpercent accuracy. Minimizing the residual determines the bin values, while any remaining pattern offers concrete guidance to the need for a third bin or not.

We propagated the residuals from the parametrization to cosmological parameter bias and showed they are negligible, at below the effective 0.1σ level in joint confidence contours, for next generation data of the characteristics of the DESI galaxy redshift survey.

As importantly, the method lays out a clear path for interpreting the bin parametrization results in terms of the physical signature of the cosmological gravity theory. Based on the trend of values, their steepness, magnitude, and any need for an early bin or scale dependence, this approach can guide the search for the laws of cosmic gravity in the appropriate direction.

Future work includes whether such a method can be fruitful for weak gravitational lensing: like cosmic growth it relies on a modified Poisson equation, with $G_{\rm light}(a)$ instead of $G_{\rm matter}(a)$, but with a different kernel. If it too can be parametrized for the observables in a low dimensional manner, then next generation surveys will – even in a model independent manner – have excellent capabilities to explore cosmic gravity.

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